MUKAVEMET

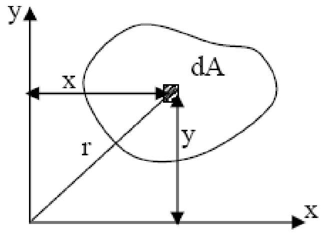
EYLEMSİZLİK MOMENTİ

Statik- M.TOPCU

ALAN ve KÜTLE ATALET MOMENTLERİ

6.1 Giriş ve Tanım

Atalet ; Direnim , karşı koyma. Bir cismin atalet momenti geometrik olarak dizaynda cismin (eğilme, burulma vb) zorlanmalara karşı direncinin bir ölçütüdür.



$r^2 = x^2 + y^2$

$J = I_o = \int r^2 dA$

$I_x = \int y^2 dA$ ⟶ *A alanının x ekseni etrafında atalet momenti (ikinci momenti)*

$I_y = \int x^2 dA$ ⟶ *A alanının y ekseni etrafında atalet momenti*

$I_o = \int r^2 dA$ ⟶ *A alanının kutupsal atalet momenti*

$I_{xy} = \int xy dA$ ⟶ *A alanının çarpım atalet momenti*

$I_o = \int r^2 dA = \int (x^2 + y^2) dA = \int x^2 dA + \int y^2 dA = I_x + I_y = I_o$

I_x , I_y ve I_o *her zaman pozitiftir,* I_{xy} $\pm$ *olabilir.*

I_x , I_y , I_o , I_{xy} =(L^4) ⟶ cm^4 , m^4 dir.

$$i_x = \sqrt{\frac{I_x}{A}} > 0 \quad I_x \text{ için atalet yarı çapı}$$

$$i_x = \sqrt{\frac{I_y}{A}} > 0 \quad I_y \text{ için atalet yarı çapı}$$

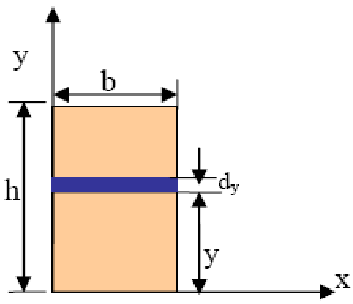
I_{xy} *için atalet yarıçapı yoktur.*

$$i_o = \sqrt{\frac{I_o}{A}} > 0 \quad I_o \text{ için atalet yarı çapı}$$

$$İ_o = \sqrt{\frac{I_x}{A} + \frac{I_y}{A}} = \sqrt{i_x^2 + i_y^2}$$

$$i_x , i_y , i_o > 0 \quad (L) \quad (mm,cm,m)$$

Örnek 6.1a *Dikdörtgenin tabanından geçen eksene göre atalet momenti*

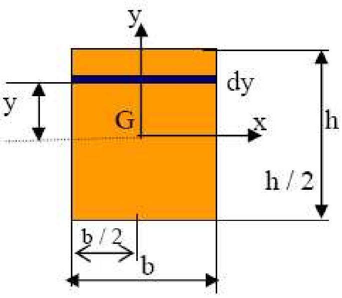


$dA = b.dy$

$$I_x = \int y^2 dA = b.\int_0^h y^2 dy = \frac{b.h^3}{3}$$

$$I_y = \frac{h.b^3}{3}$$

Örnek 6.1b *Dikdörtgenin ağırlık merkezinden geçen eksene göre atalet momenti*



$I_x = \int y^2 dA \quad dA = b.dy$

$$I_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 dy = b.\left|\frac{y^3}{3}\right|_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{b.h^3}{12}$$

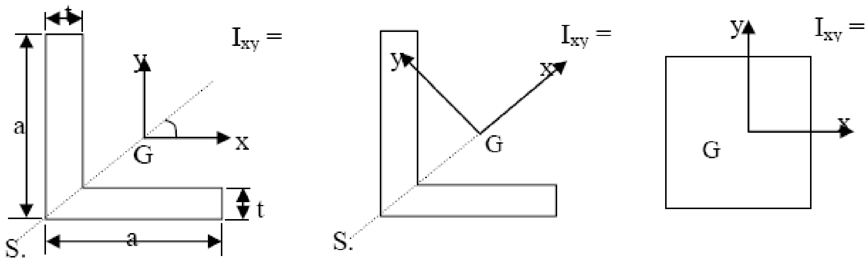
Benzer şekilde

$$I_y = \frac{h.b^3}{12}$$

Cismin tabanından geçen eksene göre ve ağırlık merkezinden geçen eksenlere göre atalet momentleri farklıdır. Ağırlık merkezinden geçen eksene göre atalet momenti minimum atalet momentidir.

Ağırlık merkezinde ; $I_{xy} = \int x.y.dA = 0$, $i_x = \sqrt{\frac{\frac{b.h^3}{12}}{b.h}} = \frac{h}{\sqrt{12}}$, $i_y = \frac{b}{\sqrt{12}}$, $i_o = \sqrt{\frac{h^2}{12} + \frac{b^2}{12}}$

Eğer kesit bir simetri eksenine sahip ve xy eksen takımının eksenlerinden birisi bu simetri ekseniyle çakışacak şekilde seçilirse ve buna ek olarak xy eksen takımının orijini G ağırlık merkezinde ise kesitin çarpım atalet momenti $I_{xy} = 0$' dır. (SE: Simetri ekseni)



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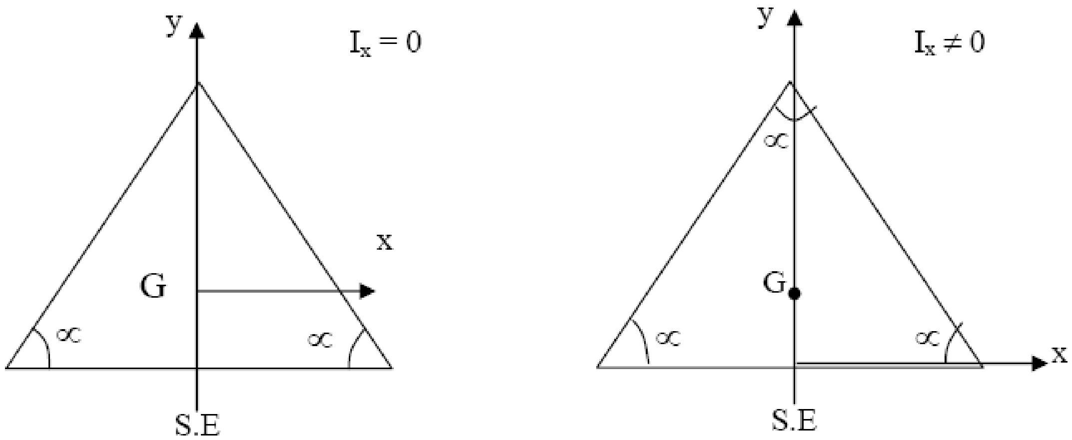
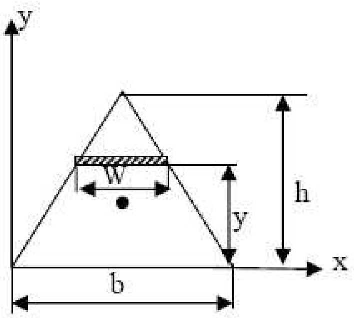


Örnek 6.2 *Üçgenin alan atalet momentinin hesabı*



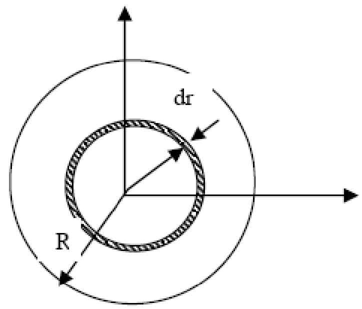
$I_x = \int y^2 dA$ dA = w.dy

$\frac{w}{b} = \frac{h-y}{h}$ w = $\frac{b}{h}(h-y)$

$$I_x = \int_0^h y^2\left(\frac{b}{h}h - y\right)dy = \int_{\frac{-h}{3}}^{\frac{2h}{3}} y^2 dA = \frac{bh^3}{36}$$

$$I_x = \frac{b.h^3}{12}$$

***Örnek 6.3** Daire kesit için atalet momenti*



$dA = 2\pi\, rdr \qquad I_o = \int r^2 dA = \int_0^R r^2 2\pi r dr$

$I_x = I_y \qquad I_o = 2\pi \frac{r^4}{2^4}\Big|_0^R = \frac{\pi R^4}{2}$

$I_o = I_x + I_y$

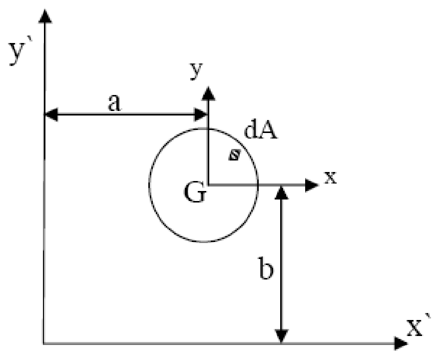
$I_x = \frac{I_o}{2}$

$I_x = \frac{\pi . R^4}{4} = I_y$

$$I_x = I_y = \frac{\pi R^4}{4} \Longrightarrow D = 2R \quad I_x = \frac{\pi . D^4}{64} = I_y$$

$$I_o = \frac{\pi R^4}{2} \Longrightarrow D = 2R \quad I_o = \frac{\pi D^4}{32}$$

6.2 Eksenlerin kaydırılması (paralel olarak)

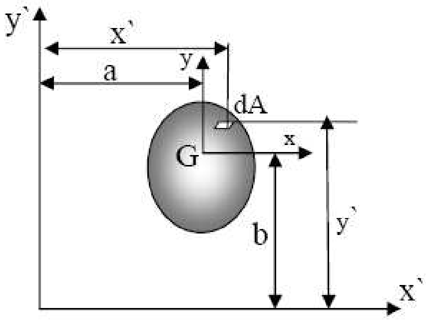


$I_X = \int y^2 dA$, $I_y = \int x^2 dA$, $I_{Xy} = \int xydA$ } biliniyor $\quad I_X` = \int y`^2 dA$, $I_y` = \int x`^2 dA$, $I_{xy}` = \int x`y`dA$

$S_x = \int y.dA = 0$

$S_y = \int x.dA = 0$

(a , b) G (x`, y`) takımındaki koordinatları

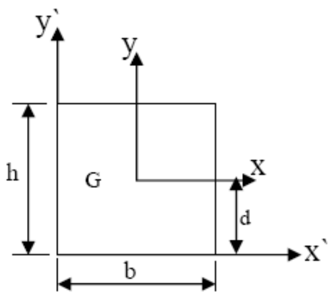


$$x` = a + x \qquad I_{x`} = \int y`^2 dA$$
$$y` = b + y \qquad I_{x`} = \int (y + b) dA$$
$$I_{x`} = \int (y^2 + 2yb + b^2) dA$$
$$I_{x`} = \int y`^2 dA + 2b\int y.dA + b^2 \int df$$
$$I_{x`} = I_x + A.b^2 > 0$$

$$I_{y`} = \int x`^2 dA$$
$$I_{y`} = \int (x + a)^2 dA$$
$$I_{y`} = I_y + a^2.A > 0$$
$$I_{x`y`} = \int x`y`dA$$
$$I_{x`y`} = \int (x + a)(y + b) dA = I_{xy} + a.b.A > 0$$

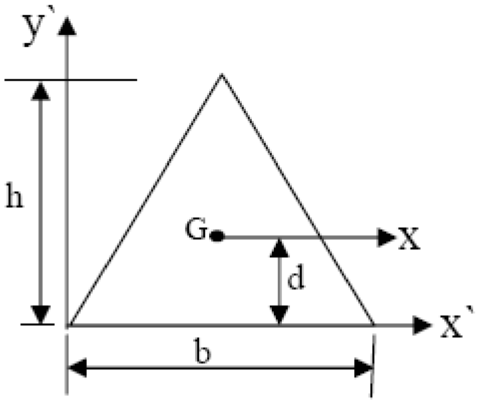
Bir cismin ağırlık merkezinden geçen eksenlere paralel herhangi bir eksene göre atalet momenti, cismin ağırlık merkezinden geçen eksene göre atalet moment ile bu paralel iki eksen arasındaki uzaklığın karesinin, cismin alanıyla çarpımının toplamına eşittir.

Örnek 6.4 *paralel eksenler teoreminin uygulaması*



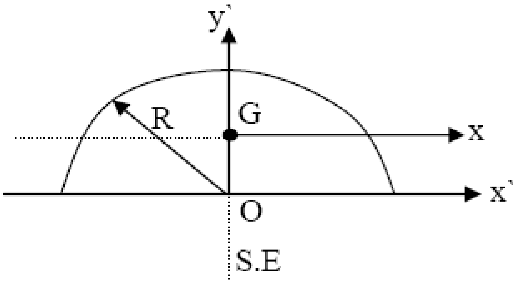
$$I_{x`} = I_{XG} + d^2A$$
$$d = \frac{h}{2}$$
$$I_{x`} = \frac{b.h^3}{12} + \left(\frac{h}{2}\right)^2 b.h = \frac{b.h^3}{3}$$

Örnek 6.5



$$I_{x`} = I_{XG} + d^2A$$
$$\frac{b.h^3}{12} = I_{XG} + \left(\frac{h}{3}\right)\frac{b.h}{2}$$
$$I_{XG} = \frac{b.h^3}{36}$$

Örnek 6.6 1 / 2 dairenin atalet momentinin hesabı



$$I_x = I_y = \frac{\pi.R^4}{4} \quad (\text{ Tüm daire })$$

$$I_{x`} = I_{y`} = \frac{\pi.R^4}{8} \quad (\text{ Yarım daire })$$

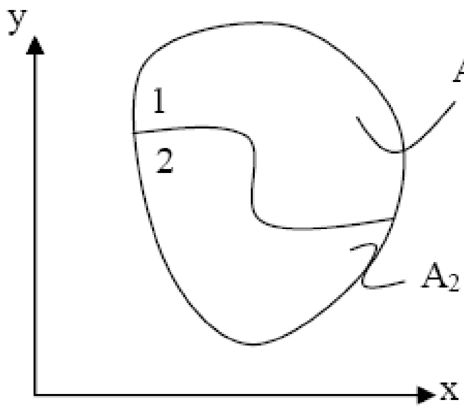
$$I_{x`} = I_{XG} + d^2A$$

$$\frac{\pi.R^4}{4} = I_{XG} = + \left(\frac{4.R}{3\pi}\right)^2 . \frac{\pi.R^2}{2}$$

$$I_{XG} = \frac{\pi.R^4}{8} - \frac{16.R^4}{18\pi}$$

$$I_{XG} = \frac{\pi.R^4}{8} . \left[1 - \frac{64}{9\pi^2}\right]$$

6.3 Bileşik Cisimlerin Atalet Momentleri ;



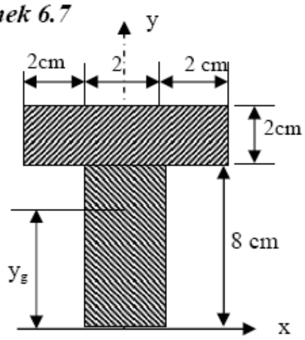
$$A = A_1 + A_2$$

$$I_x = I_{x1} + I_{x2}$$

$$I_y = I_{y1} + I_{y2}$$

$$I_{xy} = I_{xy1} + I_{xy2}$$

Örnek 6.7



a) Verilen profil kesitte I_x , I_y ve I_{xy} atalet momentlerini
b) Cismin ağırlık merkezinden geçen eksenlere göre atalet momentleri

a-) $I_x = I_{x1} + I_{x2}$

$$= \frac{6.2^3}{12} + 9^2.12 + \frac{2.8^3}{12} + 4^2 16$$

$$I_y = I_{y1} + I_{y2}$$

$$I_y = \frac{6.2^3}{12} + \frac{2^3 8}{12}$$

$$I_{xy} = 0$$

b-) $x_g = 0$

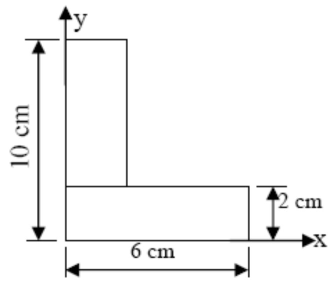
$$y_g = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{9.12 + 4.16}{12 + 16} = \frac{43}{7} \cong 6\,\text{cm}$$

$$I_{x'} = \frac{6.2^3}{12} + 3^2.12 + \frac{2.8^3}{12} + 2^2.16 = 261\text{cm}^4$$

$$I_{y'} = \frac{2.6^3}{12} + \frac{8.2^3}{12} = 5.33\text{cm}^4$$

$$I_{X'Y'} = 0$$

Örnek 6.8 *Verilen profil kesitin ağırlık merkezinden geçen eksenlere göre atalet momentlerini hesaplayınız*



$$X = \frac{1.16 + 3.12}{16 + 12} \cong 1.85$$

$$Y = \frac{6.16 + 1.12}{16 + 12} \cong 3{,}85$$

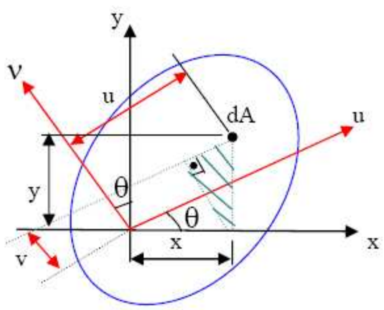
$$I_{x'} = \frac{2.8^3}{12} + (2{,}15)^2.16 + \frac{6.2^3}{12} + (2{,}85)^2.12 = 159\text{cm}^2$$

$$I_{y'} = \frac{2.8^3}{12} + (0{,}85)^2.16 + \frac{2.6^3}{12} + (1{,}15)^2.12 = 148.76\text{cm}^2$$

$$\begin{aligned} I_{x'y'} &= I_{x1y1} + I_{x2y2} \\ &= x_1 y_1 A_1 + x_2 y_2 A_2 \\ &= (-0.85).(2{,}15).16 + (1{,}15).(-2{,}85).12 \\ &= 68.57\ \text{cm}^4 \end{aligned}$$

6.4 Eksenlerin Döndürülmesi (Asal eksenler ve Asal atalet momentleri)

Atalet momentlerinin seçilen eksenlere göre yer aldıkları ve bu eksenlerin değişmesiyle tabii olarak değiştiklerini daha önce görmüştük. Şimdi eksenlerin döndürülmesi halinde atalet momentlerinin nasıl değiştiğini inceleyelim.



$$u = x.\cos\theta + y.\sin\theta$$

$$v = y.\cos\theta - x.\sin\theta$$

$$I_v = \int v^2 dA = \int (y.\cos\theta - x.\sin\theta)^2 dA$$

$$I_u = \cos^2\theta \int y^2 dA - 2\sin\theta\cos\theta \int xy dA + \sin^2\theta \int x^2 dA$$

1)- $I_u = I_x.\cos^2\theta - 2.I_{xy}.\sin\theta.\cos\theta + I_y.\sin^2\theta$

$I_u = \int v^2 dA = \int (x.\cos\theta - y.\sin\theta)^2 dA = \cos^2\theta \int x^2 dA + 2.\sin\theta.\cos\theta \int x.y.dA + \sin^2\theta \int y^2 dA$

2)- $I_v = I_x.\cos^2\theta + 2.I_{xy}.\sin\theta.\cos\theta + I_y.\sin^2\theta$

3)- $I_{uv} = \int u v.dA = \int (x.\cos\theta + y.\sin\theta)(y.\cos\theta - x.\sin\theta)$

$I_{uv} = I_x.\sin\theta.\cos\theta - I_y.\sin.\theta.\cos\theta + I_{xy}.(\cos^2\theta - \sin^2\theta)$

$2\sin.\theta.\cos\theta = \sin.2\theta$

$\cos^2\theta - \sin^2\theta = \cos.2\theta$ dönüşümleri yapılırsa

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}.\cos.2\theta - I_{xy}.\sin 2\theta$$

$$I_v = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}.\cos.2\theta + I_{xy}.\sin 2\theta$$

$$I_{uv} = \frac{I_x - I_y}{2}.\sin.2\theta - I_{xy}.\cos 2\theta$$

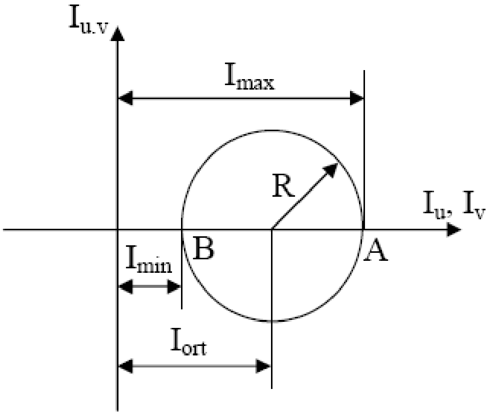
$I_{uv} = 0$ ise

$$\tan.2\theta = -\frac{2.I_{xy}}{I_x - I_y}$$

Eğer I_u , I_v , I_{uv} den θ' lı terimler yok edilirse

$$\left[I_u-\left(\frac{I_x+I_y}{2}\right)\right]^2+I_{uv}=\left(\frac{I_x-I_y}{2}\right)^2+I_{xy}^{\ 2} \qquad \text{(çember denklemi)}$$

$$(x-a)^2+(y-b)^2=r^2$$



$I_{max} = I_{ort} + R$

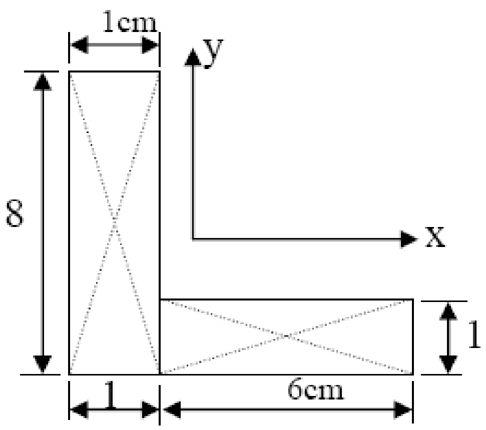
$I_{min} = I_{ort} - R$

Atalet momentinin maksimum ve minumum değerine asal atalet momentleri, Bunların bulunduğu eksenlere asal eksenler denir. Asal eksenler üzerinde $I_{xy} = 0$ ' dır.

$$I_{ort}=\frac{I_x+I_y}{2} \quad , \qquad R=\sqrt{\left(\frac{I_x-I_y}{2}\right)^2+I_{xy}^{\ 2}}$$

$$I_{max\ ,\ min}=I_{1,2}=\frac{I_x+I_y}{2}\pm\sqrt{\left(\frac{I_x-I_y}{2}\right)^2+I_{xy}^{\ 2}}$$

Örnek 6.9



I_x , I_y , I_{xy} = ?

I_{max} , I_{min} = ?

$$x=\frac{x_1A_1+x_2A_2}{2}=1{,}65\ \text{cm}$$

$$y=\frac{x_1A_1+x_2A_2}{2}=1{,}65\ \text{cm}$$

$$I_x = \frac{1.8^3}{12} + (1,35)^2.8 + \frac{5.1^3}{12} + (2,15)^2.5 = 80,6cm^4$$

$$I_y = \frac{8.1^3}{12} + (1,15)^2.8 + \frac{1.5^3}{12} + (1,85)^2.5 = 38,7cm^4$$

$$I_{xy} = x_1.y_1.A_1 + x_2.y_2.A_2 = 8.(-1,15).1,35 + 5.(1,85)(-2,15) = -32,3\ cm^4$$

$$\tan 2\propto = -\frac{2.32,3}{80,6 - 38,7}$$

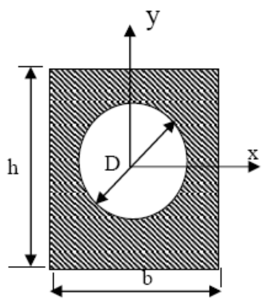
$$I_{max\ ,\ min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}{}^2}$$

$$= \frac{I80,6 + 38,7}{2} \pm \sqrt{\left(\frac{80,6 - 38,7}{2}\right)^2 + (-32,3)^2}$$

$$I_{max} = 60 + 36 = 96cm^4$$

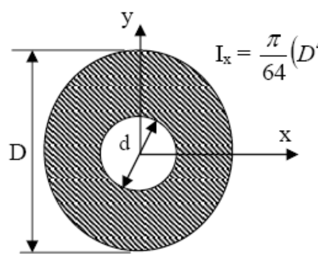
$$I_{min} = 60 - 36 = 24cm^4$$

Bir kesitte delik varsa dolu kesitin atalet momentinden deliğin atalet momenti çıkarılır.



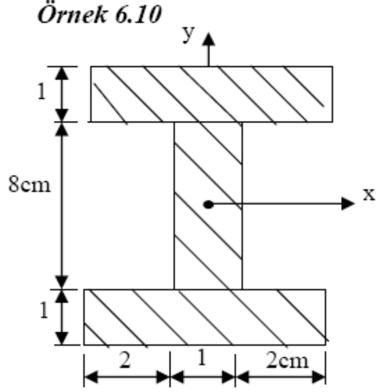
$$I_x = \frac{b.h^3}{12} - \frac{\pi.D^4}{64}$$

$$I_y = \frac{h.b^3}{12} - \frac{D.\pi^4}{64}$$



$$I_x = \frac{\pi}{64}\left(D^4 - d^4\right)$$

Örnek 6.10



$I_{xy} = 0$

$I_x = ?$

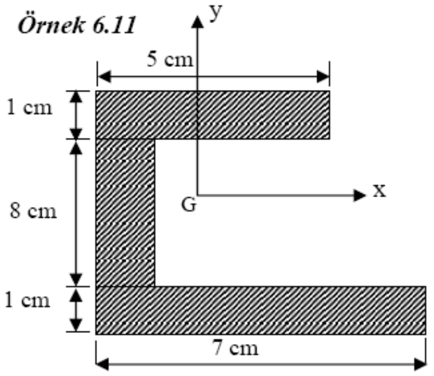
$I_y = ?$

$$I_x = \frac{1.8^3}{12} + \left(\frac{5.1^3}{12} + (4,5)^2.5\right).2 = 243cm^4$$

$I_y =$

$$\frac{8.1^3}{12} + 2.\left(\frac{5^3 1}{12}\right) veya \frac{10.5^3}{12} - 2\left(\frac{8.2^3}{12} + (1,5)^2.16\right) = 21cm^4$$

Örnek 6.11



Verilen profil kesitte ağırlık merkezinden geçen x, y eksen takımına göre atalet momentlerini hesaplayınız.

$$x = \frac{x_1A_1 + x_2A_2 + x_3A_3}{A_1 + A_2 + A_3} = \frac{2,5.5 + 0,5.8 + 3,5.7}{20} = 2,0 \text{ cm}$$

$$y = \frac{y_1A_1 + y_2A_2 + y_3A_3}{A_1 + A_2 + A_3} = \frac{9,5.5 + 5.8 + 0,5.7}{20} = 4,5 \text{ cm}$$

$$I_x = \frac{5.1^3}{12} + 5^2 5 + \frac{1.8^3}{12} + (0,5)^2.8 + \frac{1^3.7}{12} + 4^2.7 = 282.6 cm^4$$

$$I_y = \frac{1.5^3}{12} + 10,55^2.5 + \frac{8.1^3}{12} + (1,5)^2.8 + \frac{1.7^3}{12} + (1,5)^2.7 = 74,6 cm^4$$

$$I_{xy} = x_1y_1A_1 + x_2y_2A_2 + x_3y_3A_3 = 0,5.5.5 + (-1,5)(0,5).8 + (1,5)(-4).7 = -35,5 \text{ cm}^4$$

$$\tan 2\propto = \frac{-35,5.2}{282,6 - 7,6}$$

$$I_{max,\,min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^{\ 2}}$$

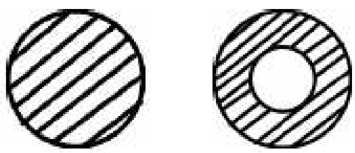
Dairesel Kesit

$$I_P = \int_A r^2 dA$$

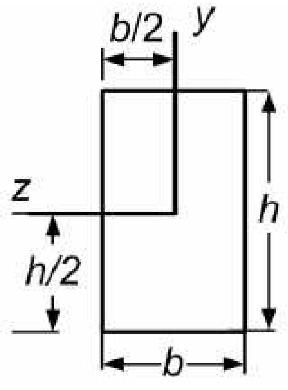
$$I_P = \frac{\pi}{32} D^4 \qquad I_P = \frac{\pi}{2} R^4$$

$$I_P = \frac{\pi}{32} D^4 - \quad Soild$$

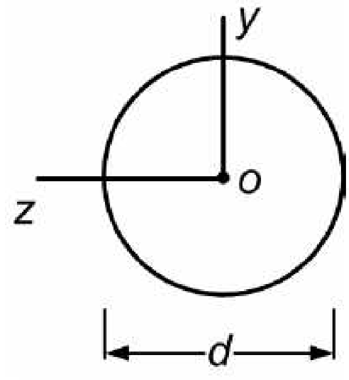
$$I_P = \frac{\pi}{32}\left(D_o^{\ 4} - D_i^4\right) - \quad hollow$$



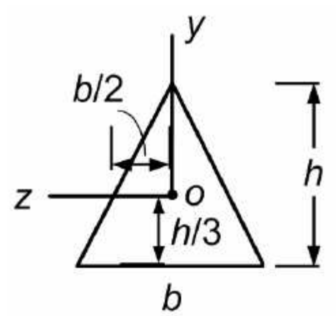
$$S = \frac{I}{y_{max}}$$



$$I_{zz} = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6}$$

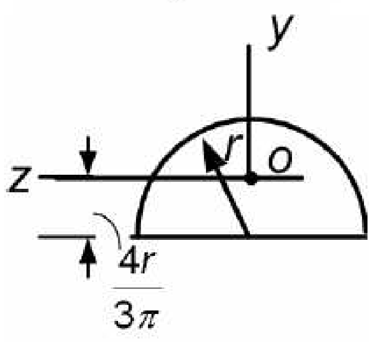


$$I_{zz} = \frac{\pi}{64}d^4 \qquad S = \frac{\pi d^3}{32}$$

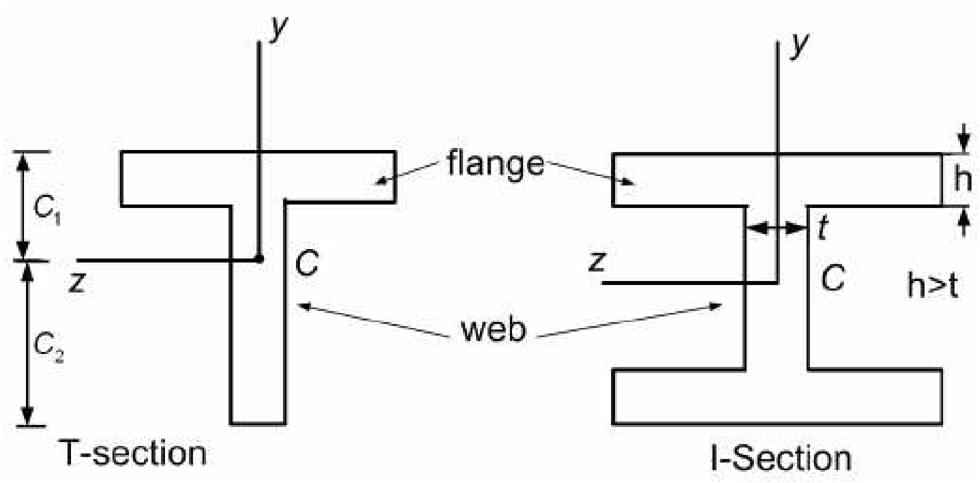


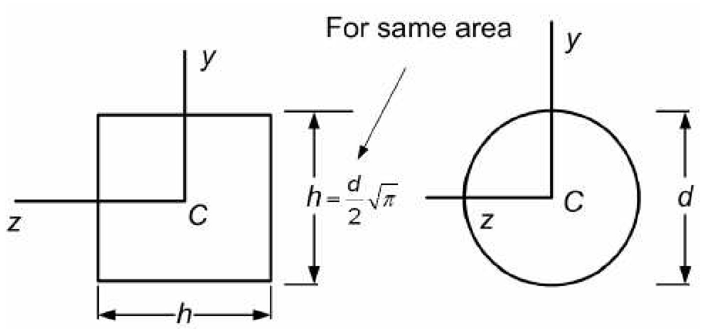
$$I_{zz} = \frac{bh^3}{36}$$

$h = \sqrt{3}\,b/2$ *for eqilateral triangle*

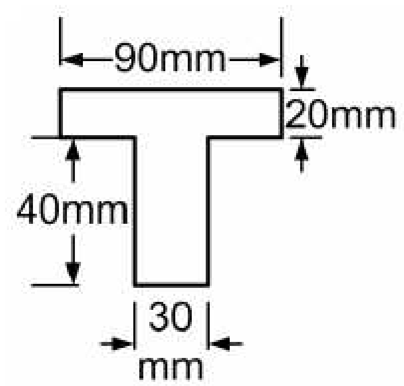


$$I_{zz} = 0.1098r^4$$





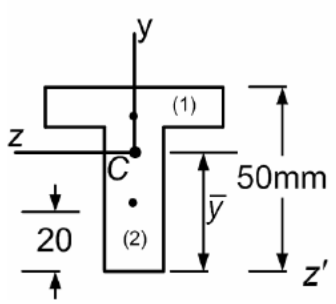
$$\frac{S_{square}}{S_{circle}} = 1.18$$



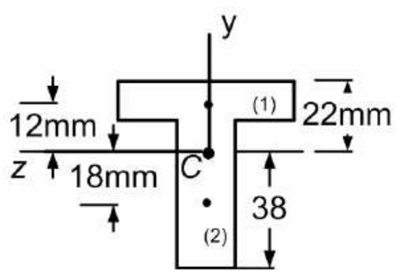
Centroid :-

	$A\ mm^2$	$\overline{y}$	$\overline{y}A\ mm^3$
1	$20\times90=1800$	50	90×10^3
2	$40\times30=1200$	20	24×10^3

$A=\Sigma A=3000 \quad \Sigma\overline{y}A=114\times10^3$



$$A\overline{y}=\Sigma\overline{y}A \Rightarrow \overline{y}3000=114\times10^3 \Rightarrow \overline{y}=38mm$$



$$I_{zz}=I=\Sigma\left(\overline{I}+Ad^2\right)s$$

$$=\Sigma\left(\frac{bh^3}{12}+Ad^2\right)$$

$$=\frac{1}{12}90\times20^3+1800\times12^2+\frac{1}{12}\times30\times40^2+1200\times18^2$$

$$I_{zz}=I=868\times10^3\ \ mm^4=868\times10^{-9}m^4$$