WORKSHEET 8

Course: Mat101E
Content: Applications of Integrals

1. A plane slices a ball of radius a into two pieces. If the plane passes b units away from the centre of the ball (where $b < a$), find the volume of the smaller piece, using the method of slicing and also the method of cylindrical shells.

2. Find the volume of the solid generated by rotating the region $0 \leq y \leq 1 - x^2$ about **(a)** the x-axis, and **(b)** the line $y = 1$.

3. Find the volume of the solid generated by revolving the triangular region with vertices $(0, -1)$, $(1, 0)$ and $(0, 1)$ about **(a)** the line $x = 2$, and **(b)** the line $y = 2$.

4. Find the volume of the solid generated by revolving the region bounded by the curve $y = 1 + \sin x$ and the lines $y = 1$, $x = 0$ and $x = \pi$ about the x-axis.

5. Write the definite integral that calculates the volume of the solid, generated by revolving the region enclosed by the curves $y = |x|$ and $y = \sqrt{1 - x^2}$ about the line $x = 1$.

6. Write the definite integral that calculates the volume of the solid, generated by revolving the region bounded by the curve $y = \ln x$ and the lines $x = e$ and $y = 0$ about the y-axis, using shell method.

7. Write the definite integral that calculates
   (a) The area or the region enclosed by the curves $y = 2 - \sqrt{2x - x^2}$; and $y = -x^2 + 2$.
   (b) The volume of the solid, generated by revolving the region bounded by the curves $y = 2 - \sqrt{2x - x^2}$; and $y = -x^2 + 2$ about the axis $x = -3$, using shell method.
   (c) The volume of the solid, generated by revolving the region bounded by the curves $y = 2 - \sqrt{2x - x^2}$; and $y = -x^2 + 2$ about the axis $x = -3$, using washer method.

8. Find the volume of the solid generated by revolving the region between the x-axis and the curve $y = x^2 - 2x$ about **(a)** the x-axis ; **(b)** the y-axis ; **(c)** the line $y = -1$; **(d)** the line $x = 2$; **(e)** the line $y = 2$

9. Find the volume of the solid generated by revolving the region bounded by the parabola $y^2 = 4x$ and the line $y = x$ about **(a)** the x-axis ; **(b)** the y-axis ; **(c)** the line $y = 4$; **(d)** the line $x = 4$; **(e)** the line $x = -1$

10. Find the volume of the solid obtained by revolving the region bounded by the curve $y = \sin x$, the line $y = 1$ and the y-axis in the first quadrant about the line $y = 1$.

11. Find the volume of the solid, generated by revolving the region bounded by the curve $y = \sin x$ and the lines $y = 1/2$ and $x = 0$ about the x-axis, using shell method.

12. Find the volume of the solid generated by revolving the region bounded by the curves $y = e^x$, $y = e^{-x}$ and $x = 2$ about the line $x = -1$.

13. Find the volume of the solid generated by revolving the region bounded on the left by the parabola $x = y^2 + 1$ and on the right by the line $x = 5$ about **(a)** the x-axis ; **(b)** the y-axis ; **(c)** the line $x = 5$.

14. Find the volume of the solid generated by rotating the region $0 \le y \le 1 - x^2$ about **(a)** the x-axis, and **(b)** the line $y = 1$.

15. The region in the second quadrant bounded above by the curve $y = -x^3$ below by the x-axis and on the left by the line $x = -1$ is revolved about

    a)the y-axis

    b)the line $x = -1$

    c)the x-axis

    d)the line $y = 1$

    to generate a solid.Find the volumes of the solids by using

    i)Washer/Disk and

    ii)Shell Method.

16. Find the volume of the solid which lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis between these planes are squares whose bases run from the semicircle $y = -\sqrt{1 - x^2}$ to the semicircle $y = \sqrt{1 - x^2}$.

17. Find the volume of the solid generated by revolving the area under the curve $y = \dfrac{1}{\sqrt{4 - x^2}}, \quad -1 \le x \le 1$ about the x-axis. (Disk)

18. Find the volume of the solid generated by revolving the region bounded by the curves $y = 2\cos x$ and $y = \sec x$ on the interval $-\pi/4 \le x \le \pi/4$ about the x-axis. (Washer)

19. Set up an integral to find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 1$ about the line a) $y = 2$ b) $y = -2$ c) $x = 3$ by using both shell and disk (washer) method.

20. Let R be the region in the first quadrant that is bounded by the lines $y = 1$, $x = 1$ and the curve $y = \ln x$. Find the volume of the solid generated by revolving the region R about

    (a) the x-axis, (b) the line $y = 1$, (c) the y-axis.

21. Write the definite integral that calculates the length of the curve
    $y = \int_0^x \sqrt{\cos \frac{2t}{3}}\, dt$ from $x = 0$ to $x = 3\pi/4$.

22. Find the length of the curve $y = \dfrac{x^3}{12} + \dfrac{1}{x}$ from $x = 1$ to $x = 4$.

23. Find the length of the following curves:

    (a) $y = \ln(1 - x^2), \quad 0 \le x \le 1/2$

    (b) $y = \ln(\cos x), \quad 0 \le x \le \pi/3$

    (c) $x = \dfrac{y^{3/2}}{3} - \sqrt{y}, \quad 1 \le y \le 9$

    (d) $y = \int_{-2}^{x} \sqrt{3t^2 - 1}\, dt, \quad -2 \le x \le -1$

24. Find a curve through the origin whose length is $L = \int_0^4 \sqrt{1 + \frac{1}{4x}}\, dx$.

25. Find the length of the parametric curve

$$x = 8\cos t + 8t\sin t, \quad y = 8\sin t - 8t\cos t, \qquad 0 \le x \le \pi/2.$$