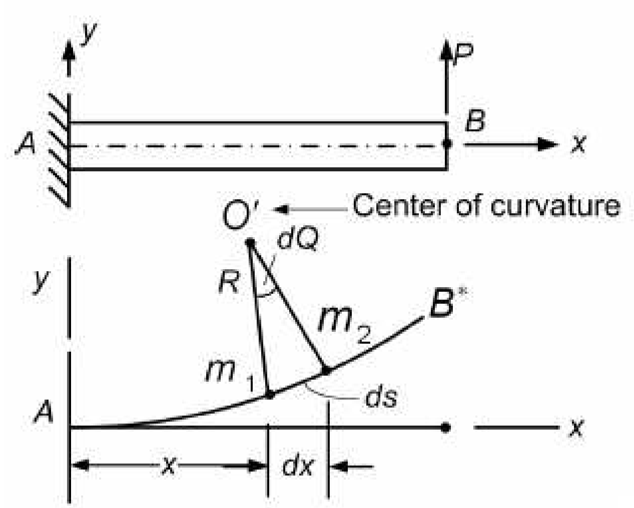
MUKAVEMET

EĞİLME

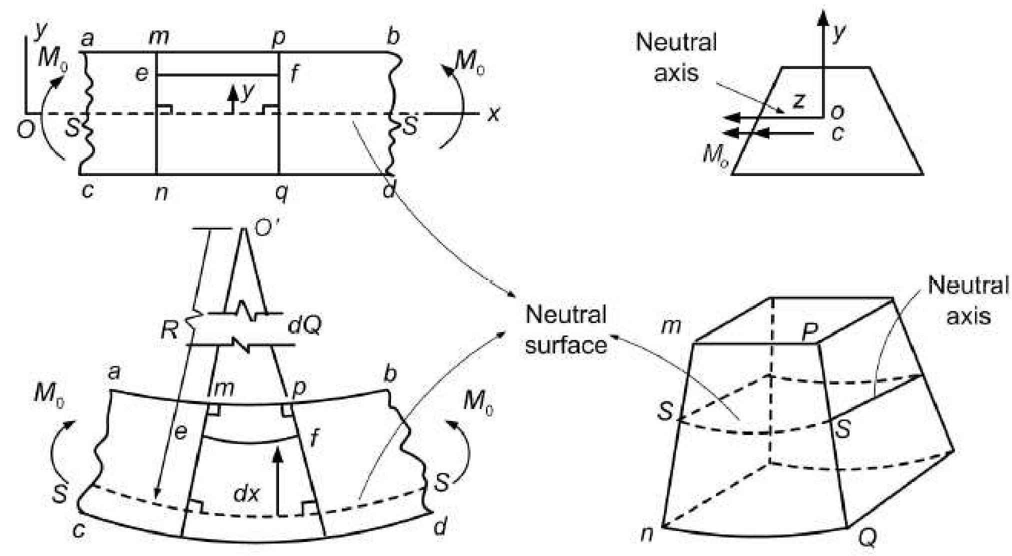


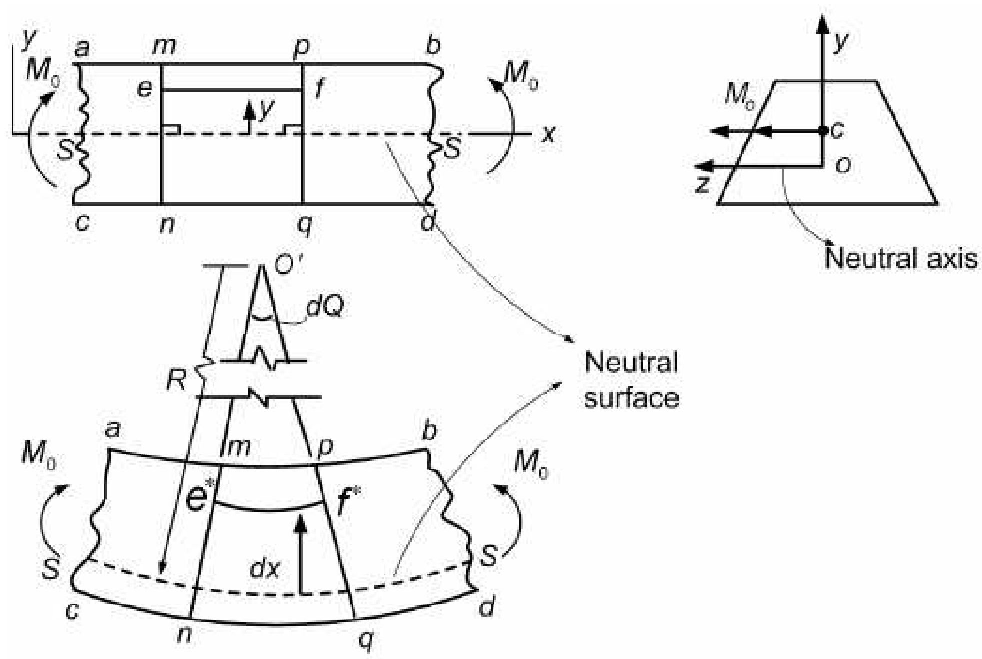
$O-$ Center of curvature

$R-$ Radius of curvature

$k = \frac{1}{R} =$ Curvature

$$k = \frac{1}{R} = \frac{dQ}{dS}$$





$$\in_x = \frac{e^* f^* - \overline{ef}}{\overline{ef}} = \frac{(R-y)dQ - dx}{dx} = \frac{-y}{R}$$

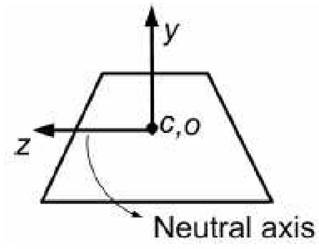
$$\therefore \ \in_x = -\frac{y}{R} \quad \Rightarrow \quad \in_x = -ky$$

Moment – Curvature relationship

$$M = -\int_A \sigma_x y dA$$

$$M = +\int_A \frac{Ey}{R} y dA$$

$$M = \frac{E}{R}\int_A y^2 dA$$

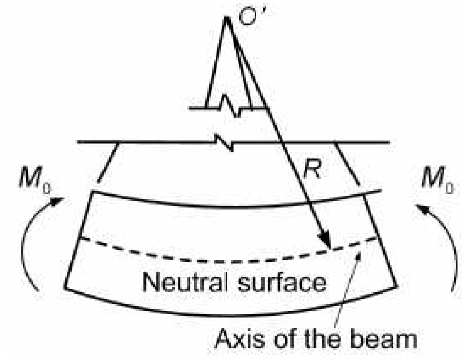


$\int_A y^2 dA = I_{zz} =$ Moment of inertia of cross-sectional area about neutral axis

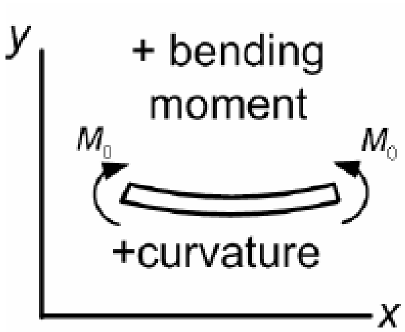
$$\therefore \quad M = \frac{EI}{R}$$

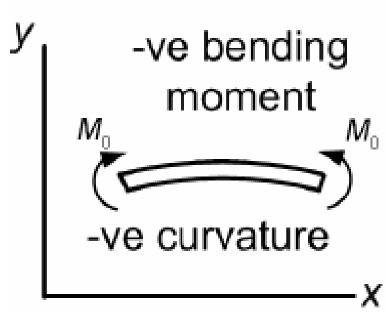
$$k = \frac{1}{R} = \frac{M}{EI}$$

$$k = \frac{1}{R} = \frac{M_0}{EI}$$



Moment-Curvature relation

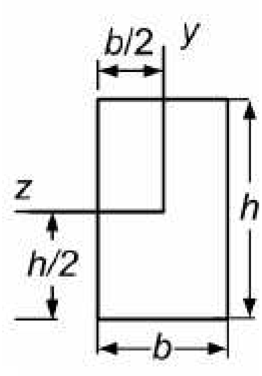




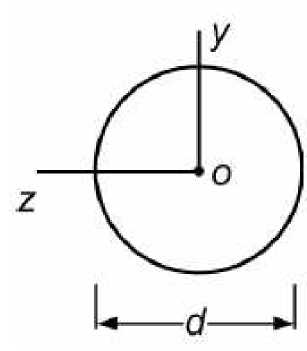
$$\sigma_x = -Eky$$

and $k = \dfrac{M}{EI}$

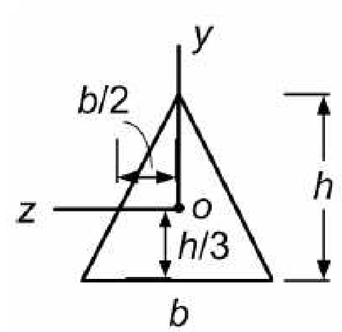
$$\therefore \quad \sigma_x = -\frac{My}{I}$$ - flexure formula.



$$I_{zz} = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6}$$

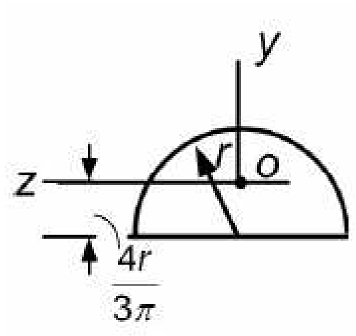


$$I_{zz} = \frac{\pi}{64}d^4 \qquad S = \frac{\pi d^3}{32}$$

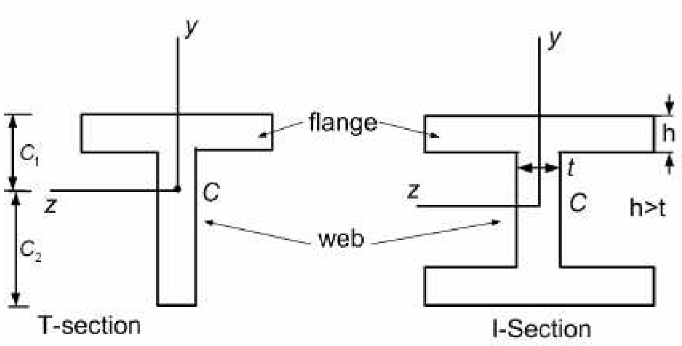


$$I_{zz} = \frac{bh^3}{36}$$

$h = \sqrt{3}\,b/2$ *for eqilateral triangle*



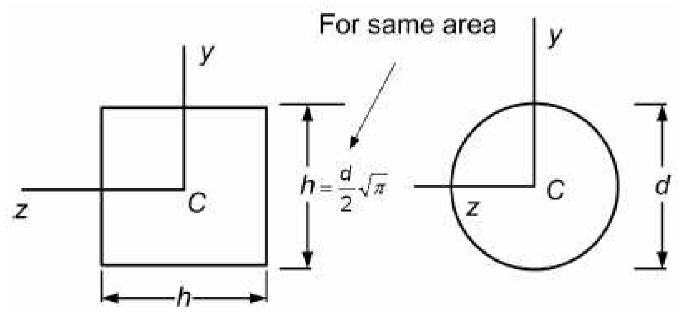
$$I_{zz} = 0.1098r^4$$



$$\sigma_{max} = \frac{M}{S}$$

$$S = \frac{I}{y_{max}}$$

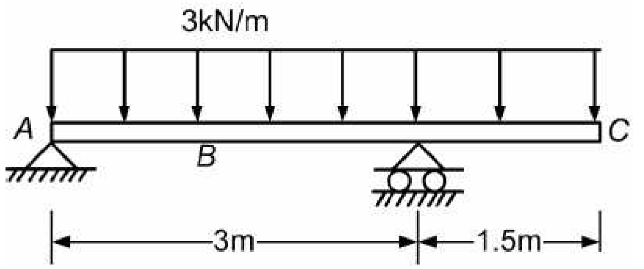
$$M = \sigma_{alllow} S$$

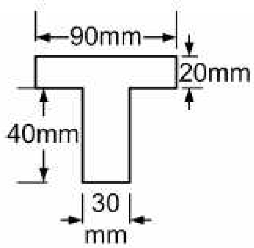


$$\frac{S_{square}}{S_{circle}} = 1.18$$

Problem

Determine the maximum tensile and compressive stresses in the beam due to the uniform load.



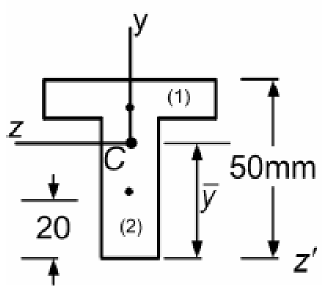


Solution

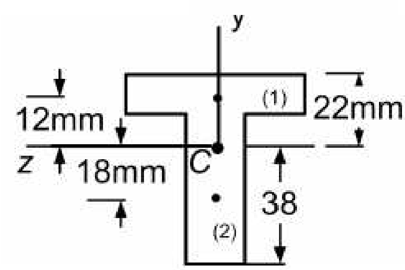
Centroid :-

	$A\ mm^2$	$\overline{y}$	$\overline{y}A\ mm^3$
1	$20\times 90=1800$	50	90×10^3
2	$40\times 30=1200$	20	24×10^3

$A=\Sigma A=3000 \quad \Sigma\overline{y}A=114\times 10^3$



$$A\overline{y}=\Sigma\overline{y}A \Rightarrow \overline{y}3000=114\times 10^3 \Rightarrow \overline{y}=38mm$$



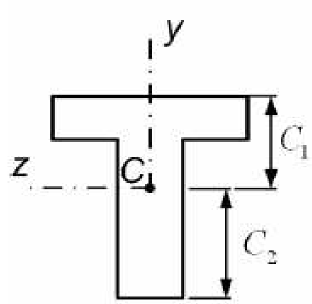
$$I_{zz}=I=\Sigma\left(\overline{I}+Ad^2\right)s$$

$$=\Sigma\left(\frac{bh^3}{12}+Ad^2\right)$$

$$=\frac{1}{12}90\times 20^3+1800\times 12^2+\frac{1}{12}\times 30\times 40^2+1200\times 18^2$$

$$I_{zz}=I=868\times 10^3\ \ mm^4=868\times 10^{-9}m^4$$

$C_1=22mm$ and $C_2=38mm$



$$\sigma_x=-\frac{My}{I}$$

$$\sigma_{max}=\frac{M}{S}:S=\frac{I}{y_{max}}$$

At maximum +ve bending moment i.e at (D)

$$S_1 = \frac{I}{C_1} = \frac{868\times10^{-9}}{22\times10^{-3}} = 39.45\times10^{-6}$$

$$S_2 = \frac{I}{C_2} = \frac{868\times10^{-9}}{38\times10^{-3}} = 22.84\times10^{-6}$$

at D:

$$\sigma_{t_{max}} = \frac{M}{s_2} = \frac{1.898}{22.84\times10^{-6}}$$

$$\sigma_{t_{max}} = 83.1 MPa$$

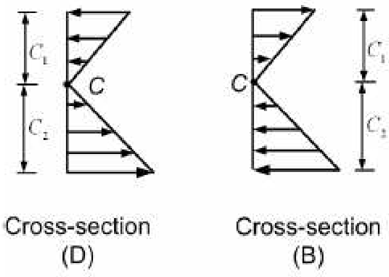
$$\sigma_{C_{max}} = \frac{M}{s_1} = \frac{1.898}{39.45\times10^{-6}}$$

$$\sigma_{C_{max}} = 48.11 MPa$$

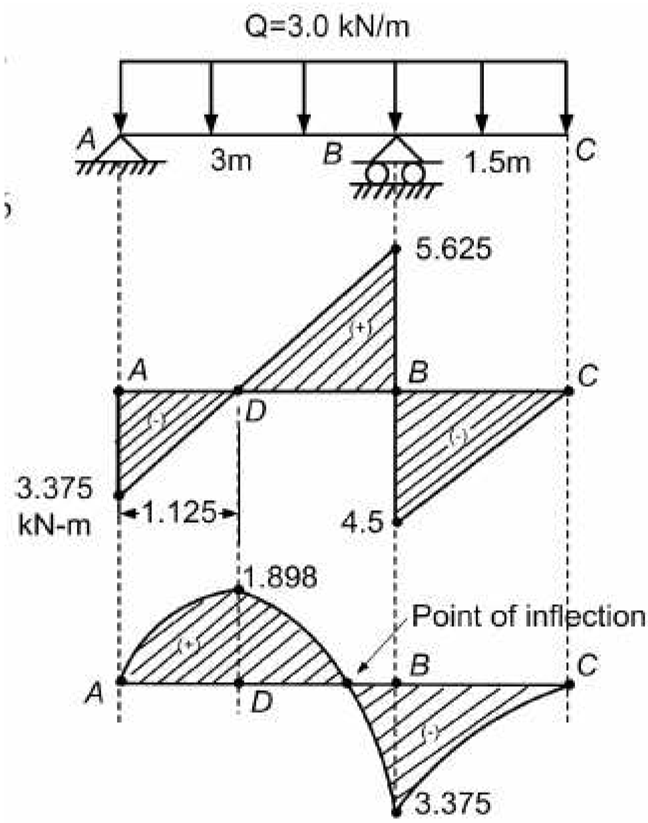
At maximum -ve moment i.e at (B)

$$\sigma_{t_{max}} = \frac{M}{s_1} = \frac{3.375}{39.45 \times 10^{-6}} = 85.55\ MPa$$

$$\sigma_{C_{max}} = \frac{M}{s_2} = \frac{3.375}{22.84 \times 10^{-6}} = 147.8\ MPa$$

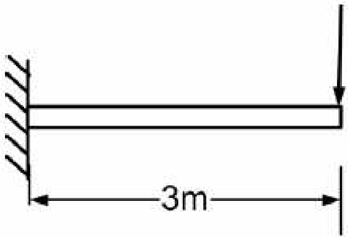


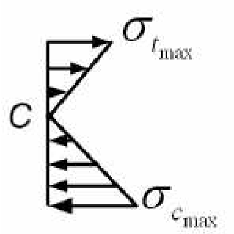
$$\sigma_{t_{max}} = 85.55 \quad and \quad \sigma_{C_{max}} = 147.8\ MPa$$



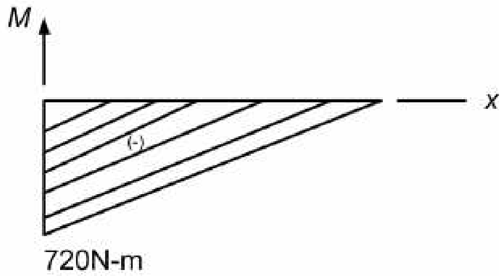
Problem

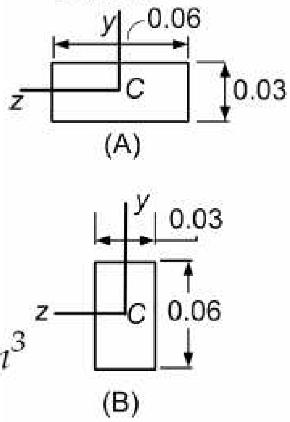
a wooden member of length $L = 3\text{m}$ having a rectangular cross-section 3 cm × 6 cm is to be used as a cantilever with a load $P = 240\text{N}$ acting at the free end. Can the member carry this load if the allowable flexural stress both in tension and in compression is $\sigma_{allow} = 50$ Mpa ?





Solution





$$M_{max} = 720 \text{ N-m}$$

$$S_A = \frac{1}{12}\frac{0.06 \times 0.03^3}{0.015} = 9 \times 10^{-6} m^3$$

$$\sigma_{t_{max}} = \sigma_{C_{max}} = \frac{M}{S_A} = \frac{PL}{S_A}$$

$$\sigma_{t_{max}} = \sigma_{C_{max}} = \sigma_{allow}$$

$$P_{alow} = \frac{\sigma_{allow} \times S_A}{L} = 150N$$

$$S_B = \frac{1}{12}\frac{0.03 \times 0.06^3}{0.03} = 1.8 \times 10^{-5} m^3$$

$$P_{alow} = \frac{\sigma_{allow} \times S_B}{L} = 300N$$

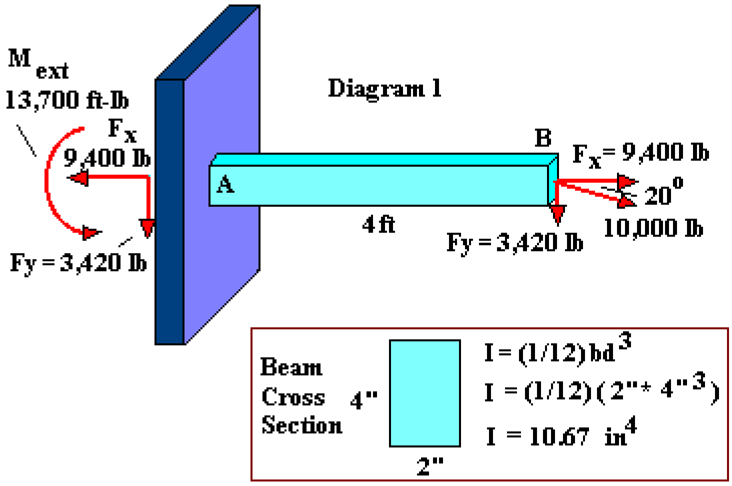
$\therefore$ The beam can carry $P = 240N$ only when oriented as in (B)

Topic 8.1: Special Topics I - Combined Stress

Up to this point we have considered only or mainly one type of applied stress acting on a structure, member of a structure, beam, shaft, rivet, or weld. Many situations involve more than one type of stress occurring simultaneously in a structure. These problems can become relatively complicated. We will look at several examples of relatively simple combined stress problems.

One of the principles we will apply in these examples is the principle of superposition, that is, the resultant stress will be the algebraic sum of the individual stresses - at least in the case of similar stresses acting along the same line, such as the axial stress due to an axial load and a bending stress. We look at this type of problem in the example below.

Example. A four foot long cantilever beam (shown in Diagram 1) is attached to the wall at point A, and has a load of 10, 000 lb. acting (at the centroid of the beam) at angle of 20^o below the horizontal. We would like to determine the maximum axial stress acting in the beam cross section.



Solution:

We first apply static equilibrium conditions to the beam and determine the external support reactions, and the external moment acting on the beam at point A. Notice we have resolved the **10,000 lb**. load into its perpendicular x and y components. The horizontal component of the load (**9,400 lb**.) produces a normal horizontal axial stress in the beam. The vertical component of the load (**-3,420 lb**.) causes a torque about point A (**13,700 ft-lb**) to act on the beam (balanced by the external moment). The resulting internal bending moment(s) in the beam produces an axial bending stress. **The total axial stress at a point in the beam will be the sum of the normal axial stress and the axial bending stress**.

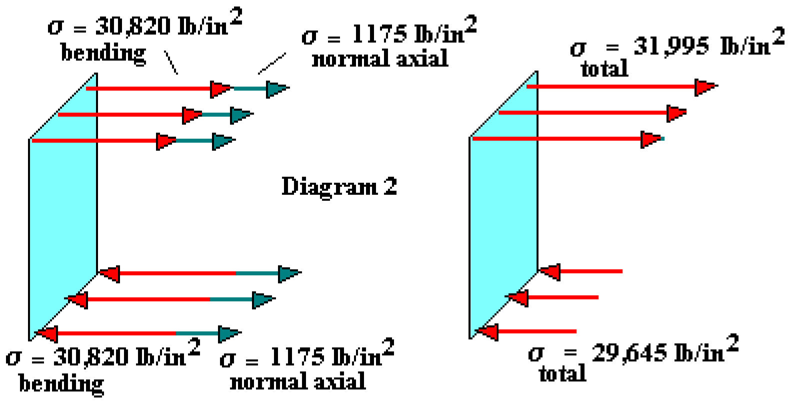
The Normal Axial Stress = Force/Area = 9,400 lb. / (2" x 4") = 1175 lb/in^2. We note that this stress will be tensile and constant through out the length of the beam. So the maximum normal Axial Stress is **1175 lb/in²**, and is the same everywhere in the beam.

The maximum bending stress occurs at the outer edge of the beam, and at the point in the beam where the bending moment is a maximum. In the cantilever beam, the maximum bending moment occurs at the wall and is equal to the (negative of) external bending moment. (**M = -13,700 ft-lb. = -164,400 in-lb.**) We can then calculate the maximum bending moment by:

Maximum Bending Stress = M y/I = (164,400 in-lb.)(2")/(10.67 in⁴) = 30,820 lb/in².

Since the bending moment was negative, this means that the top of the beam (above the centroid) is in tension, and the bottom on the beam is in compression.

We can now combine (sum) the axial stress at the very top and bottom of the beam to determine the maximum axial stress. We see in the beam section in Diagram 2, that the stresses at the top of the beam are both tensile, and so add to a **total tensile stress of 30,820 lb./in² + 1,175 lb./in² = 31,995 lb./in²**. At the bottom of the beam, the bending stress is compressive and the normal axial stress in tensile so the resultant bottom **stress is - 30,820 lb./in² + 1,175 lb./in² = -29,645 lb./in² (compression).**



We will now look at several additional examples of combined stresses.

Select :

[Topic 8.1a: Combined Stress - Example 1]

[Topic 8.1b: Combined Stress - Example 2]

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Topic 8.1a: Combined Stress - Example 1

A loaded beam (shown in Diagram 1) is pinned to the wall at point A, and is supported by a rod DB, attached to the wall at point D and to the beam at point B. The beam has a load of 6,000 lb. acting downward at point C. The supporting rod makes an angle of 25° with respect to the beam. The beam cross section is a W8 x 24 I-Beam, with the characteristics shown in Diagram 1. We would like to determine the maximum axial stress acting in the beam cross section.

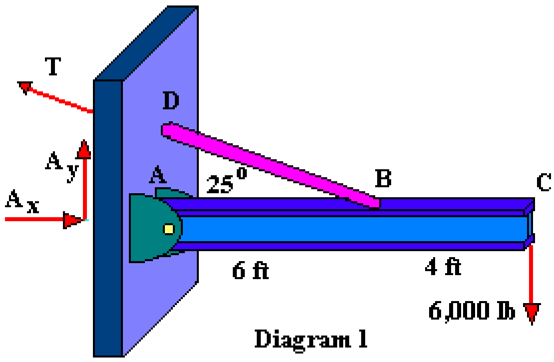
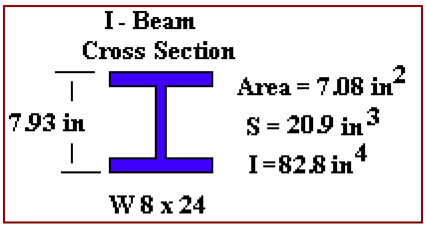


Diagram 1



Solution:

We first apply static equilibrium to the beam and determine the external support reactions acting on the beam at point A.

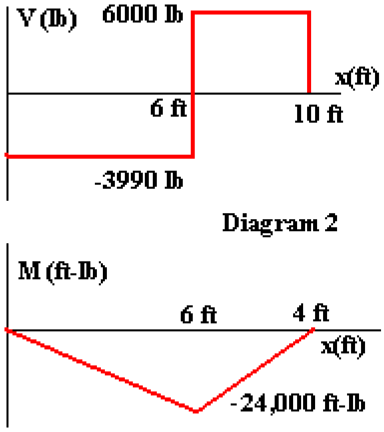
Sum of Force$_x$ = A_x - T cos 25° = 0

Sum of Force$_y$ = A_y + T sin 25° - 6,000 lb. = 0

Sum of Torque$_A$ = -6,000 lb (10ft) + T cos 25° (2.8 ft.) = 0 (where 2.8 ft. = distance from A to D)

Solving: T = 23,640 lb.; A_x = 21,430 lb., and A_y = -3990 lb. (A_y acts downward)

We next draw the Shear Force and Bending Moment Diagrams, and use the Bending Moment Diagram to determine the Maximum Bending Stress in the beam. (See Diagram 2.)



We next will consider the axial stress due to the horizontal force acting on the beam. In section AB the beam is in compression with horizontal axial force of 21,430 lb. (Due to the force A_x and the horizontal component of the force in rod DB.) For beam section BC, there is no horizontal axial force due to an external horizontal force. That is, section AB is in compression, but section BC is not experiencing normal horizontal stress, since it is to the right of where the support rod is attached. (However, there is a horizontal bending stress due to the bending moment, which is in turn due to the vertical loads being applied. This will be considered in a moment.)

The compressive horizontal axial stress in section AB is given simply by:

F/A = 21,430 lb. / 7.08 in^2 = 3,030 lb/in^2. (We have considered the force to act along the centroid of the beam.)

There is a bending stress also acting in the beam. The maximum bending stress occurs at the outer edge of the beam, and at the point in the beam where the bending moment is a maximum. From our bending moment diagram, we see that the maximum bending moment occurs at 6 feet from the left end, and has a value of **-24,000 ft-lb. = -288,000 in-lb.** (The negative sign indicating that the top of the beam is in tension and the bottom of the beam is in compression.) We can then calculate the maximum bending moment by:

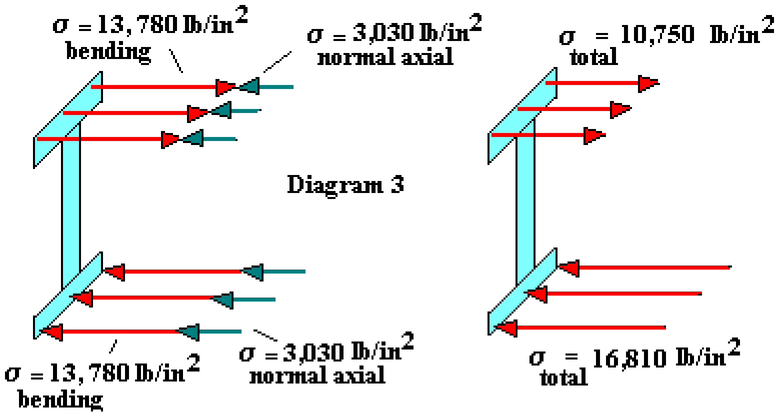
Maximum Bending Stress = M / S Where S is the section modulus for the beam. In this example S = 20.9 in^3. Then:

Maximum Bending Stress = (288,000 in-lb.)/20.9 in^3 = 13,780 lb/in^2.

Since the bending moment was negative, the top of the I-Beam will be in tension, and the bottom of the beam will be in compression.

The total axial stress at a point in the beam will be the sum of the normal axial stress and the axial bending stress. (See Diagram 3)

We can now combine (sum) the axial stresses at the very top and bottom of the beam to determine the maximum axial stress. We see in the beam section (at 6 ft from left end) in Diagram 3, that the stresses at the bottom of the beam are both compressive, and so add to a **total compressive stress of 13,780 lb./in² + 3,030 lb./in² = 16, 810 lb./in²**. At the top of the beam, the bending stress is tensile and the normal axial stress in compressive so the resultant bottom **stress is: +13,780 lb./in² - 3,030 lb./in² = 10,750 lb./in² (tension).**



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[Topic 8.1: Combined Stress]

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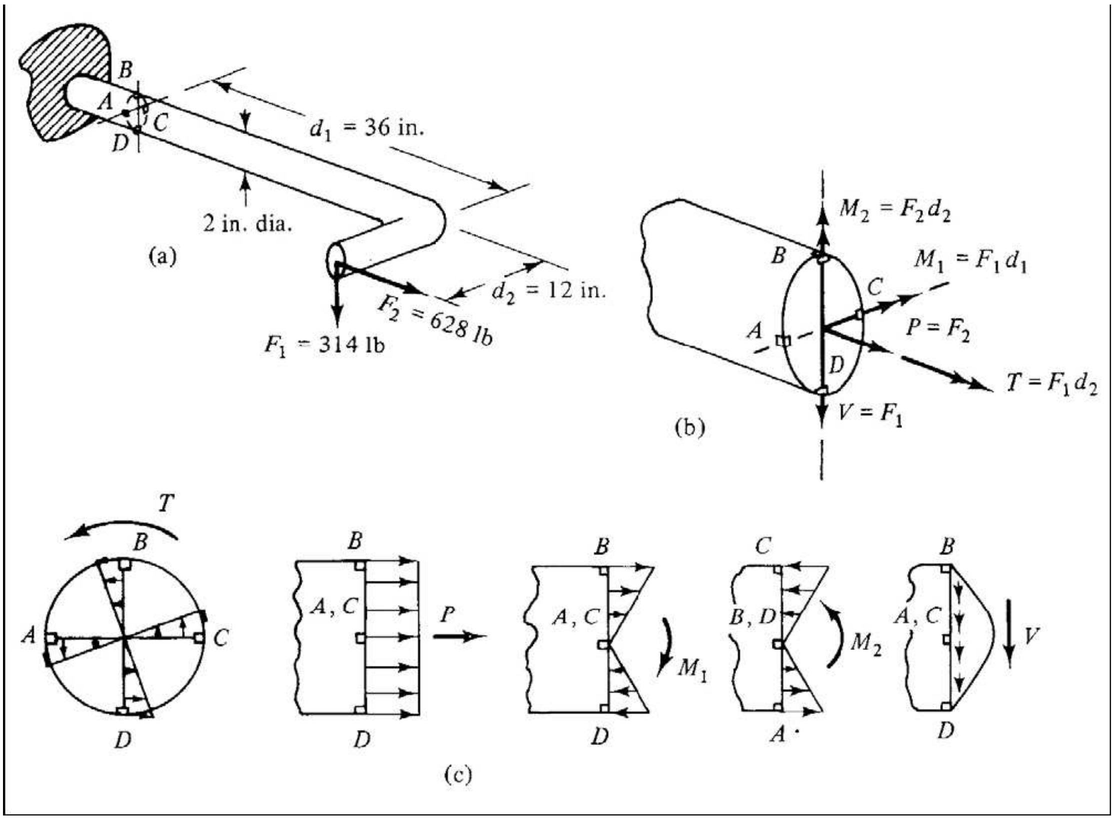
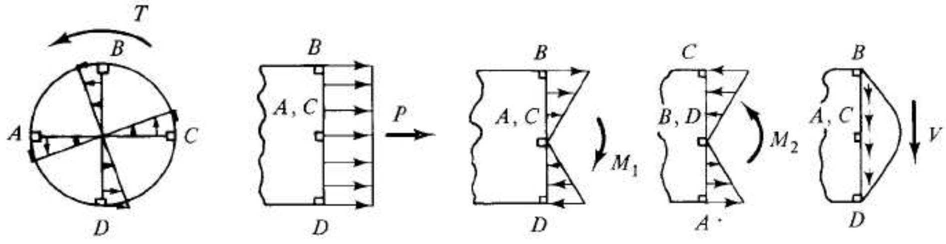
Stresses Under Combined Loads:

To this point we have considered the response of members subjected to the separate
effects of axial loads, torsion, bending and uniform pressure. However, in many cases
structural members are required to resist more than one type of loading. The stress
analysis of a member subjected to **combined loadings** can usually be performed by
superimposing the stresses due to each load acting separately. Superposition is
permissible if the stresses are linear functions of the loads and if there is no interaction

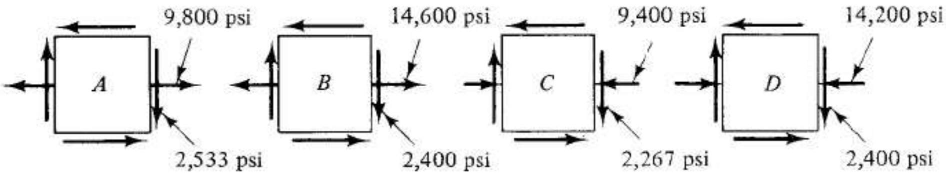
effect between the various loads (i.e. the stresses due to one load are not affected by
the presence of any other loads).



Torsion	$\tau = \frac{Tr}{J}$	2,400 psi
Bending (M_1)	$\sigma = \frac{M_1}{S}$	14,400 psi
Bending (M_2)	$\sigma = \frac{M_2}{S}$	9,600 psi
Axial	$\sigma = \frac{P}{A}$	200 psi
Transverse Shear	$\tau = \frac{4V}{3A}$	133 psi

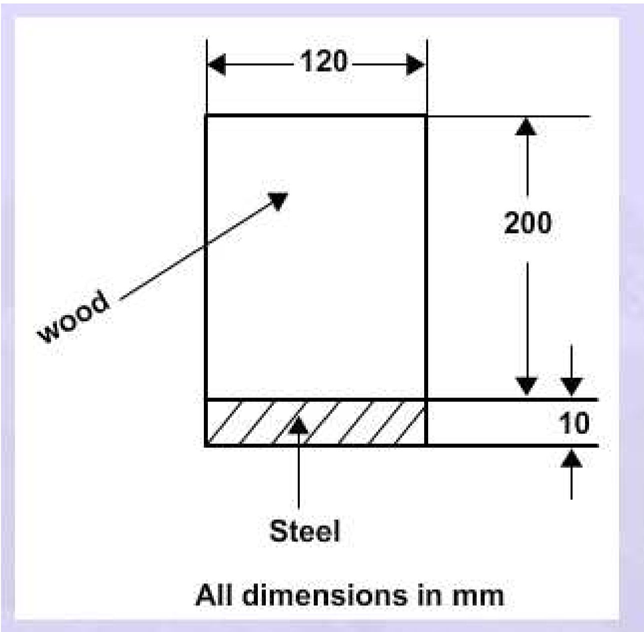


(c)



Problem 11: Beams of Composite Cross section

The composite beam shown in figure is made up of two materials. A top wooden portion and a bottom steel portion. The dimensions are as shown in the figure. Take young's modulus of steel as 210 GPa and that wood as 15 GPa. The beam is subjected to a bending moment of 40 kNm about the horizontal axis. Calculate the maximum stress experienced by two sections.



Solution:

The solution procedure involves hiding an equivalent dimension for one of the materials keeping the other as reference.

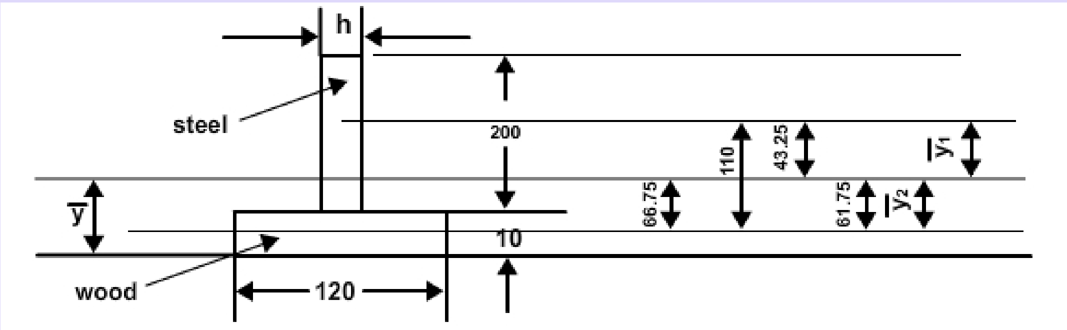
Let us take, Reference Material as steel.

Ratio of modulii,

$$r = \frac{E_{wood}}{E_{steel}} = \frac{15}{210} = .00714$$

$$= 7.14 \times 10^{-2}$$

The equivalent section is as shown in following Figure.



The equivalent thickness of wood sections, h

$= 120 \times r$

$= 120 \times 7.14 \times 10^{-2}$

$= 8.568$ mm

Distance of the neutral axis from the bottom of the beam,

$$\bar{y} = \frac{(120\times10)\times5 + (200\times8.568)\times(100+10)}{(120\times10)+(200\times8.568)}$$

$= 66.75$ mm

$$\text{Moment of Inertia } I = \sum \frac{b_i d_i^3}{12} + A_i \bar{y}_i^2$$

$$= \frac{8.568 \times (200)^3}{12} + (8.568 \times 200)\times(43.25)^2 + \frac{(120)\times(10)^3}{12} + (120)\times(10)\times(61.75)^2$$

$= 13.5 \times 10^6 mm^4$

Stresses in beams

$$(\sigma_{steel})_{max} = \frac{M.y}{I}$$

$$= \frac{40 \times 10^{3} \times 66.75 \times 10^{-3}}{13.5 \times 10^{6} \times \left(10^{-3}\right)^{4}}$$

$$= 197.78 \text{ MPa}$$

$$(\sigma_{wood})_{max} = (\sigma_{steel})_{max} \times r$$

$$= 14.12 \text{ MPa}$$