MÜHENDİSLİK MEKANİĞİ

8. HAFTA **Doğrusal Hareket**

Birimler:

		SI		U.S.	
Mass	M	kilogram	kg	Slug	
Length	L	meter	m	Feet	ft
Force	F	Newton	N	Pound	lb
Time	T	second	S	second	sec

F=ma:

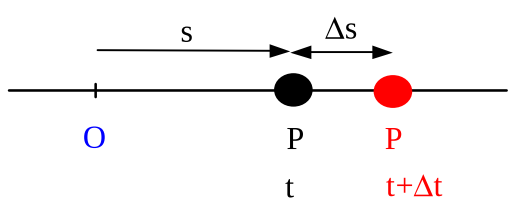
1 N = (1 kg) (1 m/s^2)
1 lb= (1 slug) (1 ft/sec^2)

Örneğin W=10 lb ve g=32.2 ft/sec^2

$$m = \frac{W}{g} = \frac{10}{32.2} \text{slugs}$$

Birim dönüşümleri:

L lb	=	4.4482 N
1 slug	=	14.5938 kg
1 ft	=	0.3048 m
1 ft	=	12 in
1 mile	=	5,280 ft
1 kip	=	1,000 lb
1 ton	=	2,000 lb



Zaman: ***t***
Pozisyon: **s**
Hız: ***v***

$$v \equiv \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \dot{s}$$

İvme: ***a***

$$a \equiv \frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \dot{v} = \ddot{s}$$

Lemma:

$$a \equiv \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} \quad \longrightarrow \quad \boxed{a = v\frac{dv}{ds} \text{or } \ddot{s} = \dot{s}\frac{d\dot{s}}{ds}}$$

İntegrasyon metodu:

1-Ivme zamanın fonksiyonu: *a=f(t)*

$$f(t) = \frac{dv}{dt} \xrightarrow{\text{First separate variables}} f(t)dt = dv \xrightarrow{\text{Then integrate}} v = \int f(t)dt + C = g(t)$$

$$g(t) = \frac{ds}{dt} \xrightarrow{SV} g(t)dt = ds \xrightarrow{I} s = \int g(t)dt + D = h(t)$$

C ve D integrasyon sabitleridir.

Örnek 1

$a = t^2$, verilmiş hız ve konum fonksiyonlarını hesaplayın

Çözüm

$$a = \frac{dv}{dt} = t^2 \xrightarrow{\text{seperate the variables}} dv = t^2 dt$$

Integrasyon alınırsa

$$v = \frac{t^3}{3} + C$$

Elde edilir

$$v = \frac{ds}{dt} \xrightarrow{\text{seperate the variables}} ds = vdt$$

integrasyonla

$$s = \int vdt + D$$

$$= \int \left(\frac{t^3}{3} + c \right) dt + D = \frac{t^4}{12} + Ct + D$$

Konum fonksiyonu elde edilir.

2-İvme hızın fonksiyonu: *a=f(v)*

$$f(v) = \frac{dv}{dt} \xrightarrow{SV} dt = \frac{dv}{f(v)} \xrightarrow{I} t = \int \frac{1}{f(v)} dv + C = g(v)$$

v=k(t), alınırsa

$$k(t) = \frac{ds}{dt} \xrightarrow{SV} k(t)dt = ds \xrightarrow{I} s = \int k(t)dt + D = h(t)$$

Lemma kullanılarak konum elde ediliyor

$$f(v) = v\frac{dv}{ds} \xrightarrow{SV} ds = \frac{v}{f(v)}dv \xrightarrow{I} s = \int \frac{v}{f(v)}dv + C = \tilde{g}(v)$$

Örnek 2

$$a = f(v) = -5v^2,$$

Konum fonksiyonunu hesaplayın.

Çözüm:

$$a = \frac{vdv}{ds} = -5v^2$$

$$ds = \frac{vdv}{-5v^2}$$

İntegrasyon ile elde edilir.

$$s = \int \frac{1}{-5v}dv + c = -\frac{1}{5}\ln v + c$$

3-Ivme konumun fonksiyonu: *a=f(s)*

$$f(s) = v\frac{dv}{ds} \xrightarrow{SV} f(s)ds = vdv \xrightarrow{I} \frac{v^2}{2} = \int f(s)ds + C \rightarrow v = g(s)$$

$$g(s) = \frac{ds}{dt} \rightarrow dt = \frac{ds}{g(s)} \rightarrow t = \int \frac{1}{g(s)}ds + D \rightarrow t = h(s)$$

Örnek 3:

$$a = f(s) = -3s,$$

Zamanı konum fonksiyonu cinsinden elde edin.

Çözüm:

$$a = v\frac{dv}{ds} = -3s \xrightarrow{sv} vdv = -3sds$$

Integrasyonla

$$\frac{v^2}{2} = -\frac{3}{2}s^2 + C$$

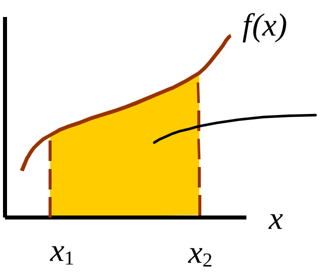
Bağıntısı bulunur

$$v = g(s) = \sqrt{-3s^2 + C_1}, C_1 = 2C$$

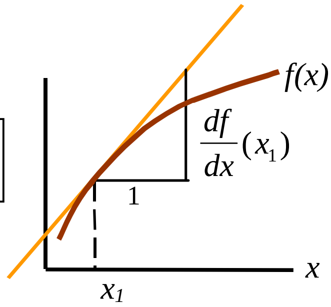
$$v = \frac{ds}{dt} \rightarrow dt = \frac{ds}{v} = \frac{ds}{\sqrt{-3s^2 + C_1}}$$

$$t = \int \frac{1}{\sqrt{-3s^2 + C_1}}ds + D$$

MATEMATİK HATIRLATMALAR



$$Area = \int_{x_1}^{x_2} f(x)dx$$



Bazı türevler:

$$\frac{d}{dx}(c) = 0, \quad \frac{d}{dx}(x) = 1, \quad \frac{d}{dx}(x^2) = 2x, \quad \frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, \quad \frac{d}{dx}(e^x) = e^x, \quad \frac{d}{dx}(\sin(x)) = \cos(x), \quad \frac{d}{dx}(\cos(x)) = -\sin(x),$$

Türevin değişim kuralı:

$$\frac{df[U(x)]}{dx} = \frac{df(U)}{dU}\frac{dU}{dx}$$

Bazı integraller:

$$\int dx = x, \quad \int x dx = \frac{x^2}{2}, \quad \int x^n dx = \frac{x^{n+1}}{n+1}, \quad \int \frac{1}{x} dx = \ln(x),$$
$$\int e^x dx = e^x, \quad \int \sin(x) dx = -\cos(x), \quad \int \cos(x) dx = \sin(x)$$

Değişim kuralı

U(x), fonksiyonu verilmiş ise

$$dU = \frac{dU}{dx} dx$$

x ten *U'* ya dönüşüm formu

$$\int_{x_1}^{x_2} f(U) dx = \int_{U(x_1)}^{U(x_2)} \frac{f(U)}{\frac{dU}{dx}} dU$$

Şeklindedir

Kısmi integrasyon:

$$\int U dV = UV - \int V dU$$

Örnek 4:

$$\mathrm{U(x)} = 2\pi \mathrm{x}$$

veriliyor

$$f(U) = \sin(U)$$

$$\frac{df[U(x)]}{dx}$$

Bağıntısını hesaplayın

Çözüm:

Değişim kuralında

$$\frac{\mathrm{df(U)}}{\mathrm{dU}} = \cos(\mathrm{U}) = \cos(2\pi \mathrm{x})$$
,

$$\frac{dU}{dx} = 2\pi$$,

Sonuç aşağıdaki gibi hesaplanır

$$\frac{df[U(x)]}{dx} = \frac{df(U)}{dU}\frac{dU}{dx} = 2\pi\cos(2\pi x)$$

Örnek 5:

$$f(x) = \sin(2\pi x)$$

veriliyor,

$$\int_{x_1}^{x_2} f(x)dx$$.

Bağıntısını hesaplayın

Çözüm:

$$U = 2\pi x, \text{ so } f(U) = \sin(U)$$

ve

$$\frac{dU}{dx} = 2\pi$$

$$\int_{x_1}^{x_2} f[U(x)]dx$$

sonuç

$$= \int_{x_1}^{x_2} \sin(2\pi x)dx = \int_{2\pi x_1}^{2\pi x_2} \frac{\sin(U)}{2\pi}dU$$

$$= \frac{-\cos(U)}{2\pi}\Bigg|_{2\pi x_1}^{2\pi x_2} = \frac{-1}{2\pi}(\cos(2\pi x_2) - \cos(2\pi x_1))$$

Örnek 6:

$f(x) = x\cos x$,

veriliyor

$\int f(x)dx$.

Integralini hesaplayın

Çözüm:

$U = x, V = \sin x$,

$$f(x) = U(x)\frac{dV(x)}{dx}$$

$dU = dx, dV = \cos x dx$,

$$\int UdV = UV - \int VdU$$

$$\int x\cos x dx = x\sin x - \int \sin x dx = x\sin x + \cos x$$

ÖRNEKLER

1-Yol- Zaman bağıntısı $x = -\frac{1}{3}t^3 + 2t^2 + 2$ **şeklinde verilen maddesel noktanın hız-zaman ve ivme –zaman ifadelerini bulunuz. t=5sn de konumu, hızı, ivmeyi ve gidilen toplam yolu hesaplayınız.**

2- Bir maddesel noktanın hareketi $x = 2t^2 - 4t$ **ve** $y = 2(t-1)^2 - 4(t-1)$

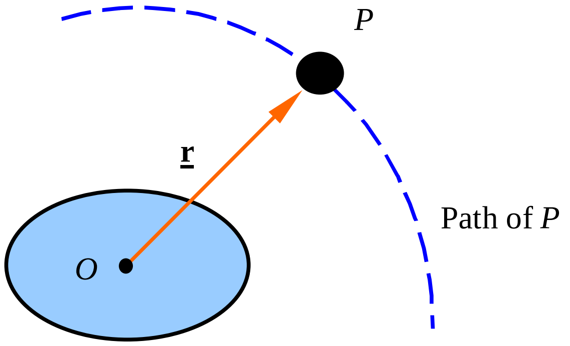
Denklemleriyle veriliyor x ve y m cinsinden t saniye cinsinden
t=1 sn ve t=3 sn de hız ve ivmeyi bulunuz.

3- Doğrusal hareket yapan bir maddesel noktanın ivmesi a=1.5k^2x^2 **olarak verilmiştir.t=0 da x=1, v=k olduğuna göre yol-zaman ve hız zaman ifadelerini bulunuz**

4- Doğrusal hareket yapan bir maddesel noktanın ivmesi a=6-3t olarak verilmiştir. t=0 da v=3m/sn x=0 olduğuna göre; hız-zaman, konum zaman ifadelerini çıkarınız t=4sn için maddesel noktanın hızını ve gittiği toplam yolu bulunuz.

Düzlemsel Hareket

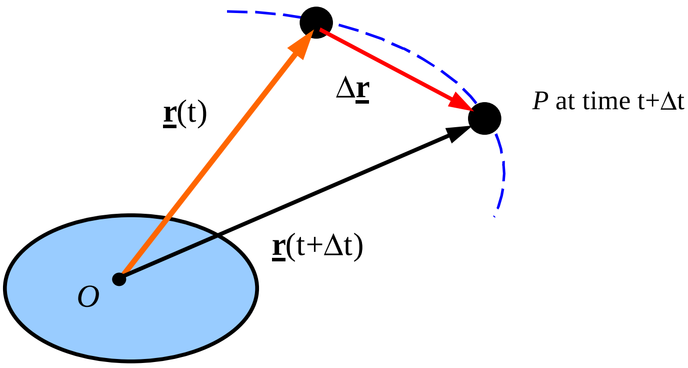
Position vector: $\underline{\mathbf{r}}$



Reference Frame

Velocity: $\underline{\mathbf{v}}$

$$\underline{\mathbf{v}} \equiv \frac{d\underline{\mathbf{r}}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \underline{\mathbf{r}}}{\Delta t} = \dot{\underline{\mathbf{r}}}$$

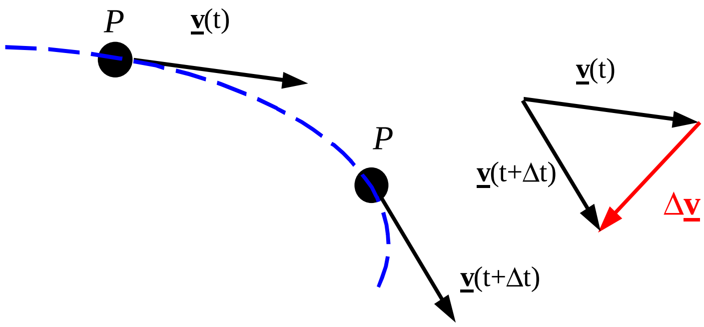


Not: Hız parçacığın yörüngesine teğettir.

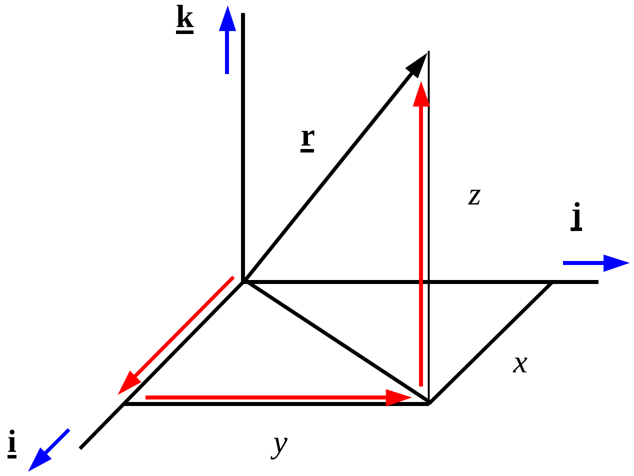
Hız: $v = |\underline{\mathbf{v}}|$

İvme: $\underline{\mathbf{a}}$

$$\underline{\mathbf{a}} \equiv \frac{d\underline{\mathbf{v}}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \underline{\mathbf{v}}}{\Delta t} = \dot{\underline{\mathbf{v}}} = \ddot{\underline{\mathbf{r}}}$$



KARTEZYEN KOORDİNATLAR



Konum: $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$

Hız: $\underline{v} = v_x\underline{i} + v_y\underline{j} + v_z\underline{k} = \dot{x}\underline{i} + \dot{y}\underline{j} + \dot{z}\underline{k}$

İvme: $\underline{a} = a_x\underline{i} + a_y\underline{j} + a_z\underline{k} = \dot{v}_x\underline{i} + \dot{v}_y\underline{j} + \dot{v}_z\underline{k} = \ddot{x}\underline{i} + \ddot{y}\underline{j} + \ddot{z}\underline{k}$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Örnek 1: Karteziyen koordinatlar

$\underline{\mathbf{r}} = t^2\underline{\mathbf{i}} + 2t^3\underline{\mathbf{j}} + \underline{\mathbf{k}}$, konum bağıntızı verilmiş hız ve ivme vektörlerini ve şiddetlerini hesaplayın

Çözüm:

$$\underline{\mathbf{v}} \equiv \frac{d\underline{\mathbf{r}}}{dt} = 2t\underline{\mathbf{i}} + 6t^2\underline{\mathbf{j}}$$

$$\underline{\mathbf{a}} = 2\underline{\mathbf{i}} + 12t\underline{\mathbf{j}}$$

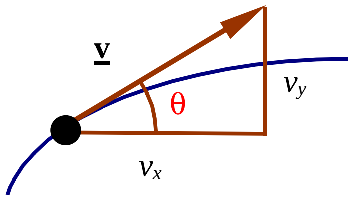
$$r \equiv \sqrt{x^2 + y^2 + z^2} = \sqrt{t^4 + 4t^6 + 1}$$

$$v = \sqrt{4t^2 + 36t^4} = 2t\sqrt{1 + 9t^2}$$

$$a = \sqrt{4 + 144t^2} = 2\sqrt{1 + 36t^2}$$

İki boyutlu düzlemde: Teğet hızın iki bileşeni vardır.

$$\tan(\theta) = \frac{v_y}{v_x}$$



Örnek 2: Karteziyen koordinatlar

$$\underline{\mathbf{a}} = x\underline{\mathbf{i}} + v_y\underline{\mathbf{j}} + 5t\underline{\mathbf{k}}.$$

Başlangıç koşulları t=0

$x = 0,\ y = 0,\ z = 0, v_x = 1, v_y = 1, v_z = 0$. Olarak verilmiş hız ve pozisyonu hesaplayın

Çözüm:

(i) *x*-yönü

$$a_x \equiv \frac{dv_x}{dt} = v_x \frac{dv_x}{dx} = x$$

First separate variables :

$$v_x dv_x = xdx$$

Then integrate :

$$\int v_x dv_x = \int xdx + C$$

$$\frac{v_x^2}{2} = \frac{x^2}{2} + C$$

Using the initial conditions :

at t = 0, x = 0 and $v_x = 1$, we can get

C = 1/2, so $v_x = \frac{dx}{dt} = \sqrt{x^2 + 1}$

by seperating variable and integrating, we get

$$t = \int \frac{dx}{\sqrt{x^2 + 1}} + D = \text{arcsinh}(x) + D$$

Using initial condition :

at t = 0, x = 0, we can get D = 0, so

t = arcsinh(x)

(ii) y-yönü

$$a_y \equiv \frac{dv_y}{dt} = v_y \text{, seperating the variable}$$

$$\int \frac{dv_y}{v_y} = \int dt + C$$

$$\ln v_y = t + C \text{, using initial condition :}$$

$$\text{at t = 0, } v_y = 1 \text{, we can get C = 0}$$

$$v_y = e^t = \frac{dy}{dt}$$

$$y = \int e^t dt + D = e^t + D$$

$$\text{using initial condition :}$$

$$\text{at t = 0, y = 0, we can get D = -1, so}$$

$$y = e^t - 1$$

(iii) *z*-yönü

$$a_z = 5t$$

$$v_z = \int 5t dt + C = \frac{5}{2}t^2 + C$$

$$\text{using initial condition :}$$

$$\text{at t = 0, } v_z = 0 \text{, we can get C = 0, so}$$

$$v_z = \frac{5}{2}t^2$$

$$z \equiv \int v_z dt = \int \frac{5}{2}t^2 dt + D = \frac{5}{6}t^3 + D$$

$$\text{using initial condition :}$$

$$\text{at t = 0, z = 0, we can get D = 0, so}$$

$$z = \frac{5}{6}t^3$$

Örnek 3: Karteziyen koordinatlar

$\underline{\mathbf{v}} = 3t\underline{\mathbf{i}} + 5\underline{\mathbf{j}}$,

yörüngenin θ açısını bulun.

Çözüm:

$$\theta = \arctan(\frac{v_y}{v_x}) = \arctan(\frac{3t}{5})$$

Motion with constant acceleration

$a_y = \text{constant} \equiv \frac{dv_y}{dt}$

$v_y = \int a_y \, dt$ *subscript y for y direction*

$= a_y t + \text{const}$

$v_y = v_{y0} + a_y t$ (i) *subscript 0 for t = 0*

$y = \int v_y \, dt$

$= \int v_{y0} + a_y t$

$= v_{y0}t + \frac{1}{2} a_y t^2 + \text{constant}$

$y = y_0 + v_{y0}t + \frac{1}{2} a_y t^2.$ (ii)

Eliminate t from (i) and (ii)

$y - y_0 = v_{y0}t + \frac{1}{2} a_y t^2 = ... \Rightarrow$

$2a_y(y - y_0) = v_y^2 - v_{y0}^2$ (iii)

Example. Joe runs at constant speed 6 ms^{-1} towards a stationary bus. When he is 30 m from it, the bus accelerates away at 2 $m.s^{-2}$. Can he overtake it?

Given: $v_J = 6\ ms^{-1}$, $v_{b0} = 0$, $a_b = 2\ m.s^{-2}$, $a_J = 0$,

$x_{b0} - x_{J0} = 30$ m

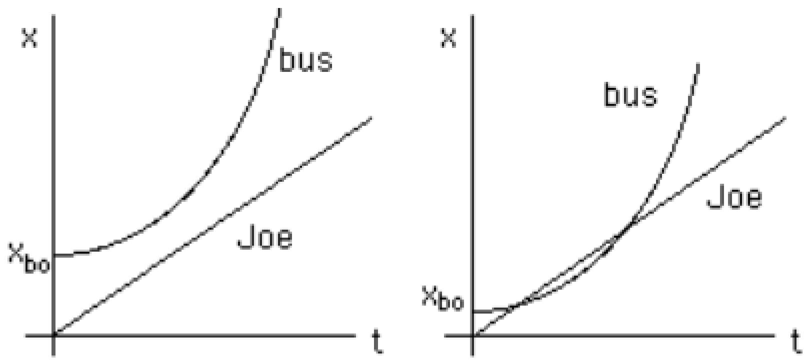
Translate the question: Does $x_J = x_b$ at any t?

$$x_b = x_{b0} + v_{b0}t + \frac{1}{2}a_bt^2 \qquad \text{(i)}$$

$$x_J = x_{J0} + v_{J0}t + \frac{1}{2}a_Jt^2 \qquad \text{(ii)}$$

Eliminate x_{Jo} by choice of origin, x_{bo} = 30 m

Draw a diagram or two



$x_b = x_J$ substitute (i) and (ii)

$$\frac{1}{2}a_bt^2 - v_{Jo}t + x_{bo} = 0$$

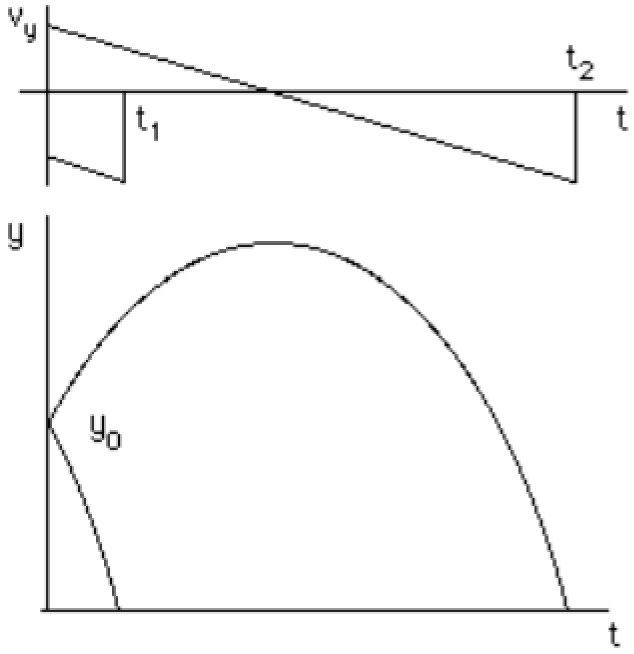
$$t = \frac{v_{Jo} \pm \sqrt{v^2_{Jo} - 2a_bx_{bo}}}{a_b}$$ *are the units correct?*

Put in numbers

$\sqrt{-ve}$ $\therefore$ no solutions, $\therefore$ no overtaking.

Example. Ball 1 thrown vertically up at 5 ms^{-1} from 20 m above ground. Simultaneously, ball 2 thrown vertically down at 5 ms^{-1} from 20 m above ground. What are their speeds when they hit the ground, and the interval between collisions?

$v_{o1} = 5\ ms^{-1}$ $\quad v_{o2} = -5\ ms^{-1}$ $\quad y_o = 20$ m



When y = 0, t = ?

Use (ii) to get t_1 and t_2. Use (i) to get v_{y1} and v_{y2}.

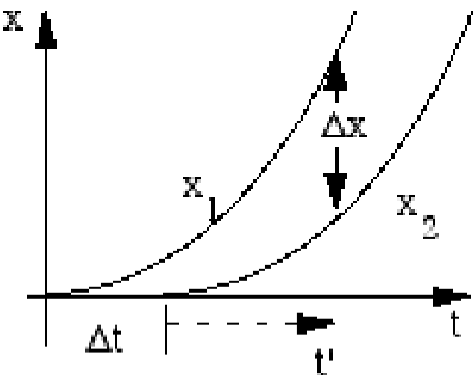
How much do you lose if you miss the gun?

Two runners at different times (Δt apart). During the (constant) acceleration phase, when are they Δx apart ?

$$x_1 = \frac{1}{2}at^2$$

$$x_2 = \frac{1}{2}a(t')^2$$

$$t' = t - \Delta t$$



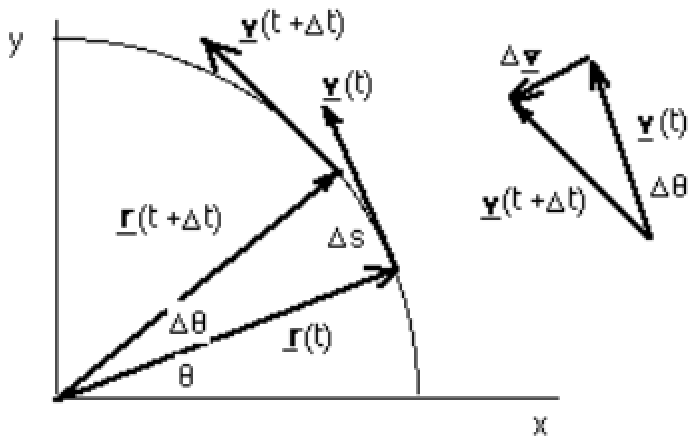
$$x_1 - x_2 = \frac{1}{2}at^2 - \frac{1}{2}a(t-\Delta t)^2$$

{solve for t}

Uniform circular motion

Write $\theta = \omega t$. where ω = const

ω is the **angular velocity**



As Δt and $\Delta\theta \to 0$, $\Delta\underline{\mathbf{v}} \to$ right angles to $\underline{\mathbf{v}}$

$$\therefore\ \underline{\mathbf{a}} \equiv \lim_{\Delta t \to 0}\left(\frac{\Delta \underline{\mathbf{v}}}{\Delta t}\right) \qquad \left(/\!/\ -\underline{\mathbf{r}} \quad \text{i.e. centripital acceleration}\right)$$

As $\Delta\theta \to 0$, $\Delta s \to r\Delta\theta$ (*arc becomes straight line*)

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$

Again, use arc ≅ str line of triangle, here for $\Delta \underline{\mathbf{v}}$:

$$|\Delta \underline{\mathbf{v}}| \cong |v\Delta\theta| \qquad \text{(n.b.: } |\Delta \underline{\mathbf{v}}| \neq \Delta|\underline{\mathbf{v}}| \text{)}$$

$$\lim_{\Delta t \to 0} \quad |d\underline{\mathbf{v}}| = |v d\theta|$$

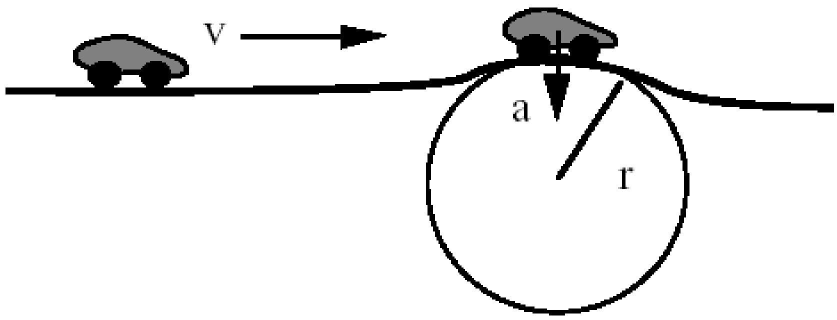
$$|\underline{\mathbf{a}}| \equiv \frac{|d\underline{\mathbf{v}}|}{dt}$$

$$= v\frac{d\theta}{dt} = v\omega$$

$$a = \frac{v^2}{r} = \omega^2 r \qquad \text{but } \underline{\mathbf{a}} \; // \; -\underline{\mathbf{r}}$$

so $\underline{\mathbf{a}} = -\omega^2 \underline{\mathbf{r}}$

Example. Car travelling a v goes over hill with vertical radius r = 30 m (>> height of car) at summit. Assume it doesn't slow down. How high must v be for the car to lose contact with the ground at the summit?

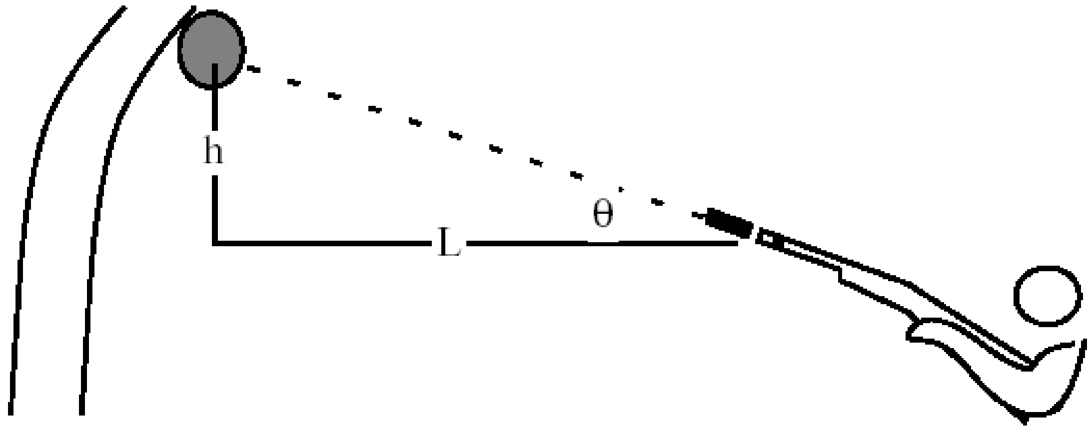


If gravity is only downwards force, then if $a_{centrip} > g$, g is not enough acceleration for circular motion.

$$a = \frac{v^2}{r} > g$$

$$v > \sqrt{rg} = 17 \text{ m.s}^{-1} = 62 \text{ kph}$$

Question. A man shoots at a coconut. At the instant that he fires, the coconut falls. What happens



obvious method (c for coconut, b for bullet)

$$h = L \tan\theta$$

$$y_c = h - \frac{1}{2}gt^2 \qquad\qquad y_b = v_{yo}t - \frac{1}{2}gt^2$$

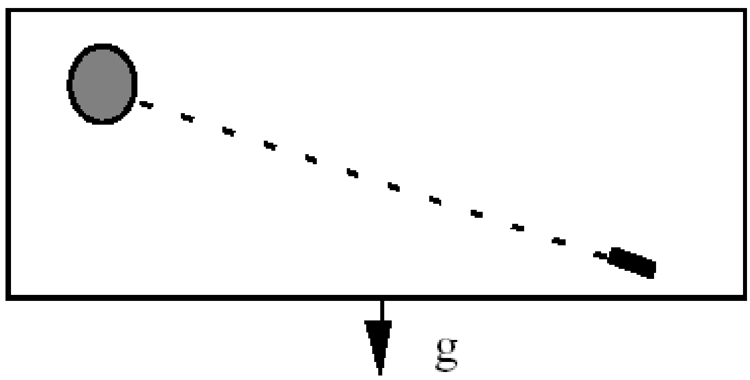
$$y_c - y_b = h - v_{yo}t = h - v_o \sin\theta . t$$

$$L = t\, v_o \cos\theta \quad \rightarrow \quad T = \frac{L}{v_o \cos\theta}$$

$$y_c - y_b = h - \frac{v_o \sin\theta . L}{v_o \cos\theta} = h - L\tan\theta$$

$$= 0$$

Alternatively: consider a frame of reference falling with g.



Projectiles

Without air, $a_y = -g = \text{constant}$. $a_x = 0$

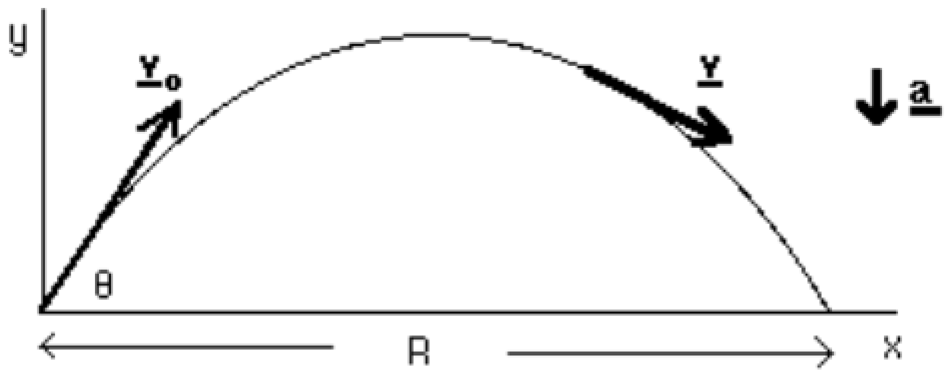
(Galileo: independence of horiz. & vert motion)

(ii) $\rightarrow$ $y = y_o + v_{yo}t - \frac{1}{2}gt^2$.

(ii) $\rightarrow$ $x = x_o + v_x t$.

Choose axes so that $x_o = y_o = 0$ and eliminate t:

$$y = v_{yo}\left(\frac{x}{v_x}\right) - \frac{1}{2}g\left(\frac{x}{v_x}\right)^2 \qquad (*)$$



y = 0 when x = 0 or R

$$(*) \Rightarrow \qquad v_{yo}\left(\frac{R}{v_x}\right) = \frac{1}{2}g\left(\frac{R}{v_x}\right)^2 \qquad (**)$$

$v_{yo} = v_o \sin\theta$, $v_x = v_o \cos\theta$ $\rightarrow$ $R = R(v_o, \theta)$

Set $\frac{\partial R}{\partial \theta} = 0$ to obtain θ for maximum range.

$$v_{yo}\left(\frac{R}{v_x}\right) = \frac{1}{2}g\left(\frac{R}{v_x}\right)^2 \qquad (**)$$

solve (**) for R:

$$R = \frac{2v_x v_{yo}}{g} \qquad (\textit{check units})$$

$$= \frac{2v_0 \sin\theta . v_0 \cos\theta}{g}$$

$$= \frac{v_0^2 \sin 2\theta}{g}$$

$$\frac{\partial R}{\partial \theta} = \frac{2v_0^2 \cos 2\theta}{g}$$

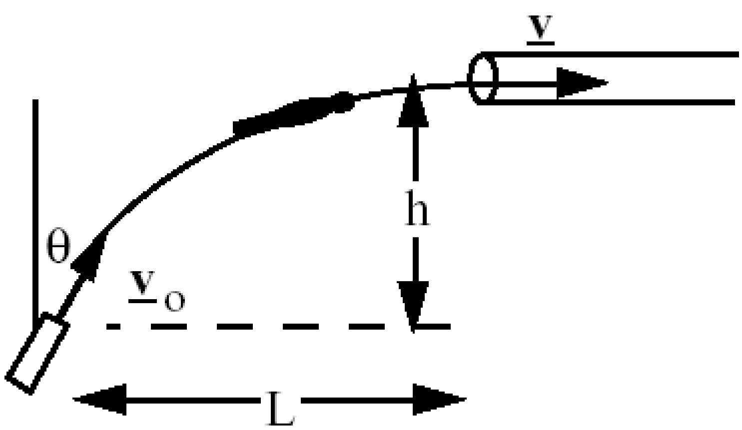
$$\frac{\partial R}{\partial \theta} = 0 \text{ when } 2\theta = 90°$$

Example The human cannon of Circus Oz has a muzzle velocity v_o. For their next trick, they will fire the human canonball into a horizontal teflon tube at height h above the canon mouth. To avoid damage to the canonball, he must arrive with purely horizontal velocity. Calculate the postion of the canon and its angle to the vertical.

i) *draw a diagram*

ii) *put in symbols for quantities*

iii) *translate question*



given h, v_o and final $v_y = 0$

Find L and θ

Relate h, v_y and v_{yo}. v_{yo} depends on θ.

During flight, the acceleration is –g upwards. The desired v_y is zero, so

$$0 = v_y^2 = v_{yo}^2 + 2a_y(\Delta y) = v_o^2 \cos^2\theta - 2gh$$

$$\therefore \quad v_o^2 \cos^2\theta = 2gh$$

$$\therefore \quad \cos\theta = \frac{\sqrt{2gh}}{v_o}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2gh}}{v_o}\right)$$

Find time of flight. $v_x t = L$.

$$0 = v_y = v_{yo} + a_y t$$

$$\therefore \quad t = \frac{v_o \cos\theta}{g}$$

$$L = v_o\, t \sin\theta.$$

$$= v_o \sin\theta \,\frac{v_o \cos\theta}{g}$$

$$\left(\text{<optional>} = \frac{v_o^2}{2g} \sin 2\theta \right)$$

$$\text{where } \theta = \cos^{-1}\left(\frac{\sqrt{2gh}}{v_o}\right)$$ *simplify optional*