

- 1- The length of certain bricks(x) is a random variable with a mean of **8** inches and a standard deviation of **0.1** inch, and the thickness of the mortar(y) between two bricks is a random variable with a mean of **0.5** inch and a standard deviation of **0.03** inch. What is the mean and the standard deviation of the length of a wall made of **50** of these bricks laid side by side, if we can assume that all the random variables involved are independent?

$$z = \sum x_i + \sum y_j \quad \text{where } z \text{ is RV (the length of a wall) } i = 1 \text{ to } 50 \text{ and } j = 1 \text{ to } 49$$

$$E\{z\} = 50 \cdot 8 + 49 \cdot 0.5 = 424.5 \text{ inches}$$

$$\text{var}(ax+by) = a^2 \text{var}(x) + b^2 \text{var}(y) + 2ab \text{cov}(x,y) \quad (\text{if } x \text{ and } y \text{ are dependent})$$

$$\text{var}(ax+by) = a^2 \text{var}(x) + b^2 \text{var}(y) \quad (\text{if } x \text{ and } y \text{ are independent})$$

$$\text{var: variance, } \sigma^2 \quad \text{covariance } \text{cov}(X,Y) = E(XY) - \mu_x \mu_y \quad \text{std: standard deviation, } \sigma$$

$$\text{var}(z) = 50(0.1)^2 + 49(0.03)^2 = 0.5441 \quad \text{and} \quad \text{std}(z) = 0.738 \text{ inches}$$

- 2- If **heads** is success when we flip a coin, getting a **six** is a success when we roll a die, and getting an **ace** is a success when we draw a card from ordinary deck of 52 playing cards, Find the mean and the standard deviation of the total number of successes when we
- Flip a balanced(**fair**) coin (x), roll a fair die(y), and then draw a card from a well-shuffled deck(z),
 - Flip a balanced coin **three** times, roll a balanced die **twice**, and then draw a card from a well shuffled deck.

Hint: Bernoulli trials, mean = np and variance = npq where q=1-p

$$\text{a) } E\{x+y+z\} = 1/2 + 1/6 + 1/13 = 58/78 = 0.74$$

$$\text{var}(x+y+z) = 1/4 + 5/36 + 12/169 = 0.46, \quad \text{std}(x+y+z) = 0.68$$

$$n=1, \text{ H/T COIN } p=q=0.5, \text{ DIE } p=1/6, q=5/6, \text{ CARD } p=1/13, q=12/13$$

$$\text{b) } E\{3x+2y+z\} = 3 \cdot 1/2 + 2 \cdot 1/6 + 1/13 = 149/78 = 1.91$$

$$\text{var}(3x+2y+z) = 3^2 \cdot 1/4 + 2^2 \cdot 5/36 + 12/169 = 2.8766$$

$$\text{std}(3x+2y+z) = 1.6961$$

- 3- If we alternately flip a **fair** coin and an **unfair** coin (probability of getting heads is **0.45**), what are the mean and the standard deviation of the number of **heads** that we obtain in **10** successive flips of these coins?

$n=5$, 5 fair and 5 unfair flips = 10 successive

$$\mu = 5(0.5) + 5(0.45) = 4.75$$

$$\sigma^2 = 5^2(0.5)^2 + 5^2(0.45)(0.55) = 12.4375$$

$$\sigma = 3.5267$$