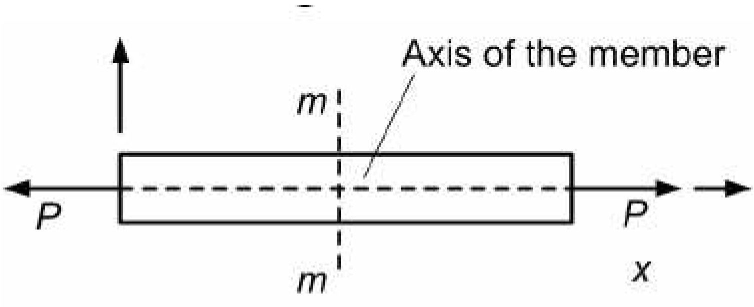
MUKAVEMET

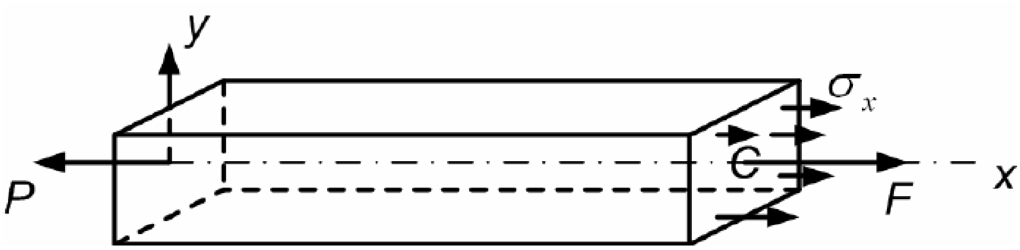
NORMAL KUVVET

Normal kuvvet etkisindeki bir çubukta

Stresses, strains and deformations

Consider a prismatic bar of constant cross-sectional area A and length L, with material properties A & v. Let the rod be subjected to an axial force “p”, which acts along x-axis.





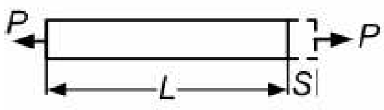
$$F = P$$

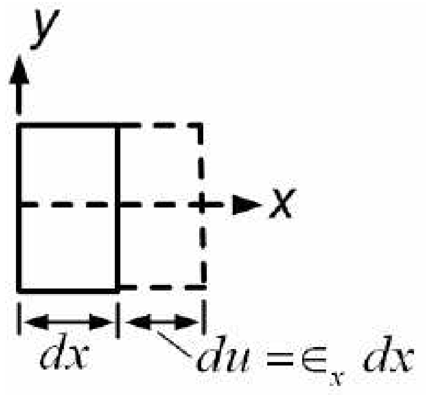
$$M_x = M_y = M_z = 0$$

$$V_y = V_z = 0$$

We know that internal resultant force

$$F = \int_A \sigma_x dA$$





Şekil değiştirme aşağıdaki gibi hesaplanabilir

$$\in_x = \frac{\sigma_x}{E} = \frac{P}{AE}$$

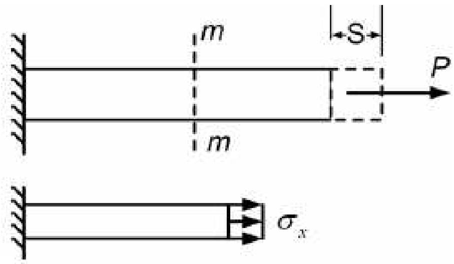
Çubuğun boy uzaması δ ise

$$u(L) - u(0) = \delta = \int_0^L \in_x \, da = \int_0^L \frac{P}{AE} dx = \frac{PL}{AE}$$

$$\sigma_x = \frac{P}{A}$$

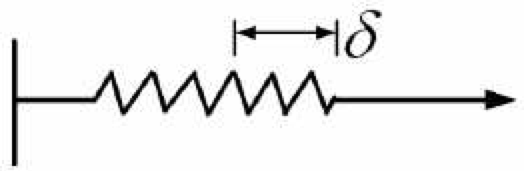
$$\delta = \frac{PL}{AE}$$

$$AE = Axial\,rigidity$$



$$\delta = \int_0^L \frac{P(x)}{A(x)E(x)} dx$$

şeklindedir



$$P = kS$$
$$S = fP$$

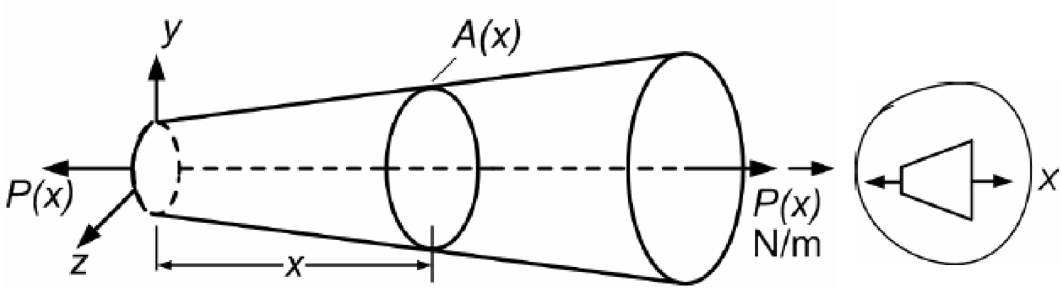
$$k = \frac{1}{f}$$

$$k = \frac{AE}{L} \qquad f = \frac{L}{AE}$$

Burada k çubuğun rijitliği, bununtersi olan f ise fleksibilitesi olarak isimlendirilir

$$\sigma_x = \frac{P}{A} \;\; \& \;\; \delta = \frac{PL}{AE}$$

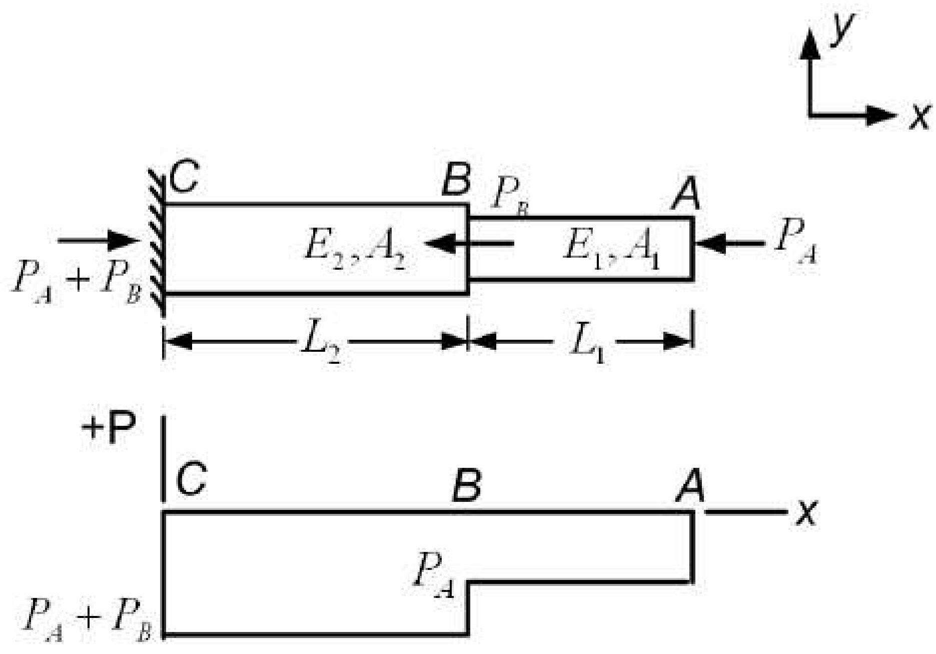
Değişgen kesitli çubuğun yer değiştirmesi



$$\sigma_x = \frac{P(x)}{A(x)} = \frac{F(x)}{A(x)}$$

$$S = \int_0^L \frac{P(x)}{A(x)E(x)} dx$$

Farklı iki malzemeden yapılmış çubuktaki yer değiştirme



$$\delta_{BC} = \frac{PL}{AE} = \frac{-(P_A + P_B)L_2}{A_2 E_2}$$

$$\delta_{AB} = \frac{PL}{AE} = \frac{-P_A L_1}{A_1 E_1}$$

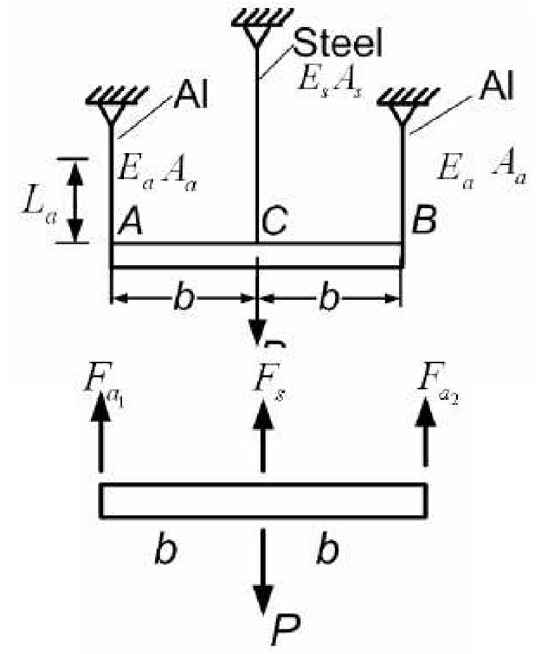
$$\sigma_{BC} = -\frac{(P_A + P_B)}{A_2}$$

$$\sigma_{AB} = -P_A / A_1$$

$$\delta_{CA} = S_{BC} + S_{AB}$$

$$\delta = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i} \qquad and \qquad \sigma_i = \frac{P_i}{A_i}$$

Esnek çubuklara asılmış rijit çubuktaki yer değiştirme



Equilibrium

$\left[\Sigma F_y = 0\right]$

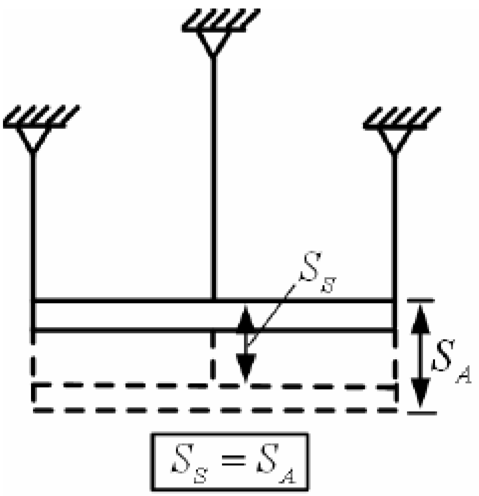
$$F_{a1} + F_{a2} + F_s - P = 0$$

$[\Sigma M_C = 0]$

$$bF_{a1} - bF_{a2} = 0$$

$$F_{a1} = F_{a2}$$

$$2F_a + F_s = P \qquad (1)$$

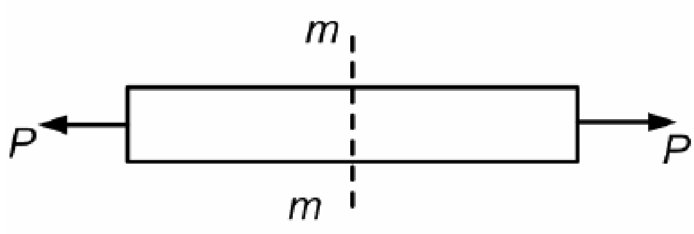


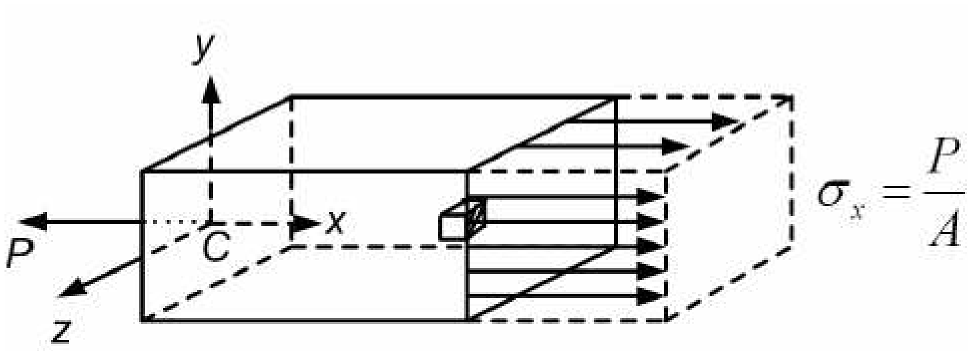
$$\delta_s = \delta_A \Rightarrow \textit{ geometric compatibility equation}$$

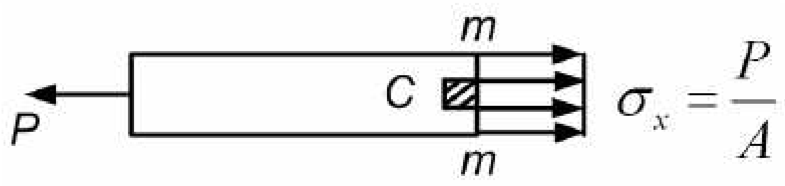
$$\delta_s = \frac{F_s L_s}{A_s E_s} \qquad \textit{and} \qquad \delta_A = \frac{F_A L_A}{E_A A_A}$$

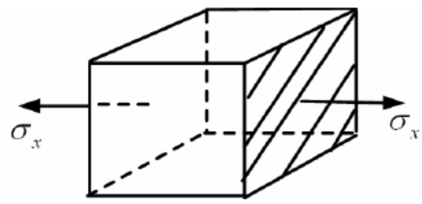
$$\delta_s = \delta_A \Rightarrow \quad \frac{F_A L_A}{E_A A_A} = \frac{F_s L_{As}}{E_s A_s} \qquad (2)$$

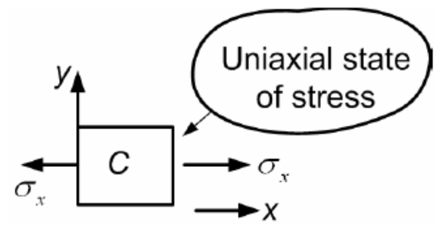
Stresses in axially loaded members









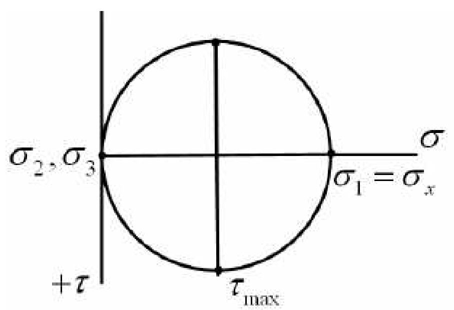


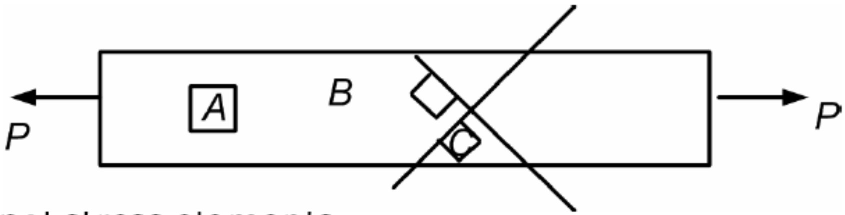
Uniaxial state stress is a special case of plane stress

$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_x & 0 \\ 0 & 0 \end{bmatrix}$$

$$\sigma_1 = \sigma_x$$

$$\tau_{max} = \left|\frac{\sigma_1}{2}\right| = \left|\frac{\sigma_x}{2}\right|$$

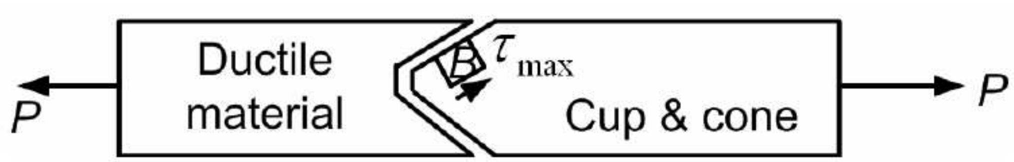




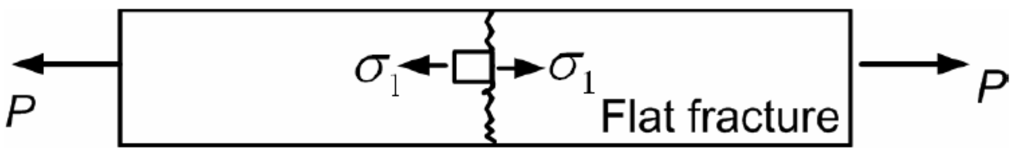
A – Principal stress elements

B,C – maximum shear stress elements.

Occurs at 45° to $x-y$ or $x-z$ planes.



Ductile material are weak in shear. They fail along τ_{max} planes.



Brittle materials weak in normal tensile stresses. They fail along σ_1 planes.

Statikçe belirsiz sistemlerde, denge denklemlerine ilaveten geometrik uygunluk koşulları yazılarak bu sistemler çözülür.

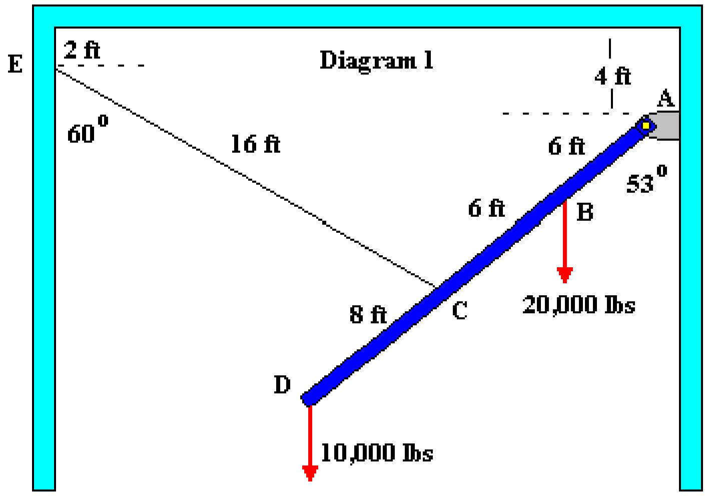
Stress: $\sigma = F / A$ (lb / in^2, or N / m^2)

Strain: $\varepsilon = \delta / Lo$ (no units)

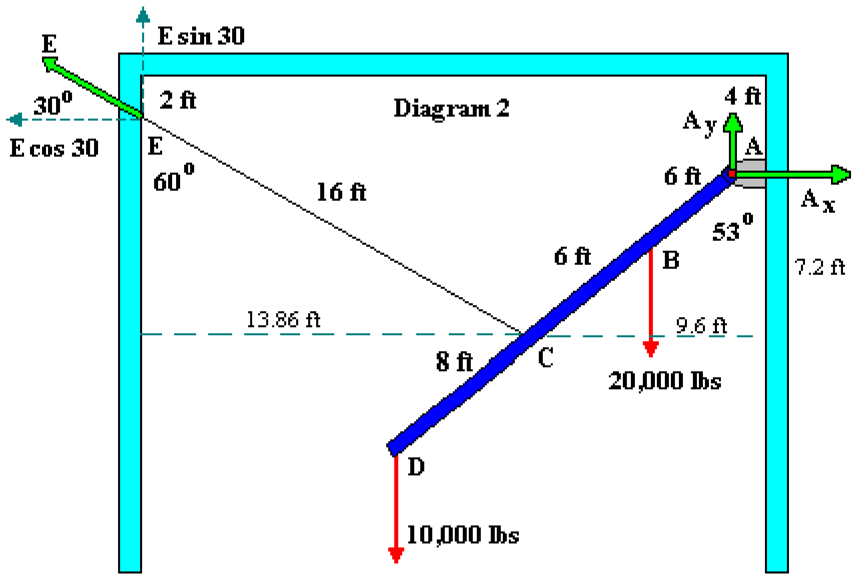
Hooke's Law: $E = \sigma / \varepsilon$ (lb / in^2, or N / m^2)

Deformation: $\delta = FL / EA$ (in. or m)

Örnek -1



Adı m 1:



$F_x = A_x - E \cos(30^o) = 0$

$F_y = A_y + E \sin(30^o) - 10{,}000 \text{ lbs} - 20{,}000 \text{ lbs} = 0$

$T_A = (20{,}000 \text{ lbs})(4.8 \text{ ft}) + (10{,}000 \text{ lbs})(16 \text{ ft}) + E \cos(30^o)(2 \text{ ft}) - E \sin(30^o)(23.46 \text{ ft}) = 0$

Bilinmeyenler: E = 25,600 lb.; A_x = 22,170 lb.; A_y = 17,200 lb.

Adı m 2.

CE = 25,600 lb.

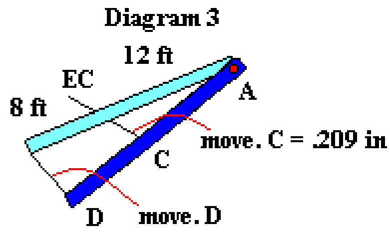
A.) Stress in CE = F/A = 25,600 lbs/(3.14*(.5 in)^2= 32,600 psi.

B.) Deformation of CE = $(FL/ EA)_{CE}$ = (25,600 lbs)(16 ft * 12 in/ft) / (30*10^6 lbs/in^2)(3.14*(.5 in)^2) = .209 in

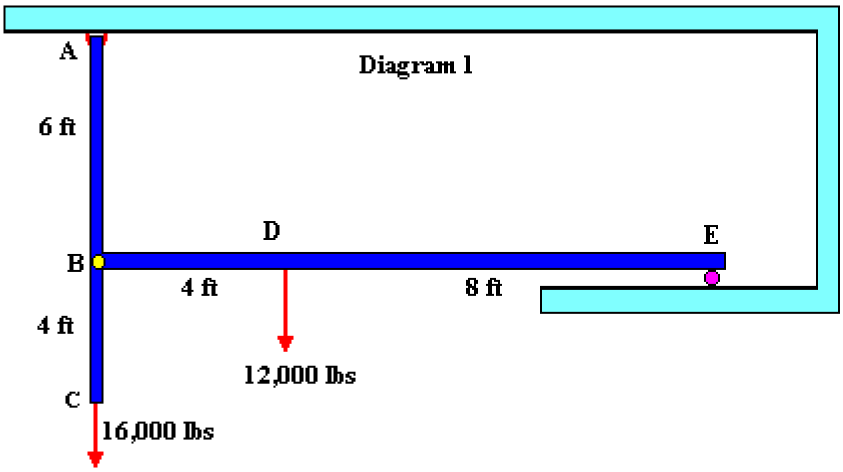
C.) Strain in CE = (Deformation of CE)/(Length of CE) = (.209"/192") = .00109

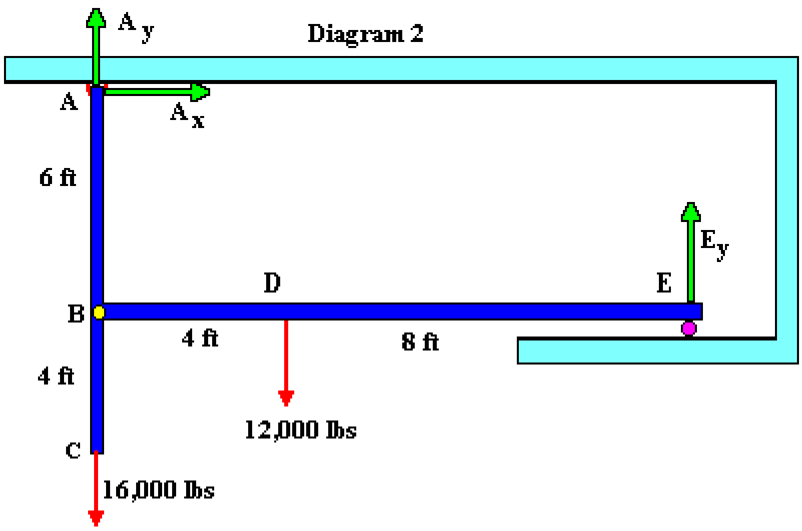
D.) Movement of point D.

[Movement of C / 12 ft = Movement of D / 20 ft] or [.209 in / 12 ft = Movement of D / 20 ft], and solving gives us **Movement of D = .348 in.**



Örnek 2





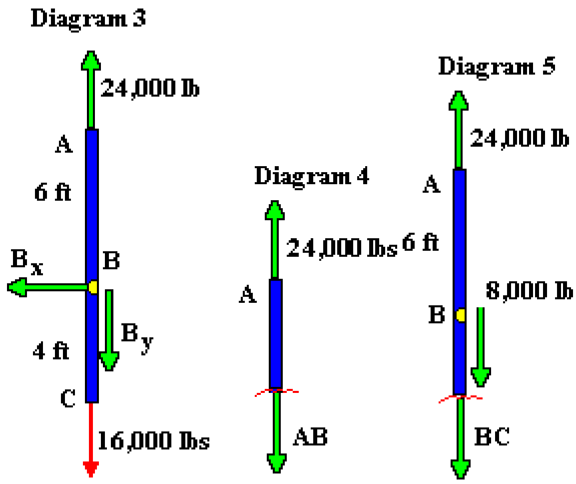
$F_x = A_x = 0$

$F_y = A_y + E_y$ -16,000 lbs - 12,000 lbs = 0

T_E = (16,000 lbs)(12 ft) + (12,000 lbs)(8 ft) - A_y(12 ft) = 0

Bilinmeyenler: **A_y = 24,000 lbs; E_y = 4000 lbs**

1: FBD of member ABC. (Diagram 3)



$F_x = B_x = 0$

F_y = 24,000 lb. - B_y - 16,000 lbs = 0

B_y = 8,000 lb.

Stress = F/A = 24,000 lb/(3.14 x .5^2) = 30,600 psi.

BC deki Stress = F/A = 16,000 lb/(3.14 x .5^2)

Stress (BC) = 20,400 psi.

delta C = Def_{AB} + Def_{BC}

C = [$(FL / EA)_{AB}$ + $(FL / EA)_{BC}$]

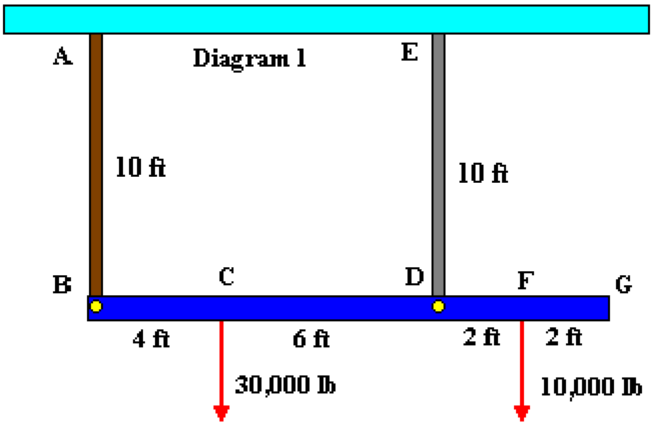
f C = [(24,000 lbs)(72 in) / (10*10^6 psi)(3.14*(.5 in)^2)]$_{AB}$ + [(16,000 lbs)(48 in) / (10*10^6 psi)(3.14*(.5 in)^2)]$_{BC}$

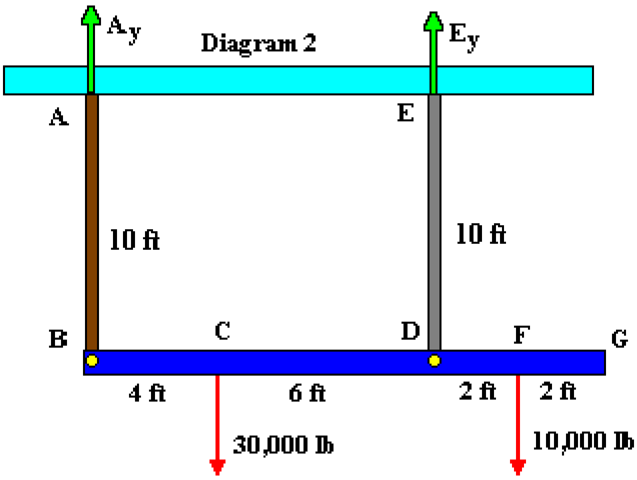
Movement of C = (.220 in) + (.0978 in) = .318 in

Örnek 3

steel : E_{st} = 30 x 10^6 psi,

Aluminum: E_{al} = 10 x 10^6 psi.





$F_y = A_y + E_y$ - 30,000 lb. - 10,000 lb. = 0

T_B = (-30,000 lb.)(4 ft) + E_y(10 ft) - (10,000 lb.)(12ft) = 0

E_y = 24,000 lb.; A_y = 16,000 lb.

F_{AB} = 16,000 lb. (tension),

F_{DE} = 24,000 lb. (tension).

Stress AB = F/A = 16,000 lbs/.5 in^2 = 32,000 psi.,

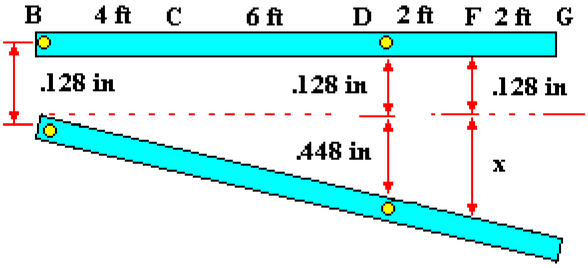
Stress DE = F/A = 24,000 lbs/.5 in^2 = 48,000 psi.

$Def_{AB} = (FL / AE)_{AB}$ = (16,000 lbs)(120 in) / (30*10^6 lbs/in^2)(.5 in^2) = .128 in

$Def_{ED} = (FL / AE)_{ED}$ = (24,000 lbs)(120 in) / (10*10^6 lbs/in^2)(.5 in^2) = .576 in

Movement of point F.

Diagram 3

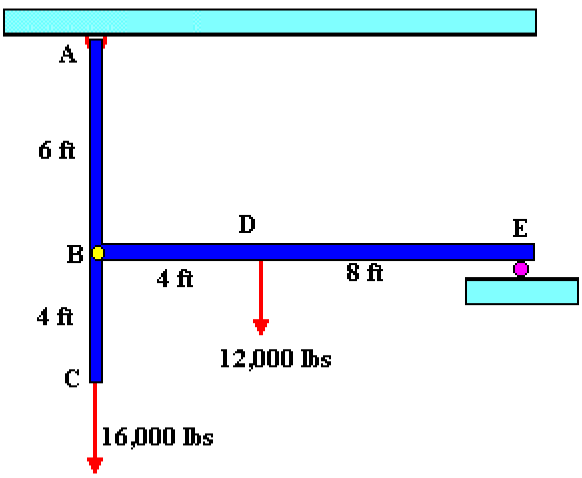


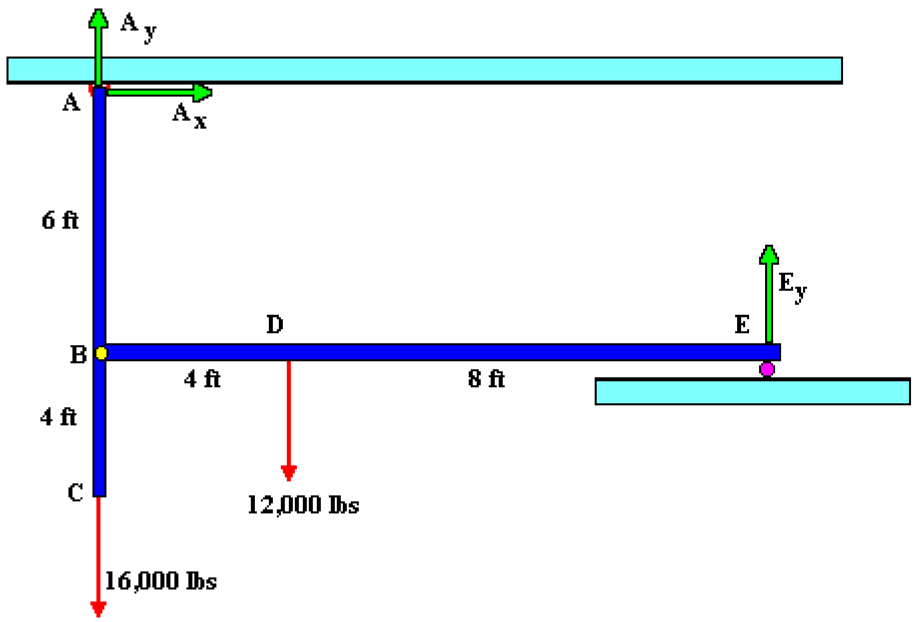
Movement of F = .128 inches + .5376 inches = .666 inches

Örnek

E_{al} = 10 x 10^6 psi

Çözüm:





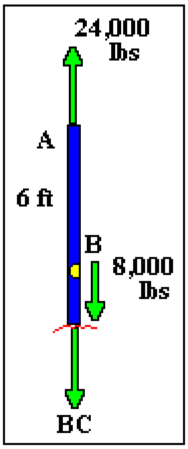
$F_x = A_x = 0$

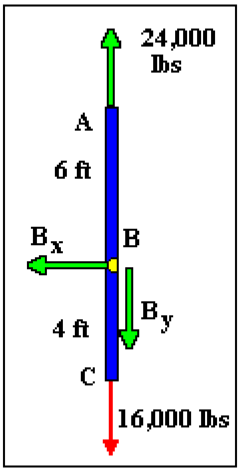
$F_y = A_y + E_y$ -16,000 lbs - 12,000 lbs = 0

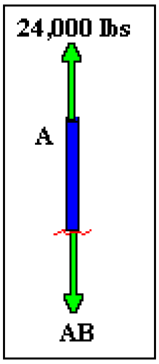
T_E = (16,000 lbs)(12 ft) + (12,000 lbs)(8 ft) - A_y(12 ft) = 0

Bilinmeyenler

A_y = 24,000 lbs; E_y = 4,000 lbs







$F_x = B_x = 0$

F_y = 24,000 lbs - B_y - 16,000 lbs = 0

Bilinmeyenler:

B_y = 8,000 lbs

$Stress_{AB}$ = F/A = 24,000 lbs/.785 in^2 = 30,600 psi

$Stress_{BC}$ = F/A = 16,000 lbs/.785 in^2 = 20,400 psi

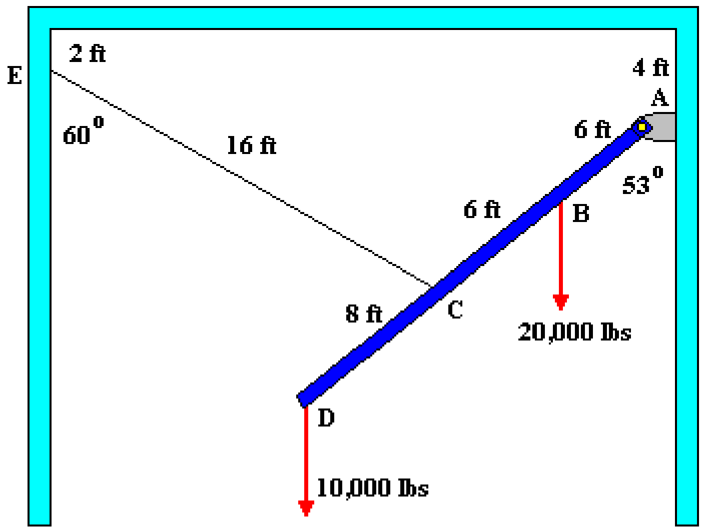
movement of C = Def_{AB} + Def_{BC}

move.C=[$(FL / EA)_{AB}$ + $(FL / EA)_{BC}$]

move.C=[(24,000 lbs)(72 in)/(10*10^6 psi)(3.14*(.5 in)^2)]$_{AB}$+[(16,000 lbs)(48 in)/(10*10^6 psi)(3.14*(.5 in)^2)]$_{BC}$

move.C=(.22 in) + (.0978 in) = .318 in

Örnek



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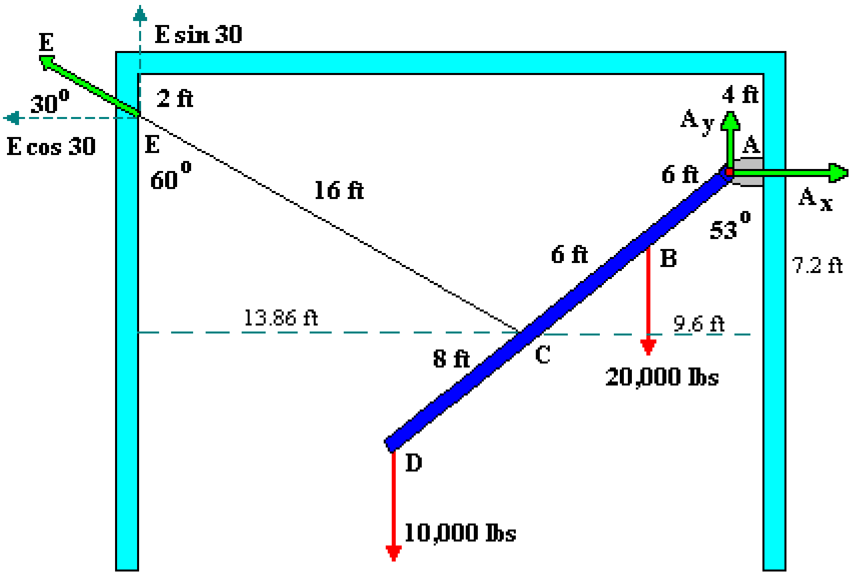
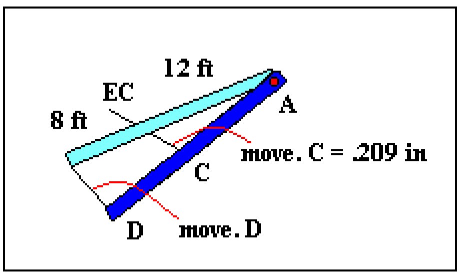


$F_x = A_x - E \cos(30^o) = 0$

$F_y = A_y + E \sin(30^o) - 10{,}000 \text{ lbs} - 20{,}000 \text{ lbs} = 0$

$T_A = (20{,}000 \text{ lbs})(4.8 \text{ ft}) + (10{,}000 \text{ lbs})(16 \text{ ft}) + E \cos(30^o)(2 \text{ ft}) - E \sin(30^o)(23.46 \text{ ft})$



$E = 25{,}600$ lbs; $A_x = 22{,}170$ lbs; $A_y = 17{,}200$ lbs

$Stress_{CE} = F/A = 25{,}600$ lbs/.785 $in^2 = 32{,}600$ psi

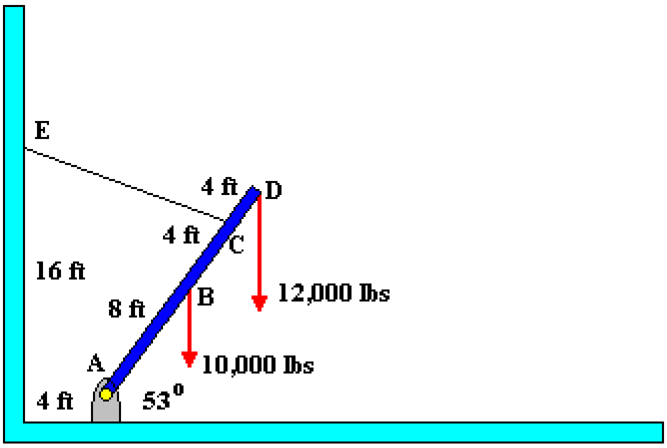
$Def_{CE} = (FL / EA)_{CE}$ = (25,600 lbs)(16 ft * 12 in/ft) / ($30*10^6$ lbs/in^2)(3.14*(.5 in)^2) = .209 in

move. C / 12 ft = move. D / 16 ft

.209 in / 12 ft = move. D / 16 ft

movement of D = .348 in

Örnek



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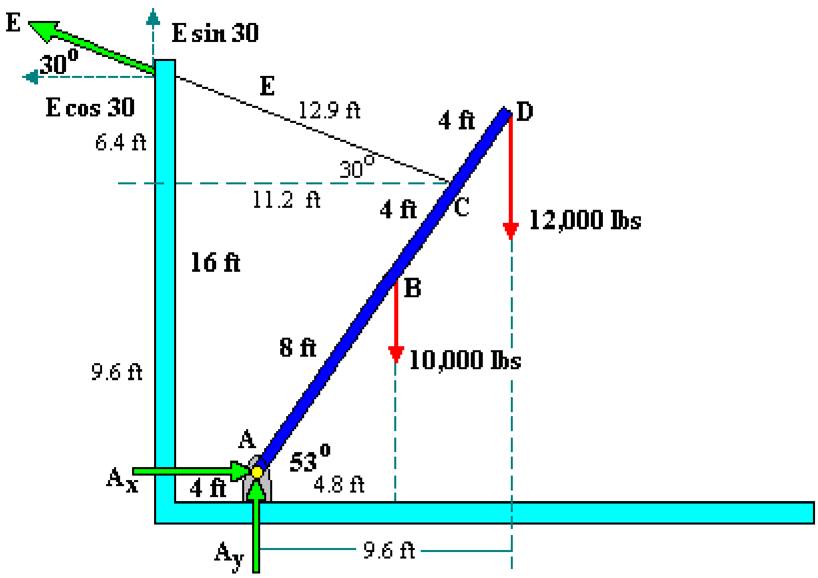


$F_x = A_x - E\cos(30^o) = 0$

$F_y = A_y + E\sin(30^o) - 10{,}000\text{ lbs} - 12{,}000\text{ lbs} = 0$

$T_A = -E\sin(30^o)(4\text{ ft}) + E\cos(30^o)(16\text{ ft}) - (10{,}000\text{ lbs})(4.8\text{ ft}) - (12{,}000\text{ lbs})(9.6\text{ ft}) = 0$

E = 13,790 lbs; A_x = 11940 lbs; A_y = 15,100 lbs

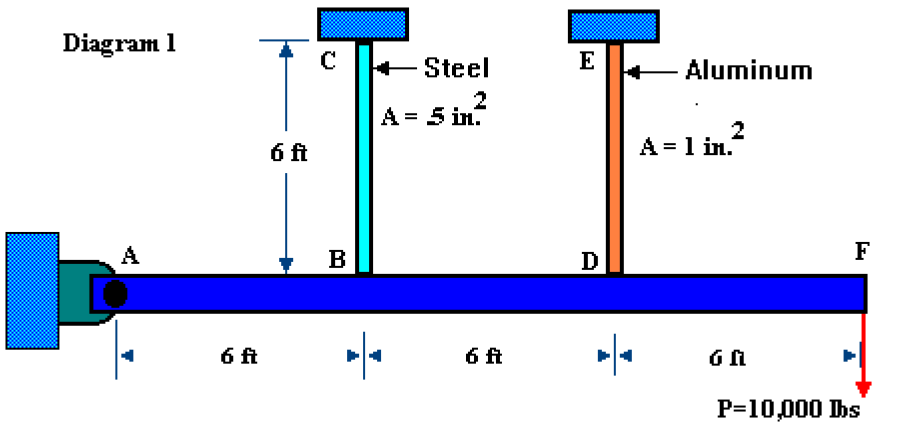
$Stress_{CE}$ = F/A = 13,790 lbs/ .785 in^2 = 17,600 psi

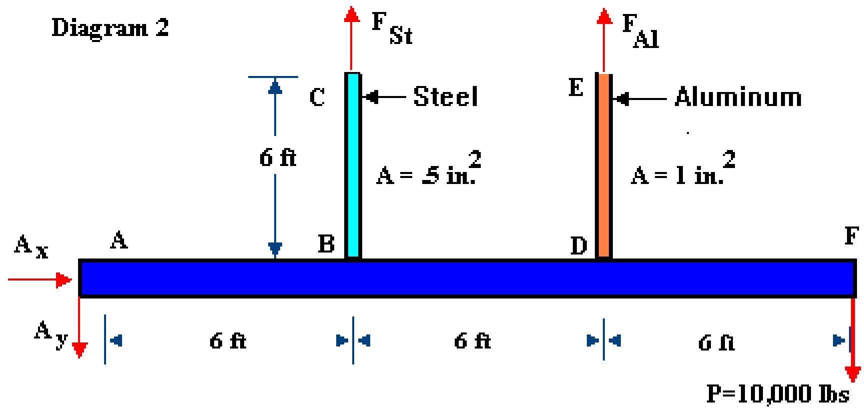
$Def_{EC} = (FL / EA)_{EC}$ = (13,790 lbs)(12.9 ft)(12 in/ft) / (30*10^6 lbs/in^2)(3.14 * .5 in^2) = .0912 in

Mov. C / 12 ft = Mov. D / 16 ft

.0912 in / 12 ft = Mov. D / 16 ft and so **Mov. D = .122 in**

Example 1:

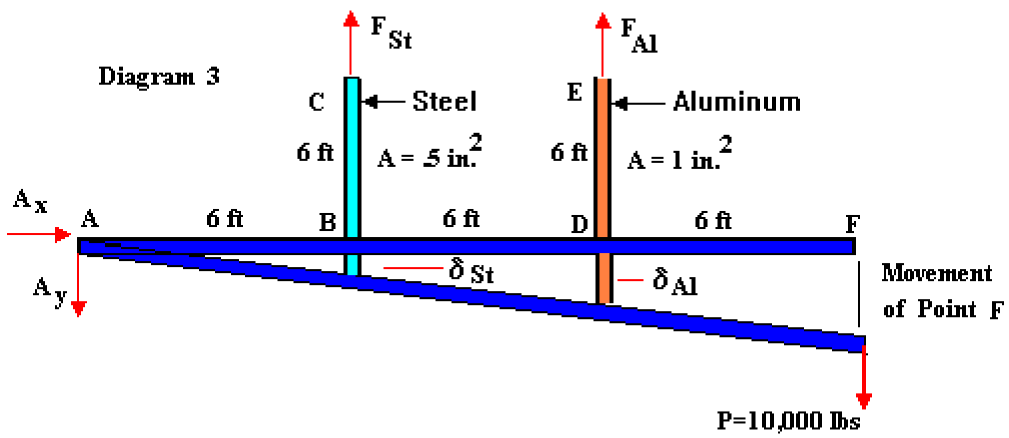




$\sum F_x = 0$: $A_x = 0$

$\sum F_y = 0$: $-A_y + F_{St} + F_{Al} - 10{,}000$ lb. $= 0$

$\sum \tau_A = 0$: $+ F_{St}$ (6 ft.) $+ F_{Al}$ (12 ft.) $- 10{,}000$ lb. (18 ft.) $= 0$



Deformation of Steel / 6 ft = Deformation of Aluminum / 12 ft

2 * Deformation of Steel = Deformation of Aluminum

or symbolically: $2\delta_{St} = \delta_{Al}$

deformation = [force in member * length of member] / [young's modulus of member * area of member], or def = FL/EA.

2 * $[FL/EA]_{St} = [FL/EA]_{Al}$

$(2 * F_{St} * 72")/(30 \times 10^6 \text{ lb/in}^2 * .5 \text{ in}^2) = (F_{Al} * 72")/(10 \times 10^6 \text{ lb/in}^2 * 1 \text{ in}^2)$

$F_{St} = .75 F_{Al}$

$\sum F_x = 0$: $A_x = 0$

$\sum F_y = 0$: $-A_y + F_{St} + F_{Al} - 10{,}000$ lb. = 0

$\sum \tau_A = 0$: + F_{St} (6 ft.) + F_{Al} (12 ft.) - 10,000 lb. (18 ft.) = 0

$(.75 F_{Al})$(6 ft.) + F_{Al} (12) - 10,000 lb. (18 ft.) = 0;

F_{Al} = 10,900 lb., and F_{St} = 8175 lb.

Ay = 10,075 lb

Stress Steel = 8175 lb/ .5 in^2 = 16, 350 lb/in^2,

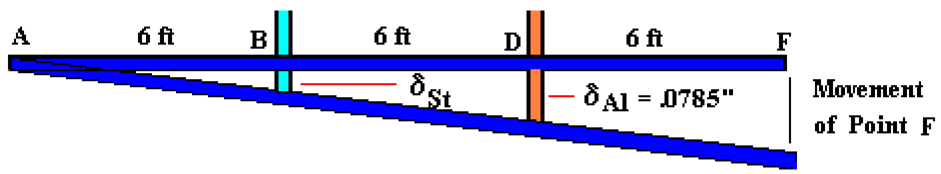
Stess Aluminum = 10,900 lb/1 in^2 = 10,900 lb/in^2.

Def. = FL/EA = (10,900 lb * 72 in.)/(10 x 10^6 lb.in^2 * 1 in^2) = .0785 in.

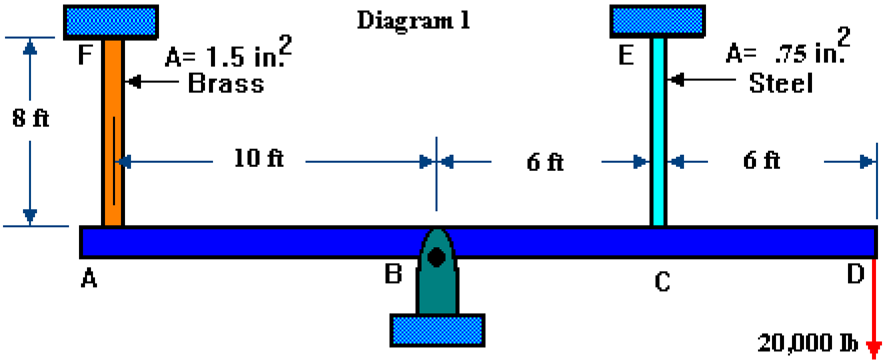
.0785 in./12 ft = Move. F/18 ft,

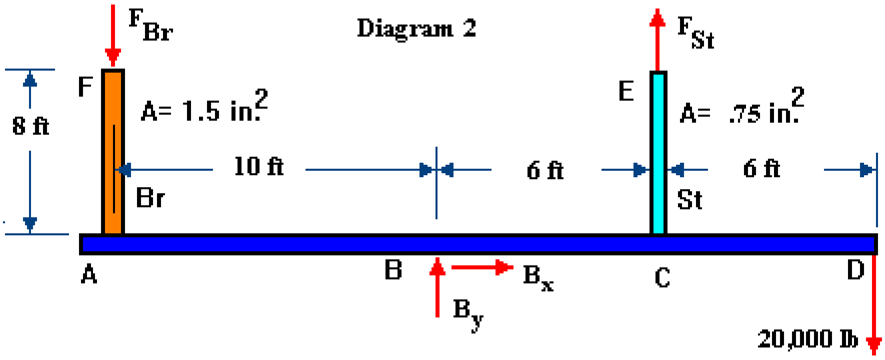
Move. F = .118 in.

Diagram 4



Örnek

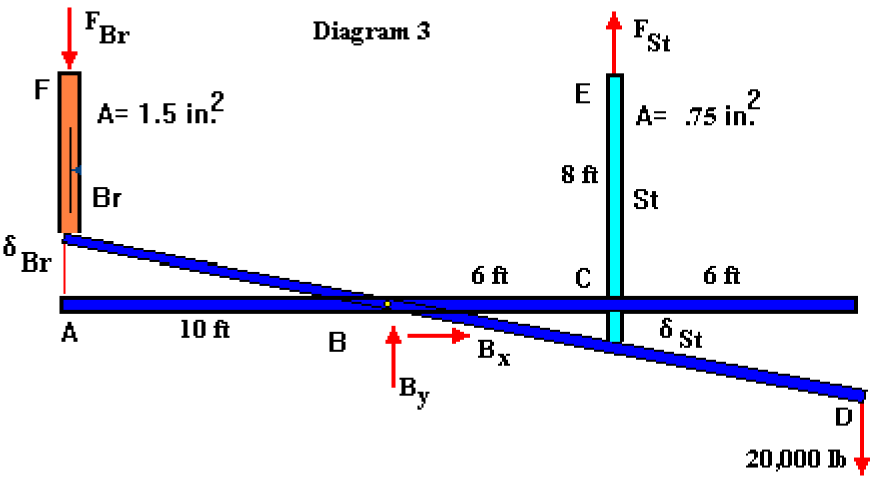




$\sum F_x = 0$: $B_x = 0$

$\sum F_y = 0$: $B_y - F_{Br} + F_{St} - 20{,}000$ lb. = 0

$\sum \tau_B = 0$: $+ F_{Br}$ (10 ft.) + F_{St} (6 ft.) - 20,000 lb. (12 ft.) = 0



Deformation of Steel / 6 ft = Deformation of Brass / 10 ft

Deformation of Steel = .6 * Deformation of Brass or symbolically: $\delta_{St} = .6\ \delta_{Br}$

deformation = [force in member * length of member] / [young's modulus of member * area of member], or def = FL/EA.

$[FL/EA]_{St} = .6\ [FL/EA]_{Br}$

$(F_{St} * 96")/(30 \times 10^6\ lb/in^2 * .75\ in^2) = .6[(F_{Br} * 96")/(15 \times 10^6\ lb/in^2 * 1.5\ in^2)]$

$F_{St} = .6\ F_{Br}$

$\sum F_x = 0$: $B_x = 0$

$\sum F_y = 0$: $B_y - F_{Br} + F_{St} - 20{,}000$ lb. = 0

$\sum \tau_B = 0$: $+ F_{Br}$ (10 ft.) + F_{St} (6 ft.) - 20,000 lb. (12 ft.) = 0

(F_{Br})(10 ft.) + (.6 F_{Br}) (6 ft.) - 20,000 lb. (12 ft.) = 0;

F_{Br} = 17,650 lb., and F_{St} = 10,600 lb.

By = 27,050 lb

Stress Steel = 10,600 lb/ .75 in^2 = 14,130 lb/in^2,

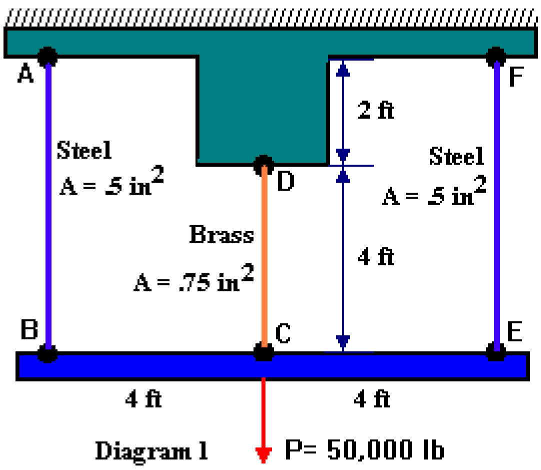
Stress Brass = 17,650 lb/1.5 in^2 = 11,770 lb/in^2.

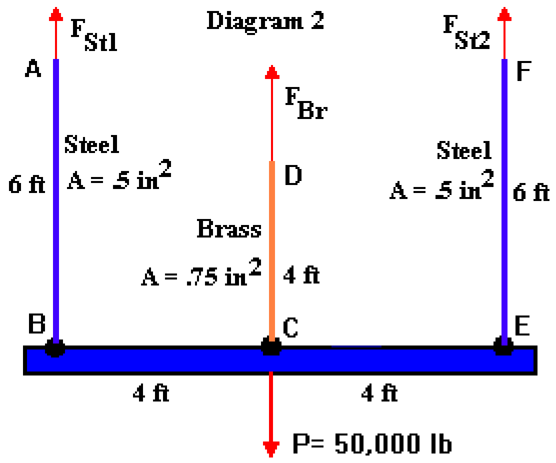
Def. = FL/EA = (10,600 lb * 96 in.)/(30 x 10^6 $lb.in^2$ * .75 in^2) = .0452

.0452 in./6ft = Move. D/12 ft,

Move. D = .0904 in.

Örnek





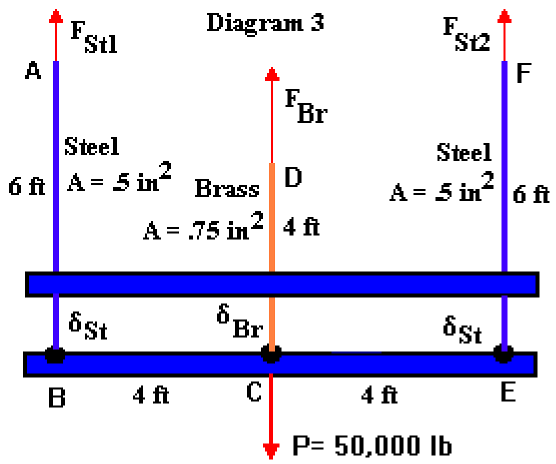
$\sum F_x = 0$:

$\sum F_y = 0$: $F_{St1} + F_{St2} + F_{Br}$ - 50,000 lb. = 0

$\sum \tau_C = 0$: - F_{St1} (4 ft.) + F_{St2} (4 ft.) = 0

$F_{st1} = F_{st2}$,

$F_{St} + F_{St} + F_{Br}$ - 50,000 lb. = 0, or 2 $F_{St} + F_{Br}$ -50,000 lb. = 0



Deformation of Steel = Deformation of Brass

or symbolically: $\delta_{St} = \delta_{Br}$

deformation = [force in member * length of member] / [young's modulus of member * area of member], or def = FL/EA.

$[FL/EA]_{St} = [FL/EA]_{Br}$

(F_{St} *72")/(30 x 10^6 lb./in^2 * .5 in^2) = (F_{Br} * 48")/(15 x 10^6 lb./in^2 * .75 in^2)

F_{St} = .89 F_{Br}

$\sum F_x = 0$:

$\sum F_y = 0$: 2 F_{St} + F_{Br} -50,000 lb. = 0

$\sum \tau_c = 0$: - F_{St1} (4 ft.) + F_{St2} (4 ft.) = 0

2 (.89 F_{Br}) + F_{Br} - 50,000 lb. = 0;

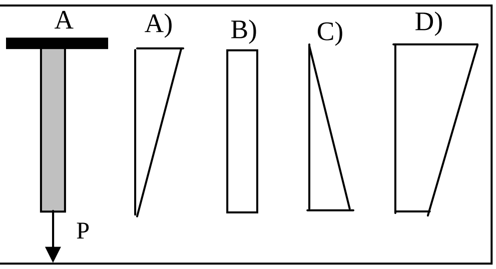
F_{Br} = 18,000 lb., and F_{St} = 16,000 lb.

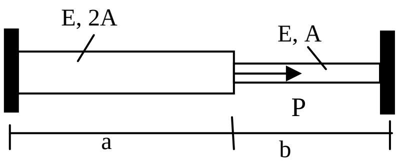
Stress Brass = 18,000 lb./ .75 in^2 = 24,000 lb./in^2,

Stress Steel = 16,000 lb./.5 in^2 = 32,000 lb./in^2.

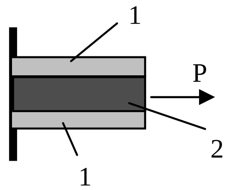
Def. = FL/EA = (18,000 lb. * 48 in.)/(15 x 10^6 lb.in^2 * .75 in^2) = .0768 in.

Şekildeki gibi A noktasından asılı bir çubuğa etkiyen P yükü ile beraber, öz ağırlığının da göz önüne alınmasıyla meydana gelen Normal kuvvet diyagramının şekli hangisi olabilir.





Şekildeki taşıyıcı sistemde A=10cm^2, E=2x10^6 N/cm^2, basınç emniyet gerilmesi 600 N/cm^2, çekme emniyet gerilmesi 1200 N/cm^2 olduğuna göre emniyetle taşınabilecek P kuvvetini bulunuz.
a= 40cm, b=60cm, $\Delta L = NL/(EA)$

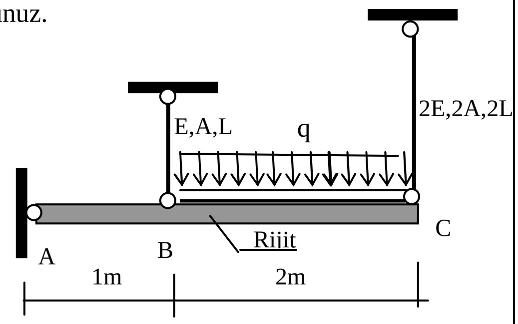


Şekilde iki malzemeden yapılmış bir çubuk P kuvvetinin etkisindedir. Buna göre aşağıdakilerin hangisi doğrudur.

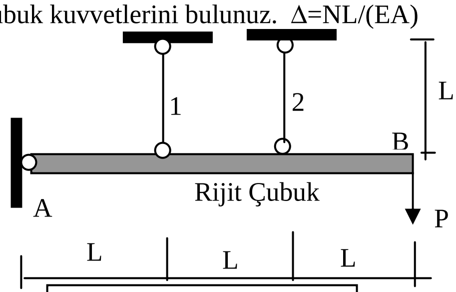
A) N1= P/2 B) N2 = P/2

C) N1= P/3 D) 2N1+N2= P

Şekildeki AC rijit çubuğu, A noktasında mafsallı bağlı, B ile C noktası arasında q yayılı kuvveti etkimektedir. L=1m, q=3kN/m ve esnek çubuktlarda emniyet gerilmesi σ_{em}=1200N/cm^2 olduğuna göre, çubuğun kesit alanını (A) bulunuz.



Şekildeki AB rijit çubuğu, A noktasında mafsallı bağlı, B noktasında P kuvveti etkimektedir. 1 ve 2 nolu esnek çubukların kesit alanları A_1= 8cm^2, A_2= 4cm^2, Elastisite modülleri E_1=E_2=E=10 kN/cm^2 , L= 1m dir. Her iki çubukta σ_{em} bilindiğine göre, P kuvvetinin uygun değerini, B noktasının düşey yer değiştirmesi ile 1 ve 2 nolu çubuk kuvvetlerini bulunuz. Δ=NL/(EA)



σ_{em}= (a +b) kN/cm^2