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Long Term Performance Prediction of a Borehole and Determination of Optimal Thermal Response Test Duration

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ABSTRACT

Capacity loss becomes a vital problem especially for long term usage of Ground Source Heat Pump Systems (GSHPS). Therefore long term prediction of heat transfer rate per unit borehole length (unit HTR value) is an important issue to reliably determine the borehole quantity in a project. In this study, constant temperature thermal response test (TRT) is considered since it has some advantages over constant heat flux TRT. An analytical model is developed to determine heat diffusivity and thermal conductivity of ground by using the experimental TRT values. Long term (3-6 months) HTR values of a borehole are predicted by using the experimental data collected during TRT in the developed model. The effect of TRT duration on long term predictions is analyzed. Variation of the predictions for HTR values at end of heating/cooling season with duration of TRT value is examined. Optimal TRT duration is determined. Results can be used for long term performance predictions as well as the optimization of TRT duration. This model can also be used for estimation of efficient borehole life time.

1. INTRODUCTION

In ground source heat pump (GSHP) applications, approximately 75-80% of the heat, which is transferred to a building, comes from the ground. For this reason, determination of ground properties and prediction of long term borehole performance are very important to avoid the capacity problems in the following years after the installation.

Thermal response test (TRT) is the best way to determine the ground properties in the application field. Especially for large scale GSHP applications, this determination is vital. In a TRT process, thermal conductivity of ground and unit heat transfer rate (HTR) are experimentally obtained. By this experimental data, variation of unit HTR value with time is calculated for both heating and cooling seasons. Then required total borehole length is determined by considering their long term performance prediction.

Generally there are two different kinds of TRT methods. One is the constant heat flux method which is cheaper and commonly used, the other is constant temperature method (CTM) which is more expensive but gives more sensitive results. In CTM, a wide range of test temperature is possible, test duration is not limited and more boreholes can be tested simultaneously.

CTM has been introduced by Wang and et al. (2009). In that study, they developed a new test method based on constant temperature. Utilizing the test data and using modified Eskilson’s G-function, they developed a method to determine thermal conductivity of ground. Then they show advantages of constant temperature TRT method.

On optimum duration of constant heat flux TRT, there are limited studies. Skouby (1998), Spitler (1999) and Sanner (2005) suggest 50 hours for test. In another study, Bujok et al. (2014) investigated effect of TRT duration on the results like thermal conductivity and borehole resistance. In the results of TRT duration of 24h and 70h, there are significant differences like 13-17%. However differences of results between 60h and 70h tests are approximately 1%. In another study by Bandyopadhyay et al. (2008), they focused on reducing TRT time. They obtained Laplace domain solutions for the equivalent single core of U-tube in grouted boreholes. They determined the averaged fluid temperature and borehole boundary temperature using Gaver-Stehfest numerical inversion algorithm and show that this method can reduce the required duration of TRT.

Signorelli et al. (2007) studied TRT duration to achieve a certain error level with 3-D finite element numerical model. They also showed the effect of borehole length, groundwater movement and heterogeneity of ground on TRT. Similar to the other studies, it has been mentioned that test duration of 50h gives sufficiently good results.

In this study, a model for long term performance prediction of a borehole is developed based on constant temperature TRT method instead of constant heat flux one. In a drilled borehole, fluid flow rate, inlet and outlet temperatures are recorded. By the recorded data, variation of unit HTR value with time is obtained and both thermal conductivity and diffusivity are determined by fitting an analytical model to experimental data. By using analytical model and the determined thermal parameters, long term performance of a borehole is predicted.

2. AN ANALYTICAL MODEL FOR BOREHOLE HEAT TRANSFER RATE

Schematic view of a borehole with a single U-tube is shown in Figure 1. Borehole thermal resistance between fluid and borehole wall depends on diameters of pipe and borehole, shank spacing and thermal properties of fluid, pipe and grout. Some analytical models are introduced for the solution of thermal resistance of a single U-tube borehole (Shonder and Beck 1999, Gu and O’neal 1998, Remund 1999, Hellström 1991). Since thermal capacity of the grout is very small in comparison with that of the large ground, time required for steady state of borehole wall temperature is typically shorter than 12h-14h. In other words, borehole wall temperature, T_b, can be assumed to be approximately constant after short time operation. Then the problem is reduced to a problem that finding the temperature distribution of ground around the borehole in terms of time and radial distance.

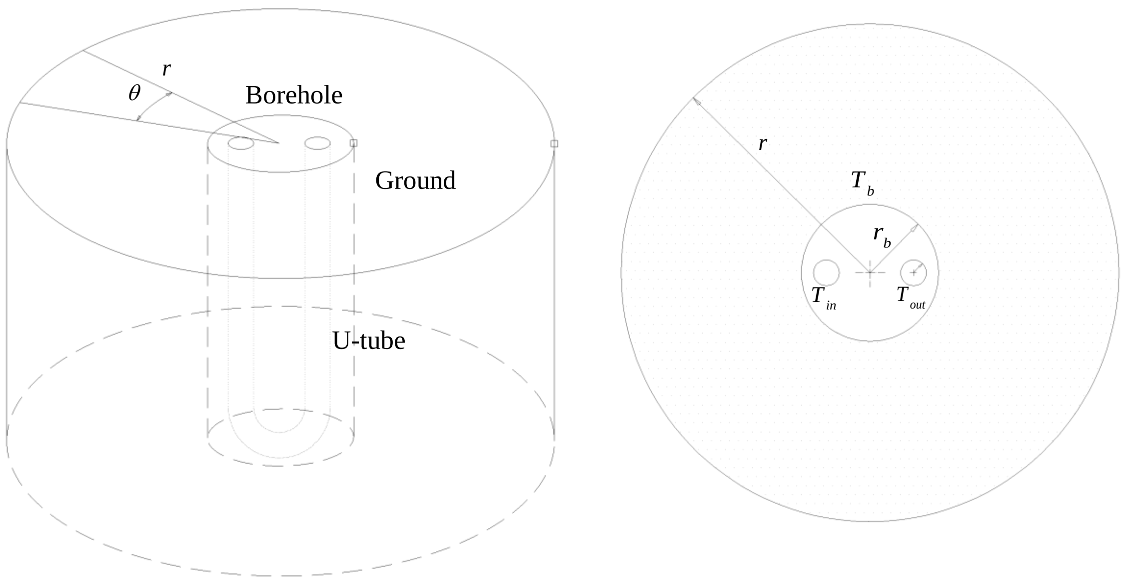

Figure 1: Schematic view of a borehole

Under the steady-state approximation for grout, borehole wall temperature is

$$T_b = \overline{T} - \frac{\dot{q}'}{2\pi k_g} ln\left(\frac{r_b}{2r_p}\right) \tag{1}$$

where T_b is borehole wall temperature, $\overline{T}$ is mean fluid temperature $\overline{T} = (T_{in} + T_{out})/2$, r_p and r_b are pipe and borehole radius respectively, k_g is thermal conductivity of grout and $\dot{q}'$ is HTR value per unit borehole length (unit HRT). $\dot{q}'$ can experimentally be determined by

$$\dot{q}' = \dot{m} c_p (T_{in} - T_{out}) \tag{2}$$

where $\dot{m}$ is flow-rate, c_p is the heat capacity of fluid (brine water) and T_{in} is inlet temperature to ground and T_{out} is outlet temperature from the ground.
By considering the ground as homogeneous and isotropic medium, vertical and angular dependencies of temperature are ignored. Then to find the temperature distribution around the borehole, 1D heat conduction equation in cylindrical coordinates should be solved,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

where α is thermal diffusivity defined by $\alpha = k / \rho c_p$, k and ρ are thermal conductivity and density of ground respectively. Boundary and initial conditions of the problem are:

$$T(r_b, t) = T_b \tag{4a}$$

$$T(r, 0) = T_\infty \tag{4b}$$

$$T(\infty, t) = T_\infty \tag{4c}$$

Equation (3) is simplified by using the following dimensionless quantities:

$$\theta = \frac{T - T_b}{T_\infty - T_b}; \quad \tilde{r} = \frac{r}{r_b}; \quad \tilde{t} = \frac{\alpha t}{r_b^2} \tag{5}$$

as

$$\frac{\partial^2 \theta}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}}\frac{\partial \theta}{\partial \tilde{r}} = \frac{\partial \theta}{\partial \tilde{t}} \tag{6}$$

Dimensionless boundary and initial conditions are

$$\theta(1, \tilde{t}) = 0 \tag{7a}$$

$$\theta(\tilde{r}, 0) = 1 \tag{7b}$$

$$\theta(\infty, \tilde{t}) = 1 \tag{7c}$$

Equation (6) is solved under conditions given by (7), then the following expression is obtained (Ozisik 1993):

$$\theta(\tilde{r}, \tilde{t}) = \int_{\beta=0}^{\infty} \frac{\beta e^{-\beta^2 \tilde{t}} [J_0(\beta \tilde{r}) Y_0(\beta) - Y_0(\beta \tilde{r}) J_0(\beta)]}{J_0^2(\beta) + Y_0^2(\beta)} d\beta . \int_{r'=1}^{\infty} r' [J_0(\beta r') Y_0(\beta) - Y_0(\beta r') J_0(\beta)] dr' \tag{8}$$

Second integral in Eq. (8) is solved analytically and it becomes the following simple form:

$$\theta(\tilde{r},\tilde{t}) = \frac{-2}{\pi}\int_{\beta=0}^{\infty} \frac{e^{-\beta^2\tilde{t}}\left[J_0(\beta\tilde{r})Y_0(\beta) - Y_0(\beta\tilde{r})J_0(\beta)\right]}{\beta\left(J_0^{\,2}(\beta) + Y_0^{\,2}(\beta)\right)} d\beta \tag{9}$$

2.1. HEAT TRANSFER RATE PER UNIT BOREHOLE LENGTH

Heat transfer rate of a borehole per unit length (unit HTR) is expressed by

$$\dot{q}' = -2\pi k r_b \left.\frac{dT}{dr}\right|_{r=r_b} \tag{10}$$

In dimensionless form, Eq.(10) can be rewritten

$$\dot{q}' = -2\pi k\left(T_\infty - T_b\right)\left.\frac{d\theta}{d\tilde{r}}\right|_{\tilde{r}=1} \tag{11}$$

Dimensionless unit HTR value is

$$\tilde{\dot{q}}' = \frac{\dot{q}'}{2\pi k\left(T_b - T_\infty\right)} = \left.\frac{d\theta}{d\tilde{r}}\right|_{\tilde{r}=1} \tag{12}$$

By using Eqs. (9) and (12), dimensionless unit HTR value are obtained as follows:

$$\tilde{\dot{q}}' = \left.\frac{d\theta}{d\tilde{r}}\right|_{\tilde{r}=1} = \frac{2}{\pi}\int_{\beta=0}^{\infty} \frac{e^{-\beta^2\tilde{t}}\left[Y_0(\beta)J_1(\beta) - J_0(\beta)Y_1(\beta)\right]}{\left[J_0^{\,2}(\beta) + Y_0^{\,2}(\beta)\right]} d\beta \tag{13}$$

In this expression, integral has no analytical solution. By numerical solution for each $\tilde{t}$, unit HTR value can be calculated. By fitting the experimental results to Eq. (13), thermal conductivity (*k*) and thermal diffusivity (α) can be found. Fitting process is represented by Eq.(14), left hand side is obtained from Eq. (13) while the right hand side is calculated from experimental data

$$\tilde{\dot{q}}'\left(\frac{t\alpha}{r_b^{\,2}}\right) = \frac{\dot{q}'(t)}{2\pi k\left(T_b - T_\infty\right)} \tag{14}$$

2.2. A SIMPLE REPRESENTATIVE FORM OF UNIT HTR EXPRESSION

As it is seen from Eq. (13), $\tilde{\dot{q}}'$ includes infinite integration of Bessel functions which have inherent oscillations. Because of that, numerical integration of Bessel functions takes a long time. To make the solution of Eq.(14) easier and faster, a simple and analytical representative expression should be used. After the k and α are obtained by solution of Eq.(14), these values can be used in Eq. (13) to make long term predictions for HTR values. Therefore we can call the representative expression as a function used to accelerate the solution.

For a borehole, variation of $\tilde{\dot{q}}'$ with $\tilde{t}$ is shown in Figure 2 for the range of $\tilde{t}$ =0 and $1.5\text{x}10^6$.

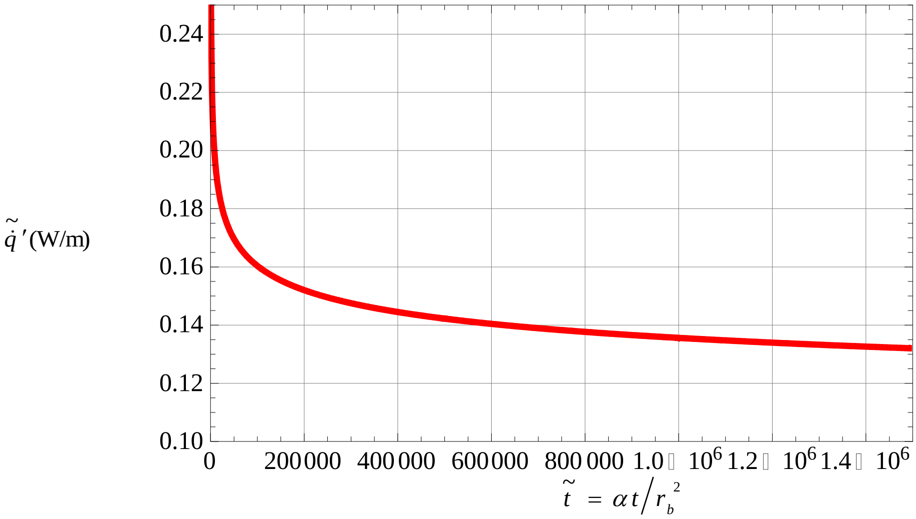

Figure 2: Variation of dimensionless unit HTR value with dimensionless time

In logarithmic coordinates, this graphic becomes like in Figure 3;

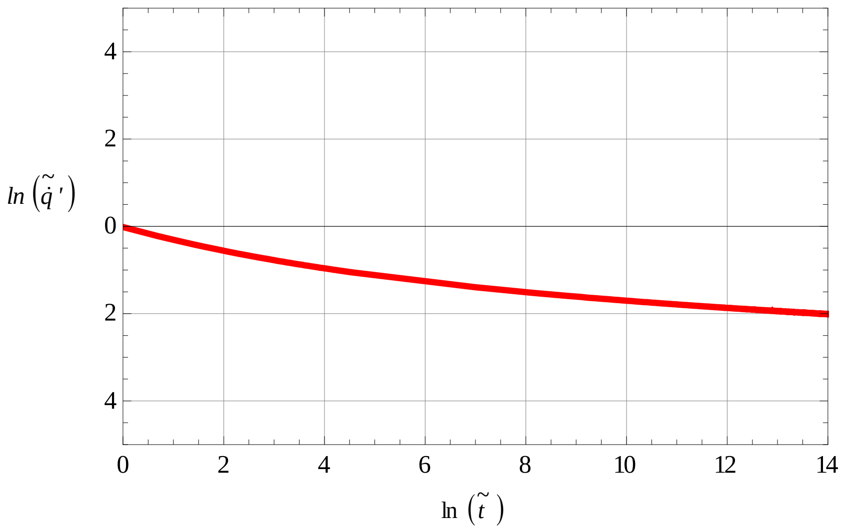

Figure 3: $\ln\left(\tilde{\dot{q}}'\right) - \ln\left(\tilde{t}\right)$ graphic

When a cubic polynomial function is fitted to the curve given in Figure 3; the following expression is obtained

$$ln\left(\tilde{\dot{q}}'\right) = -0.0004 \;\; ln^3(\tilde{t}) + 0.0156 \;\; ln^2(\tilde{t}) - 0.2840 \;\; ln(\tilde{t}) - 0.0492 \tag{15}$$

Then, dimensionless unit HTR value is

$$\tilde{\dot{q}}' = exp\left[-0.0004 \;\; ln^3(\tilde{t}) + 0.0156 \;\; ln^2(\tilde{t}) - 0.2840 \;\; ln(\tilde{t}) - 0.0492\right] \tag{16}$$

In term of explicit form of $\tilde{t}$, it is rewritten as

$$\tilde{\dot{q}}' = exp\left[-0.0004 \;\; ln^3(t\alpha / r_b^2) + 0.0156 \;\; ln^2(t\alpha / r_b^2) - 0.2840 \;\; ln(t\alpha / r_b^2) - 0.0492\right] \tag{17}$$

A comparison of Eqs. (13) and (16) is given in Figure 4. It is seen that the representative function, Eq.(17), (dashed lines) perfectly overlap with the true expression, Eq.(13). Thus the representative equation can be used for solution of Eq.(14) to accelerate the fitting process. Standard deviation between the true expression and representative one can be found as follows:

$$SD = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\tilde{\dot{q}}'_{Eq.(13),i} - \tilde{\dot{q}}'_{Eq.(16),i}\right)^{2}} = 2.83\text{x}10^{-4}\ (\text{W/m}) \tag{18}$$

The Mean Absolute Percentage Error (MAPE) is also calculated as:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\tilde{\dot{q}}'_{Eq.(16),i} - \tilde{\dot{q}}'_{Eq.(13),i}}{\tilde{\dot{q}}'_{Eq.(16),i}}\right| = 0.0011 \tag{19}$$

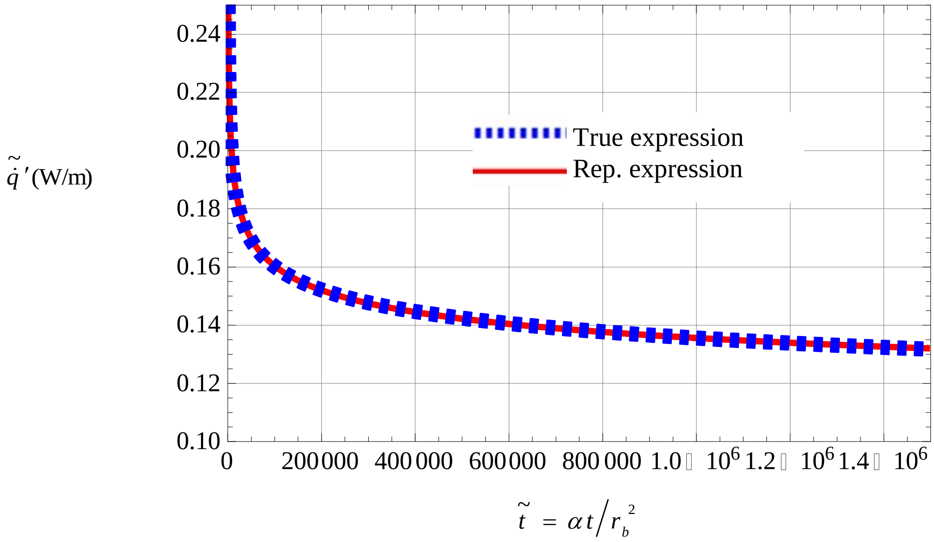

Figure 4: Comparison of the results of Eqs. (13) and (16)

Thus Eq.(16) can be used instead of Eq.(13) during the fitting process to obtain thermal conductivity and thermal diffusivity of ground. Then the obtained values are used in true expression, Eq.(13), to determine the long term unit HTR values of borehole. Fitting process and prediction process are symbolically given in Figure 5.

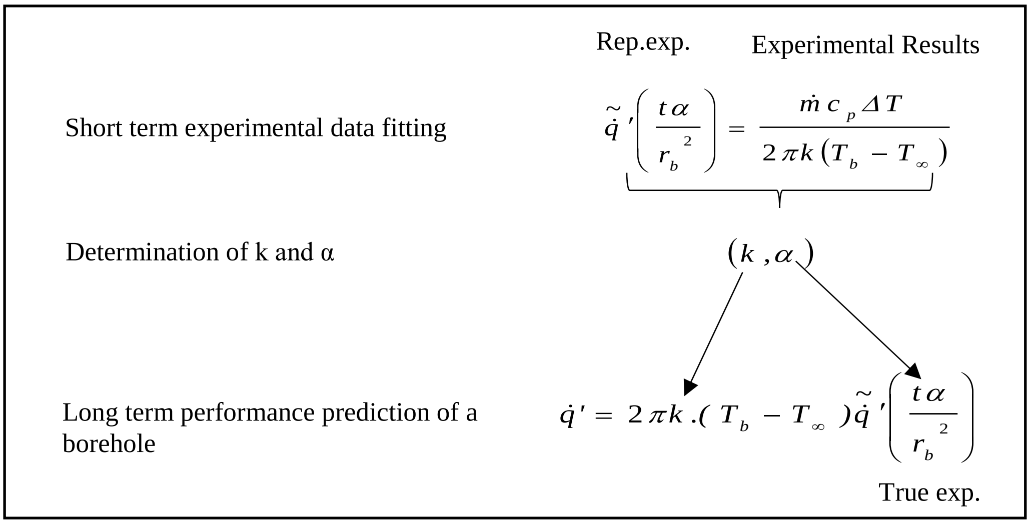

Figure 5: Fitting and prediction process

3. CONSTANT TEMPERATURE METHOD (CTM) THERMAL RESPONSE TEST SYSTEM

CTM has some important advantages like better accuracy, shorter time to achieve steady state regime and wider range for testing temperature etc. However test system is more expensive due to its temperature control requirements. Constant temperature TRT system mainly consist of an electrical resistances connected to water tank, hydraulic circulating pump, PID control unit, data logger. (shown in Fig. 6 and 7) By this system, each U-tube can be tested separately, more than one U-tube can be tested simultaneously and also test duration can be longer to get more accurate results.

In the test system, flow rate, inlet and outlet temperatures are measured and recorded in real-time for each pipe by turbine flow-meter and PT1000 temperature sensors. Properties of temperature sensor and flow meter are given in Table 1. Before the test system is operated, temperature sensors are calibrated in a calorimetric container to get the same results from each sensor for the temperature range of from 2 ^{0}C and 55 ^{0}C. Flow-meters are also calibrated by Siemens Mag5000 flow-meter.

Table 1: Specifications of Flow-meter and Temperature Sensors

Flow meter		
Nominal Diameter	15	mm
Repeatability	±0.2	%
Accuracy - Standard	±1	%
Temperature Sensor		
Type	Pt1000	
Precision	±0.15	K

Figure 6: Thermal Response Test System Pictures

To measure the HTR values of boreholes, edge of pipes are connected to the test system. After the air is purged from the system, undisturbed ground temperature has to be measured before the test is started. To determine undisturbed ground temperature as recommended by Gehlin (2002), the valves 3, 5, 6, 7 are closed (in Fig. 7) and pump is operated, circulating water temperature after 15-20 minutes gives the information about the undisturbed temperature. Later, valves 2, 3, 7 and borehole's valves are closed, mini pump and electrical resistances with PID control are operated to heat the water in the tank up to test temperature.

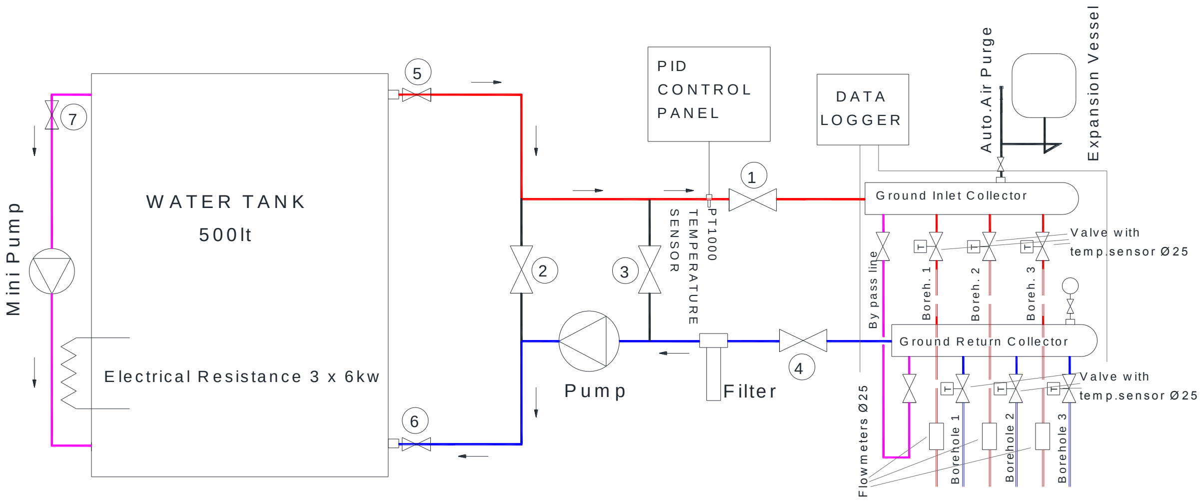

Figure 7: Constant Temperature TRT System

When the tank temperature achieved to test temperature, by-pass line and valves 2 and 3 are closed, the others are opened and then test is started. Mini pump on left hand side of the tank provide homogeneity of tank temperature. Inlet temperature is measured and controlled by PID controller.

4. FINDING THE OPTIMUM TEST DURATION

To find the optimum test duration, the longest test is performed for 240 hours. Then, using the different durations of test data, 4 months predictions of unit HTR values are calculated. Differences between the predictions based on 24, 48, 72, 96 and 120h data and the reference prediction based on 240h data are examined. Differences on HTR predictions of different test durations with that of the longest test results are given in Table 2 and Figure 8.

Table 2: Predicted results and their differences for different test duration

Test Duration	$\dot{q}'$ (after 4months)	% Difference
24	62.8	2.03
48	63.1	1.56
72	63.4	1.09
96	63.6	0.78
120	63.7	0.62
240	64.1	0

Steady-state condition inside the borehole is reached after 12h from the beginning of test. Therefore the first 12h test data are omitted due to the assumption used in analytical model. Using the data in the second 12 hours of 24h test, unit HTR value is predicted for after four months non-stop working as 62.8 W/m. However using 240h data this value is predicted as 64.1 W/m. There is about 2% difference between the result of 24h and 240h. With examining the Table 2, it can be said that 40-50 h test duration is enough to obtain reliable results.

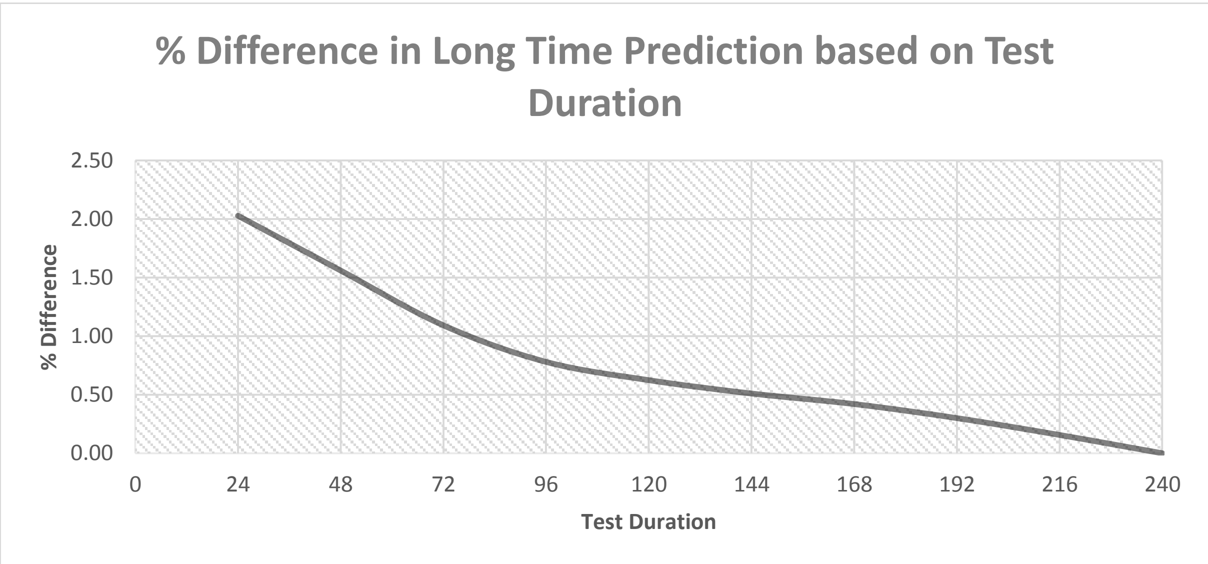

Figure 8: Variation of relative difference with test duration

5. DETERMINATION OF THERMAL PROPERTIES OF GROUND and LONG TERM PERFORMANCE PREDICTION of BOREHOLES

A sample test is operated between 28th of May and 7th of June. Test conditions are given in Table 3.

Table 3: Borehole specifications and test conditions

Borehole diameter	0.17	m
Borehole length	50	m
Total test duration	240	hours
Ground inlet temperature	40.0	°C
Ground avg. outlet temperature	37.5	°C
Flow-rate	25.4	lt/min
Average unit HTR value	88.0	W/m

During the test, variation of unit HTR value with time is given in Figure 9.

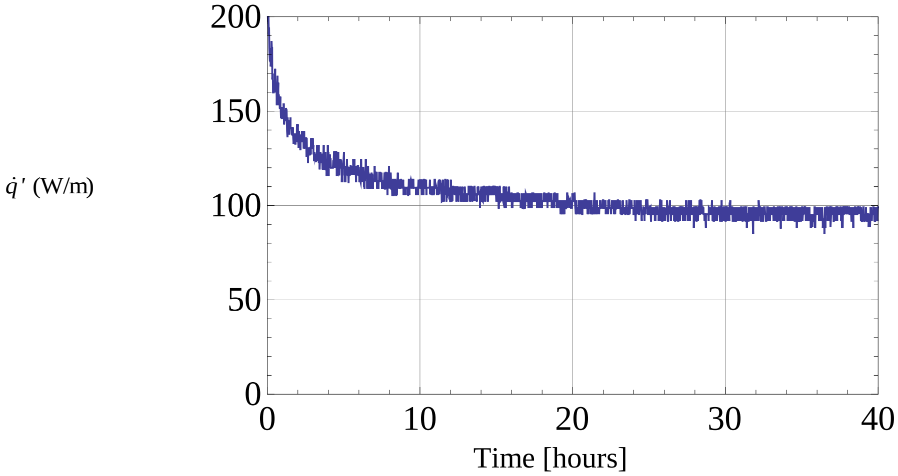

Figure 9: Test results of 40h

When Eq. (16) is fitted to the experimental data given in Fig.8, thermal conductivity and thermal diffusivity are found as k=3.8 W/mK and α=4.32x10^{-5} m^2/s respectively. These values are used in Eq. (13) and its results are compared with experimental results, Fig.10. It is seen that there is a good agreement between them. It should be noted that since there are different strata and underground water movement, obtained ground properties represent their average and effective values.

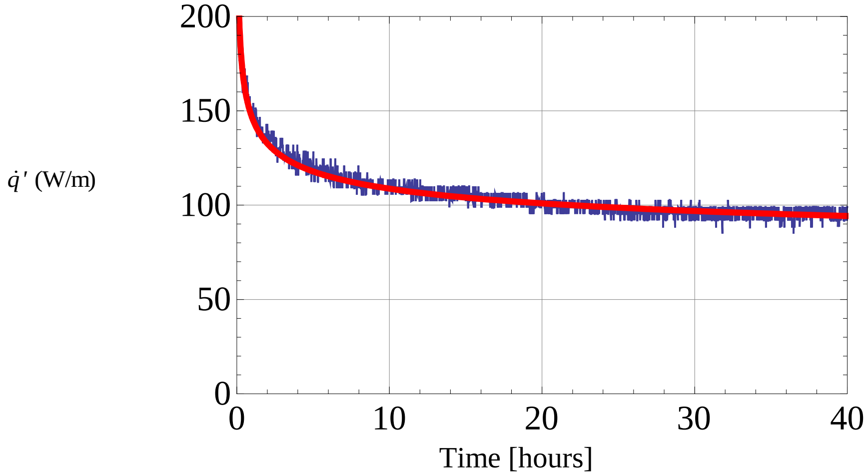

Figure 10: Experimental data (blue) and analytical results (red curve)

By using the model, a long term unit HTR value of the borehole can be predicted, Figs.11 and 12.

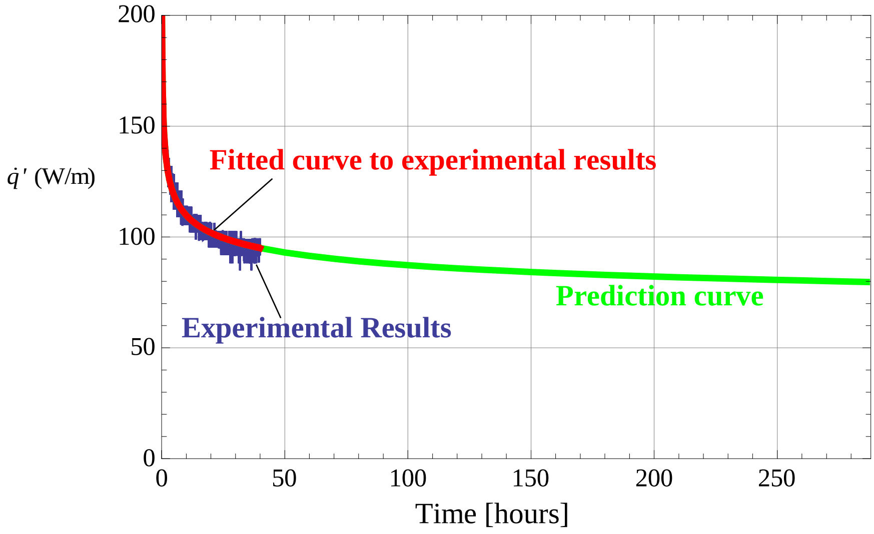

Figure 11: 12 days performance prediction of the borehole

For a location that heating period about 4 months, one year heating performance can be calculated with this results (Fig.12). If in that location the borehole is used also for cooling there will not be capacity loss in the following years. However if the borehole will be used only for heating, renovation of the ground around the borehole should be considered during design process of GSHP.

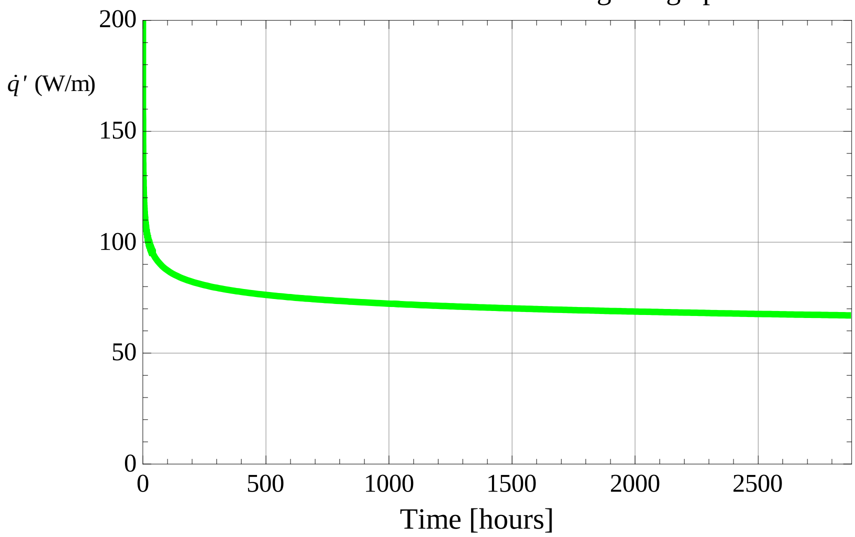

Figure 12: 4 months performance prediction of the borehole

6. CONCLUSION

An analytical model is developed based on constant temperature thermal response test to obtain thermal conductivity and diffusivity of ground and to make long term performance prediction. A test station is set up for this constant temperature TRT. Furthermore optimum test duration is investigated and found that 40-50h test duration is enough to get reliable results. Determination of the length of boreholes in a GSHP system application is a difficult problem. It is better to consider the worst case during this determination. Therefore the method used in this work can be used for long term prediction of HTR values of boreholes and determination of amount of boreholes in GSHP applications.

ACKNOWLEDGEMENT

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