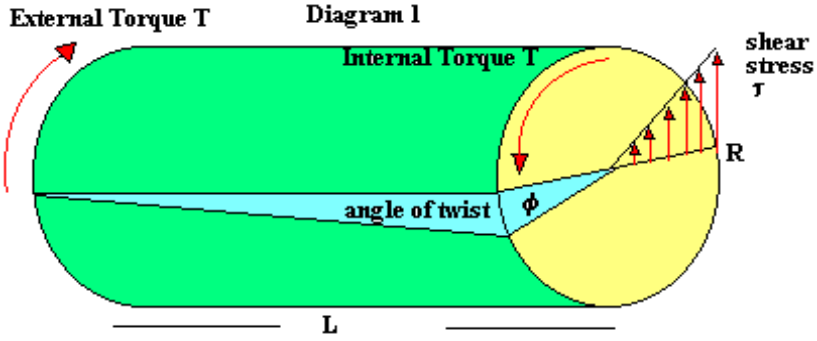
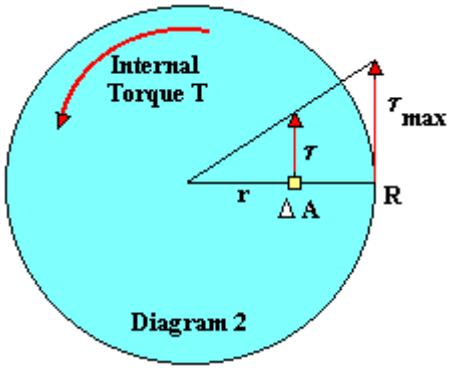
# MUKAVEMET

## BURULMA

L uzunluğunda R yarıçapında burulma çubuğu,

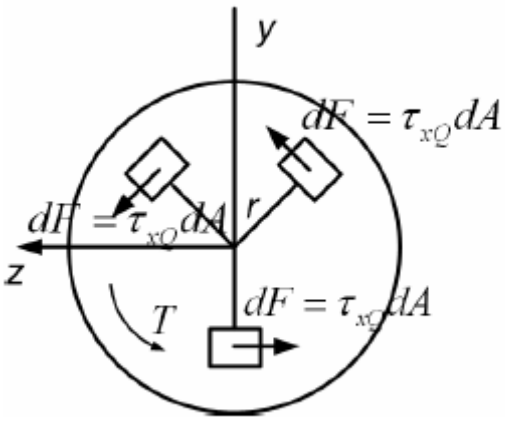



merkezden r kadar uzaklı ktaki, diferansiyel eleman kesit alanı $\Delta A$,




Kayma gerilmesi:

$\tau = (r/R)\ \tau_{max}$.



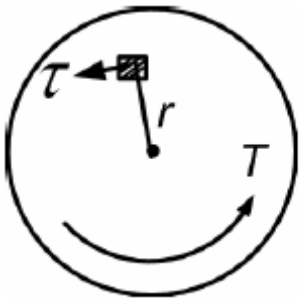

$$dF = \tau_{x\theta} dA$$

$$dT = dF \times r = \sigma_{x\theta} r dA$$

$$T = \int_A \tau_{x\theta} r dA$$

$$T = T_0$$

$$T = \int_A \tau r dA$$




$$T = \int G r \frac{d\phi}{dx} r dA$$

$$T = G \frac{d\phi}{dx} \int_A r^2 dA$$

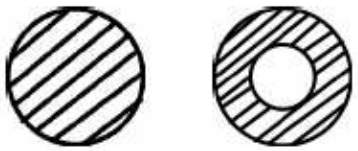
$$I_P = \int_A r^2 dA$$

$$I_P = \frac{\pi}{32} D^4$$

$$I_P = \frac{\pi}{2} R^4$$

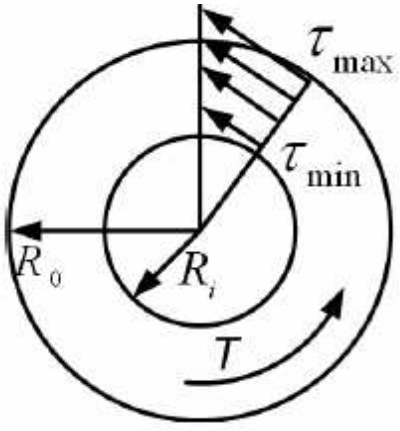
$$I_P = \frac{\pi}{32} D^4 - \text{ Soild}$$

$$I_P = \frac{\pi}{32} \left( D_o{}^4 - D_i^4 \right) - \text{ hollow}$$



$$\tau_{max} = \frac{TR_o}{I_P}$$

$$\tau_{min} = \frac{TR_i}{I_P}$$

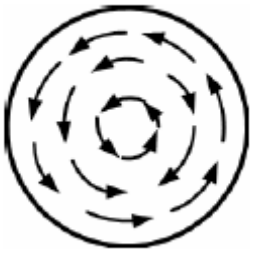


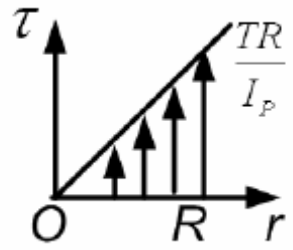
$$T = GI_P \frac{d\phi}{dx}$$

$$\frac{d\phi}{dx} = \odot = \frac{T}{GI_P}$$

$$\frac{\tau}{Gr} = \frac{T}{GI_P}$$

$$\tau = Gr\frac{d\phi}{dx} \qquad \tau = \frac{Tr}{I_P}$$






$$\tau_{max} = \tau_{xQ_{max}} = \frac{TR}{I_P}$$

Diferansiyel Kuvvet

$\Delta F = \tau \Delta A = (r / R)\, \tau_{max} \Delta A$ .

**Diferansiyel burulma momenti**

$\Delta T = r \Delta F = r\tau \Delta A = (r^2 / R)\tau_{max} \Delta A$ .

**T =** $\Sigma \Delta T = \Sigma (r \Delta F) = \Sigma (r\tau \Delta A) = \Sigma (r^2 / R)\tau_{max} \Delta A$ .

**T =** $(\tau_{max} / R) \Sigma r^2 \Delta A = (\tau_{max} / R)\, J$

$\tau$ **= T r / J**

# Örnek 1

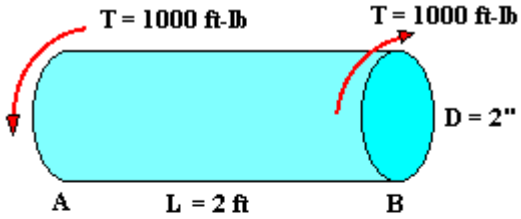


Diagram 1a

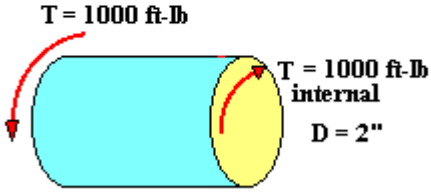


Diagram 1b

$\tau = T\,r / J$;

T = 1000 ft-lb = 12,000 in-lb.

r = 1 in.

J = $(\pi/32)\,d^4$ for a solid shaft = (3.1416/32) ($2^4 in^4$) = 1.57 $in^4$.

$\tau = T\,r / J$ = 12,000 in-lb. * 1 in./1.57 $in^4$. = 7,640 $lb/in^2$.

Halka Kesit

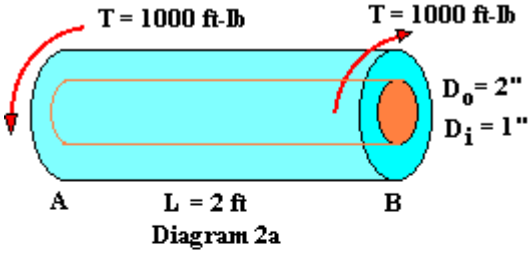


Diagram 2a

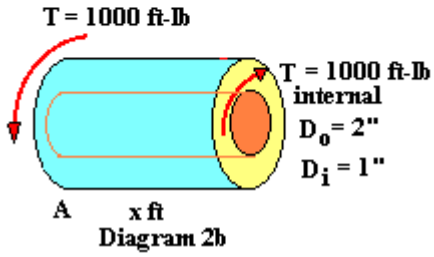


Diagram 2b

$\tau = T\,r / J$

T = 1000 ft-lb = 12,000 in-lb.

J = (3.1416/32) [$d_o^4$ - $d_i^4$] for a hollow shaft = (3.1416/32) [($2^4 in^4$) - ($1.0^4 in^4$)] =

1.47 in$^4$.

= T r / J = 12,000 in-lb. * 1 in./1.47 in$^4$. = 8,150 lb/in$^2$.

# Örnek 2

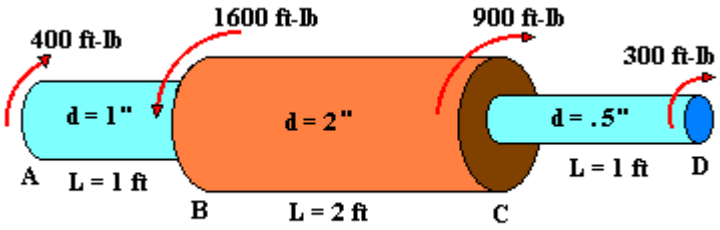
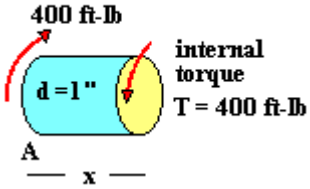



Diagram 1

Diagram 2




$\sigma$ = T r / J:

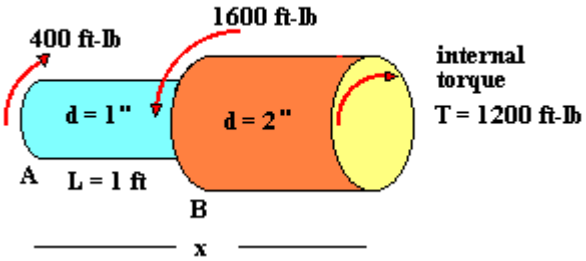
T = 400 ft-lb. = 4,800 in-lb.

r = .5 in.

J = ( $\pi$/32) d$^4$ for a solid shaft = (3.1416/32) (1$^4$in$^4$) = .098 in$^4$.

$\sigma$ = T r / J = 4,800 in-lb. * .5 in./.098 in$^4$. = 24,500 lb./in$^2$.

Diagram 3



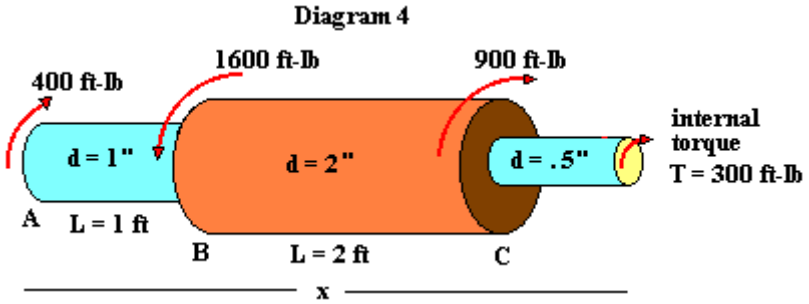

$\sigma$ = T r / J;

T = 1,200 ft-lb. = 14,400 in-lb.

r = 1 in.

J = ( $\pi$/32) d$^4$ for a solid shaft = (3.1416/32) (2$^4$in$^4$) = 1.57 in$^4$.

$\sigma$ = T r / J = 14,400 in-lb. * 1 in./ 1.57 in$^4$. = 9,170 lb./in$^2$.



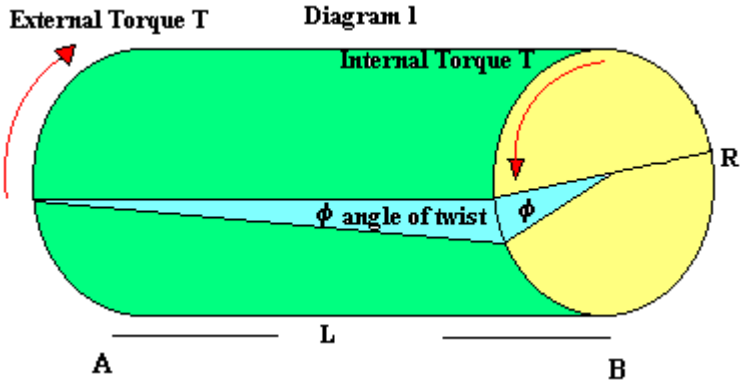

**T = 300 ft-lb. = 3,600 in-lb.**

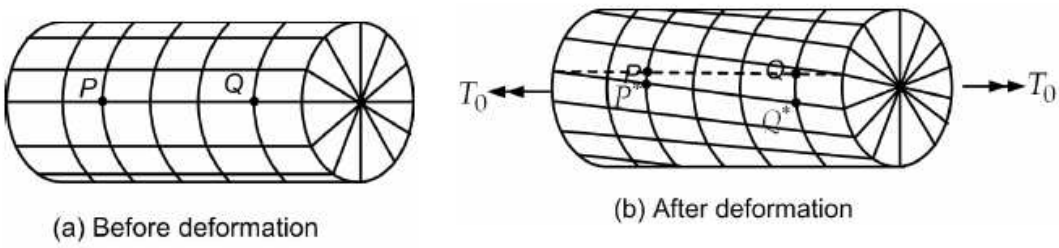
r= .25 in.

J = ( $\pi$/32) $d^4$ for a solid shaft **= (3.1416/32) ($.5^4 in^4$) = .0061 $in^4$.**

$\tau$ **= T r / J = 3,600 in-lb. * .25 in./ .0061 $in^4$. = 147,500 $lb./in^2$.**

# AÇISAL DÖNME




(a) Before deformation

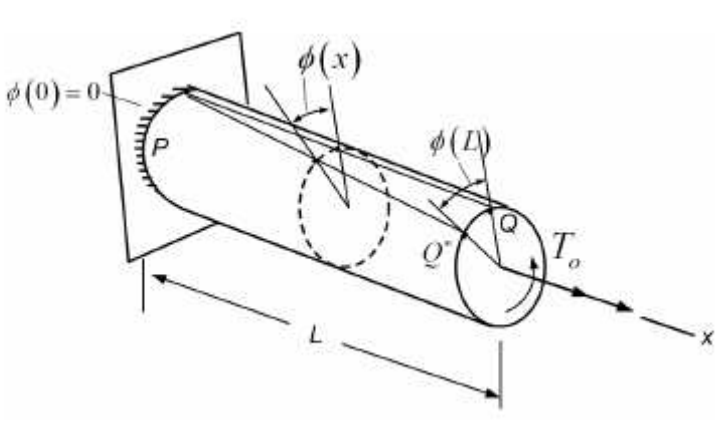
(b) After deformation

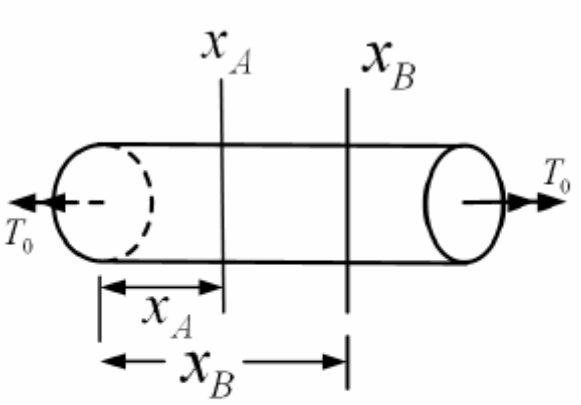
$\gamma$ **= T L / J G**

G = Kayma Modülü

$G_{steel}$ = 12 x $10^6$ $lb/in^2$,

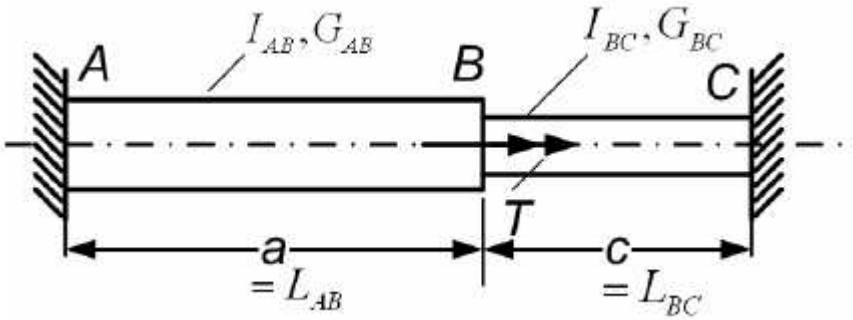
$G_{brass}$ = 6 x $10^6$ $lb/in^2$.

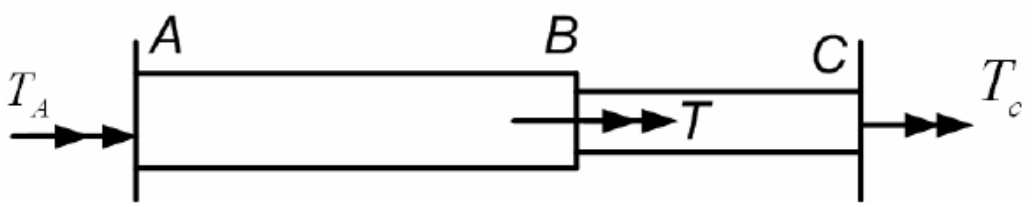



$$\odot = \frac{d\phi}{dx} = \frac{T}{GI_P}$$

$$\phi_{B/A} = \phi_B - \phi_A = \int_{x_A}^{x_B} \frac{T}{GI_P} dx$$



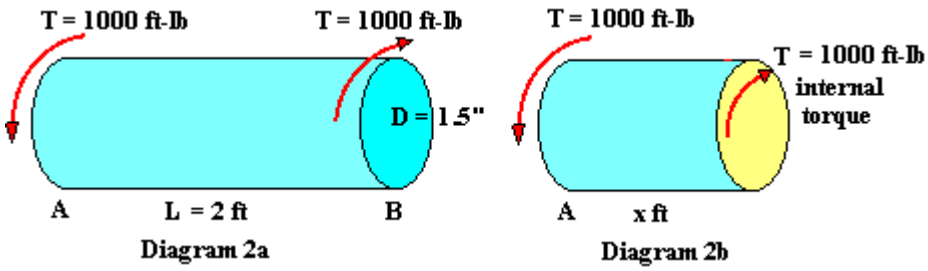



$[\Sigma M_x = 0]$ $T_A + T_C + T = 0$ (

$$\phi_{B/A} = \phi_{B/C}$$
$$Compatibility\ equation$$

$$\phi_{B/A} = \frac{T_A L_{AB}}{G_{AB} I_{P_{AB}}} \; ; \quad \phi_{B/C} = \frac{T_C L_{BC}}{G_{BC} I_{P_{BC}}}$$

$$\frac{T_A L_{AB}}{G_{AB} I_{P_{AB}}} = \frac{T_C L_{BC}}{G_{BC} I_{P_{BC}}}$$




$\phi$ = T L / J G

T = 1000 ft-lb. = 12,000 in-lb.

L = 2 ft. = 24 inches

J = ( $\pi$/32) $d^4$ for a solid shaft = (3.1416/32) ($1.5^4 in^4$) = .5 $in^4$.

$G_{steel}$ = 12 x $10^6$ lb/$in^2$

$\phi$ = T L / J G = (12,000 in-lb.* 24 in) / (.5 $in^4$ * 12 x $10^6$ lb/$in^2$) = .048 radians = 2.75°.

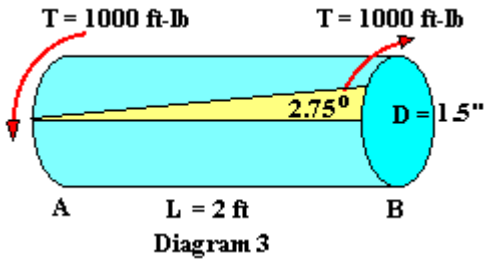


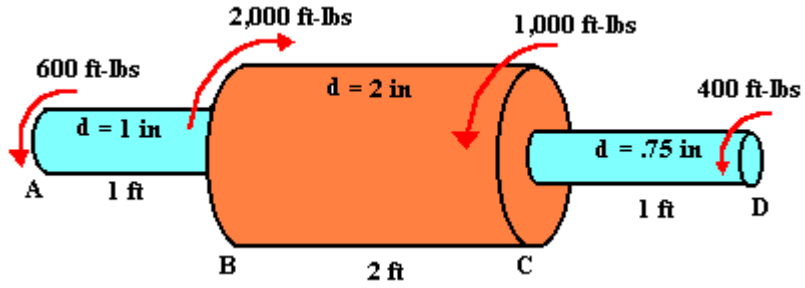
Diagram 3

$\tau = T\,r / J = 12{,}000$ in-lb. * .75 in./.5 $in^4$. = 18,000 $lb/in^2$.

## Örnek

Çeliğ in kayma modülü = 12 x 106 $lb/in^2$.

Pirincin kayma modülü = 6 x 106 $lb/in^2$.




**I. Kesit:**

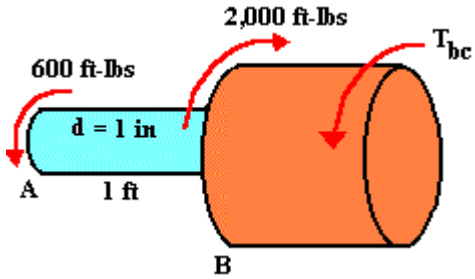
T = 600 ft-lb - $T_{AB}$ = 0; So $T_{AB}$ = 600 ft-lb

$\tau_{AB}$ = Tr / J = (600 ft-lb)(12 in/ft)(.5 in) /[3.1416 * (1 in)$^4$ / 32] = 36,700 psi

**II. kesit:**

T = 600 ft-lb - 2000 ft-lb + $T_{BC}$ = 0; So $T_{BC}$ = 1400 ft-lb

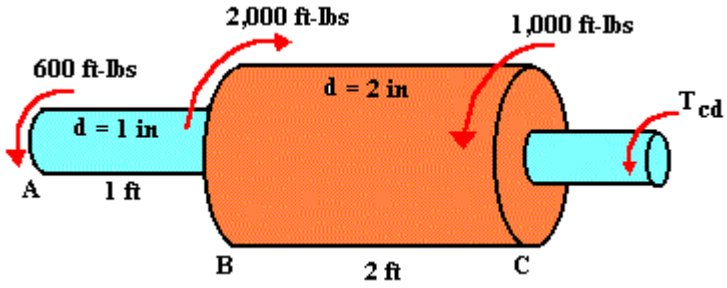
$\tau_{BC}$ = Tr / J = (1400 ft-lb)(12 in/ft)(1 in) /[3.1416 * (2 in)$^4$ / 32] = 10,700 psi




**III. kesit:**

T = 600 ft-lb - 2000 ft-lb + 1000 ft-lb + $T_{CD}$ = 0; So $T_{CD}$ = 400 ft-lb

$\tau_{CD}$ = Tr / J = (400 ft-lb) (12 in/ft)(.375 in) /[3.1416 * (.75 in)$^4$ / 32] = 57,900 psi




$\phi_{AB}$ = TL/JG = (600 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (1 in)$^4$ / 32)(12x10$^6$ lb/in$^2$)] = .0733 radians (cw)

$\phi_{BC}$ = TL/JG = (1,400 ft-lb)(12 in./ft)(2 ft)(12 in/ft)/[(3.1416 * (2 in)$^4$ / 32)(6x10$^6$ lb/in$^2$)] = .0428 radians (ccw)

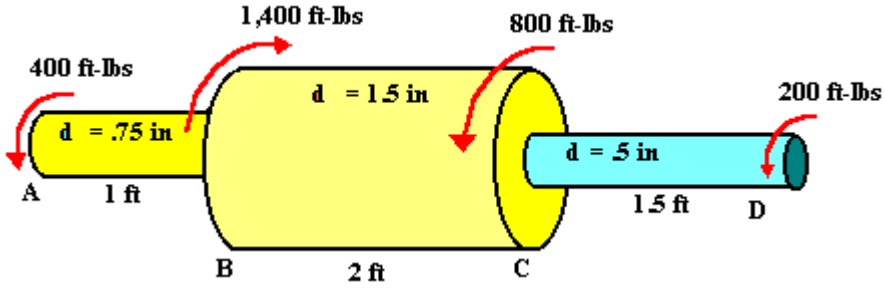
$\phi_{CD}$ = TL/JG = (400 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (.75 in)$^4$ / 32)(12x10$^6$ lb/in$^2$)] = .155 radians (ccw)

**$\phi_{Total} = -\phi_{AB} + \phi_{BC} + \phi_{CD}$ = .1245 radians (ccw)**

## Örnek

**Çeliğ in kayma modülü = 12 x 106 lb/in$^2$.**

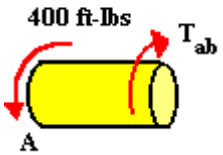
**Pirincin kayma modülü = 6 x 106 lb/in$^2$.**




**I. kesit:**

**T** = 400 ft-lb - $T_{AB}$ = 0; So $T_{AB}$ = 400 ft-lb

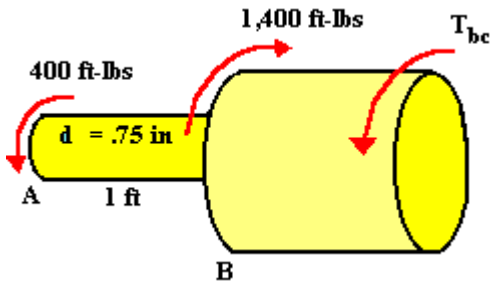
$\tau_{AB}$ = Tr / J = (400 ft-lb)(12 in/ft)(.375in) /[3.1416 * (.75 in)$^4$ / 32] = 57,900 psi




**II.kesit:**

**T**= 400 ft-lb - 1,400 ft-lb + $T_{BC}$ = 0; So $T_{BC}$ = 1,000 ft-lb

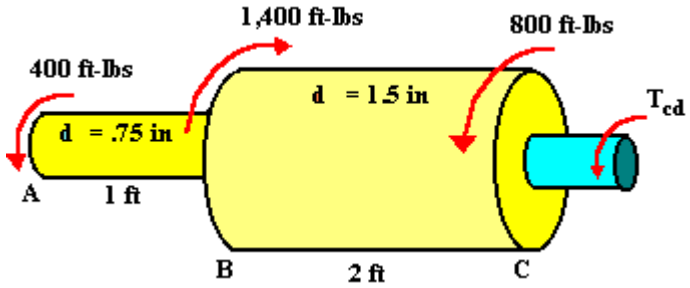
$\tau_{BC}$= Tr / J = (1,000 ft-lb)(12 in/ft)(.75 in) /[3.1416 * (1.5 in)$^4$ / 32] = 18,100 psi




III.kesit:

T = 400 ft-lb - 1,400 ft-lb + 800 ft-lb + $T_{CD}$ = 0; So $T_{CD}$ = 200 ft-lb

$\tau_{CD}$ = Tr / J = (200 ft-lb) (12 in/ft)(.25 in) /[3.1416 * (.5 in)$^4$ / 32] = 97,800 psi




$\phi_{AB}$ = TL/JG = (400 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (.75 in)$^4$ / 32)(6x10$^6$ lb/in$^2$)] = .309 radians (cw)

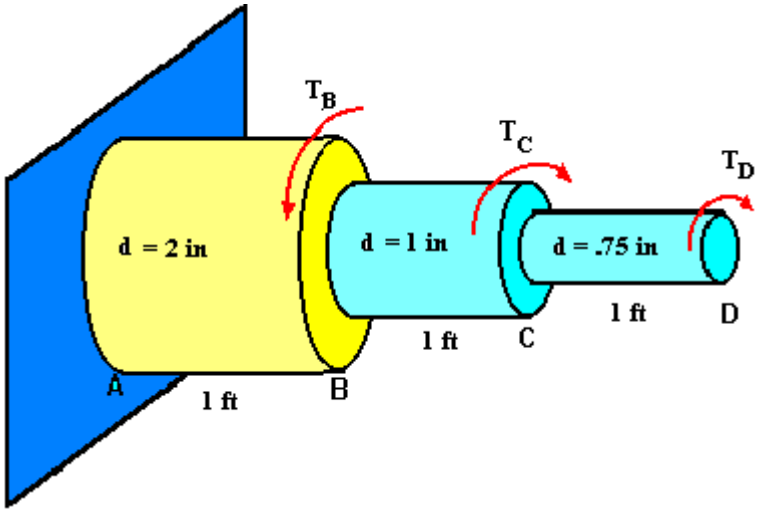
$\phi_{BC}$ = TL/JG = (1,000 ft-lb)(12 in./ft)(2 ft)(12 in/ft)/[(3.1416 * (1.5 in)$^4$ / 32)(6x10$^6$ lb/in$^2$)] = .0966 radians (ccw)

$\phi_{CD}$ = TL/JG = (200 ft-lb)(12 in./ft)(1.5 ft)(12 in/ft)/[(3.1416 * (.5 in)$^4$ / 32)(12x10$^6$ lb/in$^2$)] = .587 radians (ccw)

$\phi_{Total} = -\phi_{AB} + \phi_{BC} + \phi_{CD}$ = .375 radians (ccw)

## Örnek

$\tau_{steel}$ = 18,000 psi $\tau_{brass}$ = 12,000 psi



**Çeliğ in kayma modülü = 12 x 106 lb/in$^2$.**

**Pirincin kayma modülü = 6 x 106 lb/in$^2$.**

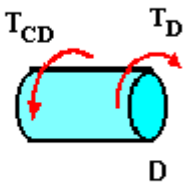
**I:**

**T** = $T_{CD} - T_D = 0$; So $T_D = T_{CD}$

$\tau_{steel} = T_{CD}\, r / J$

18,000 lb/in$^2$ = $T_{CD}$(.375in) /[3.1416 * (.75 in)$^4$ / 32]

**$T_{CD} = T_D$ = 1491 in-lb = 124 ft-lb**




**II:**

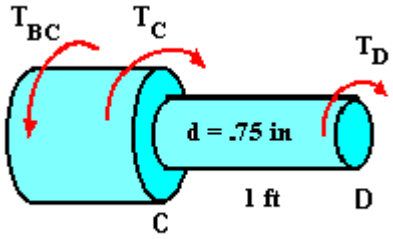
**T** = $T_{BC} - T_{CD}$ - 124 ft-lb = 0; So $T_C = (T_{BC}$ - 124 ft-lb)

$\tau_{steel} = T_{BC} r / J$

18,000 lb/in$^2$ = $T_{BC}$(.5in) /[3.1416 * (1 in)$^4$ / 32]

**$T_{BC}$ = 3534 in-lb = 295 ft-lb**

**$T_C$ = (295 ft-lb - 124 ft-lb) = 171 ft-lb**




**III:**

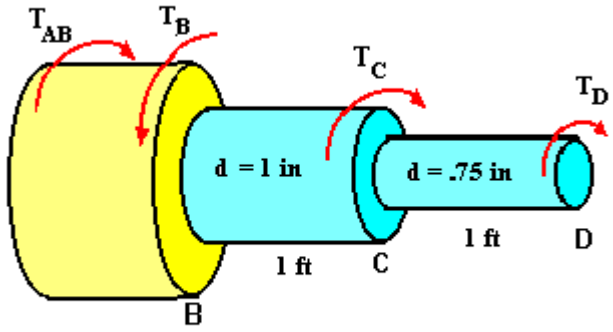
**T** = $-T_{AB} + T_B$ - 171 ft-lb - 124 ft-lb = 0; So $T_B = (T_{AB}$ + 171 ft-lb + 124 ft-lb)

$\tau_{brass} = T_{AB} r / J$

12,000 lb/in$^2$ = $T_{AB}$(1 in) /[3.1416 * (2 in)$^4$ / 32]

**$T_{AB}$ = 18,850 in-lb = 1570 ft-lb.**

**$T_B$ = (1570 ft-lb + 171 ft-lb + 124 ft-lb) = 1865 ft-lb**




$\phi_{AB}$ = TL/JG = (1,570 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (2 in)$^4$ / 32)(6x10$^6$ lb/in$^2$)] = .024 radians (ccw)

$\phi_{BC}$ = TL/JG = (245 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (1 in)$^4$ / 32)(12x10$^6$ lb/in$^2$)] = .0361 radians (cw)

$\phi_{CD}$ = TL/JG = (124 ft-lb)(12 in./ft)(1 ft)(12 in/ft)/[(3.1416 * (.75 in)$^4$ / 32)(12x10$^6$ lb/in$^2$)] = .0479 radians (cw)

**$\phi_{Total} = + \phi_{AB} - \phi_{BC} - \phi_{CD} = .06$ radians (cw)**

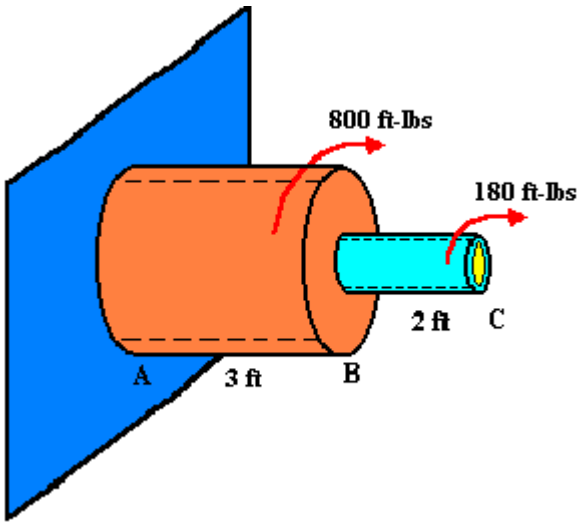
## Örnek

**Çeliğ in kayma modülü = 12 x 106 lb/in$^2$.**

**Pirincin kayma modülü = 6 x 106 lb/in$^2$.**

Dı ş çap AB = 2.5 in, iç çap AB = 2 in

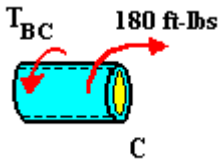
dı ş çap BC = 1 in , iççap BC = .8 in



**I:**

**T** = $T_{BC}$ - 180 ft-lb = 0; So $T_{BC}$ = 180 ft-lb

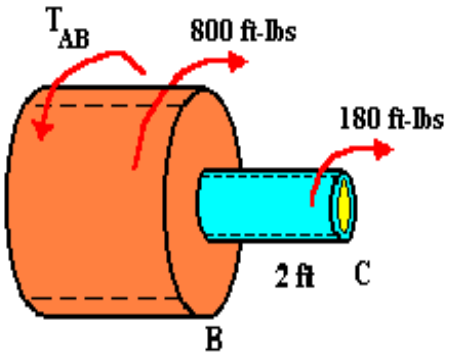
$\tau_{AB}$ = Tr / J = (180 ft-lb)(12 in/ft)(.5 in) /[3.1416 * [(1 in)$^4$ - (.8 in)$^4$] / 32] = 18,600 psi




**II:**

**T** = $T_{AB}$ - 800 ft-lb - 180 ft-lb = 0; So $T_{AB}$ = 980 ft-lb

$\tau_{AB}$ = Tr / J = (980 ft-lb)(12 in/ft)(1.25 in) /[3.1416 * [(2.5 in)$^4$ - (2 in)$^4$] / 32] = 6,490 psi




$\phi_{AB}$ = TL/JG = (980 ft-lb)(12 in./ft)(3 ft)(12 in/ft)/[(3.1416 * [(2.5 in)$^4$ - (2 in)$^4$] / 32)(6x10$^6$ lb/in$^2$)] = .0312 radians (cw)

$\phi_{BC}$ = TL/JG = (180 ft-lb)(12 in./ft)(2 ft)(12 in/ft)/[(3.1416 * [(1 in)$^4$ - (.8 in)$^4$] / 32)(6x10$^6$ lb/in$^2$)] = .0149 radians (cw)

**$\phi_{Total}$ = – $\phi_{AB}$ – $\phi_{BC}$ = –.0461 radians (cw)**

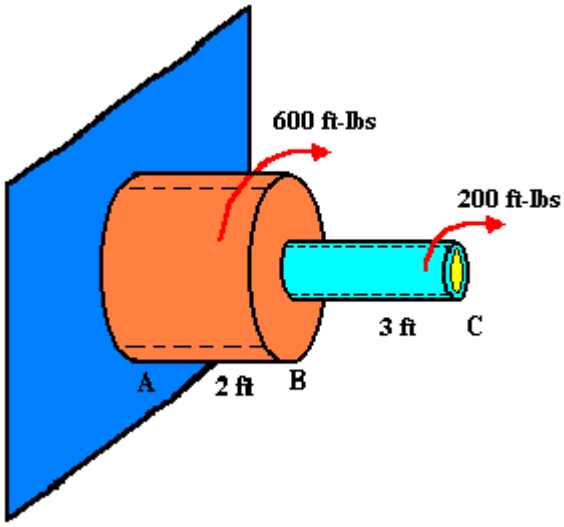
**Örnek**

**Çeliğ in kayma modülü = 12 x 106 lb/in$^2$.**

**Pirincin kayma modülü = 6 x 106 lb/in$^2$.**

Dı ş çap AB = 3.5 in, iç çap AB = 2.8 in

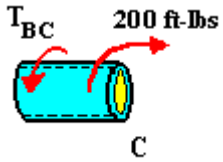
dı ş çap BC = 2 in , iççap BC = 1.6 in




**I:**

**T** = $T_{BC}$ - 200 ft-lb = 0; So $T_{BC}$ = 200 ft-lb

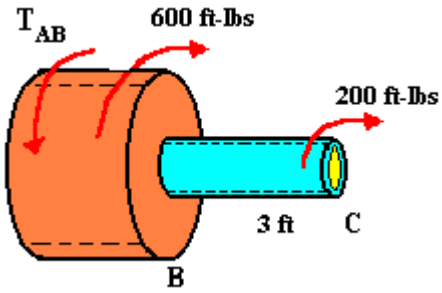
$\tau_{AB}$ = Tr / J = (200 ft-lb)(12 in/ft)(1 in) /[3.1416 * [(2 in)$^4$ - (1.6 in)$^4$] / 32] = 2,600 psi




**II:**

**T** = $T_{AB}$ - 600 ft-lb - 200 ft-lb = 0; So $T_{AB}$ = 800 ft-lb

$\tau_{AB}$ = Tr / J = (800 ft-lb)(12 in/ft)(1.75 in) /[3.1416 * [(3.5 in)$^4$ - (2.8 in)$^4$] / 32] = 1,150 psi

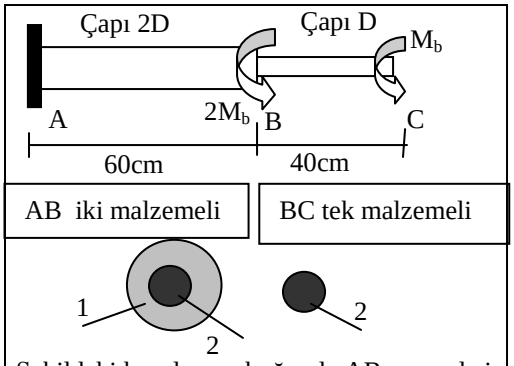



$\phi_{AB}$ = TL/JG = (800 ft-lb)(12 in./ft)(2 ft)(12 in/ft)/[(3.1416 * [(3.5 in)$^4$ - (2.8 in)$^4$] / 32)(6x10$^6$ lb/in$^2$)] = .0044 radians (cw)

$\phi_{BC}$ = TL/JG = (200 ft-lb)(12 in./ft)(3 ft)(12 in/ft)/[(3.1416 * [(2 in)$^4$ - (1.6 in)$^4$] /

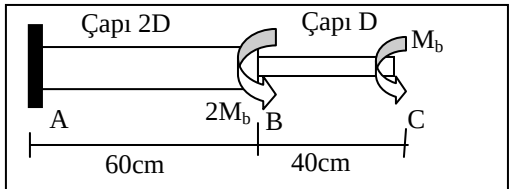
32)(6x10$^6$ lb/in$^2$)] = .0155 radians (cw)

$\phi_{Total} = -\phi_{AB} - \phi_{BC} = -.0199$ radians (cw)




Şekildeki burulma çubuğunda AB arasında iç çap D, dış çap 2D olmak üzere iki malzemeli, BC arasının çapı D ve tek malzemeli olduğuna göre emniyetle taşınabilecek $M_b$ burulma momentini bulunuz. D= 4cm, $\tau_{em}$=8kN/cm$^2$ G1=800kN/cm$^2$, R=D/2, G2=2*G1, $\varphi$=w*L

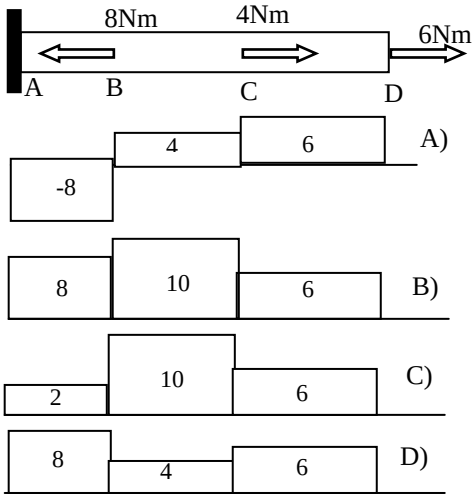
$$I_0 = \frac{\pi D^4}{32},\ \tau_{max} = \frac{M_b}{I_o}R,\ \varphi = \frac{M_b L}{GI_0}$$

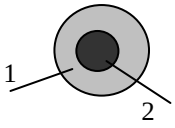



Şekildeki burulma çubuğunda AB arasının çapı 2D, BC arasının çapı D olduğuna göre emniyetle taşınabilecek $M_b$ burulma momentini bulunuz. D= 4cm, $\tau_{em}$=8kN/cm$^2$ G=800kN/cm$^2$, R=D/2

$$I_0 = \frac{\pi D^4}{32},\ \tau_{max} = \frac{M_b}{I_o}R,\ \vartheta = \frac{M_b L}{GI_0}$$

Şekildeki burulma çubuğunun moment diyagramı hangisidir.




Şekildeki iki malzemeden yapılmış olan burulma çubuğuna $M_b$ burulma momenti etkimektedir. İç çap D, dış çap 2D olmak üzere bu iki malzemenin taşıyabilecekleri momentler arasındaki ilişki nedir. $G_1 = G_2$

$$I_0 = \frac{\pi D^4}{32},\ \tau_{max} = \frac{M_b}{I_o}R,\ \varphi = \frac{M_b L}{GI_0}$$

A) $M_1=M_2$, B) $M_1=I_1*M_2/I_2$ C) $M_1=2M_2$ D) $M_1= M_2/2$