MUKAVEMET

GERİLME

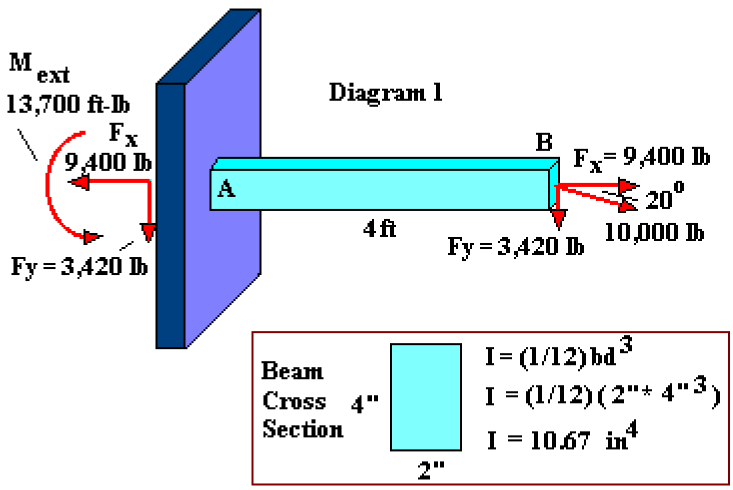
Gerilme birim alana gelen kuvvettir, (N/mm^2, kg/cm^2)

σ Normal gerilme: Çubuğ un ekseni doğ rultusunda oluş an gerilmedir.

τ kayma gerilmesi:çubuğ un eksenine dik doğ rultuda oluş an gerilmedir.

Gerilme = F/A.

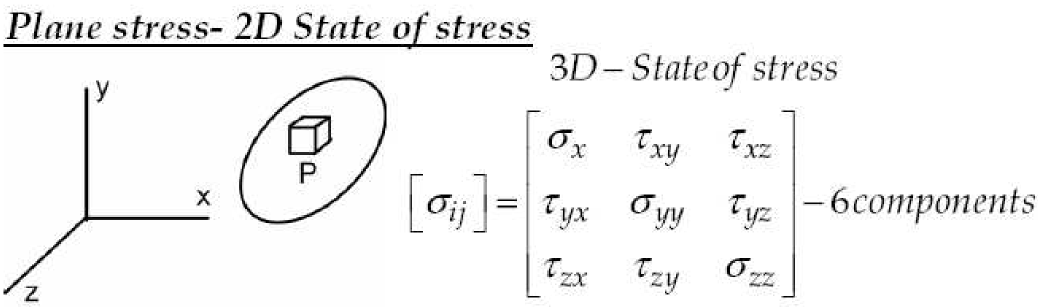
Konsol kirişte gerilmeler



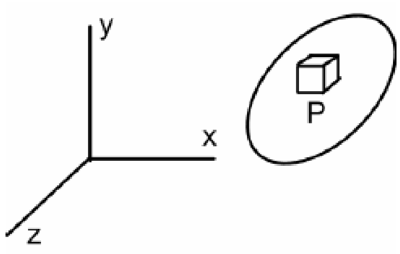
Normal Gerilme = Kuvvet/Alan = 9,400 lb. / (2" x 4") = 1175 lb/in².

Kayma Gerilmesi = Kuvvet/Alan = 3,420 lb. / (2" x 4") = 427.5 lb/in².

DÜZLEM GERİLME
3 Boyutlu gerilme hali



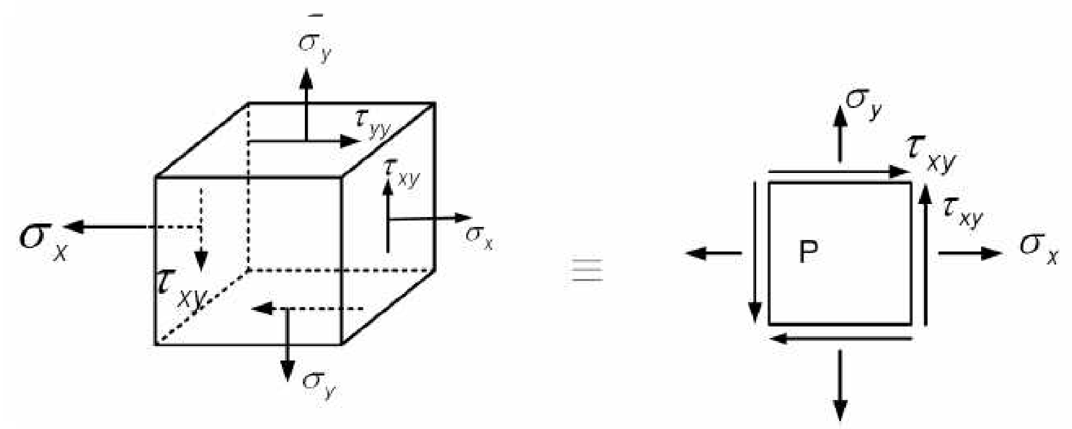
İki boyutlu gerilme hali



2D – State of stress

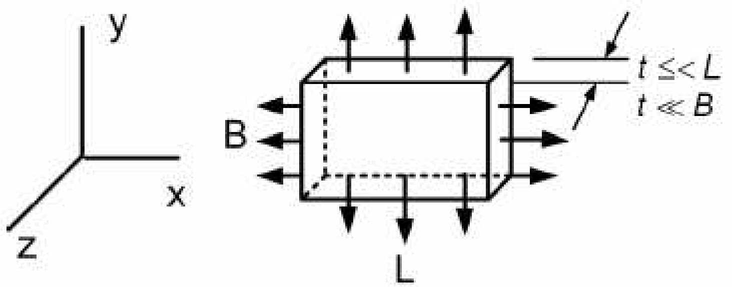
$$\left[\sigma_{ij}\right]=\begin{bmatrix}\sigma_x & \tau_{xy} & 0\\ \tau_{xy} & \sigma_y & 0\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}\sigma_x & \tau_{xy}\\ \tau_{yx=\tau_{xy}} & \sigma_y\end{bmatrix}$$

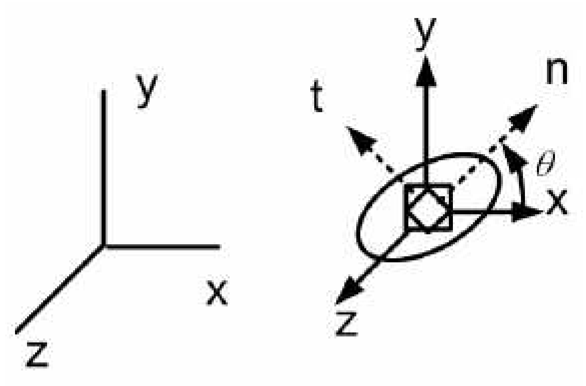
Stress components in plane xy



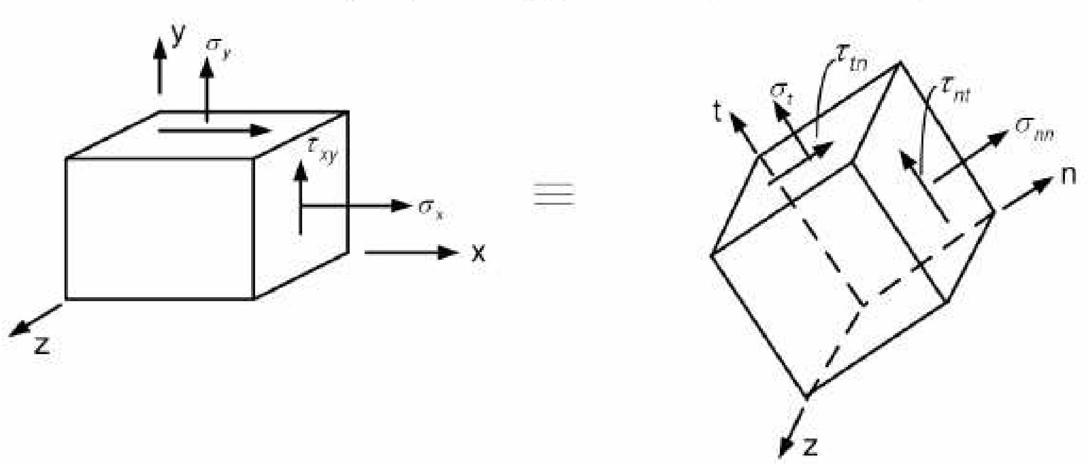
$$\left[\sigma_{ij}\right]=\begin{bmatrix}\sigma_x(x,y) & \tau_{xy}(x,y)\\ \tau_{xy}(x,y) & \sigma_{yy}(x,y)\end{bmatrix}$$

Kalınlık diğer boyutların yanında çok küçük olduğundan düzlem olarak göz önüne alınabilir.



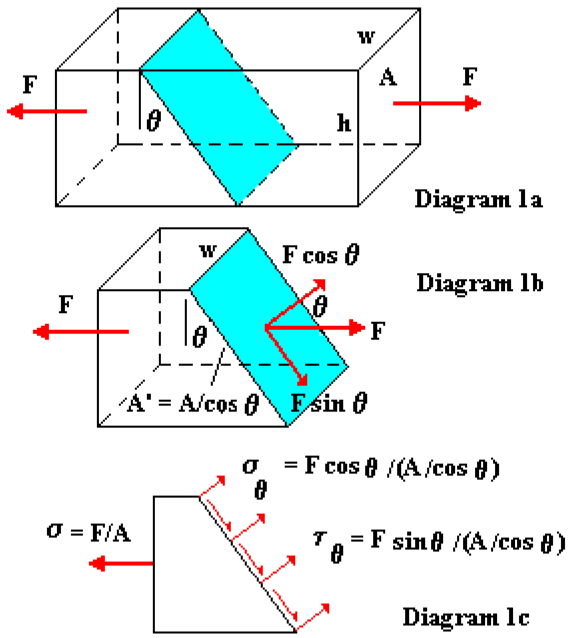


$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} = \begin{bmatrix} \sigma_{nn} & \tau_{nt} \\ \tau_{nt} & \sigma_{tt} \end{bmatrix}$$



Eğik düzlemde

A' = A/cos (teta).



Normal Gerilme = F cos (theta) / A /cos (theta) = (F/A) cos²(theta)

Kayam Gerilmesi = F sin (theta) / A /cos (theta) = (F/A) sin(theta) * cos(theta)

veya

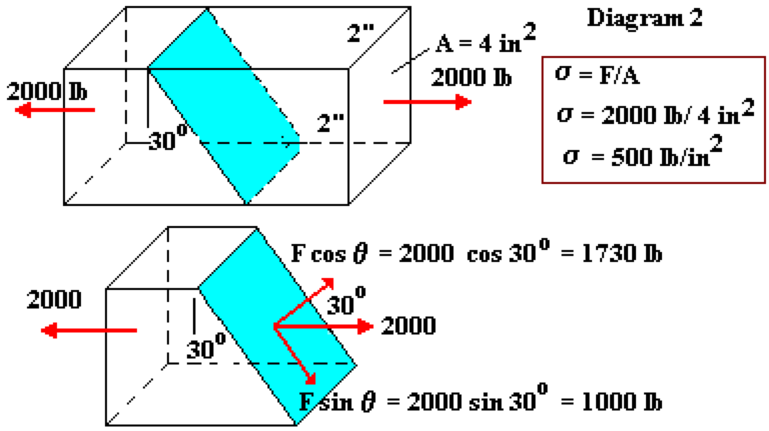
Kayma Gerilmesi = (F/2A)Sin (2*theta),

$$\sigma_\theta = \sigma \cos^2 \theta$$

$$\tau_\theta = (\sigma / 2) \sin 2\theta$$

Örnek

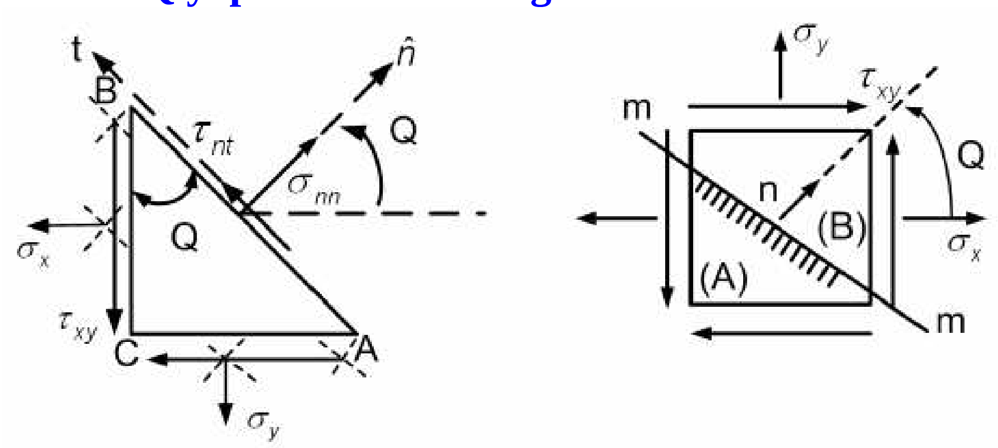
Kare kesitli kiriş te, 30 derece lik düzlemdeki gerilmeler

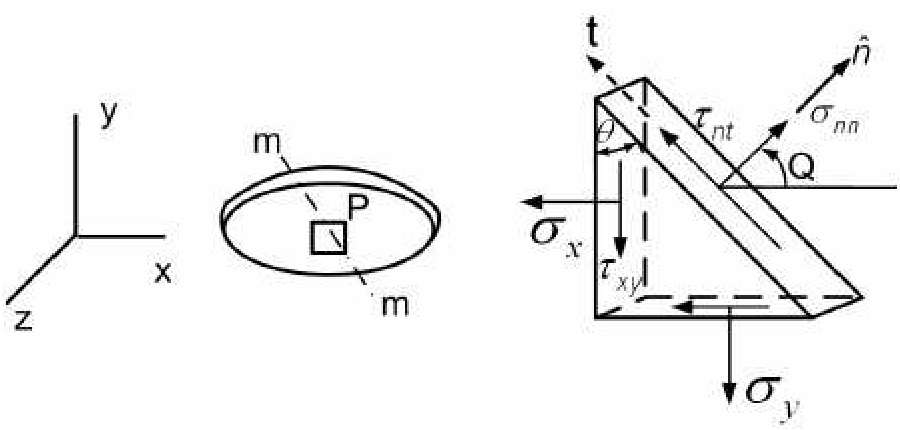


$$\sigma_{30^0} = \sigma \cos^2\theta = 500 \text{ lb/in}^2 \cos^2 30^0 = 375 \text{ lb/in}^2$$

$$\tau_{30^0} = (\sigma/2)\sin(2\theta) = (500 \text{ lb/in}^2/2)\sin(60^0) = 217 \text{ lb/in}^2$$

Q yapan düzlemdeki gerilmelerin hesabı





Diferansiyel elemanda boyutlar

$$dA = Area\ of\ AB$$

$$dACs\theta = Area\ of\ BC$$

$$dASin\theta = Area\ of\ AC$$

Denge denklemlerinden q açısı yapan düzlemdeki gerilmeler hesaplanıyor

$$\left[\Sigma F_n \nearrow + = 0\right]$$

$$\sigma_{nn} dA - \sigma_x dACos\theta Cos\theta - \tau_{xy} dACos\theta Sin\theta - \tau_{xy} dASin\theta Cos\theta -$$
$$\sigma_{yy} dASin\theta Sin\theta = 0$$

$$\sigma_{nn} - \sigma_x Cos^2\theta - 2\tau_{xy} Sin\theta Cos\theta - \sigma_{yy} Sin^2\theta = 0$$

$$\sigma_{nn} = \sigma_x Cos^2\theta + \sigma_y Sin^2\theta + 2\tau_{xy} Sin\theta Cos\theta$$

$$\sigma_{nn} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

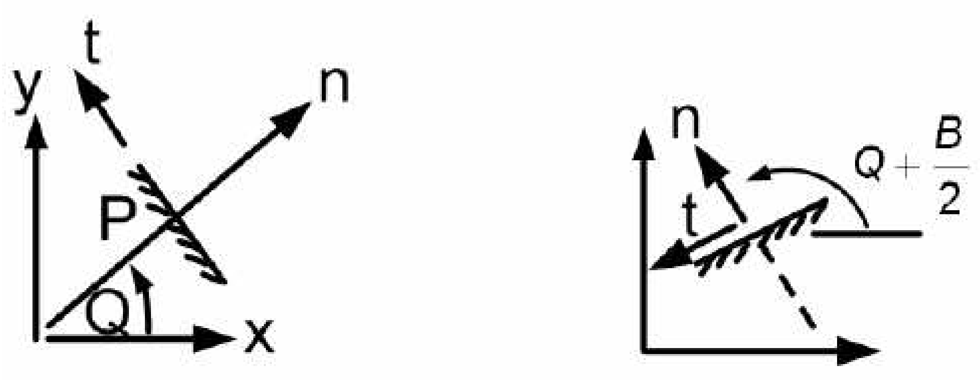
$$\left[\Sigma F_n \nwarrow + = 0\right]$$

$$\sigma_{nt} dA - \sigma_x dACos\theta Sin\theta - \tau_{xy} dACos\theta Cos\theta + \tau_{xy} dASin\theta Sin\theta -$$
$$\sigma_y dASin\theta Cos\theta = 0$$

$$\tau_{nt} = -\sigma_x Cos\theta Sin\theta + \sigma_y Sin\theta Cos\theta + \tau_{xy}\left(Cos^2\theta - Sin^2\theta\right)$$

$$\tau_{nt} = -Cos\theta Sin\theta\left(\sigma_x - \sigma_y\right) + \tau_{xy}\left(Cos^2\theta - Sin^2\theta\right)$$

$$\tau_{nt} = -\frac{\left(\sigma_x - \sigma_y\right)}{2} Sin2\theta + \tau_{xy} Cos2\theta$$



2θ cinsinden aynı ifadeler tekrar yazılıyor

$$\sigma_{nn} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

$$\sigma_t = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta - \tau_{xy} Sin2\theta$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2} Sin2\theta + \tau_{xy} Cos2\theta$$

Düzlem gerilmelerde değişmezler (invaryantlar)

Birinci değişmez

$$\sigma_n + \sigma_t = \sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'} = I_1$$

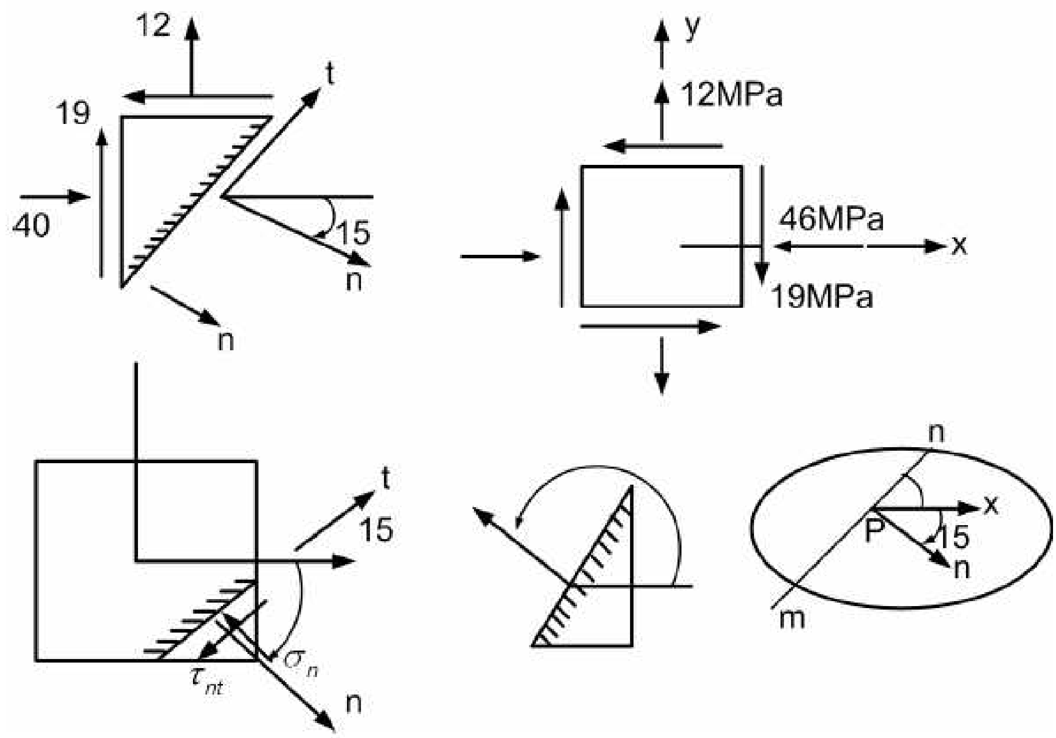
İkinci değişmez

$$\sigma_n \sigma_t - \tau_{nt}^2 = \sigma_x \sigma_y - \tau_{xy}^2 = \sigma_{x'} \sigma_{y'} - \tau_{x'y'} = I_2$$

ÖRNEK

Düzlem gerilme durumunda bir noktadaki gerilme şekilde verilmiştir
Saat yönünde 15 derece açı yapan düzlemdeki gerilmeleri bulunuz

ÇÖZÜM



$$\sigma_x = -46$$

$$\sigma_y = 12$$

$$\tau_{xy} = -19$$

$$Q = -15^\circ$$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{-46+12}{2} = \frac{-34}{2} = -17 \ MPas$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{-46-12}{2} = \frac{-58}{2} = -29 \ MPa$$

$$Sin2\theta = Sin2(-15) = -0.5; Cos2\theta = Cos2(-15) = 0.866$$

$$\sigma_n = -17 - 29 \times 0.866 + 19 \times 0.5$$

$$\left[\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta \right]$$

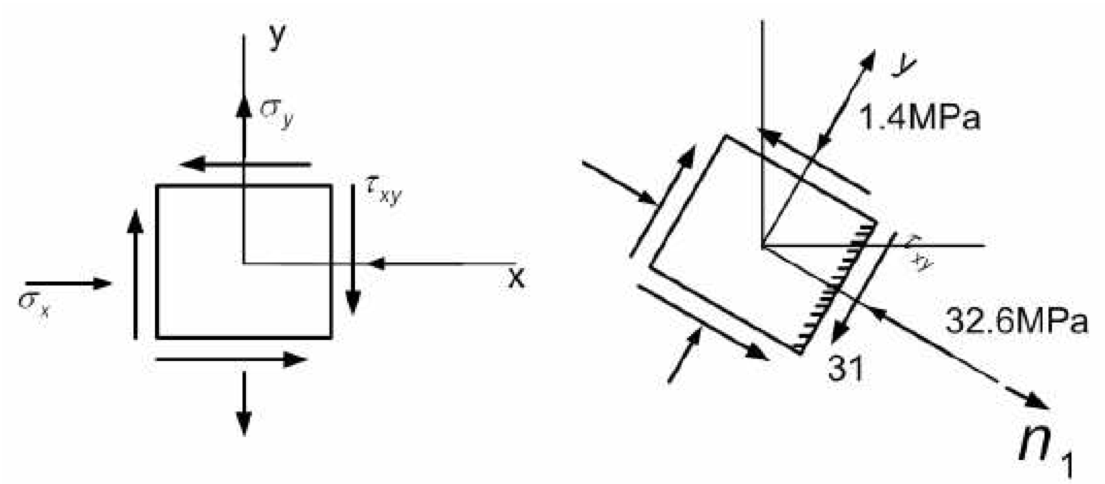
$$\sigma_{n_1} = -32.6 \ MPas$$

$\theta = -15^\circ$ in τ_{nt}

$$\left[\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2} Sin2\theta + \tau_{xy} Cos2\theta \right]$$

$$\tau_{n_1 t_1} = -29 \times 0.5 - 19 \times 0.866$$

$$\tau_{n_1 t_1} = -31 \ \ MPa$$



$$\left[\sigma_t = \sigma_{n_2} = \tau_{nt} \big|_{\theta = 75^\circ} \right]$$

$$\therefore \quad \sigma_t = -17 - 29 \cos 150 - 19 \sin 150$$

$$\sigma_t = -1.4 MPa$$

$$\left[\tau_{tn} = \tau_{n_2 t_2} = \tau_{nt} \big|_{\theta = 75^\circ} \right]$$

$$\tau_{tn} = +29 \times Sin150 - 19 \times Cos150$$
$$= 31\ MPa$$

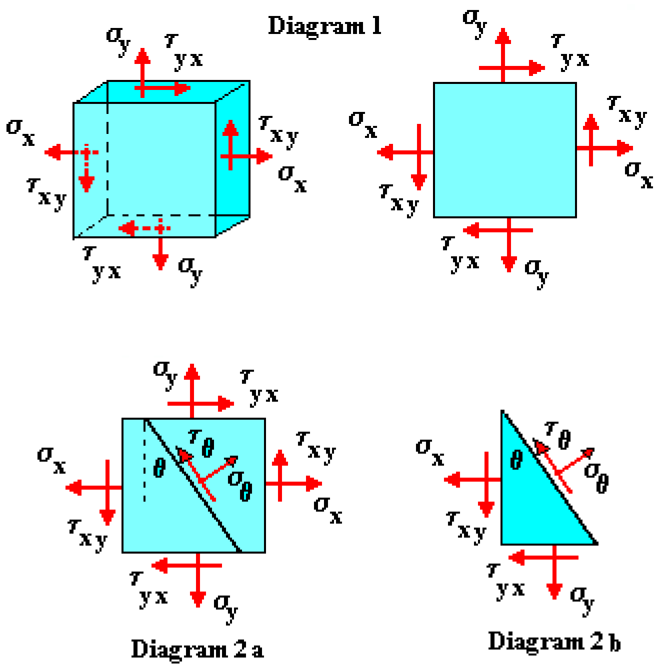
$$\theta = 145^{\circ}$$

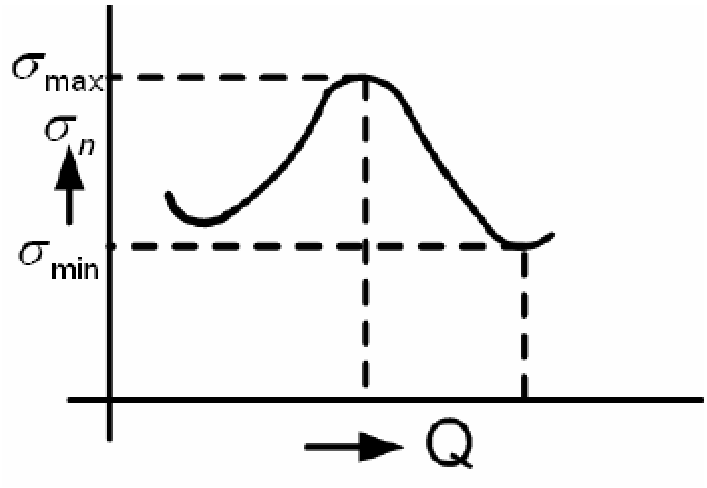
$$\sigma_n = -17 - 29Cos2 \times 165 - 19Sin2 \times 165$$
$$= -32\ MPa$$
$$\tau_{nt} = 0.29Sin330 - 19Cos330^{0}$$
$$\tau_{nt} = -31\ MPa$$

KONTROL

$$\sigma_n + \sigma_t = \sigma_x + \sigma_y = -32.6 - 1.4 = -34\ MPa = -46 + 12s$$

Asal gerilmeler





$$\sigma_n = \sigma_n(\theta) = \frac{\sigma_n + \sigma_y}{2} + \frac{\sigma_n - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

$$\sigma_n = \frac{\sigma_n + \sigma_y}{2} + \frac{\sigma_n - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

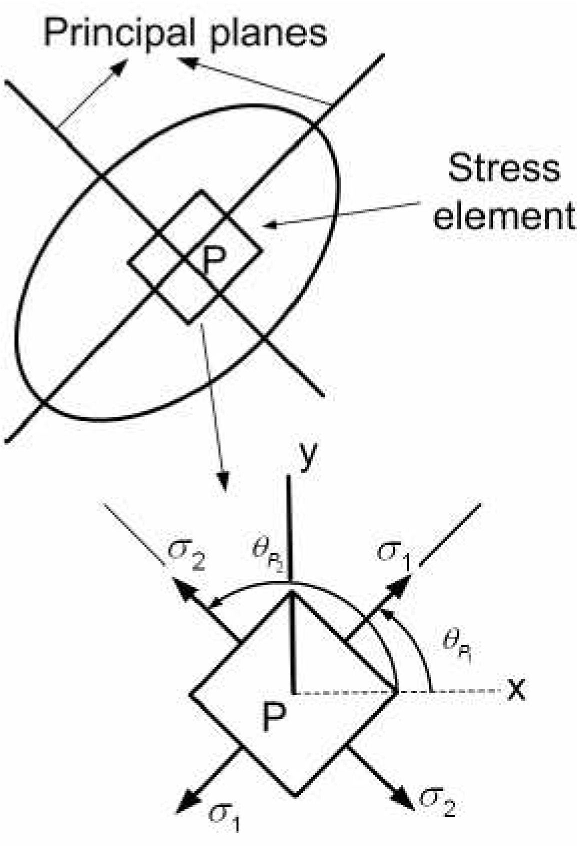
MAKSİ MUM İ Çİ N

$$\frac{d\sigma_n}{d\theta} = 0 = -\left(\sigma_x - \sigma_y\right) Sin2\theta + 2\tau_{xy} Cos2\theta$$

$$tan\,2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma \; = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$\tau_{max} = +\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

$$\sigma_n - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

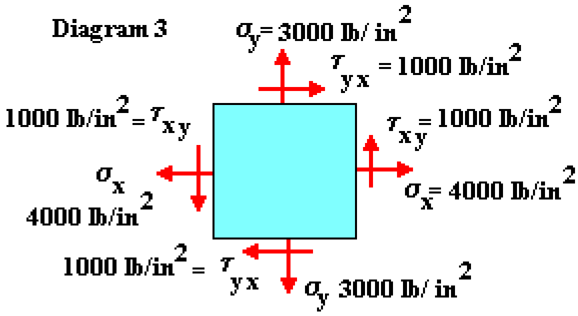
$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) Sin2\theta + \tau_{xy} Cos2\theta$$

$$\sigma_{max} = (\sigma_x + \sigma_y)/2 + \tau_{max}$$

$$\sigma_{min} = (\sigma_x + \sigma_y)/2 - \tau_{max}$$

$$\tau_{max} = (\sigma_{max} - \sigma_{min})/2$$

Örnek.



Çözüm

$\mathrm{Tan}2\theta = \tau_{xy} / [(\sigma_x - \sigma_y)/2]$ = 1000 lb/in²/[(4000 lb/in² - 3000 lb/in²)/2] = 2,

2(theta) = 63.4°, ve 243.4°, theta = 31.7°, ve 121.7°.

$\sigma_\theta = (\sigma_x + \sigma_y)/2 + [(\sigma_x - \sigma_y)/2](\cos 2\theta) + \tau_{xy}(\sin 2\theta)$ = (4000 lb/in² + 3000 lb/in²)/2 + [(4000 lb/in² - 3000 lb/in²)/2]cos(63.4°) + 1000 lb/in²*sin(63.4°)

= 3500 lb/in² + 224 lb/in² + 894 lb/in² = 4618 lb/in²

2. düzlemde

= (4000 lb/in² + 3000 lb/in²)/2 + [(4000 lb/in² - 3000 lb/in²)/2]cos(243.4°) + 1000 lb/in²*sin(243.4°)

= 3500 lb/in² - 224 lb/in² - 894 lb/in² = 2382 lb/in²

İ kinci metot formüllerden

$\sigma_{max/min} = (\sigma_x + \sigma_y)/2 +/- \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$ = (4000 lb/in² + 3000 lb/in²)/2 +/- Sqrt[{(4000 lb/in² - 3000 lb/in²)/2}² + (1000 lb/in²)²]

= 3500 lb/in² + 1118 lb/in² = 4618 lb/in²

= 3500 lb/in² - 1118 lb/in² = 2382 lb/in²

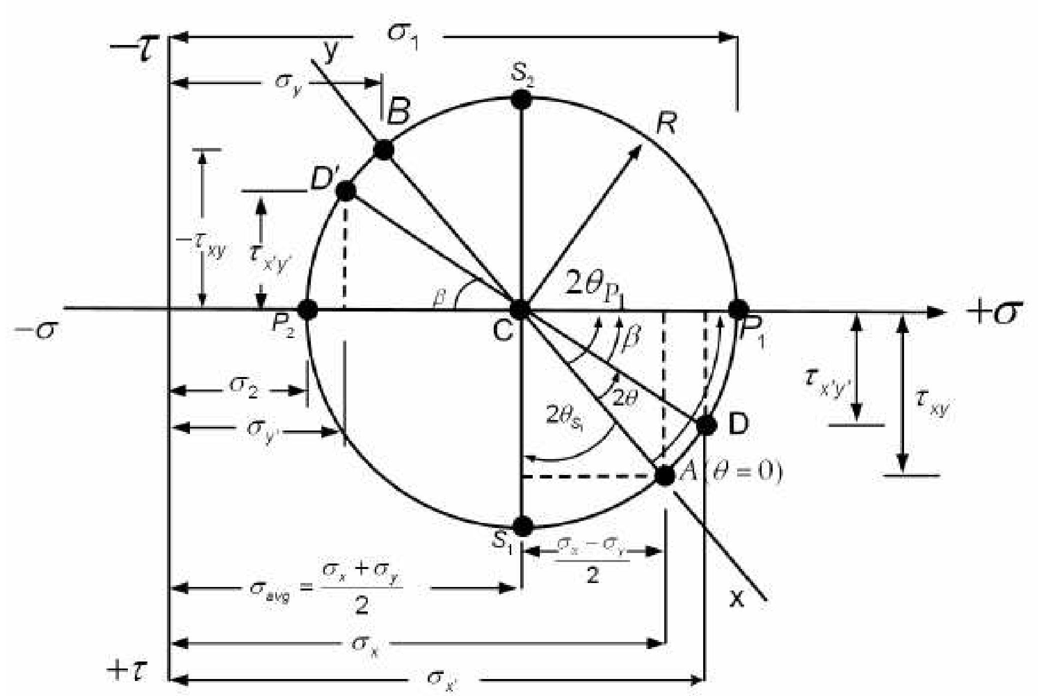
$\tau_{max} = +/- \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$ = Sqrt[{(4000 lb/in² - 3000 lb/in²)/2}² + (1000 lb/in²)²] = +/- 1118 lb/in².

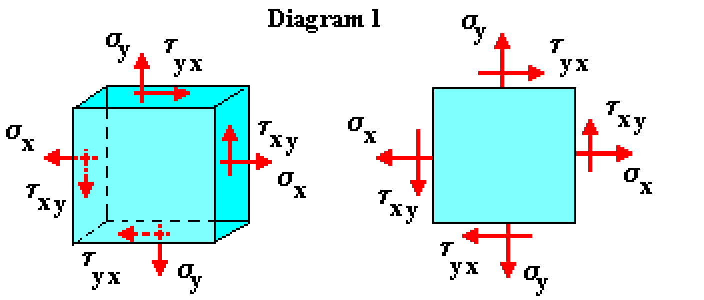
Mohr Dairesi

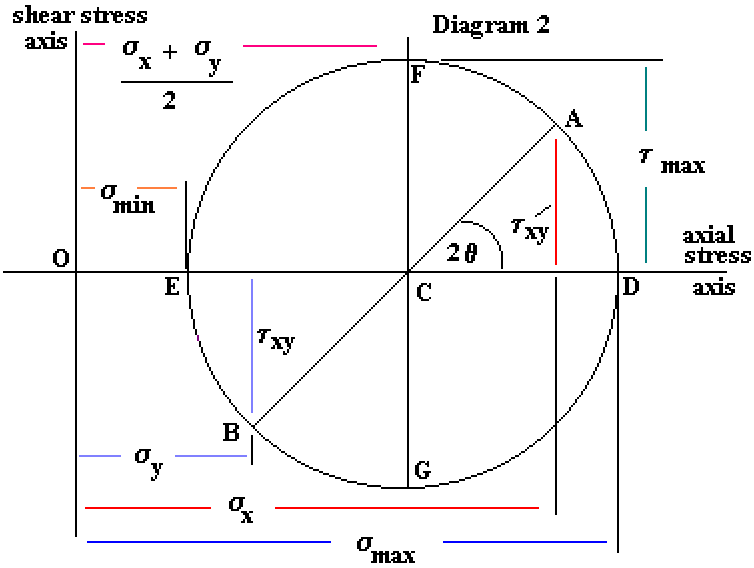
$$\left[\sigma_n - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{nt}^2 = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$(x - a)^2 + y^2 = R^2$$

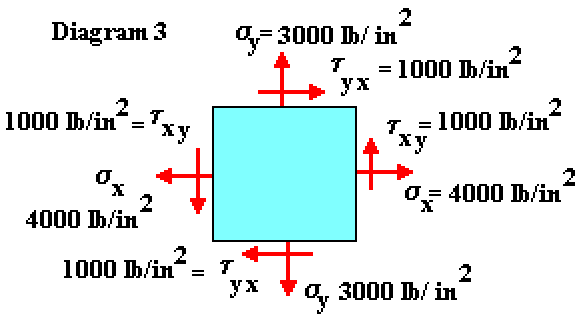






$R = \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$,

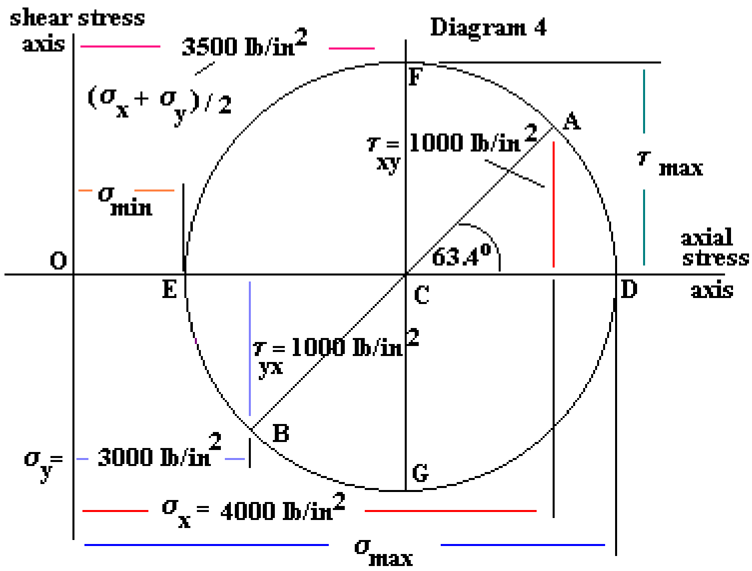
Örnek.



$R = \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2}$ = {[(4000 lb/in^2 -3000 lb/in^2)/2]^2 + (1000 lb/in^2)^2}$^{1/2}$

R = 1118 lb/in^2.

Mohr Dairesi merkezi = (4000 lb/in^2 +3000 lb/in^2)/2 = 3500 lb/in^2.



Maximum Stress = Location of Center + Radius = 3500 lb/in^2 + 1118 lb/in^2 = 4618 lb/in^2

and

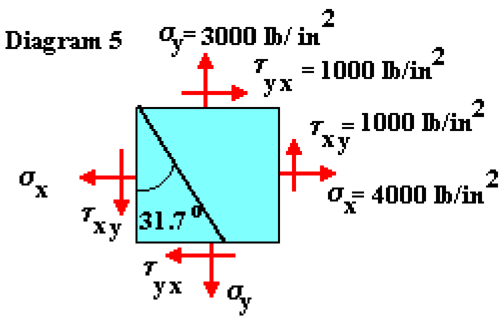
Minimum Stress = Location of Center - Radius = 3500 lb/in^2 - 1118 lb/in^2 = 2382 lb/in^2

Geometriden

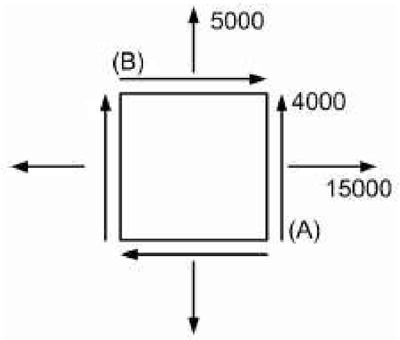
(2*theta) = (1000 lb/in^2)/(4000 lb/in^2 - 3500 lb/in^2),

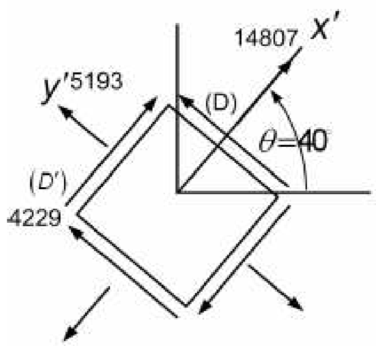
(2 theta) = 63.4^o, and 243.4^o,

theta = 31.7^o, and 121.7^o.



Mohr' circle problem





Solution:

$$\frac{\sigma_x+\sigma_y}{2}=\frac{15000+5000}{2}=10000\,MPa$$

A - (15000,4000)

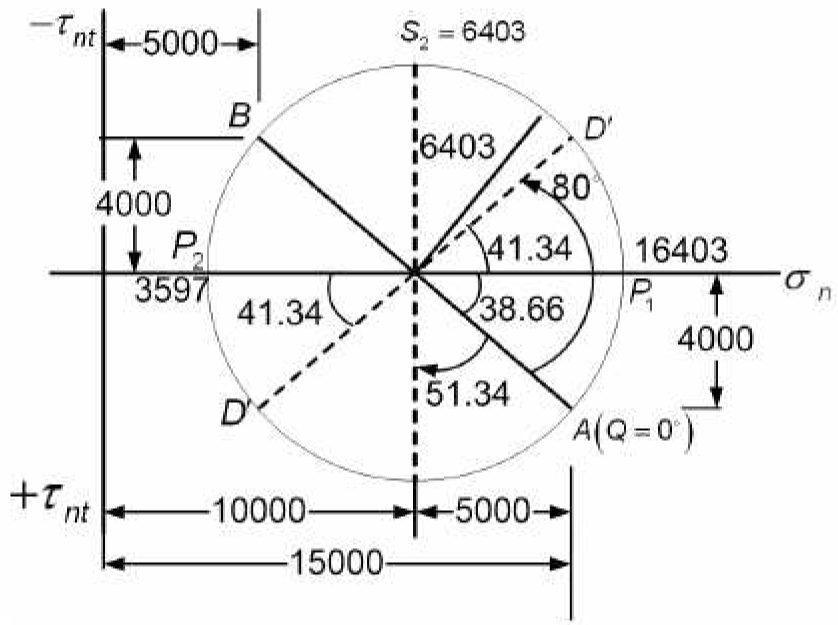
B - (5000,-4000)

$$R=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}=\sqrt{\left(\frac{15000-5000}{2}\right)^2+4000^2}$$

$$=\sqrt{5000^2+4000^2}$$

$$R=6403\,MPa$$

(a) $$\frac{\sigma_x-\sigma_y}{2}=5000$$



Point D: $\sigma_{x'} = 10000 + 6403 Cos 41.34 = 14807\ MPa$

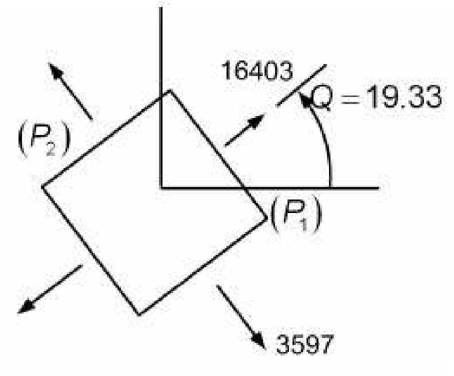
$\tau_{x'y'} = -6403 Sin 41.34 \qquad = -4229\ MPa$

Point D': $\sigma_n = \sigma_{y'} = 10000 - 6403 Cos 41.34 = 593\ MPa$

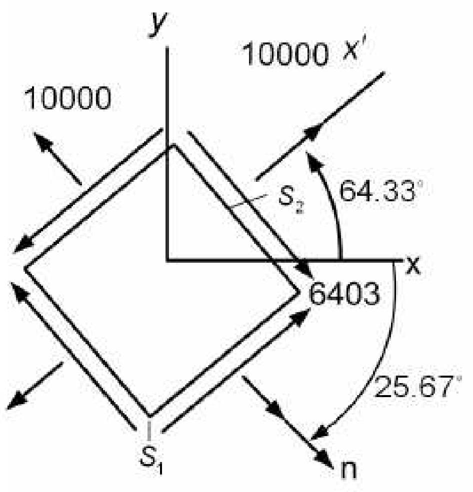
$\tau_{nt} = \tau_{x'y'} = 6403 Sin 41.34 \qquad = 4229$

b) $\sigma_1 = 16403$; $\theta_{P_1} = \dfrac{38.66}{2} = 19.33$

$\sigma_2 = 3597\ MPa$



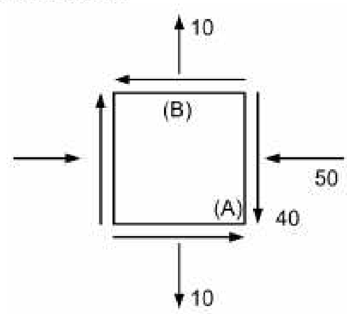
c) $\tau_{max} = 6403\ MPa - \theta_{S_1} = 25.67 = -25.67^\circ$



(2) $\theta = 45^\circ$

Principal stresses and principal shear stresses.

Solution:



$$\frac{\sigma_x + \sigma_y}{2} = \frac{-50+10}{2} = -20$$

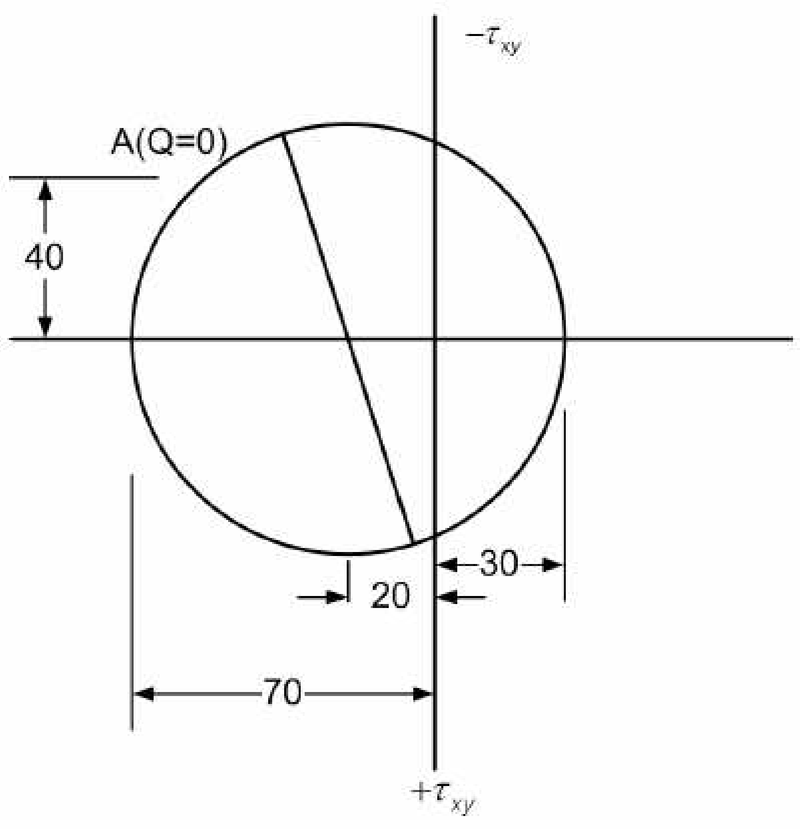
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-50-10}{2}\right)^2 + (-40)^2} = 50\,MPa$$

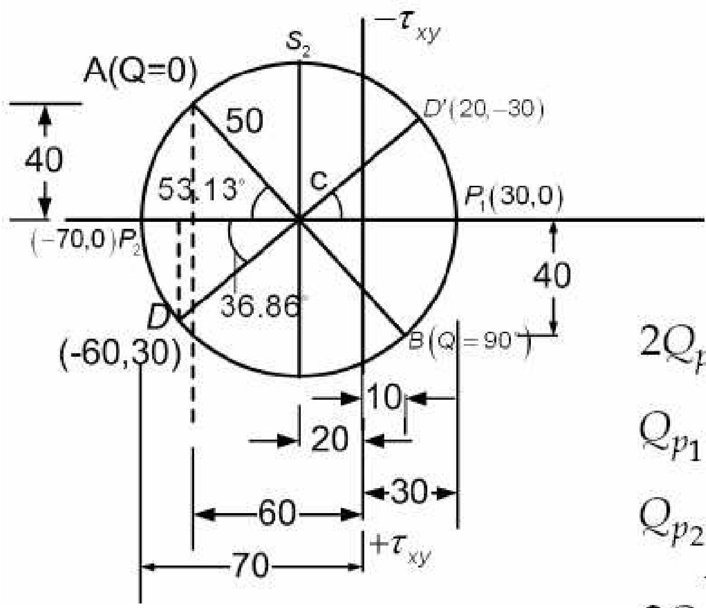
$A \rightarrow (-50, -40)$

$B \rightarrow (10, 40)$

$$p_1 = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + R = -20 + 50 = 30s$$

$$p_2 = \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - R = -20 - 50 = -70$$





$$2Q_{p_1} = 233.13$$

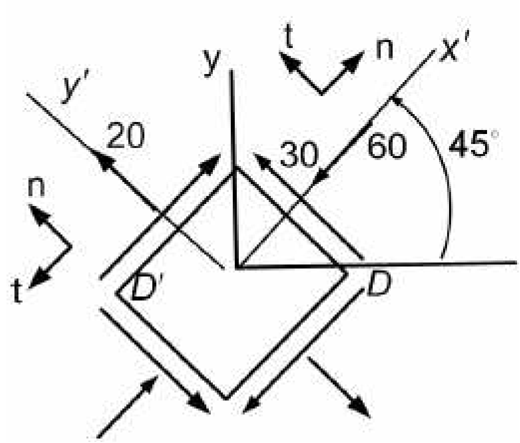
$$Q_{p_1} = 116.6^\circ$$

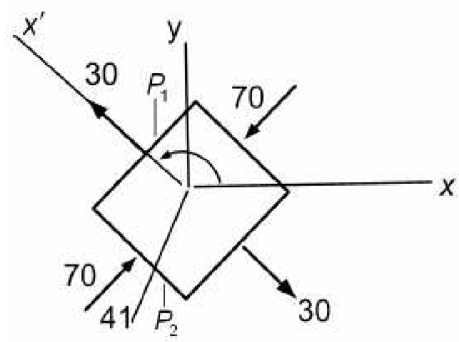
$$Q_{p_2} = 206.6^\circ$$

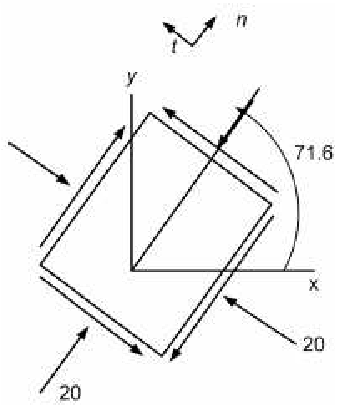
$$2Q_{s_1} = 143.13$$

$$Q_{s_1} = 71.6$$

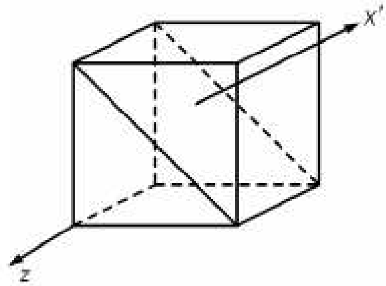
$$Q_{s_2} = 161.6$$

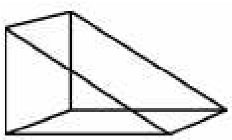


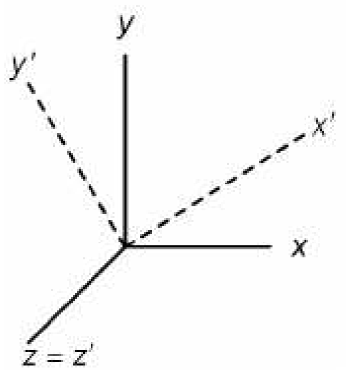




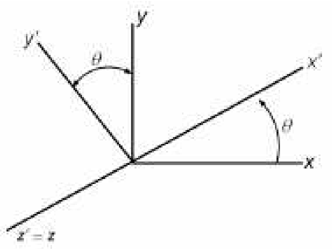
3 boyutlu uzayda gerilme transformasyonu







$$n_{x'x} = Cos\theta \quad n_{y'x} = -Sin\theta \quad n_{z'x} = 0$$
$$n_{x'y} = Sin\theta \quad n_{y'y} = Cos\theta \quad n_{z'y} = 0$$
$$n_{x'z} = 0 \quad n_{y'z} = 0 \quad n_{z'z} = 1$$



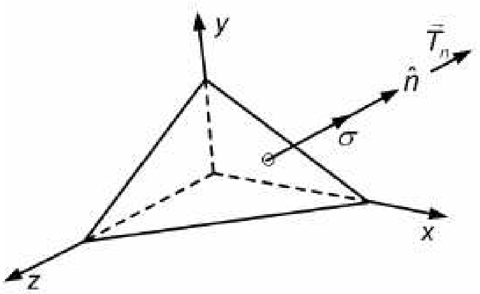
$$\sigma_{z'} = 0 : \tau_{x'z'} = 0 : \tau_{y'z'} = 0$$
$$= \sigma_z$$

$$\sigma_{x'} = \sigma_x Cos^2\theta + \sigma_y Sin^2\theta + 2\tau_{xy} Sin\theta Cos\theta$$
$$\sigma_{y'} = \sigma_x Sin^2\theta + \sigma_y Cos^2\theta - 2\tau_{xy} Sin\theta Cos\theta$$
$$\tau_{x'y'} = -\left(\sigma_x - \sigma_y\right) Sin\theta Cos\theta + \tau_{xy}\left(Cos^2\theta - Sin^2\theta\right)$$

$$\begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Principal stresses

$$n_x, n_y, n_z$$



$$\vec{T}_n = \sigma\hat{n} = \sigma\left(n_x\hat{i} + n_y\hat{j} + n_z\hat{k}\right)$$
$$\vec{T}_n = T_{nx}\hat{i} + T_{ny}\hat{j} + T_{nz}\hat{k}$$

$$T_{nx} = \sigma_x n_x + \tau_{yx} n_y + \tau_{zx} n_z$$
$$T_{ny} = \tau_{xy} n_x + \sigma_y n_y + \tau_{zy} n_z$$
$$T_{nz} = \tau_{xz} n_x + \tau_{yz} n_y + \sigma_z n_z$$

$$Tn_x = \sigma n_x \,\Big|\, Tn_y = \sigma n_y \,\Big|\, Tn_z = \sigma n_z$$

$$\left.\begin{aligned} (\sigma_x - \sigma) n_x + \tau_{yx} n_y + \tau_{zx} n_z = 0 \\ \tau_{yx} n_x + \left(\sigma_y - \sigma\right) n_y + \tau_{zy} n_z = 0 \\ \tau_{xz} n_x + \tau_{yz} n_y + (\sigma_z - \sigma) n_z = 0 \end{aligned}\right\} \text{Syst. of linear homog. eqns.}$$

$$n_x = n_y = n_z = 0 : \;\; n_x^2 + n_y^2 + n_z^2 = 1$$

$$\begin{bmatrix} \sigma_x-\sigma & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y-\sigma & \tau_{zy} \\ \tau_{zx} & \tau_{yz} & \sigma_z-\sigma \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} = (0)$$

For non trivial solution | | must be zero.

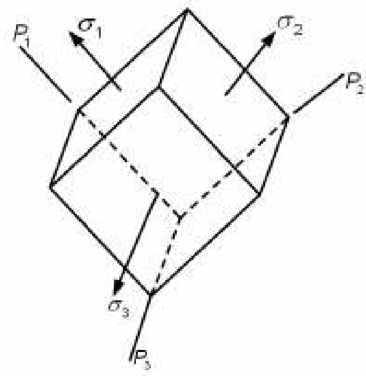
$$\sigma^3-\left(\sigma_x+\sigma_y+\sigma_z\right)\sigma^2+\left(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x-\tau_{xy}^2-\tau_{yz}^2-\tau_{zx}^2\right)\sigma$$
$$-\left(\sigma_x\sigma_y\sigma_z+2\tau_{xy}\tau_{yz}\tau_{zx}-\sigma_x\tau_{yz}^2-\sigma_y\tau_{zx}^2-\sigma_z\tau_{xy}^2\right)=0$$

This has 3- real roots $\sigma_1, \sigma_2, \sigma_3$

$$\left.\begin{aligned} &(\sigma_x-\sigma_1)n_x+\tau_{yx}n_y+\tau_{zx}n_z=0 \\ &\tau_{yx}n_x+\left(\sigma_y-\sigma_1\right)n_y+\tau_{zy}n_z=0 \\ \text{and} \qquad &n_x^2+n_y^2+n_z^2=1 \end{aligned}\right\}$$

$$\Rightarrow \quad n_x, n_y, n_z \rightarrow \sigma_1$$

$$\sigma_1>\sigma_2>\sigma_3$$



Stress invariants

$$\sigma^3-I_1\sigma^2+I_2\sigma-I_3=0 \qquad (1)$$

$$\left.\begin{aligned} I_1&=\sigma_x+\sigma_y+\sigma_z \\ I_2&=\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_x\sigma_z-\tau_{xy}^2-\tau_{yz}^2-\tau_{zx}^2 \\ I_3&=\sigma_x\sigma_y\sigma_z+2\tau_{xy}\tau_{yz}\tau_{zx}-\sigma_x\tau_{yz}^2-\sigma_y\tau_{zx}^2-\sigma_z\tau_{xy}^2 \end{aligned}\right\} \text{stress invariants}$$

$$\sigma^3-I_1'\sigma^2+I_3'=0$$

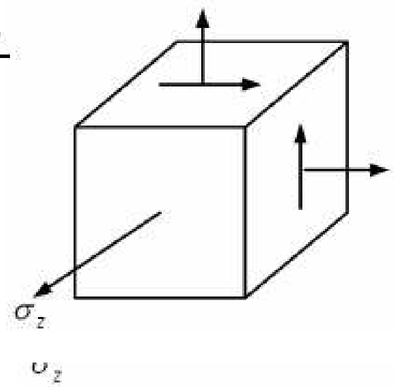
$$I_1'=\sigma_{x'}+\sigma_{y'}+\sigma_{z'} \,\Big|\, I_2'=\sigma_{x'}\sigma_{y'}+\sigma_{x'}\sigma_{z'}+\sigma_{y'z'}-\tau_{x'y'}^2-\tau_{y'z'}^2-\tau_{x'z'}^2$$

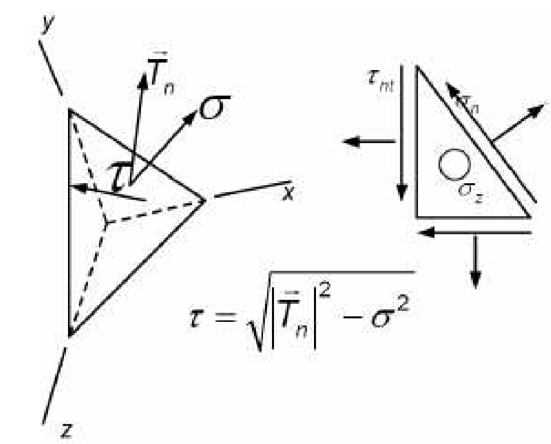
$$I_1=I_1'; \quad I_2=I_2'; \quad I_3=I_3'$$

3D	2D
$I_1 = \sigma_1 + \sigma_2 + \sigma_3$ $I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$ $I_3 = \sigma_1\sigma_{2\sigma_3}$	$I_1 = \sigma_1 + \sigma_2$ $I_2 = \sigma_1\sigma_2$ $I_3 = 0$

3-D Mohr's circle & principal shear

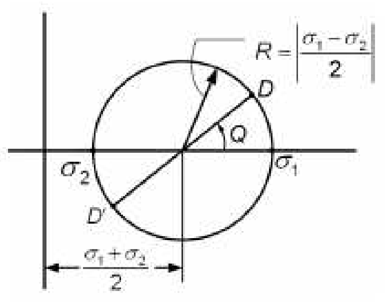
$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$





$$\tau = \sqrt{\left|\vec{T}_n\right|^2 - \sigma^2}$$

Once if you know σ_1 *and* σ_2



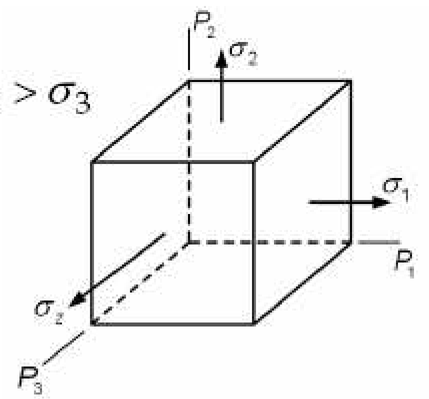
$$\tau_1 = \left|\frac{\sigma_2 - \sigma_3}{2}\right|$$

$$\sigma_1 > \sigma_2 > \sigma_3$$

$$\sigma_{\tau_1} = \frac{\sigma_1 + \sigma_3}{2}$$

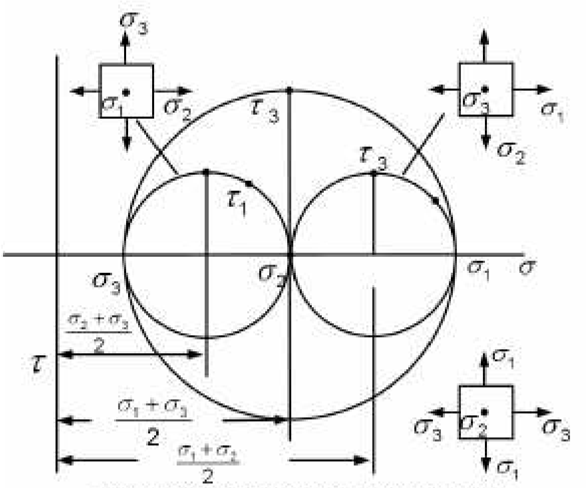
$$\tau_2 = \left|\frac{\sigma_1 - \sigma_3}{2}\right|$$

$$\sigma_{\tau_2} = \frac{\sigma_1 + \sigma_2}{2}$$



$$\tau_3 = \left|\frac{\sigma_1 - \sigma_2}{2}\right| \qquad \tau_{max} = max\left|\frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2}, \frac{\sigma_3 - \sigma_1}{2}\right|$$

$$\sigma_{\tau_3} = \frac{\sigma_1 - \sigma_2}{2}$$



The cluster of these three circles is known as 3D-Mohr's circle

Problem:

The state of stress at a point is given by

$\sigma_x = 100\ MPa,\ \sigma_y = -40\ MPa,\ \sigma_z = 80\ MPa\ and$

$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$

Determine in plane max shear stresses and maximum shear stress at that point.

$\sigma_1 = 100MPa,\ \sigma_2 = 80MPa\ \ \sigma_3 = -40MPas$

$\tau_{12} = \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{100 - 80}{2} = 10MPa$

$\tau_{13} = \dfrac{\sigma_1 - \sigma_3}{2} = \dfrac{100 + 40}{2} = 70MPa$

$\tau_{23} = \dfrac{\sigma_2 - \sigma_3}{2} = \dfrac{80 + 40}{2} = 60MPa$

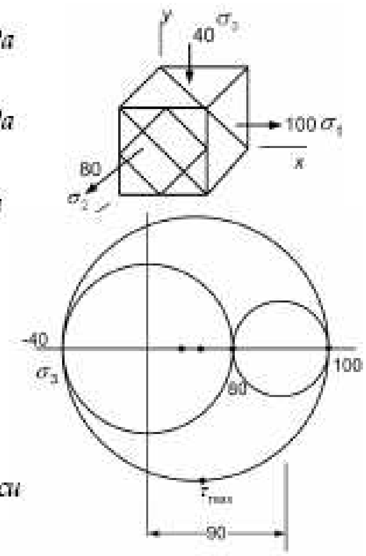
$\sigma_{12} = \dfrac{\sigma_1 + \sigma_2}{2} = 90$

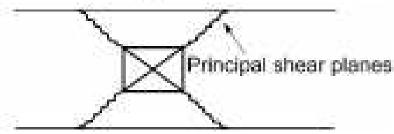
$\sigma_{13} = 30MPa$

$\sigma_{23} = 20\ MPa$

$\tau_{max} = max|\tau_{12}, \tau_{13}, \tau_{23}|$

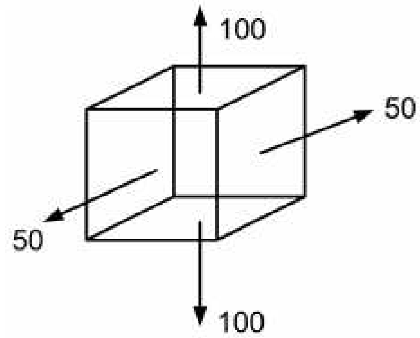
$\tau_{max} = 70MPa$ *This occu*





Problem

At appoint in a component, the state of stress is as shown. Determine maximum shear stress.

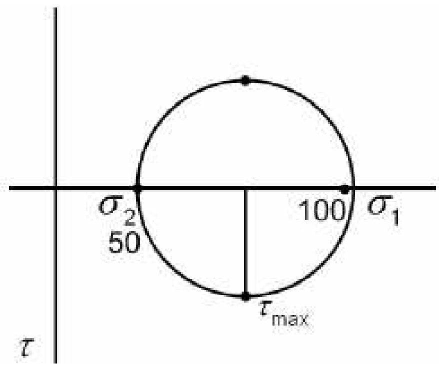


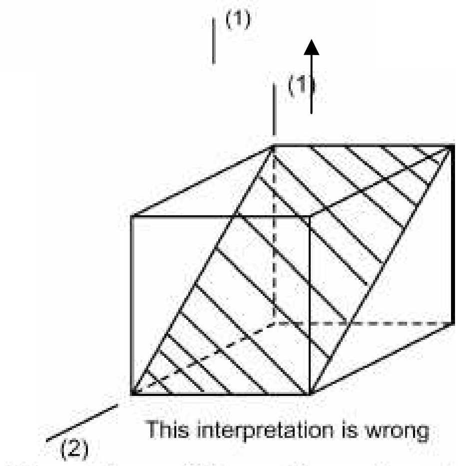
Solution:

$$\left[\sigma_{ij}\right] = \begin{bmatrix} 100 & 0 \\ 0 & 50 \end{bmatrix}$$ - plane stress problem

We can also write the matrix as $\left[a_{ij}\right] = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\sigma_1 = 100$$
$$\sigma_2 = 50$$
$$\left|\frac{\sigma_1 - \sigma_2}{2}\right| = \frac{100-50}{2} = 25$$
$$\tau_{max} = 25\ MPa$$

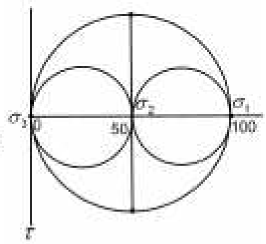


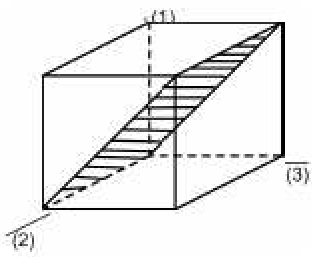


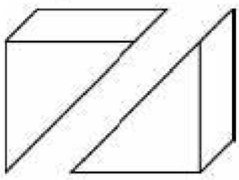
Now with $\sigma_1 = 100,\ \sigma_2 = 50,\ \sigma_3 = 0$

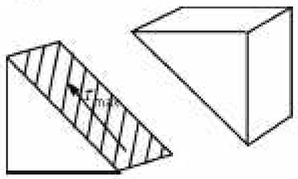
$$\tau_{max} = \left|\frac{\sigma_1 - \sigma_3}{2}\right| = 50\ MPa$$

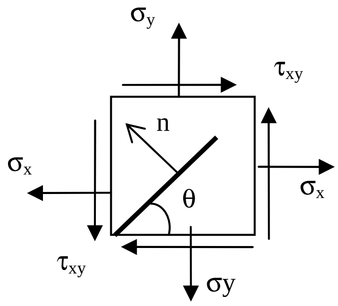
Occurs in the plane 1-3 instead of 1-2











Şekildeki gerilme durumunun tek eksenli gerilme hali olduğu bilindiğine göre, θ=45 açı yapan kesitteki σ, τ gerilmeleri ve asal gerilmelerden σ1 gerilmesi ile bilinmeyen σx gerilmeisini bulunuz.

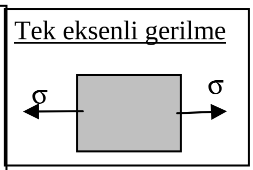
σ_y=2(a+e) N/cm^2,
τ_{xy}=4(a+e) N/cm^2.

$$\sigma = \sigma_X \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

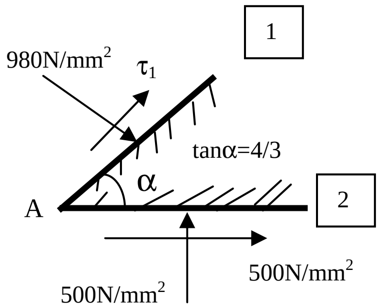
$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\sigma_{1,2} = \frac{\sigma_X + \sigma_Y}{2} \pm \sqrt{\left(\frac{\sigma_X - \sigma_Y}{2}\right)^2 + \tau_{XY}^2}\ ,$$

$$\tan 2\theta_0 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



Düzlem gerilme etkisindeki bir levhada A noktasındaki 1 ve 2 düzlemlerindeki gerilmeler şekilde verilmiştir. Bu gerilme haline ait asal gerilmelerin değer ve doğrultularını bulunuz. Verilenler ile bulunanlar Mohr çemberi üzerinde gösteriniz.



$$\sigma = \sigma_X \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\sigma_{1,2} = \frac{\sigma_X + \sigma_Y}{2} \pm \sqrt{\left(\frac{\sigma_X - \sigma_Y}{2}\right)^2 + \tau_{XY}^2}\ , \quad \tan 2\theta_0 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\cos 45 = \sin 45 = \frac{\sqrt{2}}{2}$$

<u>Değişmezler (invaryantlar)</u>

I1=$\sigma_x+\sigma_y=\sigma_n+\sigma_t$

I2=$\sigma_x\sigma_y - \tau^2xy = \sigma_n\sigma_t - \tau^2nt$

Üçeksenli gerilme halinde $\sigma_1 + \sigma_2 + \sigma_3$ değeri hangisidir

$$\begin{bmatrix} 50 & 100 & 200 \\ 100 & 150 & 300 \\ 200 & 300 & 250 \end{bmatrix}$$

A) 350 B) 450 C) 750
D) 550