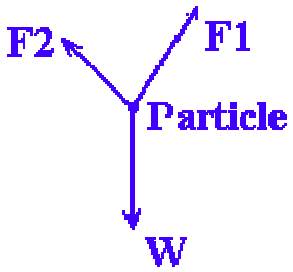
# STATİK- MUKAVEMET

## 4. Hafta

### 4.1 Rijit Cisim Dengesi

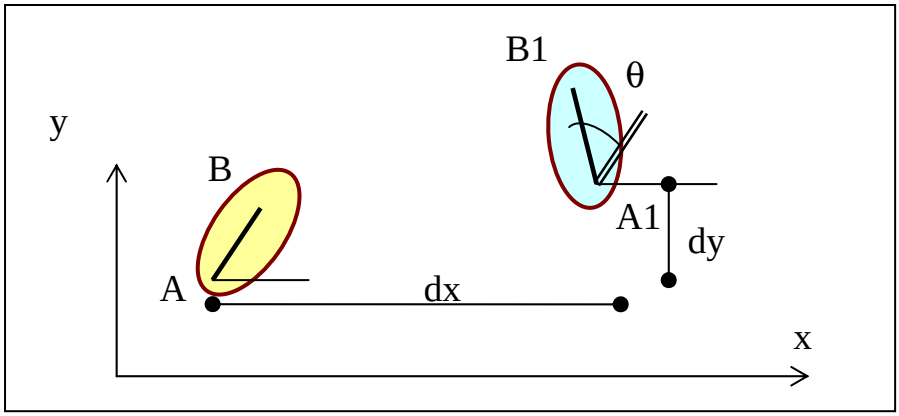
Düzlemde Parçacık dengesi




$$\sum F_x = \sum F_y - 0$$

**Düzlemde serbestlik derecesi**

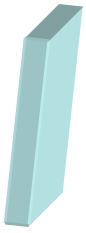
dx, dy, θ (iki öteleme , bir dönme)




**Taşıyıcı sistemler**

**-Bir boyutlu** çubuk (doğru eksenli, eğri eksenli)

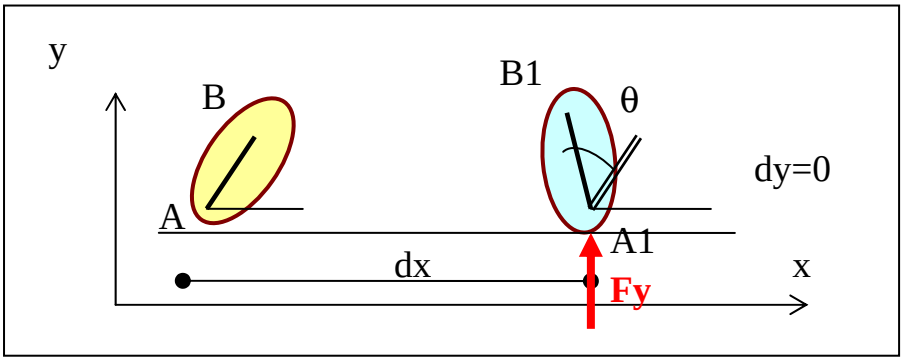
**-İki boyutlu** levha, plak



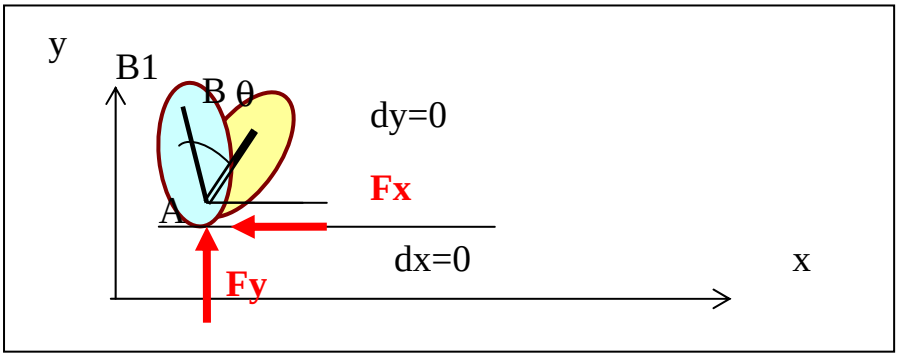
**-Üç boyutlu**

## Bağlar (mesnetler) ve bağ kuvvetleri

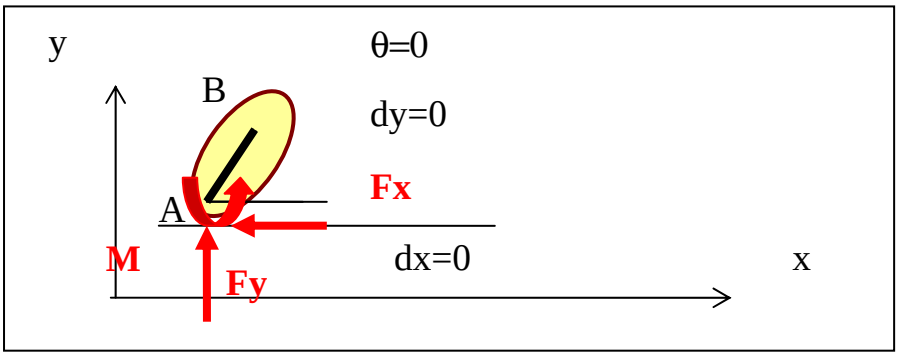
### - Bir doğrultuda hareketi kısıtlanan bağlar (kayıcı)



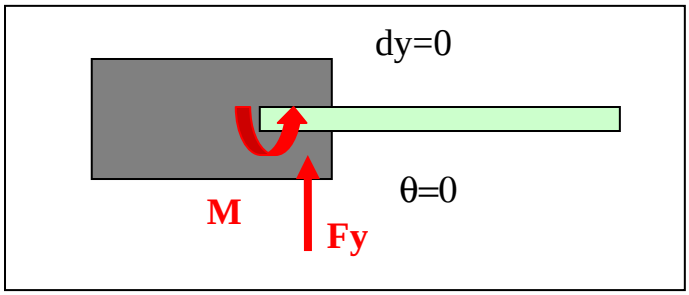

### - İki doğrultuda hareketi kısıtlanan bağlar (sabit)




### - Bütün hareketi kısıtlanan bağlar (ankastre)



**-Bir doğrultuda hareketi ve dönmesi kısıtlanan bağlar (kayıcı-ankastre mesnet)**




Rijit Cisim dengesi

$$\sum \vec{F} = 0 \quad \text{and} \quad \sum \vec{M} = 0$$

Or,

$$\sum F_x = 0 \qquad \sum M_x = 0$$
$$\sum F_y = 0 \quad \text{and} \quad \sum M_y = 0$$
$$\sum F_z = 0 \qquad \sum M_z = 0$$

## 4.2 Mesnet Tepkileri

Düzlemde 3 adet bağ kuvveti (mesnet tepkisi) vardır.

Bu 3 bilinmeyen bağ kuvveti, 3 adet denge denkleminden hesaplanabilir.

Düzlemde x-y

$$F_z = 0, \quad M_x = M_y = 0$$

**So,**

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_{zA} = 0$$

**Where, M = Moment about Z-axis at any arbitrary point A.**

**We can replace any of the 3 equations with $\sum M_{zB} = 0$, where B is again an arbitrary point.**

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_{zB} = 0$$

**or,**

$$\sum F_x = 0, \quad \sum M_{zB} = 0, \quad \sum M_{zA} = 0$$

$$\sum M_{zA} = 0, \quad \sum M_{zB} = 0, \quad \sum M_{zC} = 0$$

**Where A, B, C do not form at straight line.**

NOT:

3 bilinmeyen bulunduktan sonra, diğer denklemler kontrol için kullanılabilir.

**Ex.**

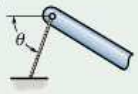
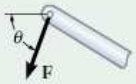
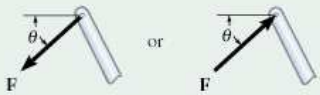
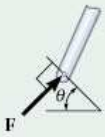
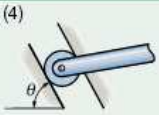
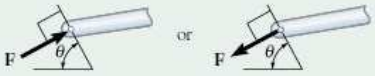
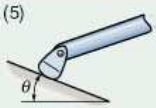
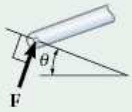
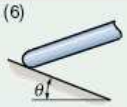
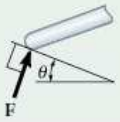
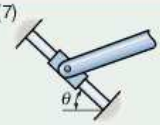
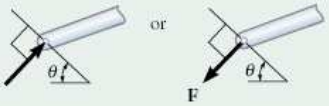
**We may use $\sum F_x - 0, \quad \sum F_y - 0, \quad \sum M_{zA} = 0$**

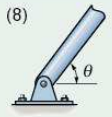
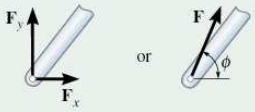
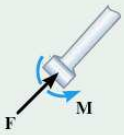
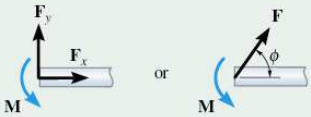
**check with $\sum M_{zB} = 0$**

**Bağ çeşitleri**

1-Kablo (cable)
2-Pandül Ayak (short link)
3-Kayıcı mesnet (roller)
4- Kayıcı mesnet (roller/pin in smooth slot)
5- Kayıcı mesnet (rocker)
6- Kayıcı mesnet (smooth surface)
7- Kayıcı mesnet (pin/collar smooth rod)
8- Sabit mesnet (smooth pin or hinge)
9- Kayıcı ankastre mesnet (fixed collar smooth rod)

## 10- Ankastre mesnet (fixed)

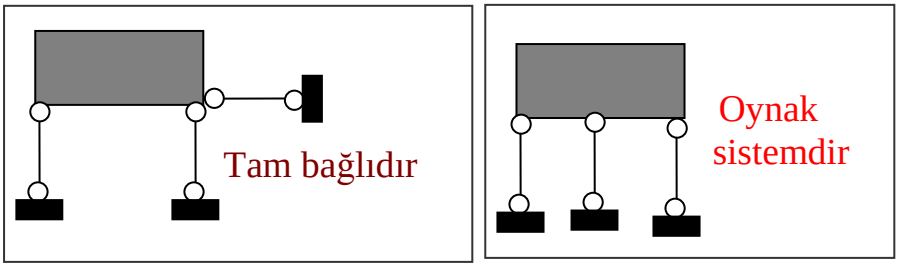
| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (1)  cable |  | One unknown. The reaction is a tension force which acts away from the member in the direction of the cable. |
| (2)  weightless link |  | One unknown. The reaction is a force which acts along the axis of the link. |
| (3)  roller |  | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |
| (4)  roller or pin in confined smooth slot |  | One unknown. The reaction is a force which acts perpendicular to the slot. |
| (5)  rocker |  | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |
| (6)  smooth contacting surface |  | One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact. |
| (7)  member pin connected to collar on smooth rod |  | One unknown. The reaction is a force which acts perpendicular to the rod. |

| Types of Connection | Reaction | Number of Unknowns |
|---|---|---|
| (8)  smooth pin or hinge |  | Two unknowns. The reactions are two components of force, or the magnitude and direction $\phi$ of the resultant force. Note that $\phi$ and $\theta$ are not necessarily equal [usually not, unless the rod shown is a link as in (2)]. |
| (9)  member fixed connected to collar on smooth rod |  | Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod. |
| (10)  fixed support |  | Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction $\phi$ of the resultant force. |

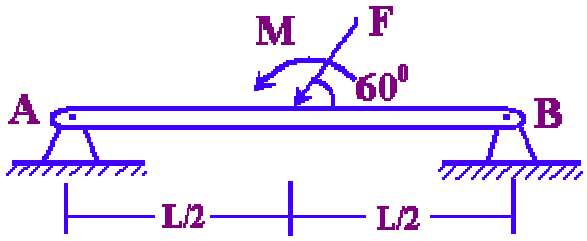
## 4.3 Statikçe Belirli (İzostatik), Statikçe Belirsiz (Hiperstatik) ve Oynak Sistemler

**Mesnetlerin bağlılık durumu**

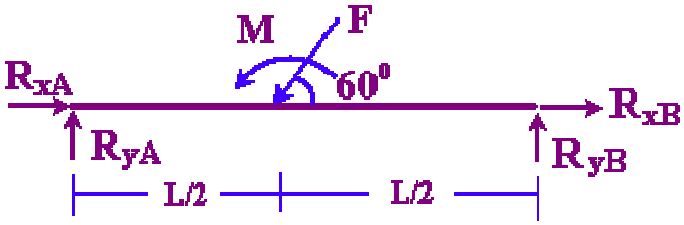
- Tam bağlı (statikçe belirli), (izostatik)
- Fazla bağlı (statikçe belirsiz) (hiperstatik)
- Eksik bağlı (oynak sistem)



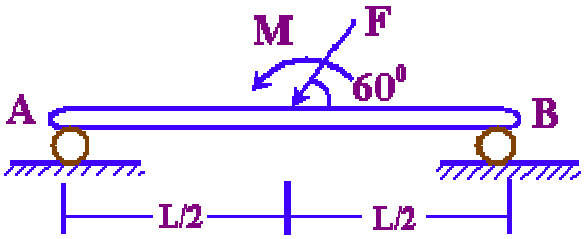

**Eğer 3 denklem, 4 bilinmeyen varsa bir bilinmeyen fazladır, bu sistem hiperstatiktir. Denge denklemleriyle çözülemez.**



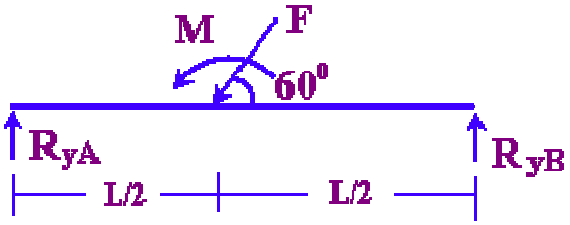

F.B.D.




**Eksik bağlı sistemde, 3 denge denklemi var, fakat 2 bağ kuvveti var, yani bir bağ eksik sistem hareket edebilir.**
**Bu sisteme oynak sistem denir**




F.B.D.

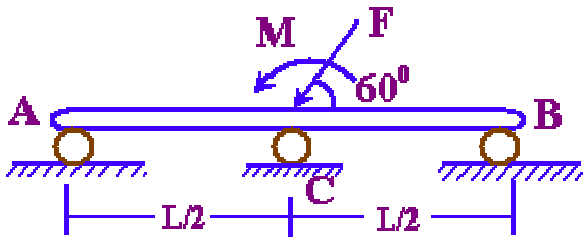



**3 equations but only 2 unknows**

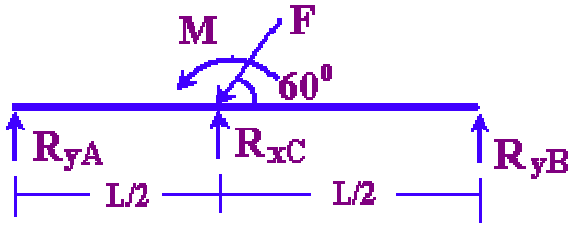
$$\Sigma F_x = F \cos 60 \neq 0$$

**So, move to left, unstable!**

**3 bilinmeyen ve 3 denklem olmasına rağmen bazı sistemler yine oynaktır. Aşağıdaki şeklin x yönünde hareketi kısıtlanmamış, sistem geometrik olarak oynak sistemdir.**




F.B.D.




**3 equations & 3 unknowns, but**

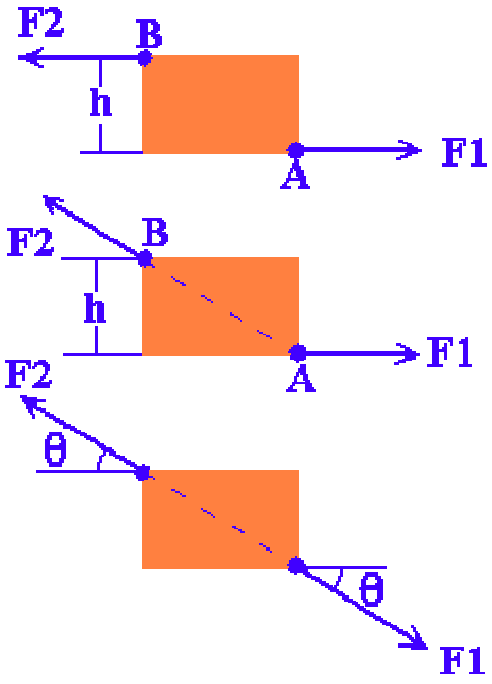
$\sum F_x = F \cos 60 \neq 0$

**Geometrically unstable!**

## 4.4 İki kuvvet etkisinde sistemin dengesi

Eğer rijit bir cisme iki kuvvet etkiyorsa

Aynı şiddette, aynı doğrultuda ve ters yönde olması gerekir.




(1). $\sum M_A \neq 0$ Since $F_2$ h

(2). $\sum M_A = 0$ but $\sum M_B \neq 0$

Since $F_1$ h

(3). $\sum M_A = 0$ $\sum M_B = 0$

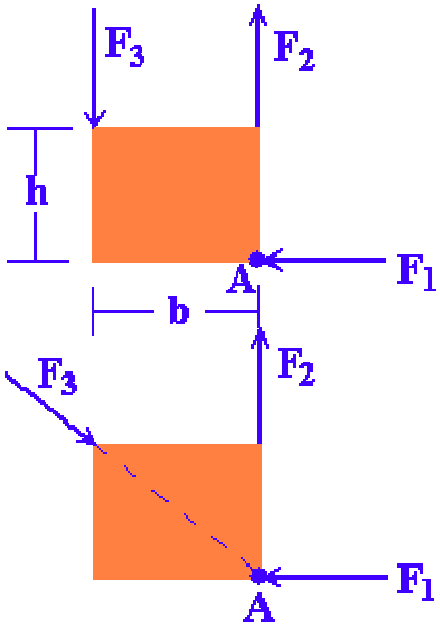
$\sum F_x = 0$ $F_1 = F_2$

$(-F \cos\theta - F \cos\theta = 0)$

## 4.5 3 kuvvet etkisinde denge

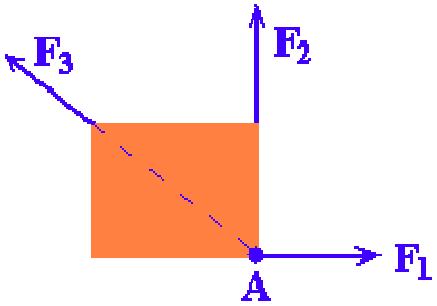
Eğer 3 kuvvet bir noktada kesişiyorsa veya 3 de paralel ise denge

yoktur.




(1). $\sum M_A \neq 0$, Since $F_3 b$

(2). $\sum M_A = 0$, O.K.

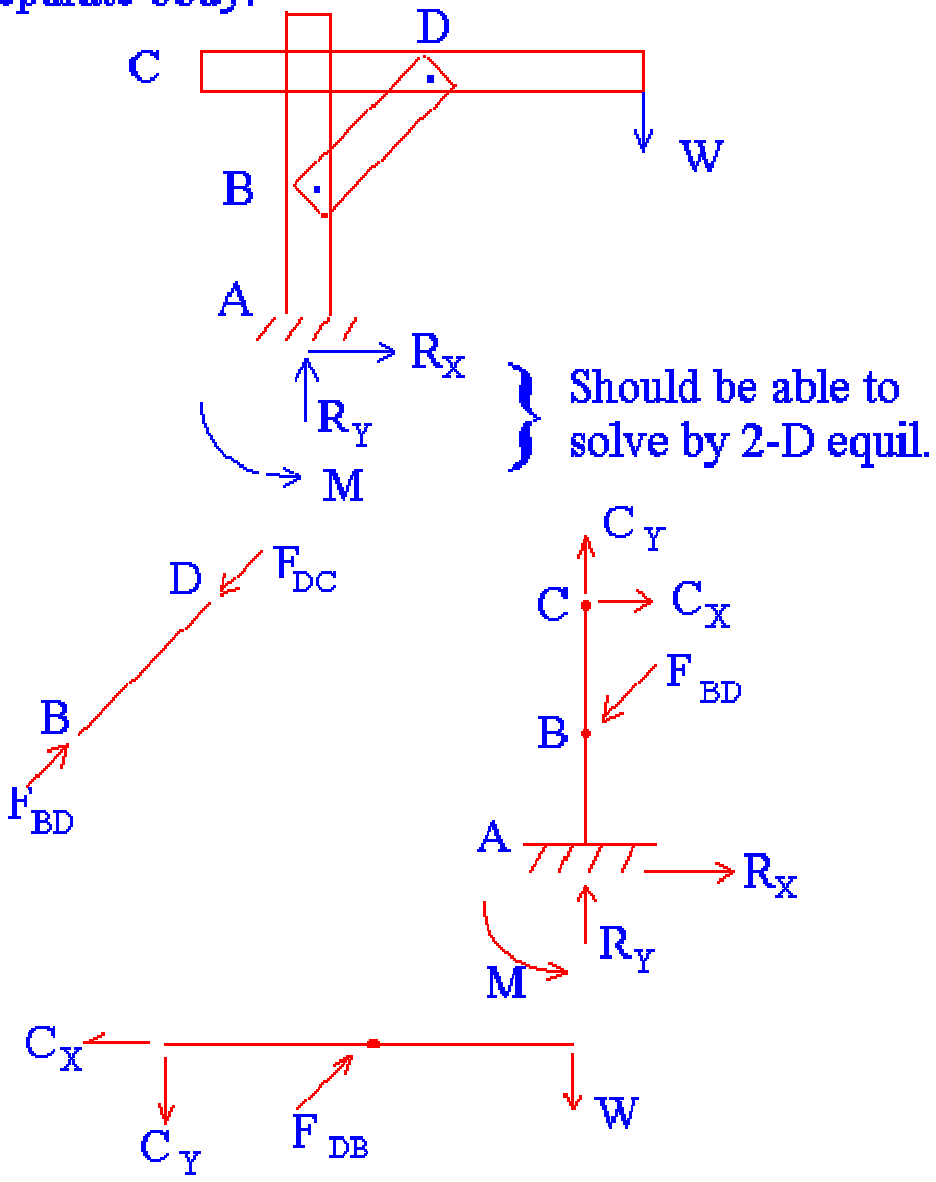



$\sum F = 0$, so not in equilibrium

## 4.7 Çok Parçalı Taşıyıcı Sistemler :

Çok parçalı taşıyıcı sistemler
f=2p-2
p:levha sayısı (çubuk sayısı)
f:bağ kuvvetleri toplamı (bilinmeyen sayısı)
izostatik için
s=3n
s:toplam statik değer sayısı
n:levha sayısı (bilinmeyen sayısı

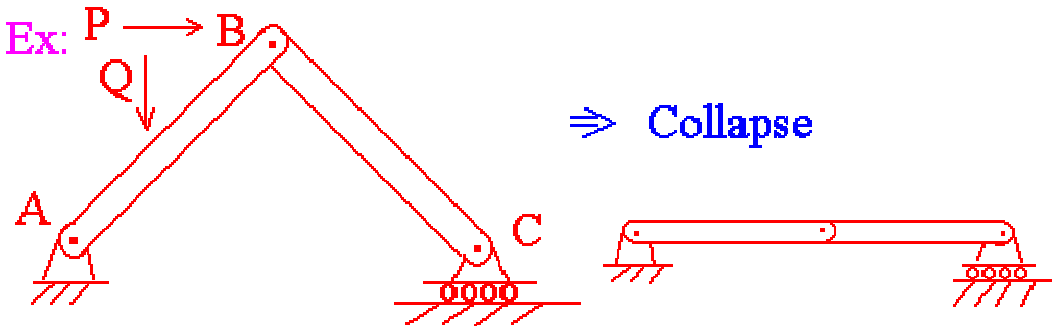
In the analysis of frames, we can separate out each member of the frame and analyze each member as a separate body.

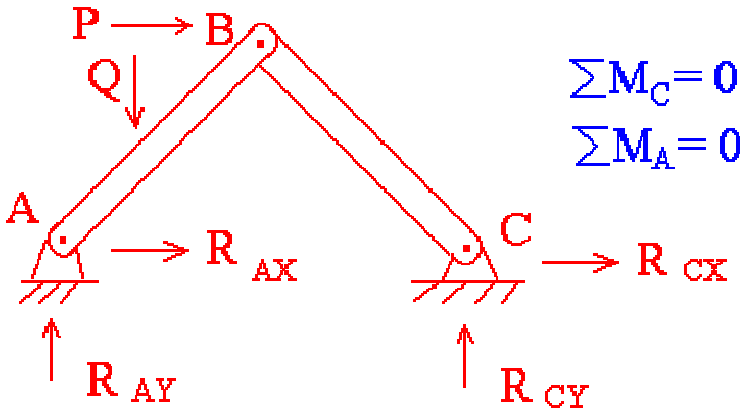



Bodies CD & AC can solved for by 2-D equil.

$$\sum F_X = \sum F_Y = \sum M = 0$$

Some frames would not be rigid if they had inadequate supports.

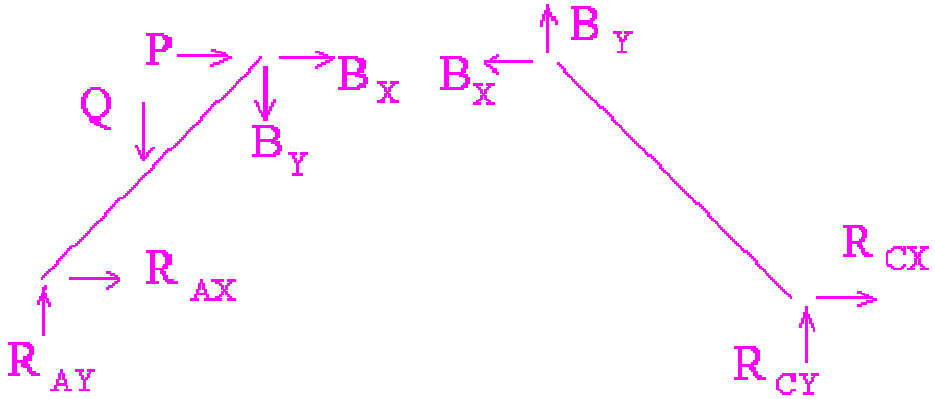



$\sum M_C = 0 \Rightarrow R_{AY}$

$\sum M_A = 0 \Rightarrow R_{CY}$

Now it is rigid. However, for the whole body we have 4 reactions & 3 EQNS. We can find the other reaction by considering the 2 members AB & AC as separate rigid bodies.




$\sum M_B = 0 \Rightarrow R_{AX}$

$\sum F_X = 0 \Rightarrow B_X$

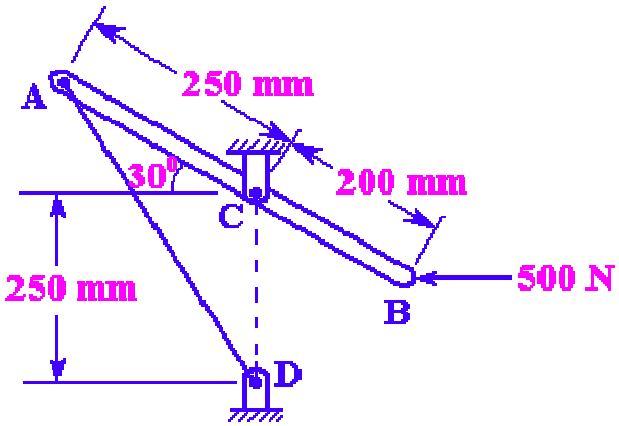
$\sum F_Y = 0 \Rightarrow B_Y$

$\sum M_B = 0 \Rightarrow R_{CX}$

Or look at total struct &

$\sum F_X = 0$

**Example:**



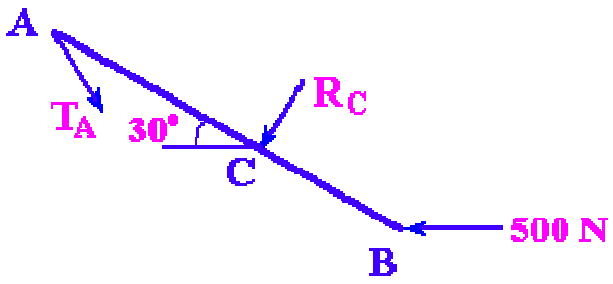

**Solve for:**

**a). Determine tension in cable**

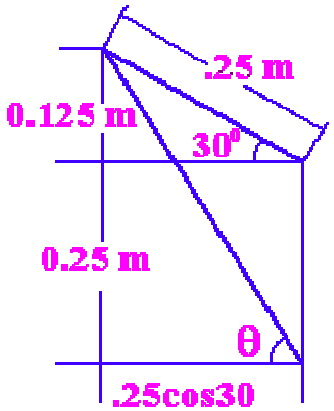
**b). Determine reaction at C .**

Adım 1:

**F.B.D.**



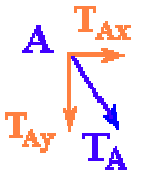

**Angle related to $T_A$**




$$\tan\theta = \frac{.375}{.2165} \Rightarrow \theta = 60^\circ$$

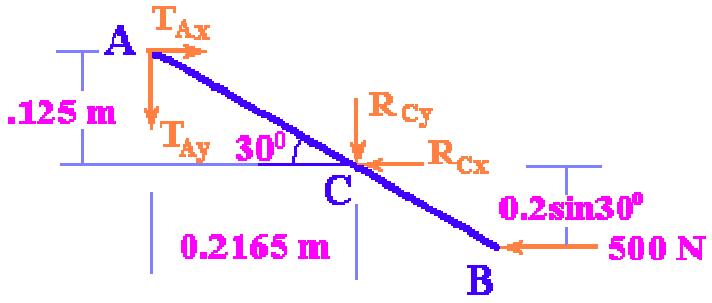
Adım 2. Kablo kuvvetinin Bulunması

**Decomposition of $T_A$**




$$T_{Ax} = T_A \cos 60 = 0.5\, T_A$$

$$T_{Ay} = T_A \sin 60 = 0.866\, T_A$$




$$\sum M_C = 0$$

$$T_{Ay}(0.2165) - T_{Ax}(0.125) - 500\,(0.2)\sin 30 = 0$$

So,

$$T_A(0.866)(0.2165) - T_A(0.5)\,(0.125) - 50 = 0$$

Then, $\underline{T_A = 400\ N}$

Adım 3. C noktasındaki tepkilerin bulunması

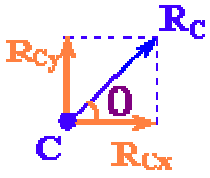
$$\sum F_y = 0$$

$$T_{Ax} - R_{Cx} - 500 - 0 \Rightarrow 0.5\,(400) - R_{Cx} - 500 = 0$$

$$R_{Cx} = \underline{-300\ N}$$

$$\sum F_y = 0$$

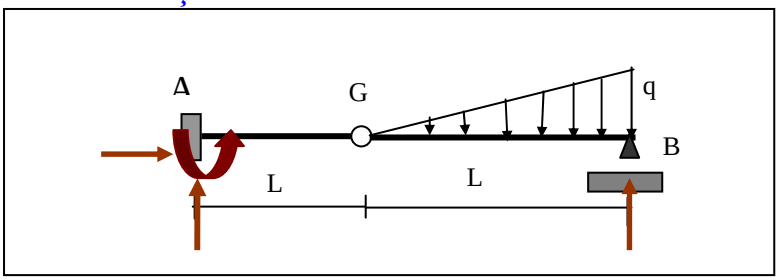
$$-T_{Ay} - R_{Cy} - 0 \Rightarrow R_{Cy} = \underline{-346.40\ N}$$

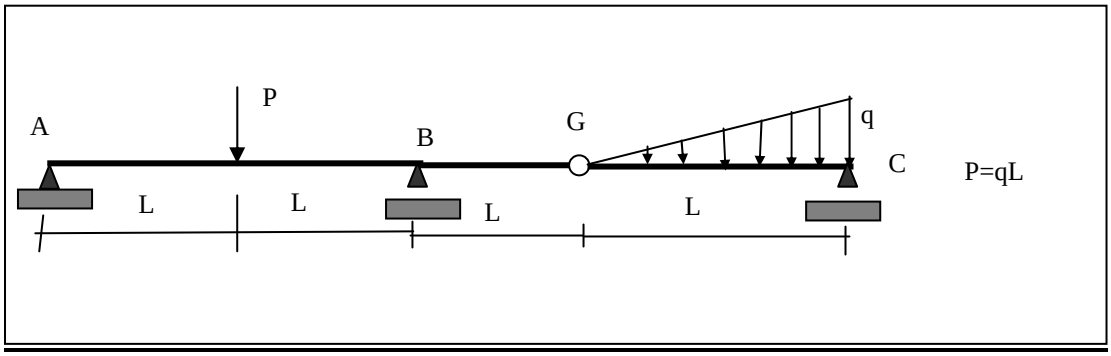


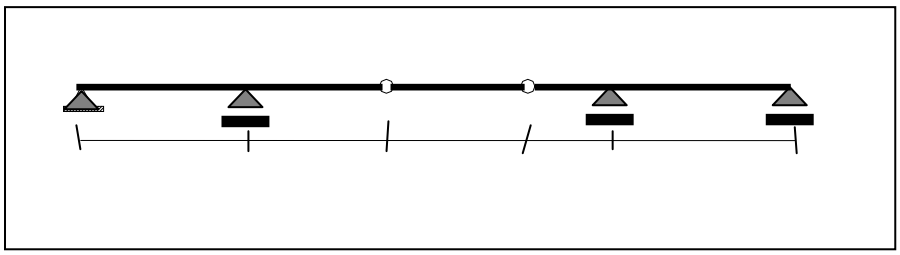

$$R_C = \sqrt{(300)^2 + (346.4)^2} = \underline{458.25\ N}$$

$$\theta = \tan^{-1}\left(\frac{346.4}{300}\right) = \underline{49.1^\circ}$$

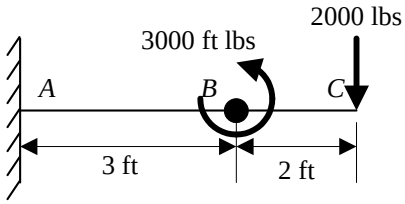
## Gerber Kirişleri



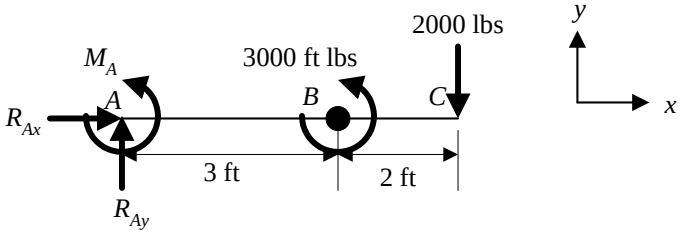



## 1). Örnek:




A Noktasındaki Bağ kuvvetleri bulunuz

FBD




$\Sigma F_x = 0$

$R_{Ax} = 0$

$\Sigma F_y = 0$

$R_{Ay} - 2000 = 0$

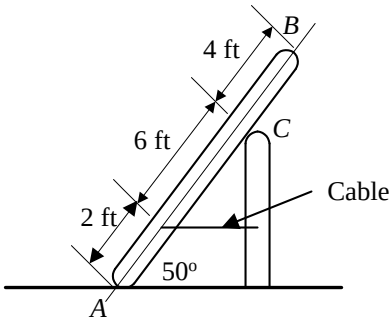
$R_{Ay} = 2000$ lbs

$\Sigma M_A = 0$

$M_A + 3000 - 2000(5) = 0$

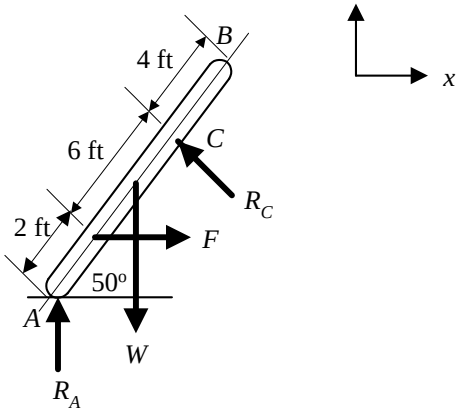
$M_A = 7{,}000$ ft lbs

2). Örnek: *AB* nin ağırlığı 250 pounds ve bütün yüzeyler kayıcı.




Kablo kuvvetini, A ve C noktasındaki mesnet tepkilerini bulunuz

FBD




$\Sigma F_x = 0$

$F - R_C \cos 40 = 0$

$F - 0.766 R_C = 0$

$\Sigma F_y = 0$

$R_A - W + R_C \sin 40 = 0$

$R_A + R_C \sin 40 = 250$

$R_A + 0.643 R_C = 250$

$\Sigma M_A = 0$

$-F(2\sin 50) - W(6\cos 50) + R_C(8) = 0$

$-1.532F + 8R_C = 964.2$

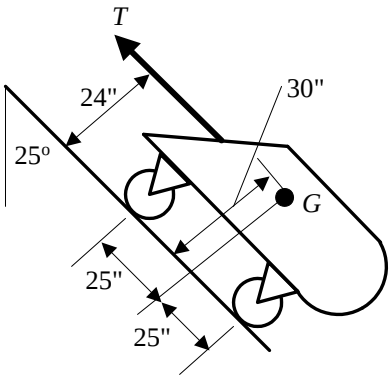
3 denklemin çözümünden:

$R_A$ = 159.2 lbs

$F$ = 108.2 lbs

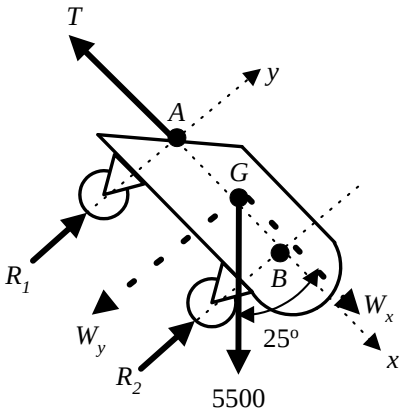
$R_C$ = 141.2 lbs

3). Örnek: 5500 lbs. ağırlığındaki araba yukarı doğru çekilmeye çalışılıyor (*G*. Ağırlık merkezi)




Koblodaki kuvveti ve tekerleklerin yaptığı kuvvetleri bulunuz.

FBD




$R_2$ , *A* noktasına göre momentten hesaplanıyor

$$\Sigma M_A = 0$$
$$R_2(50) - W_x(6) - W_x(25) = 0$$
$$50R_2 - 5500\cos 25^\circ(6) - 5500\sin 25^\circ(25) = 0$$
$$R_2 = 1760 \text{ lbs}$$

**$R_1$, B noktasına göre momentten hesaplanıyor.**

$$\Sigma M_B = 0$$
$$-R_1(50) - 5500\cos 25^\circ(6) + 5500\sin 25^\circ(25) = 0$$
$$R_1 = 564 \text{ lbs}$$

***T*: x yönündeki dengeden hesaplanıyor**

$\Sigma F_x = 0$

$-T + W_X = 0$

$T = 5500\cos 25^\circ$

$T = 4985 \ \text{lbs}$

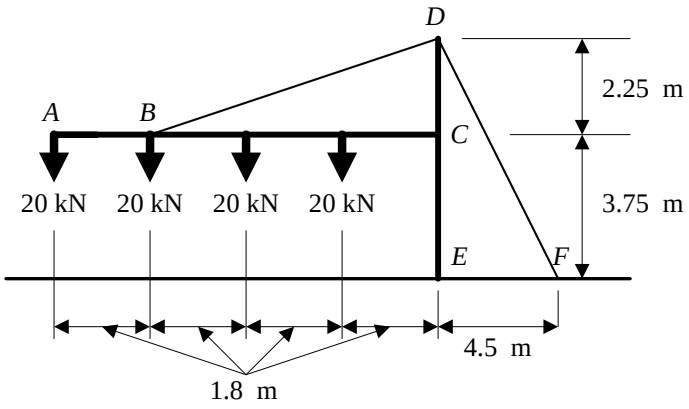
## Kontrol:

$\Sigma F_x = 0$

$564 + 1760 - 5500\sin 25^\circ = 0$

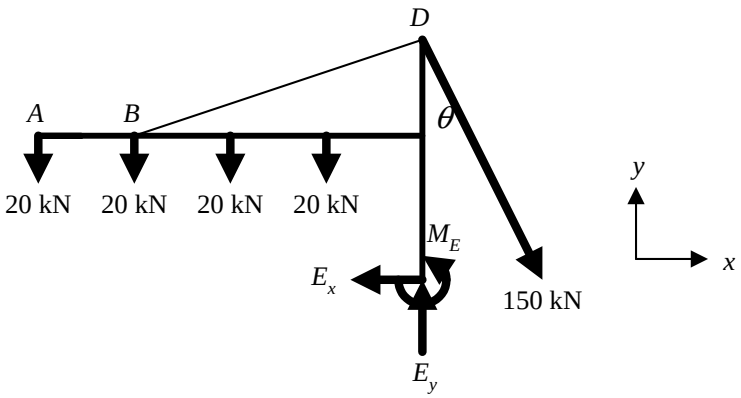
$-0.4 \approx 0$

## 4). *DF* deki gerilme 150 kN




## E noktasındaki bağ kuvvetlerini bulunuz (E ankastre mesnettir)

FBD




$DF = \sqrt{4.5^2 + 6^2}$

$DF = 7.5 \ \text{m}$

$\Sigma F_x = 0$

$-E_x + 150\sin\theta = 0$

$E_x = 150\left(\frac{4.5}{7.5}\right)$

$E_x = 90 \ \text{kN}$

$\Sigma F_y = 0$

$-80 + E_y - 150\cos\theta = 0$

$E_y = 80 + 150\left(\frac{6}{7.5}\right)$

$E_y = 200 \ \text{kN}$

$\Sigma M_E = 0$

$M_E + 20(1.8)(4) + 20(1.8)(3) + 20(1.8)(2) + 20(1.8) - 150\sin\theta(6) = 0$

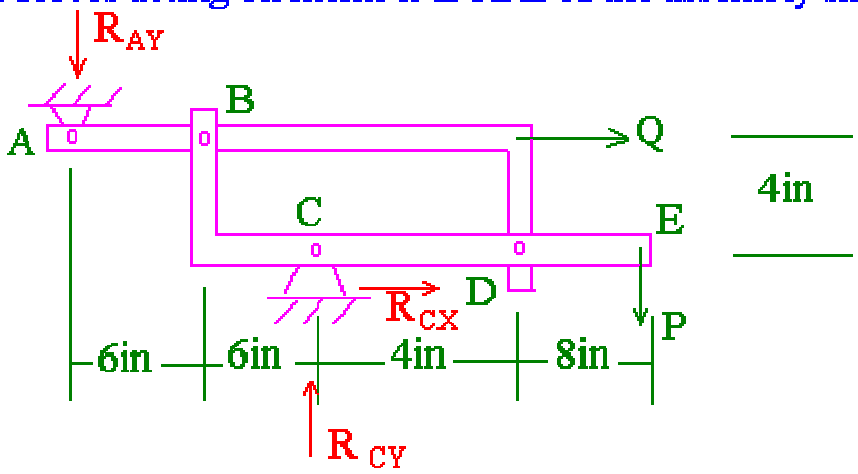
$M_E = 150\left(\frac{4.5}{7.5}\right)(6) - 360$

$M_E = 180 \ \text{kN m}$

-

## Example 2:

Knowing that P=90lb and Q=60lb, determine the components of all forces acting on member BCDE of the assembly shown.




Find rections:

$\sum M_C = 0 \quad 90(12) + 60(4) - R_{AY}(12) = 0 \Rightarrow R_{AY} = 110$ lb

$\sum F_X = 0 \quad R_{CX} = -60$ lb

$\sum F_Y = 0 \quad R_{CY} = 200$lb

Member BCDE:

$\sum F_X = 0 \quad D_X = 60$ lb

$\sum M_D = 0 \quad B_Y(10) - 200(4) - 90(8) = 0$

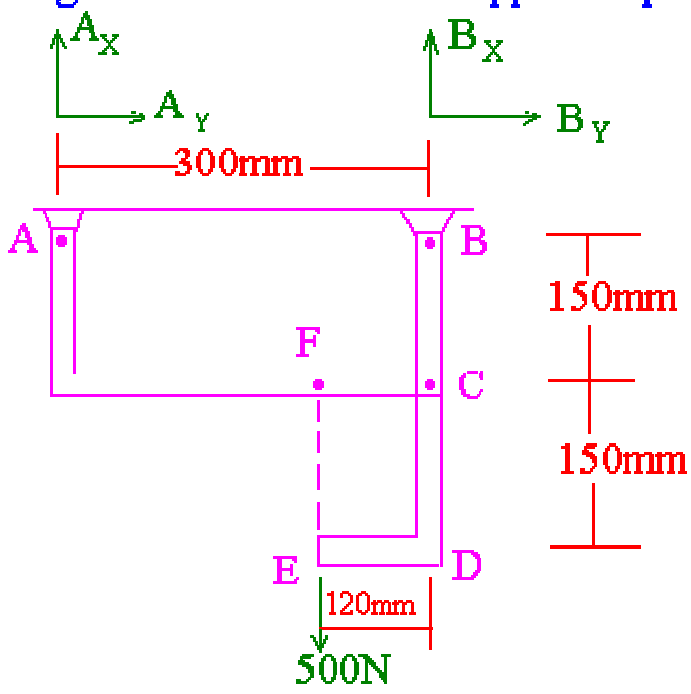
$\Rightarrow \quad B_Y = 152$ lb

$\sum F_Y = 0 \quad 200 - 90 - B_Y - D_Y = 0$

$\Rightarrow \quad D_Y = -42$ lb

## Example 6.3:

Determine the components of the reactions at A and B, (a) If the 500-N load is appllied as shown, (b) if the 500-N load is moved along its line of action and is applied at point F.

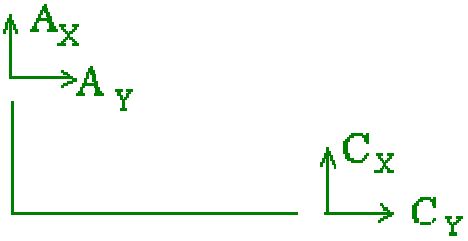



$\sum M_A = 0 \quad B_Y(0.3) - 500(0.18) = 0 \Rightarrow B_Y = 300\ N$

$\sum F_Y = 0 \quad B_Y + A_Y - 500 = 0 \rightarrow A_Y = 200\ N$

$\sum F_X = 0$ or $\sum M$ about any other point will only give us A in terms of B or vise-versa.

Look at AC:




$\sum M_C = 0 \quad A_X(0.15) + A_Y(0.3) = 0 \Rightarrow A_X = -400\ N$

Whole struct:

$A_X + B_X = 0 \Rightarrow B_X = 400\ N$