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A New Method for Constant Temperature Thermal Response Tests

Murat Aydin[1], Mustafa Onur[2], Altug Sisman[1]

[1]Istanbul Technical University, Energy Institute, 34469 Maslak, Istanbul, Turkey

[2]The University of Tulsa, Dept. of Petroleum Engineering, 800 South Tucker Drive Tulsa, Oklahoma 74104 USA
e-mails: murataydin@itu.edu.tr, mustafa-onur@utulsa.edu, sismanal@itu.edu.tr

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ABSTRACT

In ground source heat pump (GSHP) applications, determination of thermal properties such as thermal conductivity and diffusivity of ground is essential to make a reliable long term performance predictions of boreholes and system design. Constant heat-flux or constant-temperature thermal response tests (TRT) is used to determine ground's thermal properties. Constant-temperature TRT has some advantages over the constant heat-flux TRT due to its more accurate prediction, shorter test duration and compatibility with heat pump performance tests. In this study, an analytical method is developed for constant-temperature TRT to estimate thermal properties of a ground system of interest. Analytical solution of heat equation for a borehole is obtained based on Laplace transformation and heat transfer rate per unit borehole length (unit HTR) is analytically determined. Analytical expression for the variation of inverse unit HTR value with logarithm of time is fitted to experimental data to estimate thermal conductivity and diffusivity of the ground. The advantage of this method is to allow to determine thermal conductivity directly from the slope of logarithmic time dependency of inverse unit HTR value without making an estimation for heat capacity. Therefore, errors based on incorrect estimation of heat capacity can be eliminated. The method is applied for different experimental data based on different test temperatures, and the results are compared with those of other methods. A very good agreement between the results is obtained. The method can be used to make more precise long term thermal performance predictions of boreholes.

1. INTRODUCTION

In ground source applications; one of the most critical factors, which determine overall system performance, is the ground heat exchanger. Improper planning results excessive or deficient length for ground heat exchangers and causes capacity deficit or high initial cost for those systems. To avoid this kind of problems, some planning methods that based on Thermal Response Test (TRT) are used. TRT based methodologies have been used for more than 20 years.

In a TRT process, constant heat flux is injected to a borehole for a period of time, by circulating the water inside the borehole. Inlet and outlet temperatures and flowrate of testing fluid are measured during the test. From the test results thermal conductivity and thermal resistance of borehole can be predicted by using some analytical methods. First and most used analytical method is infinite line source (ILS), based on Kelvin's [1] line source method and it is developed by Ingersoll [2] for using in boreholes. ILS method assumes the borehole as an infinite line source and it gives ground thermal conductivity after a period of time. Because of ILS assume the borehole as infinite line it could not represent a borehole sufficiently. However Zeng [3] developed a Finite Line Source (FLS) model based on quasi-three dimensional ILS. The other method is cylindrical source (CS) method, formulated by Carslaw and Jeager [4] and Deerman and Kavanaugh [5] used it in the TRT analysis. In this method, the borehole is assumed as hollow cylinder and in constant heat flux temperature distribution can be calculated.

In some studies, the results of ILS and CS methods have been compared with each other [6-8]. For example, Gehlin [6] compared these two methods and obtained higher thermal conductivity results in comparison with CS. However, Philippe [7] showed the existence of a good agreement between two methods. Recently Yu [8] has compared two methods and showed that cylindrical source (CS) give 9-13% higher thermal conductivity results than that of ILS.

Furthermore numerous numeric models have been developed [9-11]. Numerical methods are relatively easy and more general to be applied in an engineering software. For instance, numerical solutions can consider more complex geometries with different underground events (groundwater flow, etc.). Furthermore with a numerical model, the long term performance of a system can be predicted more rigorously [12]; however designing process using a numerical software may be time consuming. Numerical models can be of 1, 2 or 3D. Best results can be obtained from 3D models, however in the borehole because of ratio of depth to the diameter is so high that running such a 3D numerical model needs high capacity computing power and long times. Using axial symmetry and axial layers may decrease the solution time.

There are also other methods based on analytical solutions. For instance, in case of multi U-tubes usage in a borehole solutions become more complicated, Aydin and Sisman [13]. However Carli [14] solved the problem using Capacitance Resistance Model (CaRM) similar to electrical resistance model for single and double U-tubes. In an improved study of CaRM, [15] the model include thermal capacities of materials inside the borehole and the results have an agreement with numerical ones. Some other analytical solutions have been done by Beier [16] based on Laplace method and by Bandyopadhyay [17] using Gaver Stehfest and Laplace methods. Similarly, Y.Man [18] has used the Green function methods.

The methods mentioned above have been developed for constant heat-flux tests. Over the years, however, some problems have been encountered for constant heat flux tests. One of the important problems is electrical voltage irregularities caused from the grid. The other problem is electrical cut-off stopping of the test and need to wait a period of time until ground regeneration is completed. Another disadvantage is that it needs a long time (about two full of days) for testing. The general acceptance for a duration of a test is about 48 hours. [19-21]. Recently shorter tests [22] have been tried, however, deviations of the tests are in unacceptable range. Another weakness of this method is the injection of a serious amount of heat energy and researches to reduce the used energy during a test is a continuing effort [23].

In an actual operation of a ground source heat pump, ground side's circulation pump runs when the compressor runs and it sends hot fluid (in cooling mode) or cold fluid (in heating mode) to the borehole. In designing process for the required borehole length in a project, heat load of a building, usage time, seasonal coefficient of performance (SCOP) of heat pumps are taken in account. In this process, simulating the worst possible case always guaranteed to obtain the best results. In that case, non-stop operation of ground circulating pump is assumed and always hot fluid (for cooling) or cold fluid (for heating) is pumped to the borehole. For simulating the worst possible case, constant temperature fluid is assumed to be pumped for the whole operation of time.

Furthermore in heat pump standards (EN14511-2)[24], constant temperatures are used for brine (water + antifreeze) and load sides. These values are brine return temperature from the ground and flow temperature to the load side. Therefore a thermal response test with constant temperature is very compatible with heat pump standards. In case of constant temperature TRT, fluctuations of grid does no effect the results, thermal properties can be determined in a shorter time and the conditions are closer to real heat pump working conditions. Besides, it is easy to produce constant temperature fluid by any type of simple boiler.

Constant temperature TRT has been used in the past [25, 26]. In Wang's study [25], he stated that constant temperature TRT has some important advantages like better accuracy, shorter time to achieve steady state regime and wider range for testing temperature etc. In Aydin's study [26] constant temperature TRT has also be used with CS methodology. However in these studies computation load is higher.

In this study, a simple analytical model is presented to examine constant temperature TRT data. It is shown that in a constant temperature TRT, the inverse of a unit heat transfer rate changes linearly with the logarithm of time; and hence thermal conductivity of ground can be predicted by using the slope of the inverse of unit heat transfer rate versus log of time. Furthermore using the value of inverse unit heat transfer rate at time of unit, thermal diffusivity of the ground can easily be predicted. This proposed method is applied to experimental data of boreholes for different fluid temperatures and U-tube configurations. The results of our method are compared with those of the numerical and other methods. Considering the same borehole, we applied both constant temperature and heat-flux TRTs. The estimated values of thermal conductivity are also compared. The results show that there are good agreement between the results especially for a borehole with 2U-tubes and it needs longer test durations for boreholes with 3U-tubes and 1U-tube.

2. AN ANALYTICAL MODEL FOR CONSTANT TEMPERATURE TRT

A standard borehole consists of U-tube(s) and grout as it is seen in the left view of Fig. 1. In the construction process of a borehole, first U-tube(s) are down and the remaining gaps between borehole and U-tubes are filled with thermally enhanced grout. If we assume there is no difference in thermal properties of a borehole in vertical direction, the cross-section is the same along the whole borehole. Using the equivalent radius approximation, we get a geometrical representative cross-section given on the right side of Fig.1.

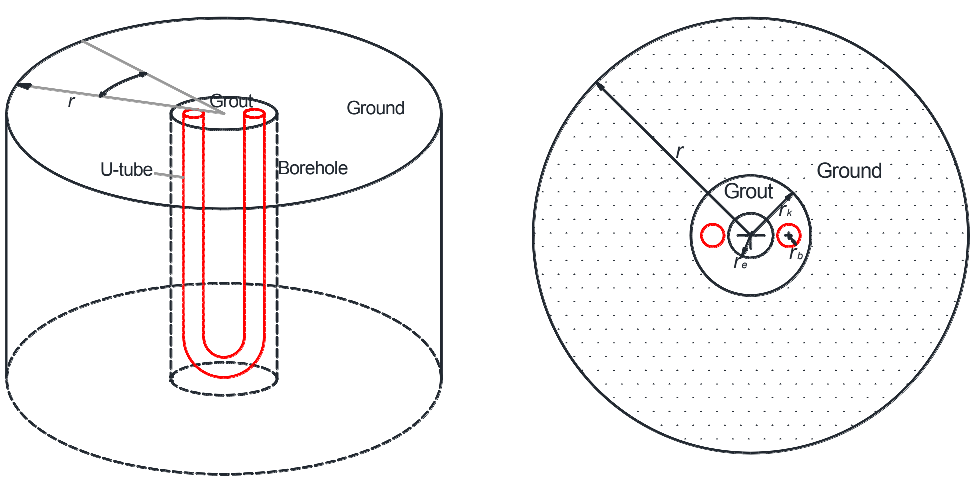

Figure 1- A sample borehole view.

Then we can consider this problem as an initial, boundary value problem (IBVP) described by the following partial differential equations and the initial and boundary conditions:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

$$T(r,t=0)=T_{\infty} \tag{2}$$

$$\lim_{r\to\infty} T(r,t)=T_{\infty} \tag{3}$$

and

$$T(r=r_e,t)=\overline{T}_{wf}=\text{constant}. \tag{4}$$

For the sake of simplicity, we use the following dimensionless variables:

$$\theta(\widetilde{r},\widetilde{t})=\frac{T(r,t)-T_{\infty}}{\overline{T}_{wf}-T_{\infty}}, \tag{5}$$

$$\widetilde{r}=\frac{r}{r_e}, \tag{6}$$

$$\widetilde{t}=\alpha\frac{t}{(r_e)^2}, \tag{7}$$

Having a homogeneous initial condition eases to find the solution when using the Laplace transformation. Using the dimensionless variables given by Eqs. 5-7 in Eqs. 1-4, we can express the IBVP in terms of the dimensionless variables as:

$$\frac{1}{\widetilde{r}}\frac{\partial}{\partial \widetilde{r}}\left(\widetilde{r}\frac{\partial \theta}{\partial \widetilde{r}}\right)=\frac{\partial \theta}{\partial \widetilde{t}}, \tag{8}$$

$$\theta(\widetilde{r},\widetilde{t}=0)=0, \tag{9}$$

$$\lim_{\widetilde{r}\to\infty}\theta(\widetilde{r},\widetilde{t})=0, \tag{10}$$

and

$$\theta(\widetilde{r}=1,\widetilde{t})=1. \tag{11}$$

We can apply Laplace transformation to solve the above IBVP. Taking the Laplace transform of Eq. 8 and using the initial condition given by Eq. 9 in the resulting equation gives:

$$\frac{1}{\widetilde{r}}\frac{d}{d\widetilde{r}}\left(\widetilde{r}\frac{d\hat{\theta}}{d\widetilde{r}}\right)-s\hat{\theta}=0, \tag{12}$$

where $\hat{\theta}$ is the Laplace transform of $\theta(\widetilde{r},\widetilde{t})$, that is, ,

$$\hat{\theta}(\widetilde{r},s)=\int_0^{\infty} e^{-s\widetilde{t}}\,\theta(\widetilde{r},\widetilde{t})\,d\widetilde{t}, \tag{13}$$

where s is the Laplace transformation variable with respect to dimensionless time. Using the boundary condition (after taking its Laplace transform) given by Eq. 10, solution of Eq. 12 is given by

$$\hat{\theta}(\widetilde{r},\widetilde{t})=AK_0\left(\sqrt{s}\,\widetilde{r}\right), \tag{14}$$

where the constant A is determined by using the Laplace transform of the boundary condition given by Eq. 11. Therefore the solution is obtained as:

$$\hat{\theta}(\tilde{r},\tilde{t})=\frac{K_0\left(\sqrt{s}\,\tilde{r}\right)}{sK_0\left(\sqrt{s}\right)}. \tag{15}$$

Inverse Laplace of Eq. 15 does not appear in the tables, but its inverse can be obtained by using the complex inversion integral formula involving integration along the complex line (Churchill's book [27]). Hence the inverse of Eq. 15 gives:

$$\theta(\tilde{r},\tilde{t})=1+\frac{2}{\pi}\int_0^{\infty}\left\{exp\left(-u^2\tilde{t}\right)\frac{J_0(\tilde{r}u)Y_0(u)-Y_0(\tilde{r}u)J_0(u)}{u\left[J_0^{\,2}(u)+Y_0^{\,2}(u)\right]}\right\}du. \tag{16}$$

Heat transfer rate per unit length of the well is determined by:

$$q'(t)=-2\pi k\left(r\frac{dT}{dr}\right)_{r=r_e} \tag{17}$$

We define a dimensionless unit heat transfer rate by the following equation:

$$\tilde{q}'(\tilde{t})=\frac{q'(t)}{2\pi k\left(\overline{T}_{wf}-T_{\infty}\right)} \tag{18}$$

Using the definitions of dimensionless temperature (Eq. 5), radial distance (Eq. 6), and Eq. 18 in Eq. 17 gives:

$$2\pi k\left(\overline{T}_{wf}-T_{\infty}\right)\tilde{q}'(\tilde{t})=-2\pi k\left(\overline{T}_{wf}-T_{\infty}\right)\left(\tilde{r}\frac{d\theta}{d\tilde{r}}\right)_{\tilde{r}=1}, \tag{19}$$

or

$$\tilde{q}'(\tilde{t})=\frac{q'(\tilde{t})}{2\pi k\left(\overline{T}_{wf}-T_{\infty}\right)}=-\left(\tilde{r}\frac{d\theta}{d\tilde{r}}\right)_{\tilde{r}=1}, \tag{20}$$

Taking the Laplace transform of Eq. 20 gives:

$$\hat{\tilde{q}}'(s)=-\left(\tilde{r}\frac{d\hat{\theta}}{d\tilde{r}}\right)_{\tilde{r}=1}, \tag{21}$$

Taking derivative of Eq. 15 with respect to $\tilde{r}$ gives:

$$\frac{d\hat{\theta}}{d\tilde{r}}(\tilde{r},\tilde{t})=\frac{-\sqrt{s}K_1\left(\sqrt{s}\tilde{r}\right)}{sK_0\left(\sqrt{s}\right)}, \tag{22}$$

Using Eq. 22 in Eq. 21 gives:

$$\hat{\tilde{q}}'(s)=\frac{\sqrt{s}K_1\left(\sqrt{s}\right)}{sK_0\left(\sqrt{s}\right)}, \tag{23}$$

To find the long time approximation (i.e., small s) of Eq.(23), we know the following asymptotic forms of the modified Bessel's functions of the second kind of order one and zero:

$$\lim_{s\to 0}\sqrt{s}K_1\left(\sqrt{s}\right)=1, \tag{24}$$

and

$$\lim_{s\to 0}K_0\left(\sqrt{s}\right)\approx -ln\left(\frac{e^{\gamma}\sqrt{s}}{2}\right). \tag{25}$$

Using Eqs. 24 and 25 in Eq. 23 gives:

$$\hat{\tilde{q}}'(s) \approx -\frac{1}{s\,ln\left(\frac{e^{\gamma}\sqrt{s}}{2}\right)} \tag{26}$$

For logarithmic functions, Najurieta [28] shows that:

$$f(t) = L^{-1}\left[\hat{f}(s)\right] \approx \left[sf(s)\right]_{s=1/e^{\gamma}t}, \tag{27}$$

Here γ=0.5772 is Euler's number. So, the inverse of Eq. 26 can be given by

$$\tilde{q}'(\tilde{t}) = L^{-1}\left[\hat{\tilde{q}}'(s)\right] = -\frac{1}{ln\left(\frac{e^{\gamma}\sqrt{1/e^{\gamma}\tilde{t}}}{2}\right)} = -\frac{1}{ln\left(\frac{\sqrt{e^{\gamma}/\tilde{t}}}{2}\right)} = \frac{1}{\frac{1}{2}ln\left(\frac{4\tilde{t}}{e^{\gamma}}\right)}, \tag{28}$$

The long time approximation given by Eq. 28 is recalled as:

$$\tilde{q}'(\tilde{t}) = \frac{1}{\frac{1}{2}ln\left(\frac{4\tilde{t}}{e^{\gamma}}\right)}, \tag{29}$$

and then taking reciprocal of Eq. 29 gives:

$$\frac{1}{\tilde{q}'(\tilde{t})} = \frac{1}{2}ln\left(\frac{4\tilde{t}}{e^{\gamma}}\right). \tag{30}$$

Using the definitions of the dimensionless variables for $\tilde{t}$ and $\tilde{q}'(\tilde{t})$ and those given by Eqs. 7 and 18, respectively, in Eq. 30, we can obtain a dimensional analogue of Eq. 30 as:

$$\frac{2\pi k\left(\overline{T}_{wf} - T_{\infty}\right)}{q'(t)} = \frac{1}{2}ln\left(\frac{4\alpha t}{e^{\gamma}(r_e)^2}\right), \tag{31}$$

which is valid for $t >> t_c = e^{\gamma}r_e^2/4\alpha$ and can be rearranged as:

$$\frac{1}{q'(t)} = \frac{1}{4\pi k\left(\overline{T}_{wf} - T_{\infty}\right)}\left[ln(t) + ln\left(\frac{4\alpha}{e^{\gamma}(r_e)^2}\right)\right] = \frac{2.303}{4\pi k\left(\overline{T}_{wf} - T_{\infty}\right)}\left[log(t) + log\left(\frac{4\alpha}{e^{\gamma}(r_e)^2}\right)\right], \tag{32}$$

This equation shows that a plot of $1/q'(t)$ vs log *t* (a semilog plot) will yield a straight line with slope *m* which is equal to

$$m = \frac{2.303}{4\pi k\left(\overline{T}_{wf} - T_{\infty}\right)}, \tag{33}$$

using the slope *m*, we can easily calculate the thermal conductivity of ground: $k = (4m\pi(\overline{T}_{wf} - T_{\infty}))^{-1}$.

Furthermore the value of $1/\tilde{q}'(\tilde{t})$ at t = 1 sec (or 1 h it does not matter, it changes only the units of thermal diffusivity) equals to:

$$a = \left[\frac{1}{q'(t)}\right]_{t=1} = m\,log\left(\frac{4\alpha}{e^{\gamma}(r_e)^2}\right). \tag{34}$$

From the value of *a*, the thermal diffusivity of ground is directly determined from:

$$\alpha = \frac{e^{\gamma}(r_e)^2\,10^{a/m}}{4} \tag{35}$$

Fig. 2 shows the application of the method based on Eq. 32 to experimental data. Thermal response tests are performed for three different constant fluid temperatures and the figure shows a semi-log plots of reciprocal of heat rates per unit length versus logarithm of time *t*.

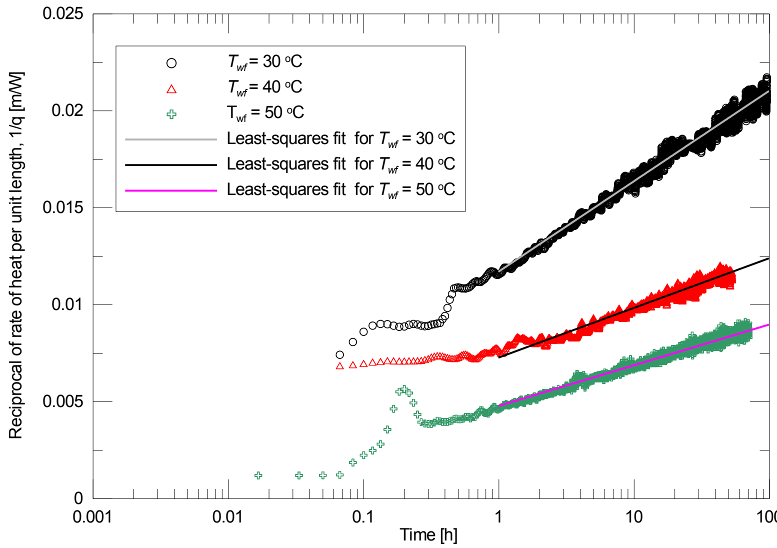

Figure 2- Semi log plots of $1/q'(t)$ versus *t* for thermal response tests performed for three different constant fluid temperatures.

Eq.32 is fitted to data of each test by least-squares (LS). In Table 1, for each test, the coefficient of regression R^2 and the slope *m* value determined from the fitted straight lines are given besides the values of average fluid temperature and the effective thermal conductivity *k* computed from the slope equation given by Eq. 33. The mean value of the effective thermal conductivity is 3.096 W/(mK) and the standard deviation is 0.246 W/(mK). The maximum deviation of the values computed from the mean is 8%, which is acceptable for all practical purposes.

Table 1. The results obtained by fitting the test data to Eq. 32.

Test No	Inlet Temperature	Average Temperature $\bar{T}_{wf}$	Slope of the LS m	Effective thermal conductivity (from eq. 33)	Coefficient of Regression R^2
	[°C]	[°C]	[m/W]	[W/(mK)]	
1	30	28.7	4.66 x 10^{-3}	3.096	0.963
2	40	37.7	2.56 x 10^{-3}	3.297	0.894
3	50	46.6	2.13 x 10^{-3}	2.808	0.971

When this method is applied to experimental data of different boreholes [26], the results given in Table 2 are obtained. In this results undisturbed formation temperature (T_∞) is 16 °C.

Table 2. Experimental results and estimated thermal conductivities for different boreholes by the method based on Eq. 32.

BHE No	Number of U Pipes	Depth	Diameter of BH	Diameter of pipe	Test Duration	Fluid inlet temp. T_g	Average Temp. $\bar{T}_{wf}$	Effective Thermal Conductivity (Eq. 33)
		[m]	[cm]	[mm]	[h]	[°C]	[°C]	[W/(mK)]
1	1	50	17	32	75	40	37.7	3.3
2	1	50	17	32	236	40	38.7	2.9
3	2	50	17	32	75	40	38.9	3.2
4	1	100	17	32	75	40	36.0	4.1
5	1	50	20	40	75	40	37.4	3.1
6	3	50	20	32	70	40	37.9	3.3

3. A COMPARISON WITH OTHER METHODS

To validate the method, a model in COMSOL [29] environment is built as shown in Fig. 3. Outer domain is chosen as sufficiently large so that the temperature around the borehole is not affected by the outer boundary. Material properties are given in Table 3. Pipe properties are taken from a catalog [30], grout properties and ground density are measured in laboratory. In drilling process it is seen that ground include greywacke and heat capacity of ground are taken from Reyes [31]. All collected data from the experiment is imported to the model, the model is solved and the unit heat load for unit borehole length is compared with the experimental one. Thermal conductivity of ground is obtained with parameter estimation method by fitting processes.

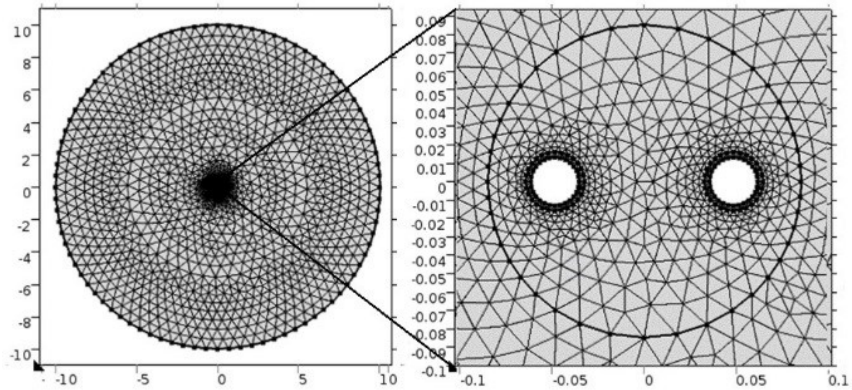

Figure 3- Numerical model and domains used in COMSOL [29].

In the model, borehole diameter is (r_k) 0.085m, outer and inner diameters of pipe are 0.016m and 0.0131 m respectively, ground domain's diameter is 10m and the shank space is 0.097m.

Table 3: Properties of domains.

		Ground	Grout	PE Pipe
Density	kg/m^3	2130	1760	959
Specific Heat Capacity	J/kgK	920	900	1900
Thermal Conductivity	W/mK	-	1.7(0.9 in borehole 3 and 6)	0.38

Results of the proposed method is compared with the results of numerical model as well as a previous method [26] that use different way to calculate thermal conductivity based on constant temperature TRT. Furthermore the thermal conductivities computed from both methods are compared with the ones obtained from conventional constant heat-flux method. All the comparisons are shown in Table 4. Since the equivalent radius approximation is just given for 1U-tube and 2U-tube, 3-tube solutions are not given in Table 4. Also as stated in Aydin's experimental study (see Aydin et al. [26]) constant heat flux tests are just applied to Boreholes 1, 4 and 5.

During the numerical modeling by COMSOL, thermal conductivity of pipes is neglected for some cases. The numbers given in parentheses in the 3rd column of Table 4 represent those cases. The numbers given in the same column without parenthesis are those for which the thermal conductivity of pipe is considered.

Table 4: Comparisons of the results of the proposed method with those of others.

BH No	Predicted Thermal Conductivities [W/mK]			
	Constant Temperature Method			Constant Heat Flux Method
	Onur's Method (Eq. 33)	Thermal conductivity of formation used in COMSOL Numerical Model $k_{grout+pipe}$=1 W/mK (k_{grout}=1.7 W/mK, k_{pipe}=0.38W/mK)	Aydin's Method [26] (with multipole r_e approximation)	Constant heat-flux Line Source Method (with 60W/m heat rate)
1	3.3	3.0 (2.3)	2.3	2.3
2	2.9	3.0 (2.2)	2.2	-
3	3.2	3.1	-	-
4	4.1	3.2 (2.5)	2.5	3.0
5	3.1	3.0 (2.4)	2.4	2.5
6	3.3	3.2	-	-

If we assume that the numerical results by COMSOL are more accurate results than the analytical ones, the proposed method introduced here gives very close results for all boreholes except the boreholes BH1 and BH4. Even for BH1 and BH4, we observe that the proposed method provides estimations of thermal conductivity better than those of other two methods. One possible explanation why the proposed method provides better estimations for the thermal conductivity of ground around the boreholes with multi U-tubes than 1U-tube is that the boreholes with multi U-tubes provide more uniform temperature distribution along the borehole and hence such configurations are more suitable for the constant temperature approximation made in the proposed method. Furthermore the thermal diffusivity of ground can be obtained from this method or it can be obtained from the observations during the drilling process. However, as it can be seen from Eq.35, to predict the thermal diffusivity equivalent radius has to be calculated. More information about equivalent radius approximation can be found from different resources [29-33].

4. CONCLUSION

In this study, a method is proposed to predict thermal properties of ground from constant temperature TRT data. The Laplace transformation method is used to obtain the solution of the governing equation for unit heat transfer rate of a borehole with constant temperature. It is shown that if the inverse of unit heat transfer rate versus log of time is considered it has a linear trend, i.e., a semi-log straight line, for long time asymptote. Similar to conventional TRT, one can estimate the thermal conductivity of ground from the slope information of this semi-log straight line. Using the value of inverse of unit heat transfer rate at a unit time ($t = 1$) , thermal diffusivity of the ground can also be predicted. In this method to determine the thermal conductivity, an estimation of the equivalent radius or heat capacity (for instance as in Aydin's method) is not required. The application of the new method proposed here to experimental data for different boreholes shows that the method gives accurate results for multi U-tubes as well as a single U-tube.

REFERENCES

[1] Thomson, W. (Lord Kelvin): Mathematical and Physical Papers, vol. 2. Cambridge University Press, London, UK, (1884), pp. 41–60.

[2] Ingersoll, L.R., Zobel, O.J., Ingersoll, A.C.: Heat Conduction with Engineering Geological and other Applications. McGraw-Hill, New York, NY, USA, (1954), pp. 325.

[3] Zeng, H., Diao, N., Fang, Z.: Heat transfer analysis of boreholes in vertical ground heat exchangers, *International Journal of Heat and Mass Transfer*, **46**, (2003), 4467-4481.

[4] Carslaw H. S., and Jaeger J. C.: Conduction of Heat in solids. Claremore Press, Oxford, UK, (1959), (Chapter XIII).

[5] Deerman , J.D. and Kavanaugh, S.P.: Simulation of Vertical U-tube Ground Coupled Heat Pump Systems using the Cylindrical Heat Source Solution, *ASHRAE Transactions*, **97(1)**, (1991), 287-295.

[6] Gehlin, S., Hellström, G., Nordell, B.: Comparison of four models for thermal response test evaluation, *ASHRAE transactions* **109 (1)**, (2003), 131-142.

[7] Philippe, M., Bernier, M., Marchio, D.: Validity ranges of three analytical solutions to heat transfer in the vicinity of single boreholes, *Geothermics*, **38**, (2009), 407-413.

[8] Yu, X., Zhang, Y., Deng, N., Wang, J., Zhang, D., Wang, J.: Thermal response test and numerical analysis based on two models for ground-source heat pump system, *Energy and Buildings*, **66**, (2013), 657-666.

[9] Yavuztürk, C.: Modeling of vertical ground loop heat exchangers for ground source heat pump systems. (PhD Thesis), Oklahoma State University, (1999).

[10] Signorelli, S., Bassetti, S., Pahud, D., Kohl, T.: Numerical evaluation of thermal response tests. *Geothermics*, **36**, (2007), 141-166.

[11] Ozudogru, T. Y., Olgun, C. G., Senol, A.: 3D numerical modeling of vertical geothermal heat exchangers, *Geothermics*, **51**, (2014), 312-324.

[12] Zanchini, E., Lazzari, S., Priarone, A.: Long-term performance of large borehole heat exchanger fields with unbalanced seasonal loads and groundwater flow, *Energy*, **38**, (2012), 66-77

[13] Aydın, M. and Sisman, A.: Experimental and computational investigation of multi U-tube boreholes, *Applied Energy*, **145**, (2015), 163-171.

[14] Carli, M.D., Tonon, M., Zarrella, A., Zecchin, R.: A computational capacity resistance model (CaRM) for vertical ground-coupled heat exchangers, *Renewable Energy*, **35**, (2010), 1537-1550.

[15] Zarrella, A., Scarpa, M., Carli, M.D.: Short time step analysis of vertical ground-couple heat exchangers: The approach of CaRM, *Renewable Energy*, **36**, (2011), 2357-2367.

[16] Beier, R. A.: Transient heat transfer in a U-tube borehole heat exchanger, *Applied Thermal Engineering*, **62**, (2014), 256-266.

[17] Bandyopadhyay, G., Gosnold, W., Mann, M.: Analytical and semi-analytical solutions for short-time transient response of ground heat exchangers, *Energy and Buildings*, **40**, (2008), 1816-1824.

[18] Man, Y., Yang, H., Diao, N., Liu, J., Fang, Z.: A new model and analytical solutions for borehole and pile ground heat exchangers, *International Journal of Heat and Mass Transfer*, **53**, (2010), 2593-2601.

[19] ASHRAE, ASHRAE Handbook: HVAC applications. Atlanta, GA, USA: ASHRAE, (2011).

[20] Gehlin, S.: Thermal Response Test: Method Development and Evaluation. PhD Thesis, Luleå University of Technology, Sweden, (2002).

[21] Sanner, B., Hellstrom, G., Spitler, J.D., Gehlin, S.E.A.: Thermal response test – current status and world-wide application. Proceedings World Geothermal Energy Congress, Antalya, Turkey, (2005).

[22] Bujok, P., Grycz, D., Klempa, M., Kunz, A., Porzer, M., Ptylik, A., Rozehnal, Z., Vojcinak, P.: Assessment of the influence of shortening the duration of TRT on the precision of measured values, *Energy*, **64**, (2014), 120-129.

[23] Raymond, J. and Lamarche, L.: Development and numerical validation of a novel thermal response test with a low power source, *Geothermics*, **51**, (2014), 434-444.

[24] EN14511-2 European Committee for Standardization, Air conditioners, liquid chilling packages and heat pumps with electrically driven compressors for space heating and cooling. No. EN 14511-2.

[25] Wang, H., Qi, C., Du, H., Gu, J.: Improved method and case study of thermal response test for borehole heat exchangers of ground source heat pump system, *Renewable Energy*, **35**, (2010), 727-733.

[26] Aydin, M.: Toprak Isı Değiştiricilerinde Yeni Bir Isıl Tepki Yöntemi Ve Performansın Parametrik İncelenmesi (Turkish), PhD Thesis, Istanbul Technical University, web.itu.edu.tr/murataydin, (2015).

[27] Churchill, R. V.: Operational Mathematics, 3rd ed. New York: McGraw-Hill, (1958).

[28] Najurieta, L.H.: A Theory of Pressure Transient Analysis in Naturally Fractured Reservoirs, *Journal of Petroleum Technology*, **32**, (1980), 1241-1250.

[29] COMSOL AB, COMSOL Version 4.2, COMSOL AB, Stockholm, Sweden, (2013).

[30] Rehau, https://www.rehau.com/download/866706/raugeo-technical-manual.pdf

[31] Reyes, A. G.: A preliminary evaluation of sources of geothermal energy for direct heat use, GNS Science Report 2007/16, 42p. (2007).

NOMENCLATURE

a	[m/W]	value of vertical axis at t=1 intercept
k	[W/(mK)]	Thermal conductivity
m	m/W/cycle	slope of $1/q'(t)$ in log scale graph
r	[m]	diameter
r_e	[m]	Equivalent diameter
r_k	[m]	Borehole diameter
r_b	[m]	Pipe diameter
$\tilde{r}$	-	Dimensionless radius
s	-	Laplace transform variable
t	[sec]	Time
$\tilde{t}$	-	Dimensionless time
T	[°C]	Temperature
T_∞	[°C]	Undisturbed ground temperature
$\overline{T}_{wf}$	[°C]	Average temperature of inlet and outlet in U-tube
T_g	[°C]	Inlet temperature to the borehole
q'	[W/m]	Unit heat flux in borehole
$\tilde{q}'$	-	Dimensionless heat flux
$\hat{\tilde{q}}'$	-	Laplace transform of $\tilde{q}'$
α	[m^2/s]	Thermal diffusivity coefficient
γ	-	Euler Gama constant 0.5772
θ	-	Dimensionless temperature
$\hat{\theta}$	-	Laplace transform of dimensionless temperature