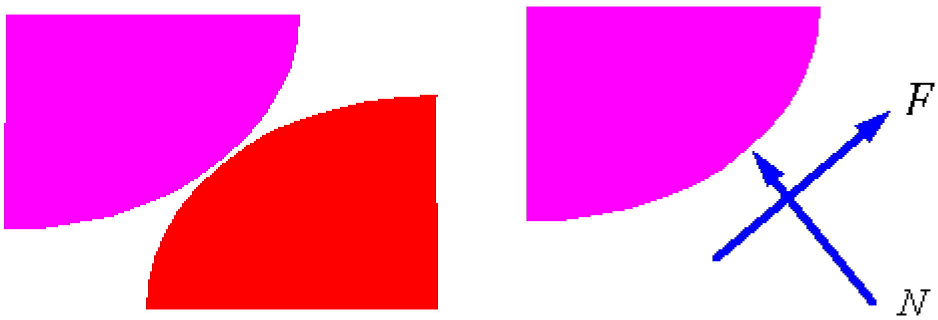
6. Hafta Sürtünme

Kuru Sürtünme

Sürtünme Kuvveti F: İki yüzey arasındaki oluşan kuvvettir.

Statik Sürtünme:N normal kuvvet μs statik sürtünme katsayısı

$$|F| \leq \mu_s N$$



Kinetik sürtünme: Eğer bir cisim diğer bir cisim üzerinde hareket ediyorsa (Kayıyorsa) bu durumda kinetik sürtünme oluşur. Kinetik sürtünme katsayısı

μκ

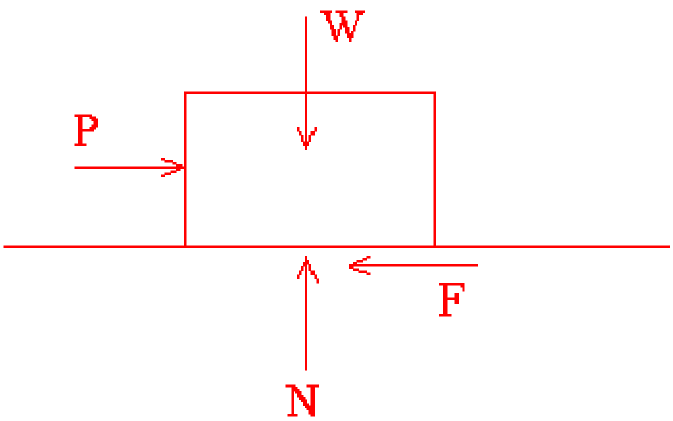
dır.

$$F = \mu_k N$$

Kinetik sürtünme katsaısı, statik sürtünme karsayısından daha küçüktür.

Up to this point we have considered most surfaces frictionless. In the real world there is no such thing as a frictionless surface. For some analysis it is OK to assume a frictionless surface since the forces caused by friction are so small. In this chapter, we will look into the forces of friction.

Block on a surface



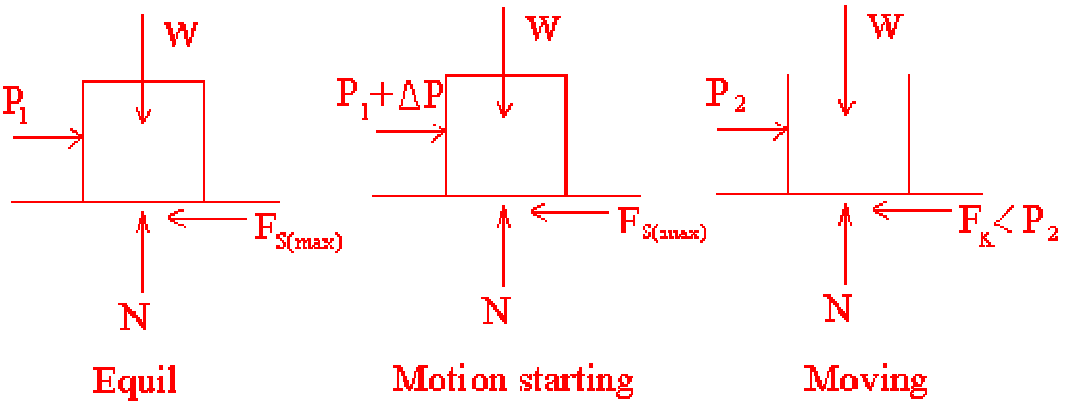
W = Weight
N = Normal Reaction Force
P = A Load
F = Friction Force

If the block is not moving, it is in equil.

So $\sum F_x = 0 \Rightarrow P = F$

As P increases, F increases to keep the body from moving. At some point slight increase in P will cause the block to move. This is the point where the friction force has been exceeded by P. It is known as the MAX static friction force. Once the body starts to move, the force P required to keep the block moving is reduced.

The friction force resisting the reduced P is known as the kinetic friction force.



Note: $P_1 > P_2$ & $F_{S\,(max)} > F_K$

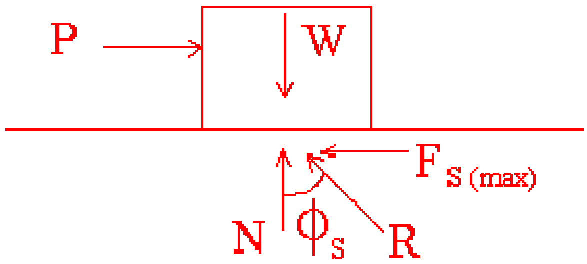
The friction forces are proportional to the normal force.

$$F_{S\,(max)} = \mu_S N \qquad F_K = \mu_K N$$

μ_S = Coeff. of static friction } Depend on surfaces in contact.
μ_K = Coeff. of kinetic friction }

Sürtünme Açısı

· The normal force and the friction force can be combined to form 1 resultant force.



When P is just about to cause motion, the friction force is MAX, $F_{S(max)} = U_S N$

$$TAN\ \phi_S = \frac{F_{S(max)}}{N} = U_S$$

ϕ_S = Angle of static friction

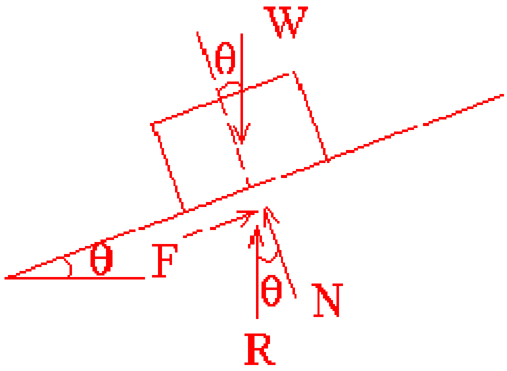
When P is causing motion ,

the friction is $F_K = U_K N$

$$TAN\ \phi_K = U_K$$

ϕ_K = Angle of kinetic friction

Eğik Düzlem



For equil, $R = W$

Forces $\perp$ to surface $W \cos\theta = N$

Forces along surface $W \sin\theta = F$

At inpending motion

$$F_{S\,(max)} = U_S N = U_S W \cos\theta = W \sin\theta$$

So $U_S = \text{TAN}\,\theta = \text{TAN}\,\phi_S$

$\theta = \phi_S$ = Angle of repose

Types of problems involving friction

(1) Forces applied to body & coeff. of friction known $\Rightarrow$ Find if body will remain at rest.

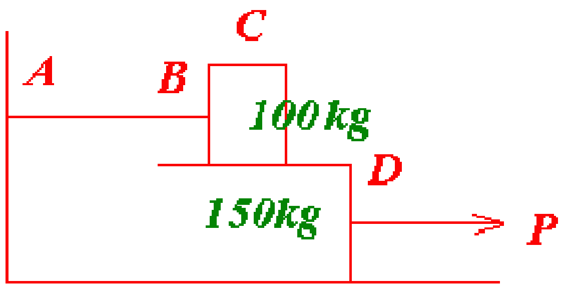
(2) Forces applied are known & motion impending $\rightarrow$ Find coeff. of friction.

(3) Motion impending & coeff. of friction known $\Rightarrow$ Solve for forces.

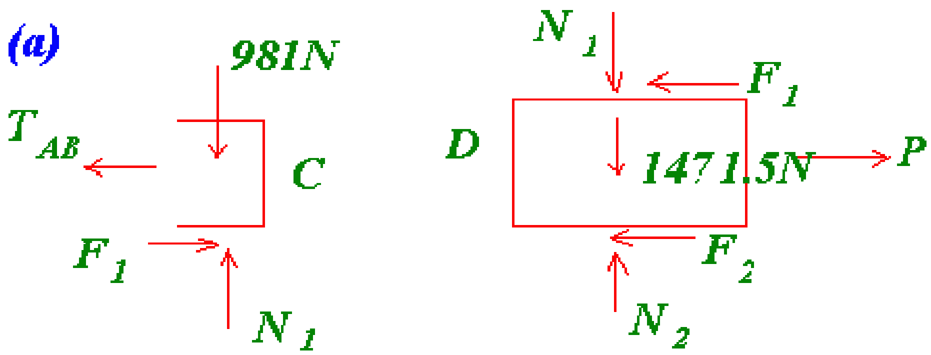
Note: Friction forces between two bodies are equal & opposite.

Örnek

8.12 The coefficients of friction are $\mu_S = 0.30$ and $\mu_K = 0.25$ between all sutfaces of contact. Determine the smallest force P required to start block D moving if
(a) block C is restrained by cable AB as shown,
(b) cable AB is removed.



(a)



Block C:

$$\sum F_Y = 0 \Rightarrow N_1 = 981\ N$$

$$F_1 = \mu_S N_1 = 0.3\ (981) = 294.3\ N$$

Block D:

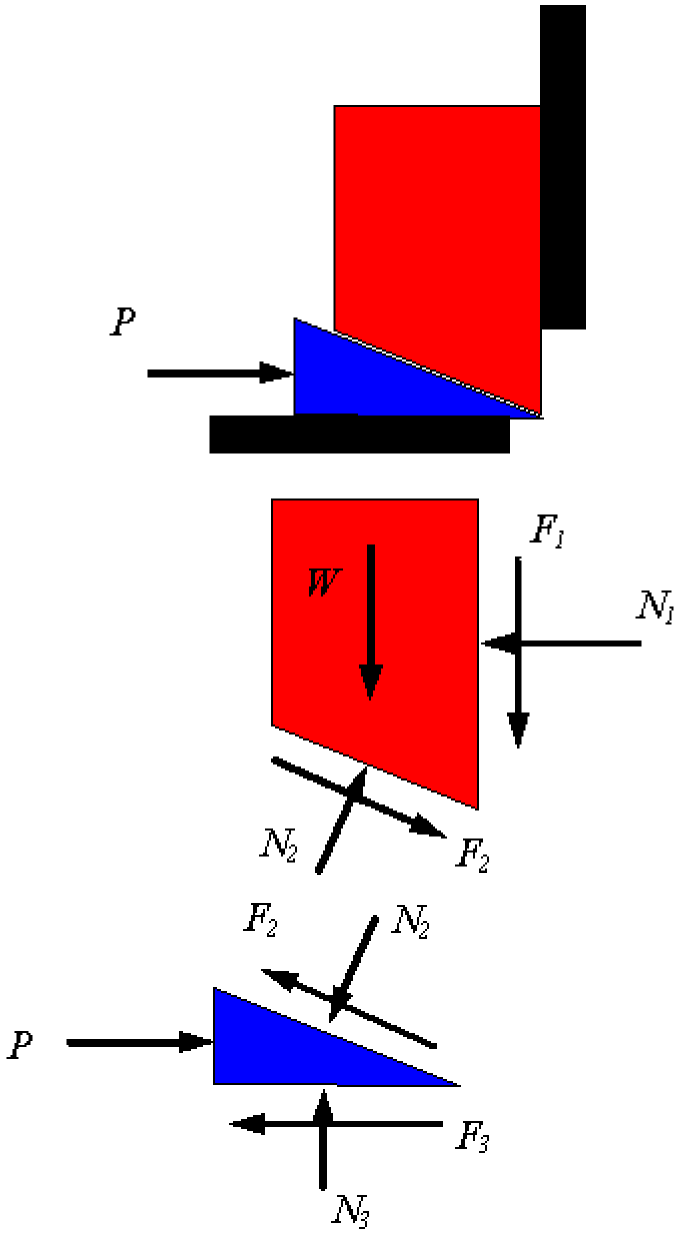
$$\sum F_Y = 0 \Rightarrow N_2 = 1471.5 + 981 = 2452.5\ N$$

$$F_2 = \mu_S N_2 = 0.3\ (2452.5) = 735.8\ N$$

$$\sum F_X = 0 \Rightarrow P = 735.8 + 294.3 = 1030\ N$$

Kama ve dişli

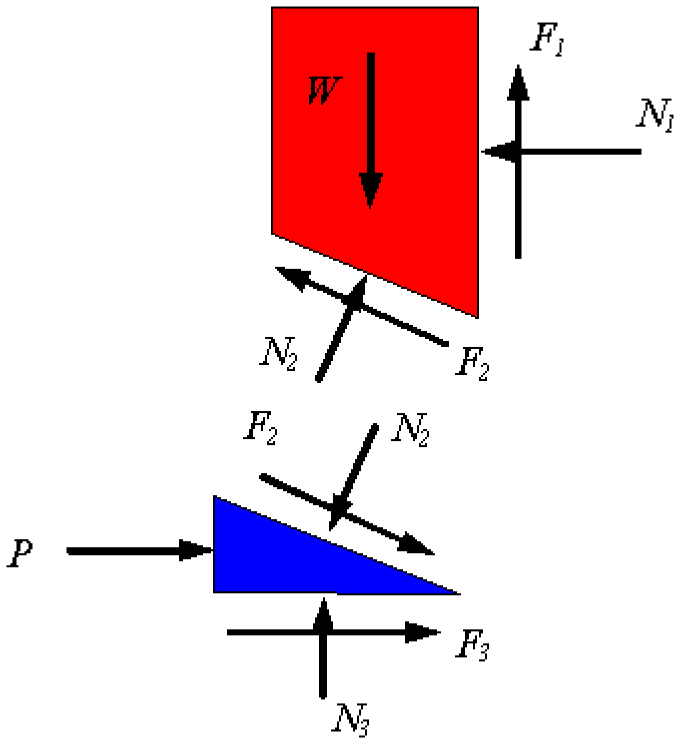
Kama: Genelde üçgen kesitli bir cisimdir



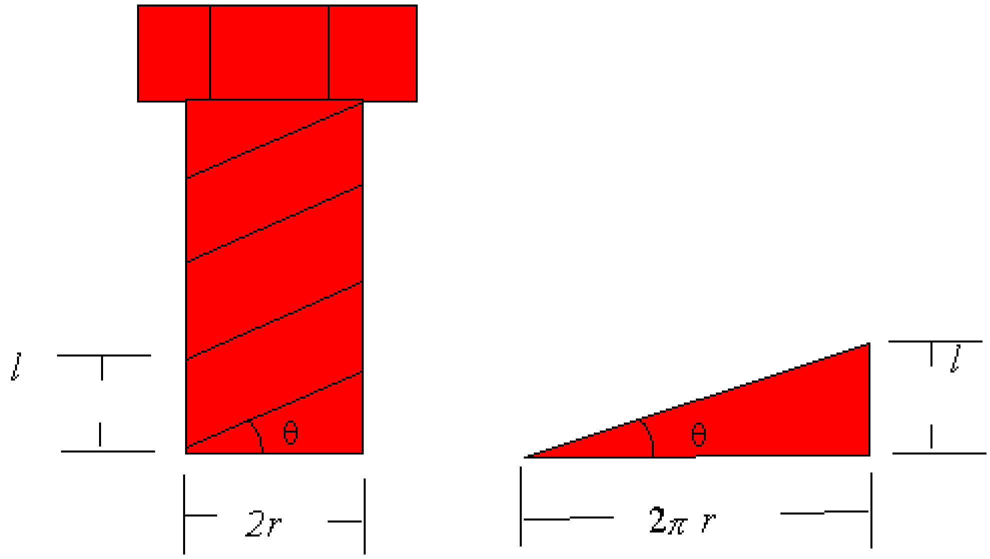
Sürtünme kuvvetleri aşağıdaki şekilde hesaplanırlar

$$F_1 = \mu N_1$$
$$F_2 = \mu N_2$$
$$F_3 = \mu N_3$$

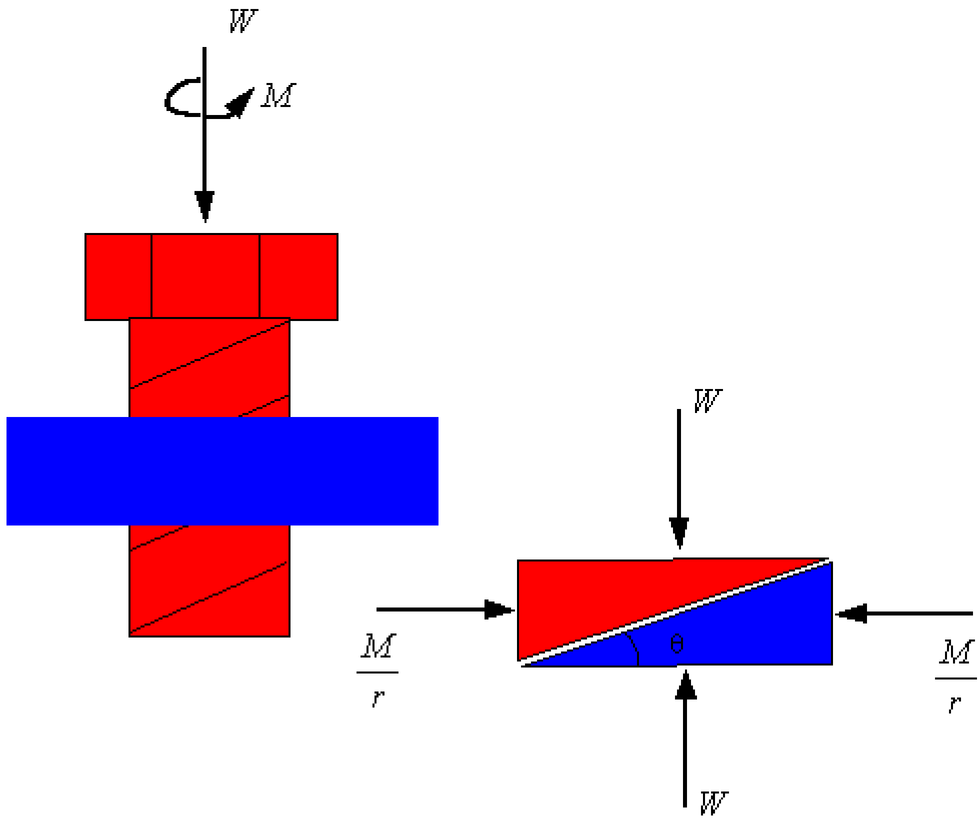
Eğer P yetersiz ise blok aşağıya doğru kayacak

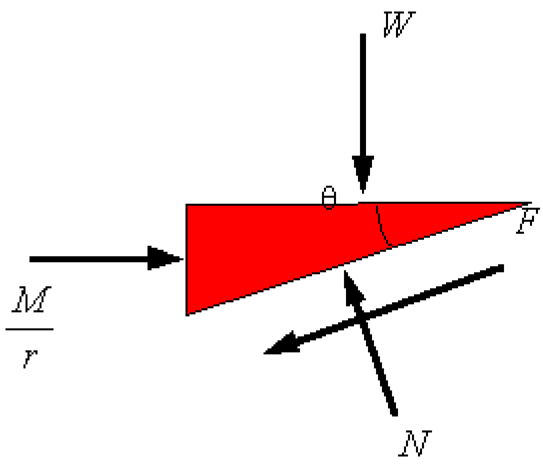


Dişli Vida: Dişli yüksekliği *l, çapı* 2*r veaçısı* θ

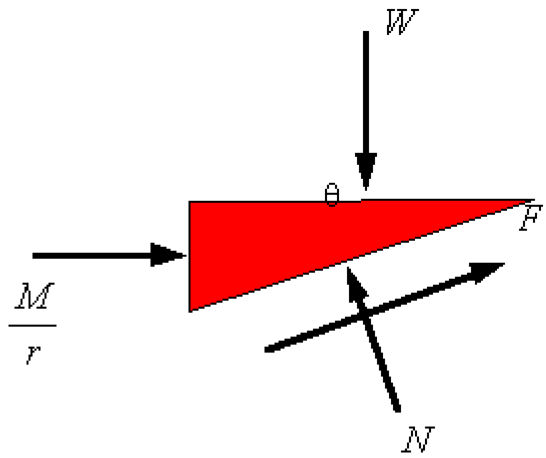


Eksenel kuvvet *W*, ve döndürme momenti *M*, *olan* dişlide serbest cisim diyagramı aşağıdaki şekildedir.





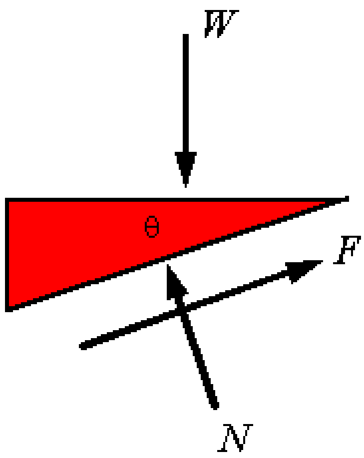
Eğer eksenel kuvvet yeterince büyük ise serbest cisim diyagramı



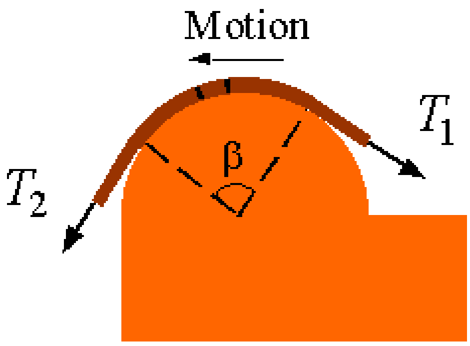
$F = \mu N$.

M=0 ise

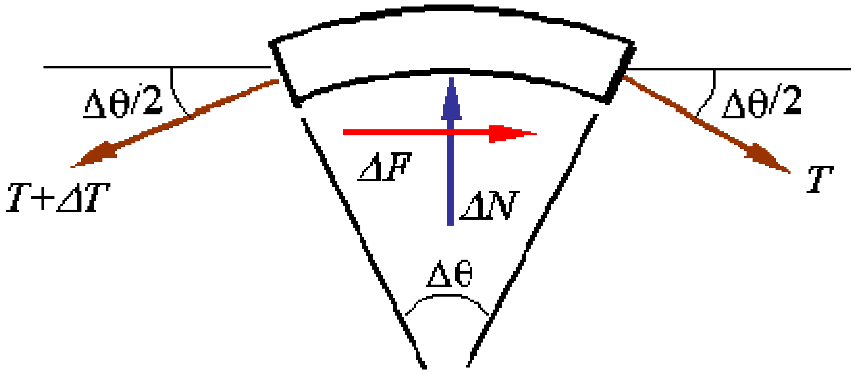
$F = \mu_s N$.



Kayış Sürtünmesi



Serbest cisim diyagramı



Denge denklemleri

$$\Delta N-(T+\Delta T)\sin(\frac{\Delta\theta}{2})-T\sin(\frac{\Delta\theta}{2})=0$$

$$\Delta F+T\cos(\frac{\Delta\theta}{2})-(T+\Delta T)\cos(\frac{\Delta\theta}{2})=0$$

$\Delta F=\mu N$, çok küçük açılar için limit durumda

$$\frac{dN}{d\theta}=T$$

$$\frac{dN}{dT}=\frac{1}{\mu}$$

Bu iki eşitlikten N elenirse

$$\frac{dT}{T} = \mu d\theta$$

Denklemi elde edilir. Bunun integralinden

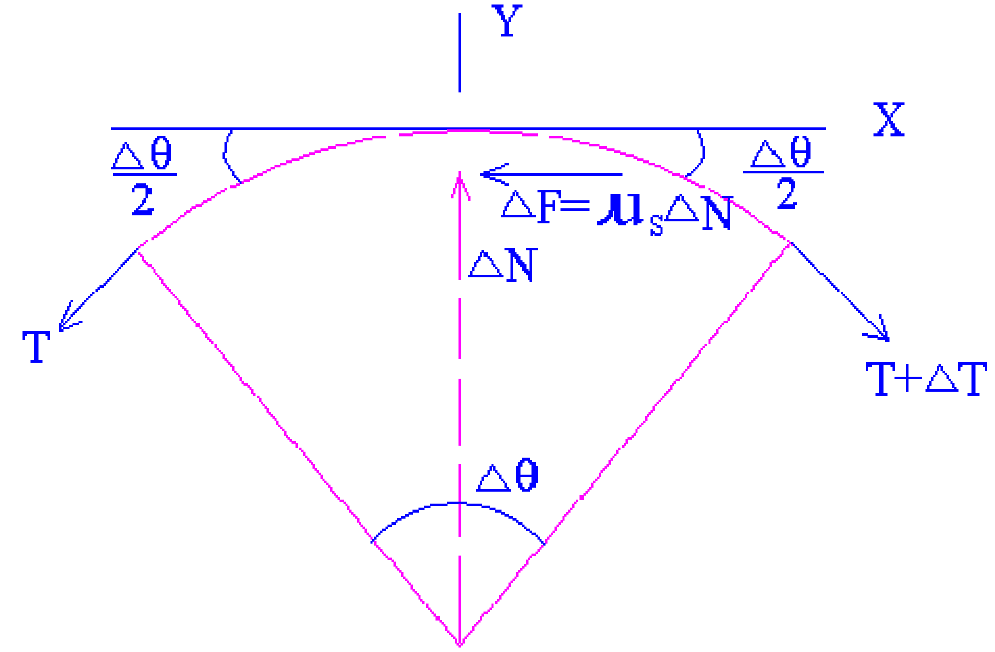
$$\ln\left(\frac{T_2}{T_1}\right) = \mu\beta$$

$$T_2 = T_1 e^{\mu\beta}$$

T2, T1 arasındaki bağıntı elde edilir.

burada β açısı radyan olarak alınacak

Consider the element PP'



$$\Sigma F_X = 0 \quad -T\cos\left(\frac{\Delta\theta}{2}\right) + (T + \Delta T)\cos\left(\frac{\Delta\theta}{2}\right) - \mu_s \Delta N = 0$$

$$\Sigma F_Y = 0 \quad -T\sin\left(\frac{\Delta\theta}{2}\right) - (T + \Delta T)\sin\left(\frac{\Delta\theta}{2}\right) + \Delta N = 0$$

$$\Delta N = (2T + \Delta T)\sin\left(\frac{\Delta\theta}{2}\right)$$

SUB ΔN in $\Sigma F_X = 0$

$$\Delta T \cos\left(\frac{\Delta\theta}{2}\right) - \mu_s (2T + \Delta T)\sin\left(\frac{\Delta\theta}{2}\right) = 0$$

Divide by $\Delta 0$

$$\frac{\Delta T}{\Delta\theta}\cos\left(\frac{\Delta 0}{2}\right) - \mu_s \left(T + \frac{\Delta T}{2}\right)\frac{\sin\left(\frac{\Delta\theta}{2}\right)}{\frac{\Delta\theta}{2}} = 0$$

As $\Delta\theta \rightarrow 0$, $\cos\left(\frac{\Delta\theta}{2}\right) \rightarrow 1$ & $\frac{\sin\left(\frac{\Delta\theta}{2}\right)}{\frac{\Delta\theta}{2}} \rightarrow 1$

Also $\frac{\Delta T}{\Delta\theta} \rightarrow \frac{dT}{d\theta}$ & $\Delta T \rightarrow 0$

$$\frac{dT}{d\theta} - \mu_s (T) = 0 \qquad \frac{dT}{T} = \mu_s d\theta$$

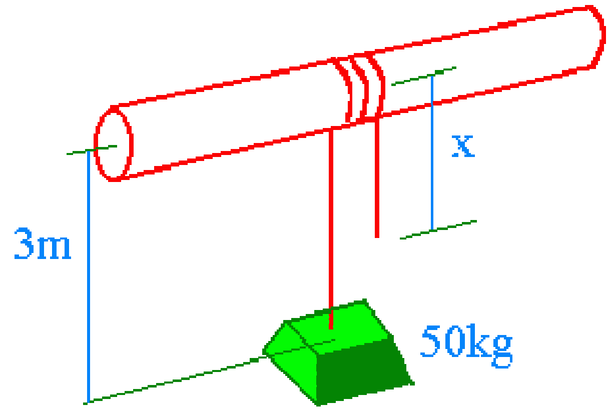
$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\beta} \mu_s d\theta$$

$$\ln\left(\frac{T_1}{T_2}\right) = \mu_s \beta$$

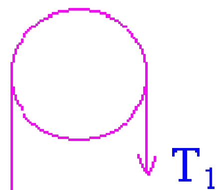
where T_1 & T_2 = Tensions in cable ($T_2 > T_1$)

Örnek

8.108 A rope having a mass per unit length of 0.6kg/m is wound 2.5 times around a horizontal rod. What length x of rope should be left hanging if a 50-kg load is to be supported? The coefficient of friciton between the rope and the rod is 0.30.



Solution:



$T_2 = [\,50 + 3\,(\,0.6\,)\,]\,9.81 = 508.16$ N

$\mu_s = 0.3 \qquad \beta = 2\,\pi\,(\,2.5\,) = 5\,\pi$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.3\,(\,5\,\pi\,)} = 111.32$$

$$T_1 = \frac{T_2}{111.32} = 4.56 \text{ N}$$

$$T_1 = x\,(\,0.6\,)\,9.81$$

So $x = 0.776$ m