INTRODUCTION TO SCIENTIFIC & ENG. COMPUTING
BIL 108E, CRN 44448

Week.7_ Arrays & Nonlinear Equation
Week.8_ Solving of Simple Equations

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01 - 04 - 2011

TENTATIVE SCHEDULE

Wk	Date	Topics
1	Feb. 11	Introduction to Scientific and Engineering Computing
2	Feb. 18	Introduction to Program Computing Environment
3	Feb. 25	Variables, Operations and Simple Plot
4	Mar. 04	Algorithms and Logic Operators
5	Mar. 11	Flow Control, Errors and Source of Errors
6	Mar. 18	Functions & Linear Algebra
7	Mar. 25	Arrays
8	Apr. 01	Solving of Simple Equations
9	Apr. 08	Polynomials Examples Exam 1
10	Apr. 15	Applications of Curve Fitting
11	Apr. 22	Applications of Interpolation
12	Apr. 29	Applications of Numerical Integration
13	May.06	Symbolic Mathematics
14	May.13	Ord Dif.Equations (ODE) solution with Built-in Functions

Lecture # 8 - Solving of Simple Equations

1- POLYNOMIALS

2- APPROXIMATION OF DATA

POLYNOMIALS

- Definition: n^{th} degree polynomial
  $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$
- Coefficients of the polynomial
  $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$
- n: degree of the polynomial

POLYNOMIALS

MATLAB toolbox: polyfun

- polyval: returs the value of the polynomial at a point x. Input arguments are vector p and vector x.

  p → $a_n, a_{n-1}, \ldots, a_1, a_0$

  x is the abscissae, where the polynomials is evaluated.

  absis

- y = polyval (p, x)

POLYNOMIALS

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  absis

- y = polyval (p, x)

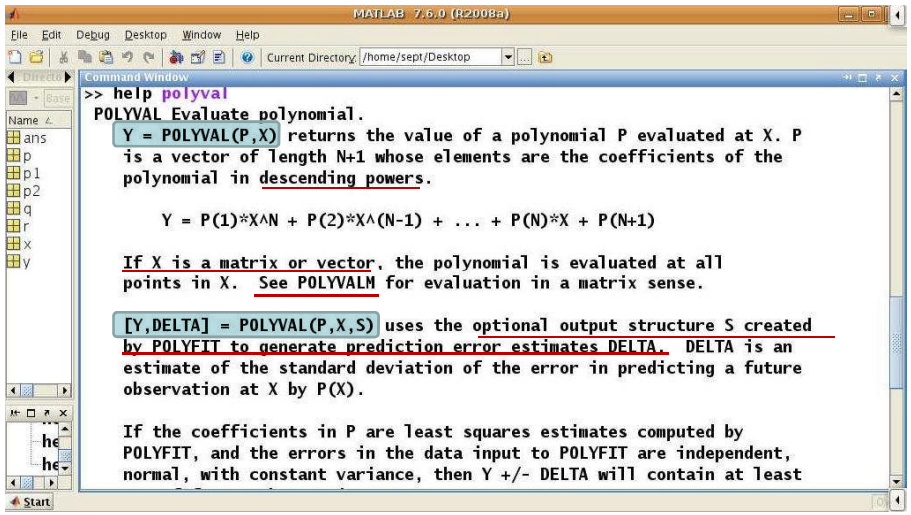
  EXAMPLE:

  Given: $p(x) = x^7 + 3x^2 - 1$,

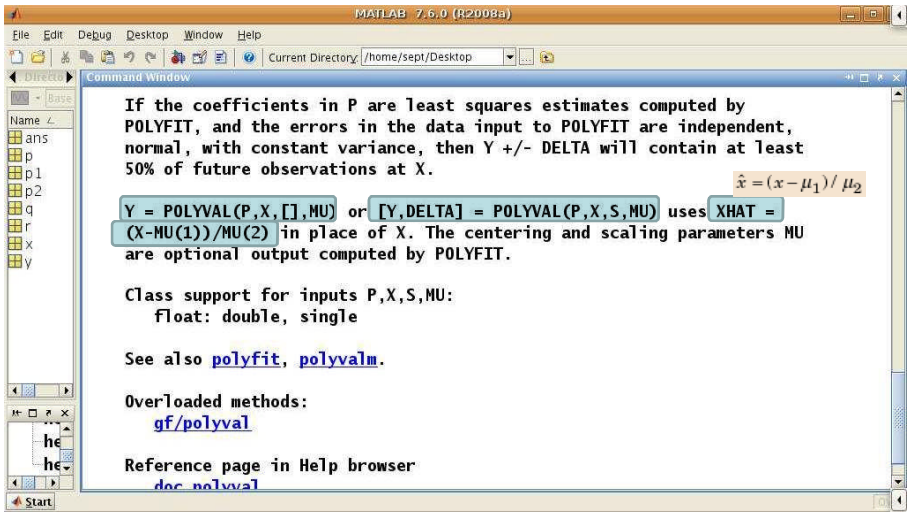
  $x_k = -1 + k/4$ for $k = 0, \ldots, 8$

  Find: Plot the given function.

POLYNOMIALS



POLYNOMIALS



POLYNOMIALS

- y = polyval(p,x,[ ],mu) or [y,delta] = polyval(p,x,S,mu) use $\hat{x} = (x - \mu_1)/\mu_2$ in place of x. In this equation $\mu_1 = \text{mean}(x)$, and $\mu_2 = \text{std}(x)$. The centering and scaling parameters $\text{mu} = [\mu_1, \mu_2]$ are optional output computed by polyfit.

Remarks

- The polyvalm (p,x) function, with x a matrix, evaluates the polynomial in a matrix sense. See polyvalm for more information.

EXAMPLE

- The polynomial $p(x) = 3x^2 + 2x + 1$ is evaluated at *x* = 5, 7, and 9 with;

```
p = [3 2 1];
polyval (p,[5 7 9])
```

which results in

ans =

86 162 262

POLYNOMIALS

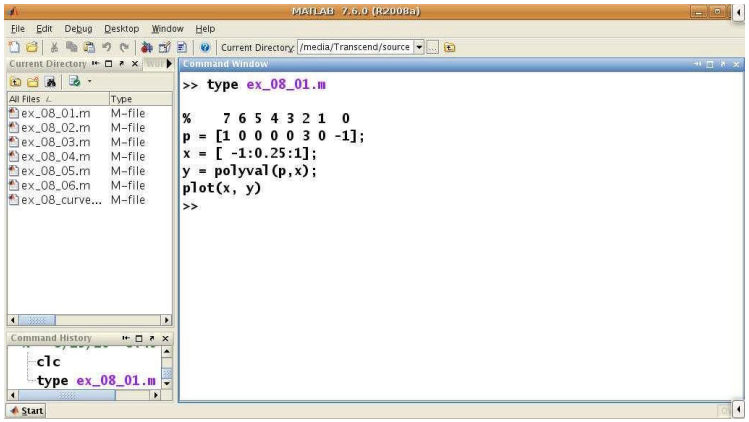
EXAMPLE cont'd.:

```
%    7 6 5 4 3 2 1  0
p = [1 0 0 0 0 3 0 -1];
x = [ -1:0.25:1];
y = polyval (p,x);
plot (x, y)
```

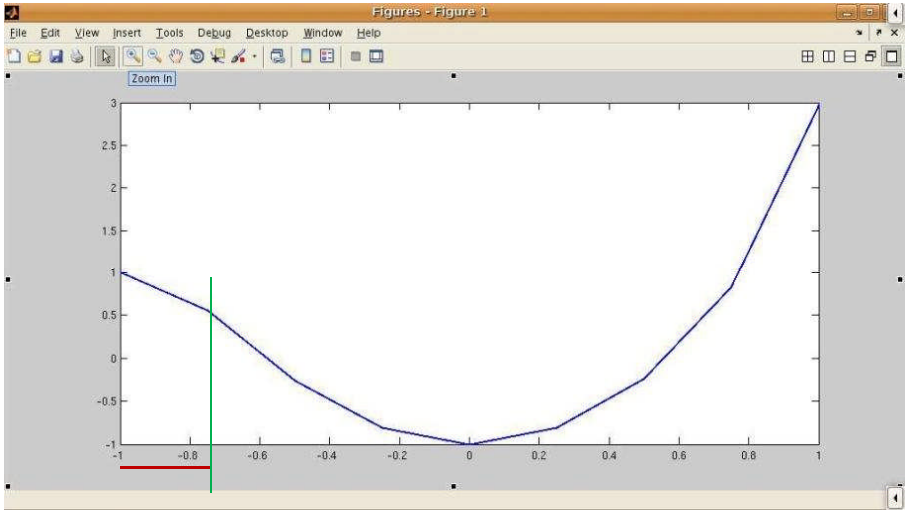

POLYNOMIALS

EXAMPLE cont'd.:

```
%    7 6 5 4 3 2 1  0
p = [1 0 0 0 0 3 0 -1];
x = [ -1:0.25:1];
y = polyval (p,x);
plot (x, y)
```



POLYNOMIALS



POLYNOMIALS

- roots: provides an approximation of the zeros of a polynomial.
  Usage: `r = roots(p)`
- poly: returns the coefficients of the polynomial, whose zeros are given.
  Usage: `p = poly(r)`
- **http://www.mathworks.com/help/techdoc/ref/roots.html**

EXAMPLE:

- Given: $p(x) = x^3 - 6x^2 + 11x - 6$
- Find: Compute the zeros of the polynomial.

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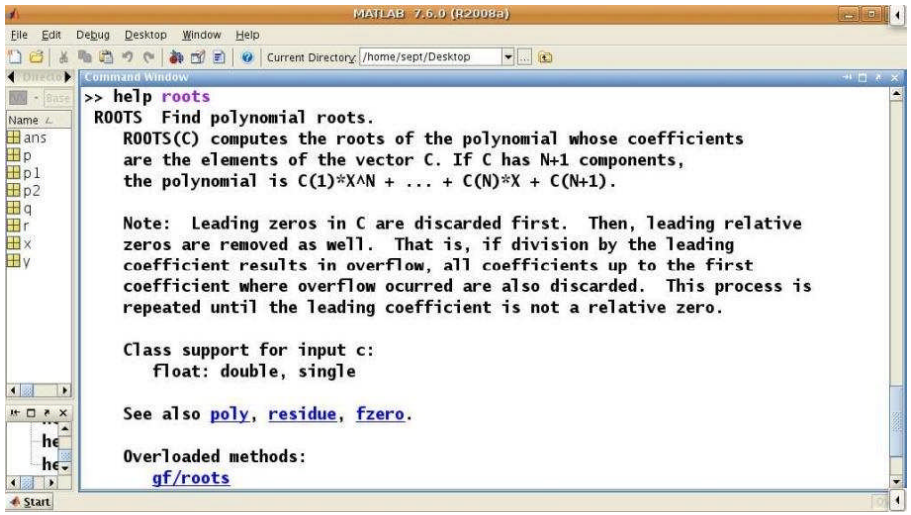
EXAMPLE:

- Given: $p(x) = x^3 - 6x^2 + 11x - 6$
- Find: Compute the zeros of the polynomial.

```
p = [1 -6 11 -6]; format long;
roots(p)
```

roots - Polynomial roots

Description: `r = roots(c)` returns a column vector whose elements are the roots of the polynomial **c**. Row vector c contains the coefficients of a polynomial, ordered in descending powers. If c has n+1 components, the polynomial it represents is $c_1 s^n + \ldots + c_n s + c_{n+1}$

Remarks

- Note the relationship of this function to `p = poly(r)`, which returns a row vector whose elements are the coefficients of the polynomial. For vectors, `roots` and `poly` are inverse functions of each other, up to ordering, scaling, and roundoff error.

Examples

$s^3 - 6s^2 - 72s - 27$ is represented in MATLAB as `p=[1 -6 -72 -27]`

The roots of polynomial are returned in a column vector by `r = roots(p)`

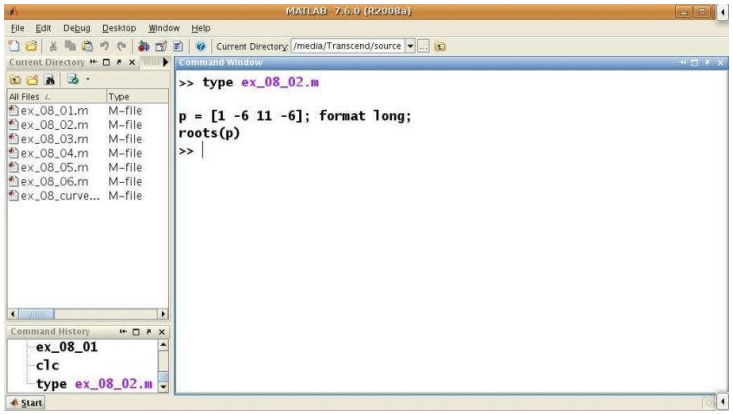
```
r =
    12.1229
    -5.7345
    -0.3884
```

POLYNOMIALS

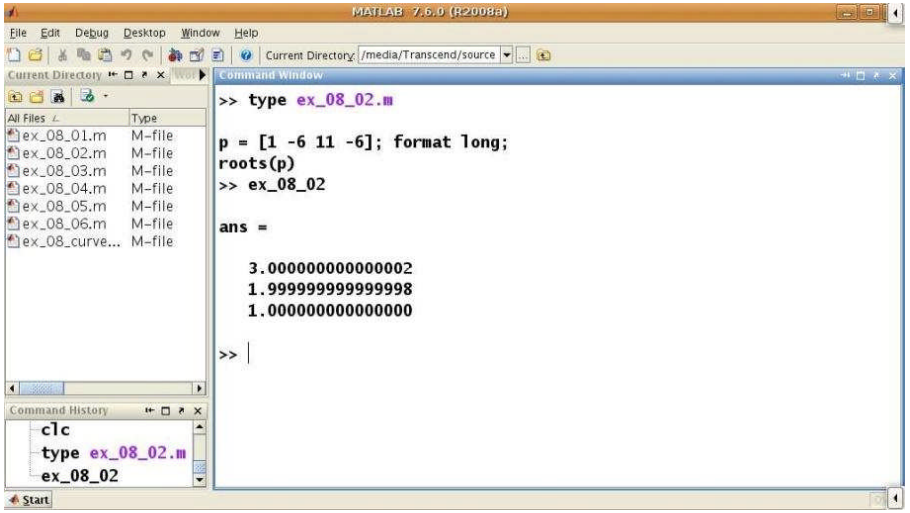
EXAMPLE:

- Given: $p(x) = x^3 - 6x^2 + 11x - 6$
- Find: Compute the zeros of the polynomial.

```
p = [1 -6 11 -6]; format long;
roots(p)
```

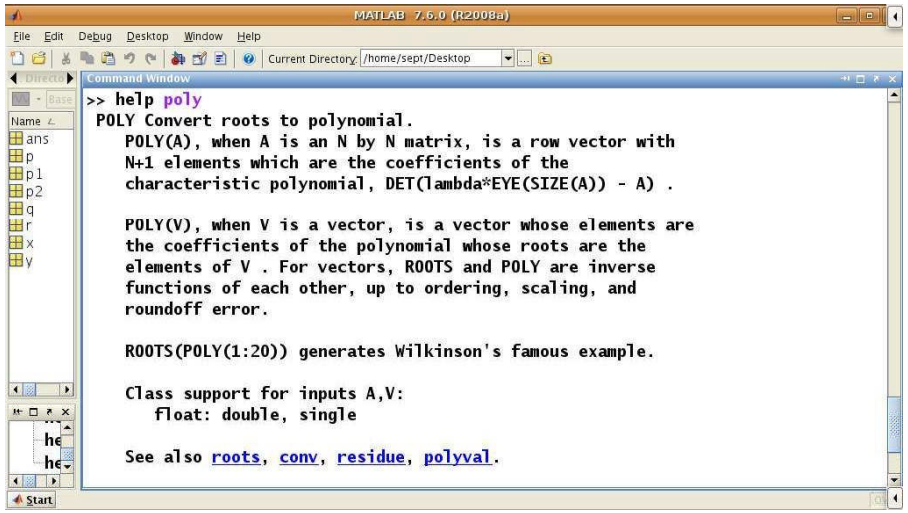


POLYNOMIALS



POLYNOMIALS

EXAMPLE: poly



POLYNOMIALS

poly - Polynomial with specified roots

Syntax `p = poly(A)`
`p = poly(r)`

Description

- `p = poly(A)` where A is an n-by-n matrix returns an n+1 element row vector whose elements are the coefficients of the characteristic polyn., **det** *(sI-A)* . The coefficients are ordered in descending powers: if a vector c has n+1 components, the polynomial it represents is $c_1 s^n + \ldots + c_n s + c_{n+1}$
- `p = poly(r)` where r is a vector returns a row vector whose elements are the coefficients of the polynomial whose roots are the elements of r.

- **Remarks**

  Note the relationship of this command to `r = roots(p)` which returns a column vector whose elements are the roots of the polynomial specified by the coefficients row vector p. For vectors, `roots` and `poly` are inverse functions of each other, up to ordering, scaling, and roundoff error.

$\det(sI - A)$

POLYNOMIALS

- **Example_1**

  A =

  1 2 3
  4 5 6
  7 8 0

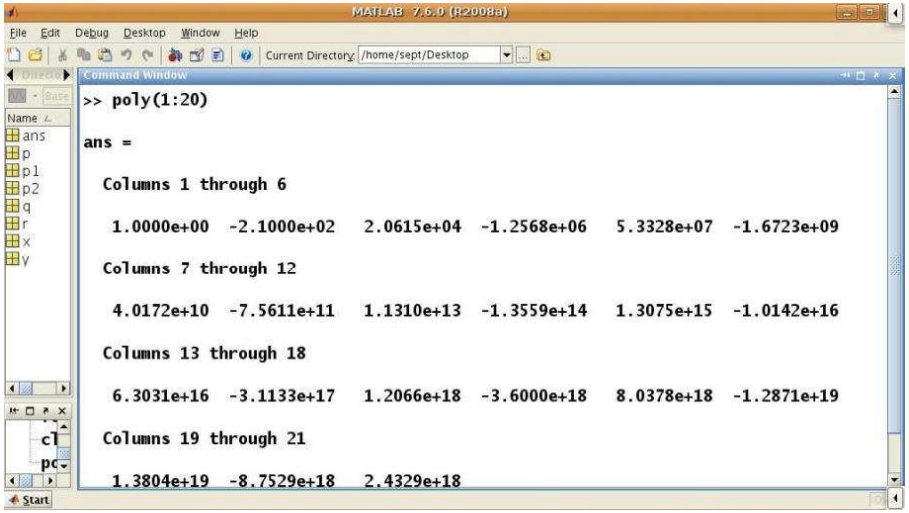
  is returned in a row vector by poly: `p = poly(A)`

```
p =
     1    -6    -72    -27
```

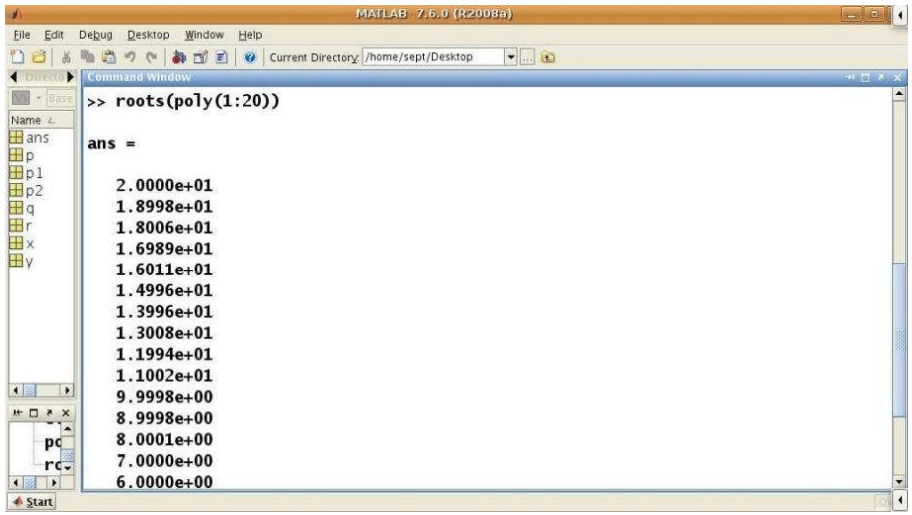
- The roots of this polynomial (eigenvalues of matrix A) are returned in a column vector by roots: `r = roots(p)`

```
r =
   12.1229
   -5.7345
   -0.3884
```


POLYNOMIALS



POLYNOMIALS



POLYNOMIALS

EXAMPLE:

The result is not always accurate.

Given: $p(x) = (x + 1)^7$

Find: Compute the zeros of the polynomial.

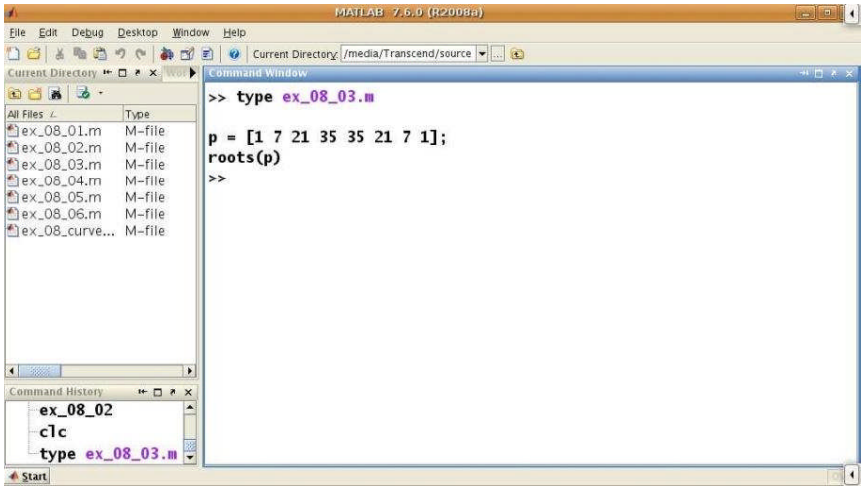
Answer: $\alpha = -1$

- In fact, numerical methods for the computation of the polynomial roots with multiplicity larger than one are particularly subject to roundoff errors.

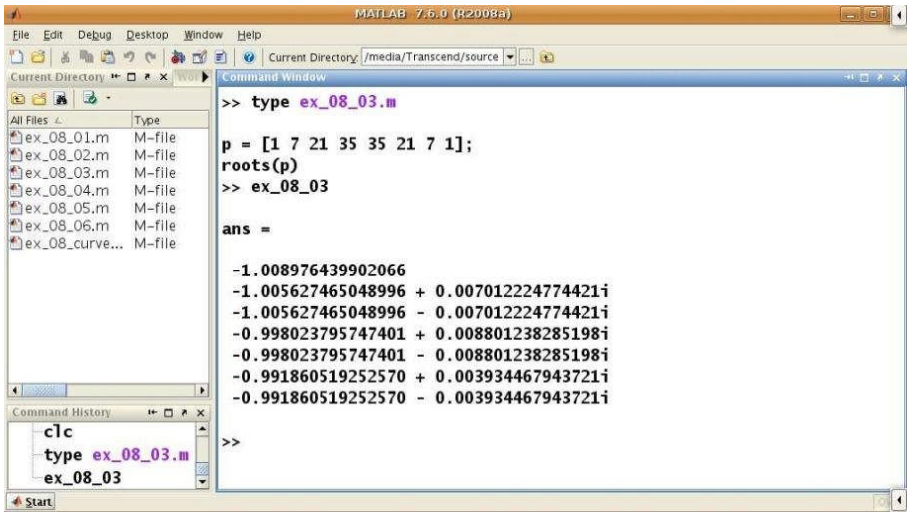
POLYNOMIALS

EXAMPLE cont'd.:

```
p = [1 7 21 35 35 21 7 1]; roots(p)
```



POLYNOMIALS



POLYNOMIALS

ADDITION AND SUBTRUCTION OF POLYNOMIALS

- Polynomial addition and subtraction is the same as vector addition/subtraction operators.
- If the order of two polynomials (size of two vectors) does not match, zero should be added in order to match size of vectors.

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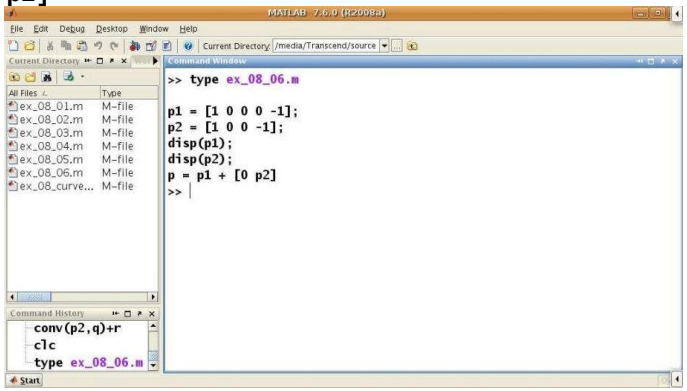
EXAMPLE:

- Given: $p_1(x) = x^4 - 1$,
  $p_2(x) = x^3 - 1$
- Find: Compute the sum of two polynomials.
- Answer: $p = x^4 + x^3 - 2$

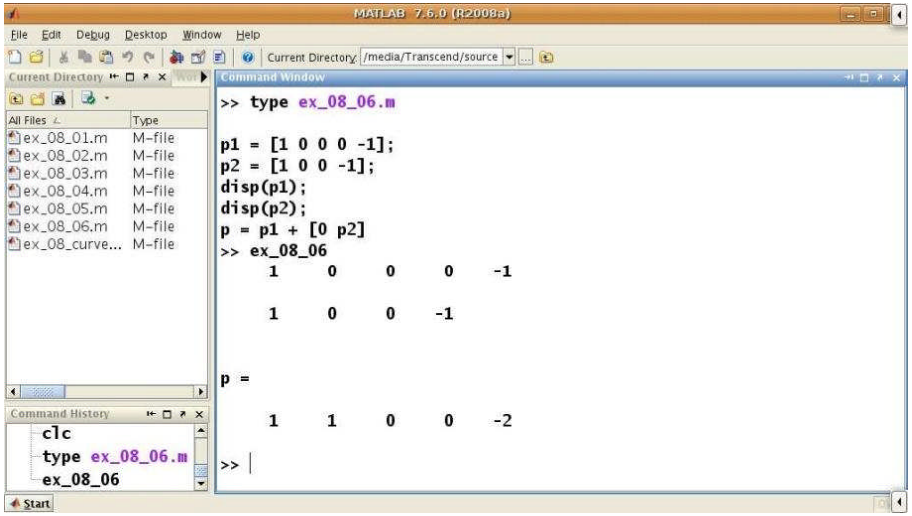
POLYNOMIALS

EXAMPLE cont'd.:

```
p1 = [1 0 0 0 -1];
p2 = [1 0 0 -1];
disp(p1);
disp(p2);
p = p1 + [0 p2]
```



POLYNOMIALS



POLYNOMIALS

MULTIPLICATION AND DIVISION OF POLYNOMIALS

- conv: returns the coefficients of the polynomial given by the product of two polynomials.

  Usage: **`conv(p1, p2)`**

- deconv: provides the coefficients of the polynomials obtained on dividing p1 by p2.

  Usage: **`[q, r]= deconv(p1, p2)`**

  q: quotient of the division,

  r: remainder of the division

$p_1(x) = q(x)\, p_2(x) + r(x)$

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  q: quotient of the division, (bölüm,oran)

  r: remainder of the division (kalanı)

$p_1(x) = q(x)\, p_2(x) + r(x)$

EXAMPLE:

Given: $p_1(x) = x^4 - 1$, $p_2(x) = x^3 - 1$

Find: Compute the product of two polynomials.

Answer: $p = x^7 - x^4 - x^3 + 1$

POLYNOMIALS

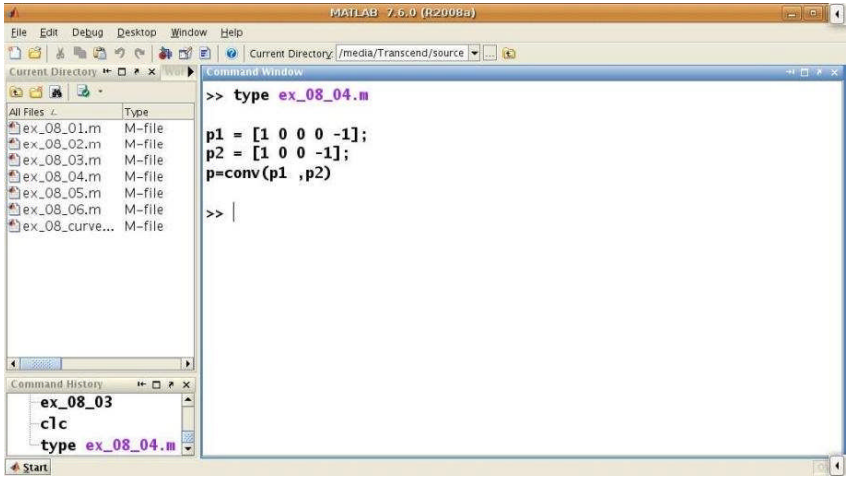
```
>> help conv
 CONV Convolution and polynomial multiplication.
    C = CONV(A, B) convolves vectors A and B.  The resulting
    vector is length LENGTH(A)+LENGTH(B)-1.
    If A and B are vectors of polynomial coefficients, convolving
    them is equivalent to multiplying the two polynomials.

    Class support for inputs A,B:
       float: double, single

    See also deconv, conv2, convn, filter and, in the signal
    Processing Toolbox, xcorr, convmtx.

    Overloaded methods:
       gf/conv

    Reference page in Help browser
       doc conv
```

Katlanma, büklüm

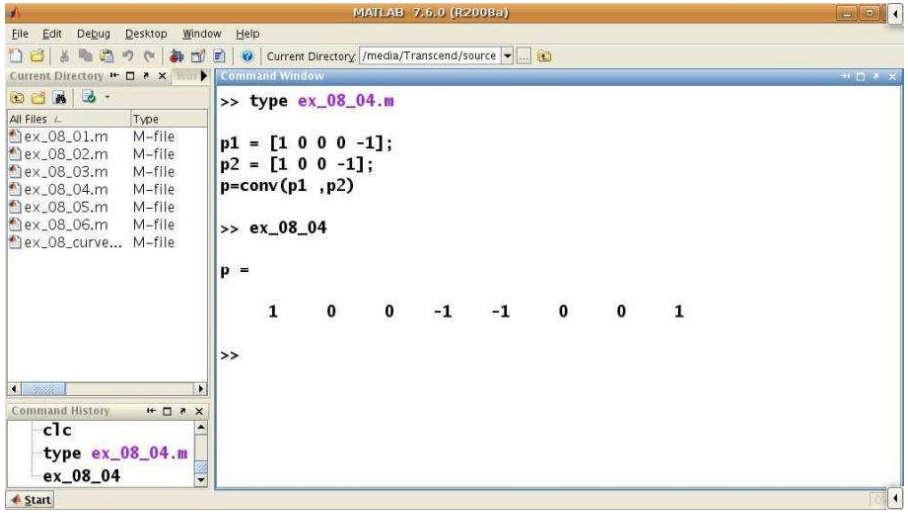
POLYNOMIALS

EXAMPLE cont'd.:

```
p1 = [1 0 0 0 -1];
p2 = [1 0 0 -1];
p=conv(p1 ,p2)
```



POLYNOMIALS



POLYNOMIALS

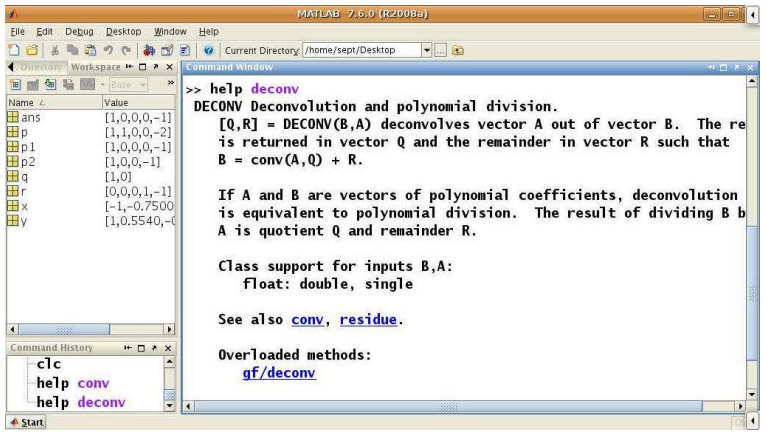
EXAMPLE:

Given: $p_1(x) = x^4 - 1$,

$p_2(x) = x^3 - 1$

Find: Compute the division of two polynomials.

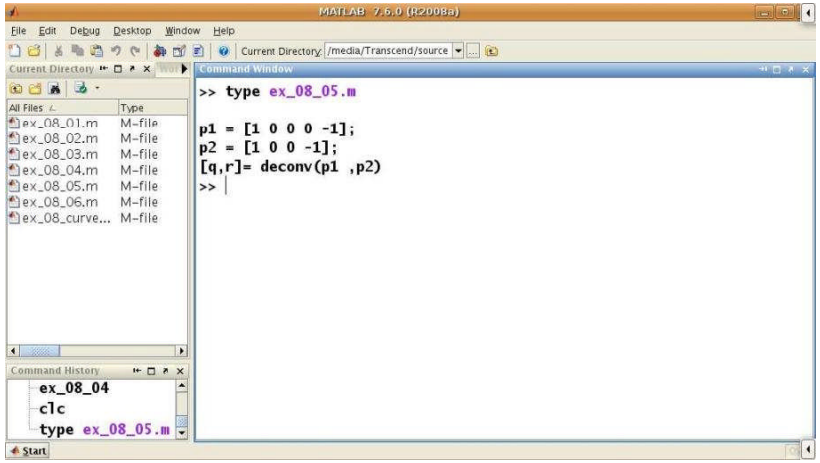
Answer: q(x) = x , r (x) = x − 1



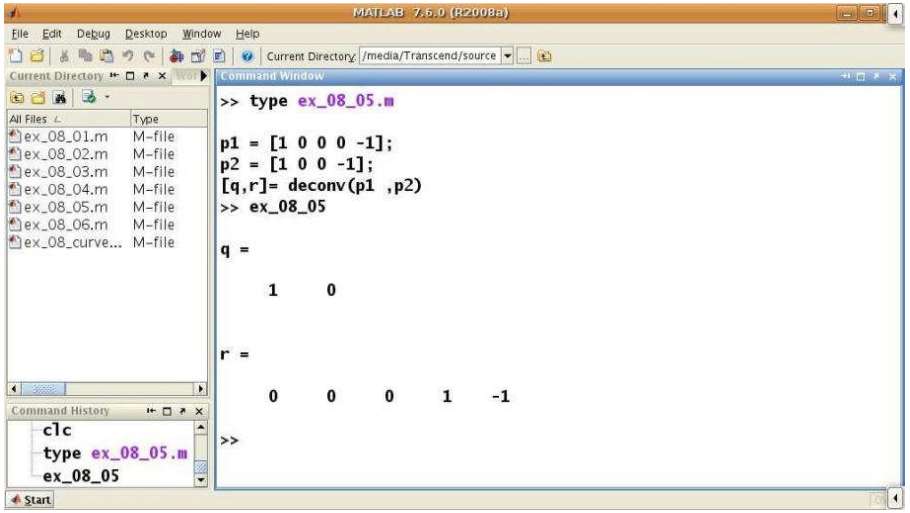
POLYNOMIALS

EXAMPLE cont'd.:

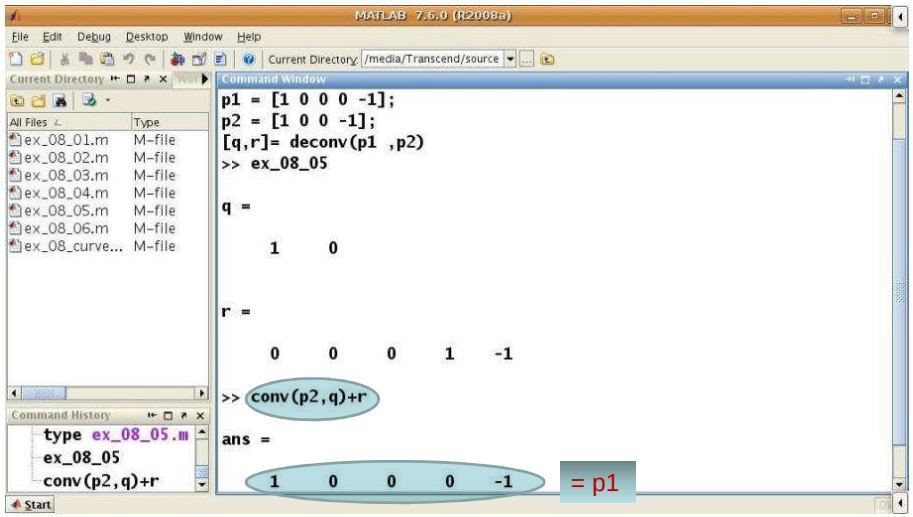
```
p1 = [1 0 0 0 -1];
p2 = [1 0 0 -1];
[q,r]= deconv(p1 ,p2)
```



POLYNOMIALS



POLYNOMIALS



POLYNOMIALS

- **`polyint`**: returns the coefficients of the primitive of the polynomial.
  Usage: **`y = polyint(p)`**,
  y: coefficients of $\int_0^x p(t)\,dt$.

  Türetilmemiş

- **`polyder`**: returns the derivative of the polynomial, whose coefficients are given by the components of the vector p.
  Usage: **`y = polyder(p)`**,
  y: coefficients of $p'(x)$

CURVE FITTING

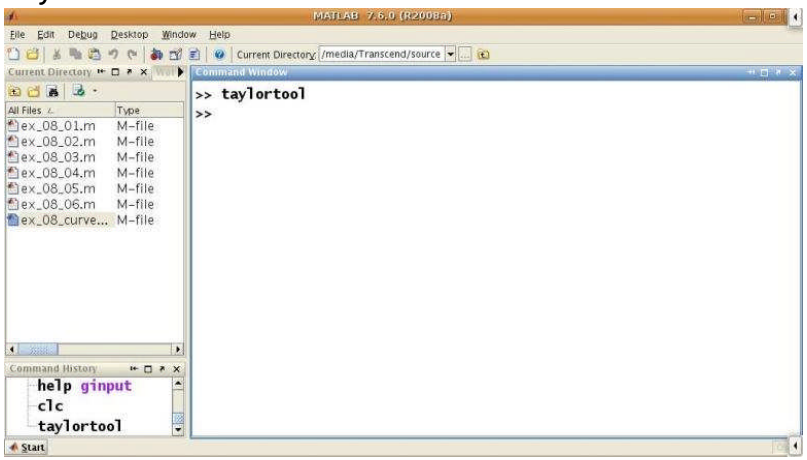
- Approximating a function f consists of replacing it by another func. f.
- A function f can be replaced in a given interval by its Taylor polynomial.
- It requires the knowledge of f and its derivatives up to the order n at a given point x_0.

CURVE FITTING

- Approximating a function f consists of replacing it by another func. f.
- A function f can be replaced in a given interval by its Taylor polynomial.
- It requires the knowledge of f and its derivatives up to the order n at a given point x_0.
- Taylor polynomial may fail to accurately represent f far enough from the point x_0.
- Use taylortool for the computation of Taylor's polynomial of arbitrary degree for any given function f .
- The agreement between the function and its Taylor polynomial is very good in a small neighborhood of x_0.

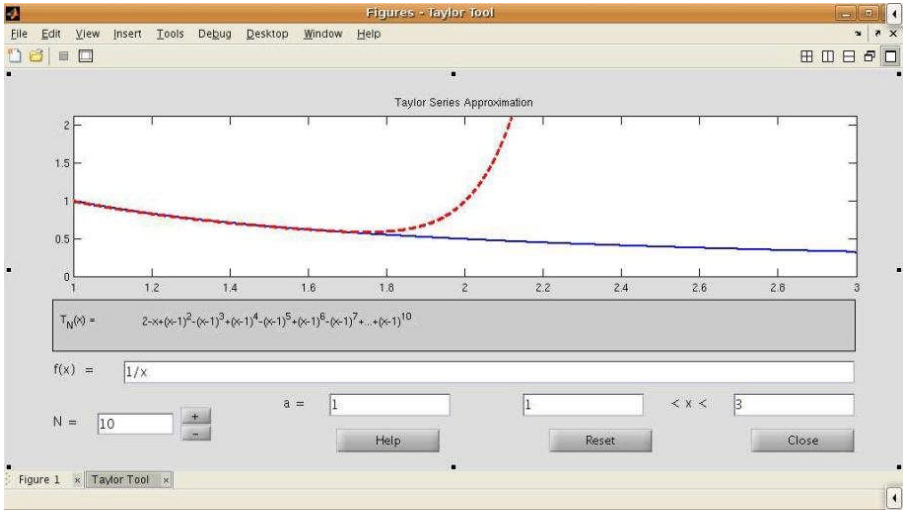
CURVE FITTING

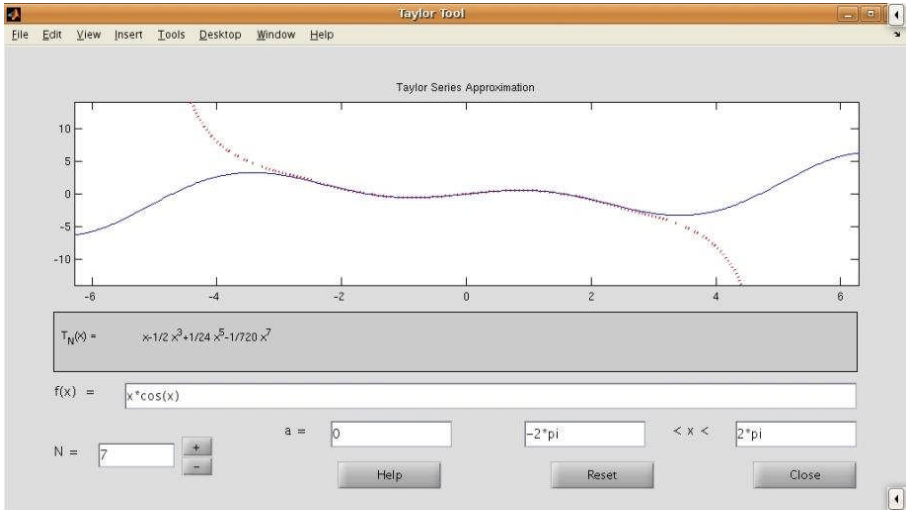
Taylortool



Comparison between the function f (x) = 1/x and its Taylor polynomial of degree 10 for the point $x_0 = 1$.

CURVE FITTING



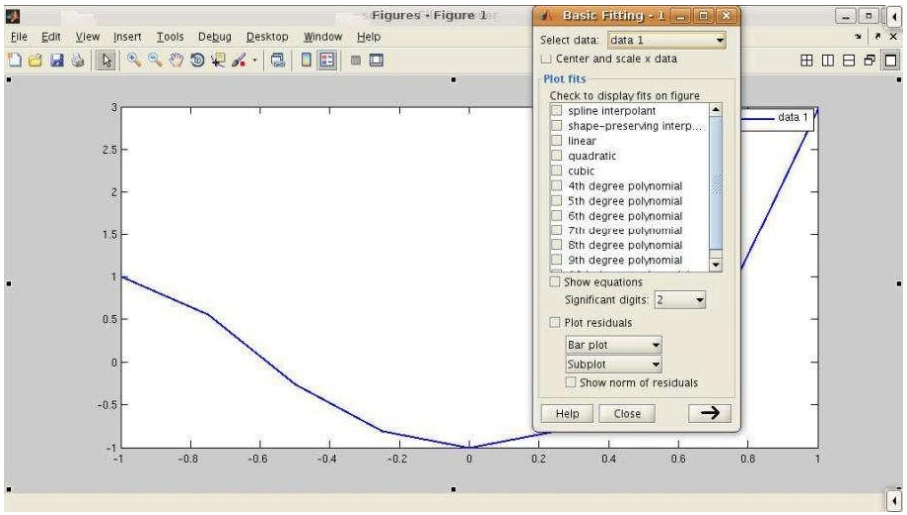


CURVE FITTING

- A function is known only through its values at some given points.
- n+1 couples (x_i,y_i) are given, i = 0,1,2,...,n+1.
- Approximate function $\tilde{f}$:

  $\tilde{f}(x_i) = y_i$, where i = 0, 1, 2, . . . , n
- $\tilde{f}$ is called *interpolant* of the set of data
- There exist different types of interpolant:
  - Polynomial interpolant
  - Trigonometric interpolant
  - Rational interpolant

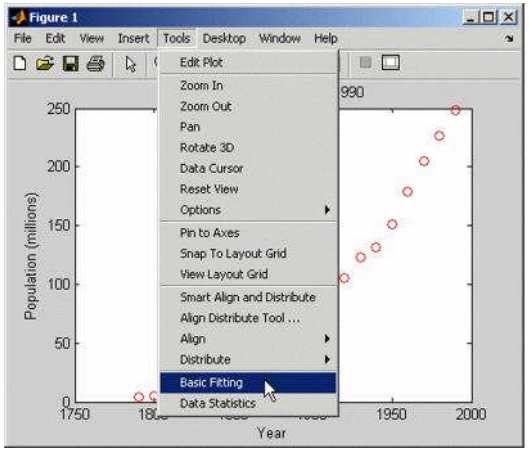
BASIC FITTING:



Opening the Basic Fitting GUI

To use the Basic Fitting GUI, you must first plot your data in a figure window, using any MATLAB plot commands that produces (only) x & y.

- To open the Basic Fitting GUI, select Tools > Basic Fitting from the menus at the top of the figure window.

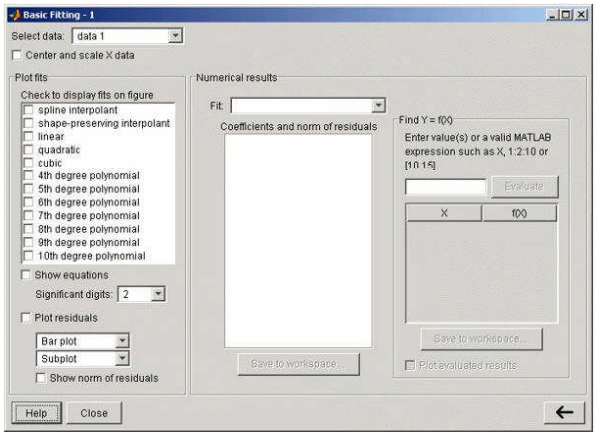


CURVE FITTING

To expand or collapse the panels, use the arrow button in the lower right corner of the interface.

The GUI consists of 3 panels:

1. For selecting a model and plotting options
2. For examining and exporting model coefficients and norms of residuals
3. For examining and exporting interpolated and extrapolated values.



CURVE FITTING

- The most common method of finding the best fit to data point is the *least squares method*.

- **`polyfit`** function uses least squares method.
- **`polyfit`** function returns the coefficients of a polynomial for a given data set (x_i , y_i).
- Usage: **`p = polyfit(x, y, n)`**

  n : degree of the polynomial.

  x and y : data set (x_i , y_i).

CURVE FITTING

EXAMPLE

- In the table below we report the values of the sea water density ρ *(in kg /m^3)* corresponding to different values of the temperature T (in degrees Celsius):

T	4°	8°	12°	16°	20°
ρ	1000.7794	1000.6427	1000.2805	999.716	998.9700

CURVE FITTING

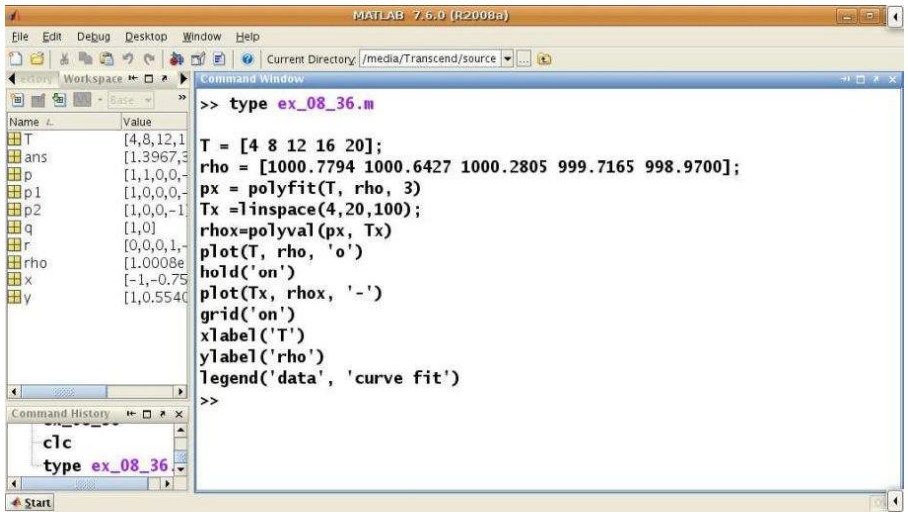
EXAMPLE

- In the table below we report the values of the sea water density ρ *(in kg /m^3)* corresponding to different values of the temperature T (in degrees Celsius):

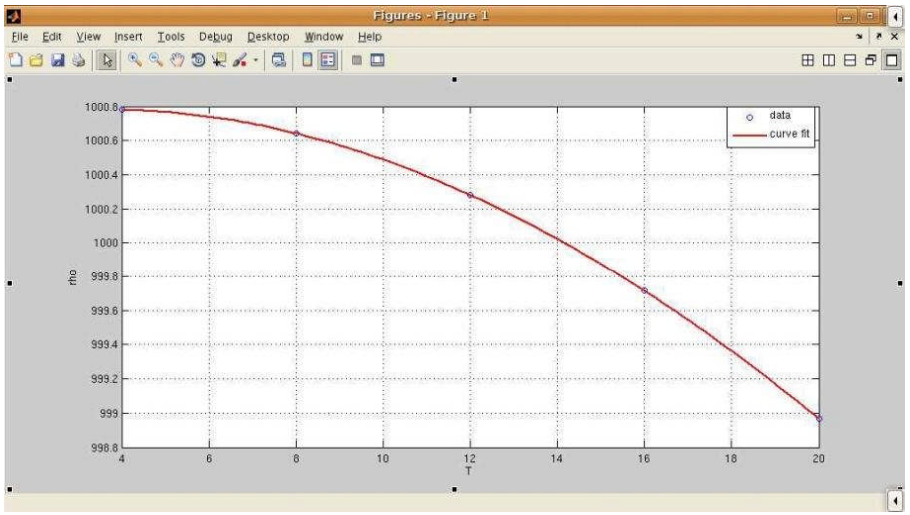
T	4°	8°	12°	16°	20°
ρ	1000.7794	1000.6427	1000.2805	999.716	998.9700

```
T =[4 8 12 16 20];
rho =[1000.7794 1000.6427 1000.2805 999.7165 998.9700];
px = polyfit(T, rho, 3)   % [ x , y , degree ]
Tx = linspace(4,20,100); % [4 , 4.1616 ,...., 20]
rhox = polyval(px, Tx)    % px func. At points Tx
plot(T, rho, 'o')         % plot T versus rho with 'o' sign.
hold('on')
plot(Tx, rhox, '-')
grid('on')
xlabel('T')
ylabel('rho')
legend('data', 'curve fit')
```

CURVE FITTING



CURVE FITTING



CURVE FITTING

LEAST SQUARES METHOD

- If the degree of the polynomial increases, interpolation does not guarantee a better approximation of a given function.
- In least - squares approximation we look for an approximant f which is a polynomial of degree m (typically, m << n) that minimizes the mean-square error $1/n \sum_{i=0}^{n} [y_i - \bar{f}(x_i)]^2$. The same minimization criterion can be applied for a class of functions that are not polynomials.

CURVE FITTING

CURVE FITTING WITH FUNCTIONS OTHER THAN POLYNOMIALS

- Rewrite the function in a first degree polynomial form.
- Power function:
  $y = b\,x^m \quad \rightarrow \quad \ln y = m \ln x + \ln b$
- Exponential function:
  $y = b\,e^{mx} \quad \rightarrow \quad \ln y = m x + \ln b$
- Logarithmic function:
  $y = m \ln x + b \rightarrow \quad y = m \ln x + b$
- Reciprocal function:
  $y = 1/(m x + b) \rightarrow \quad 1/y = m x + b$

CURVE FITTING

FUNCTION SELECTION

- For a given data it is possible to foresee which of the functions have the potential for providing a good fit.
- This is done by plotting the data using different combinations of linear and logarithmic axes.

x-axis	y-axis	function
linear	linear	$y = m\,x + b$
logarithmic	logarithmic	$y = b\,x^m$
linear	logarithmic	$y = b\,e^{mx}$
logarithmic	linear	$y = m \ln x + b$
linear	linear	$y = 1/(m\,x + b)$

CURVE FITTING

FUNCTION SELECTION

- Exponential functions can not pass through the origin.
- Exponential functions can only fit data with all positive y 's or all negative y 's.
- Logarithmic functions cannot model $x = 0$, or negative values of x .
- For the power function $y = 0$ when $x = 0$.
- The reciprocal equation cannot model $y = 0$.

CURVE FITTING

EXAMPLE:

w	0.64	0.73	0.96	1.21	1.49	1.83	2.41	3.15	3.70	4.83	6.00
t	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0

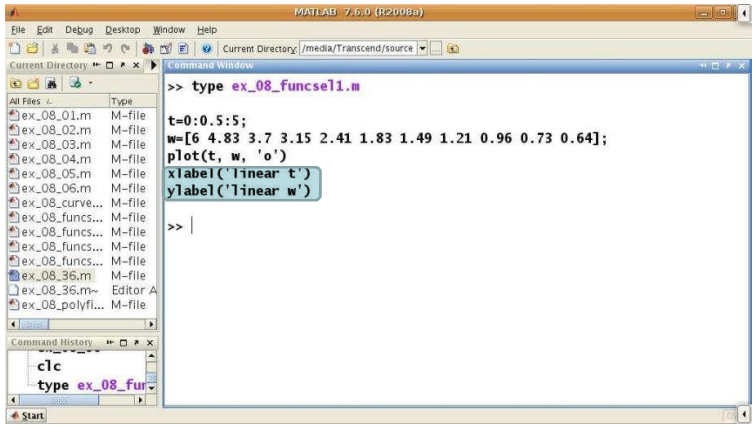
Choose the function for the given data.

CURVE FITTING

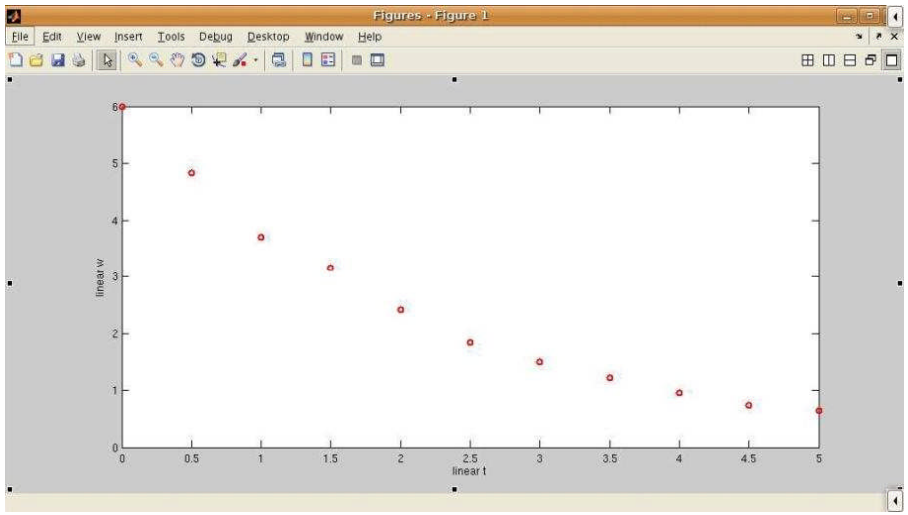
EXAMPLE:

w	0.64	0.73	0.96	1.21	1.49	1.83	2.41	3.15	3.70	4.83	6.00
t	5.0	4.5	4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.0

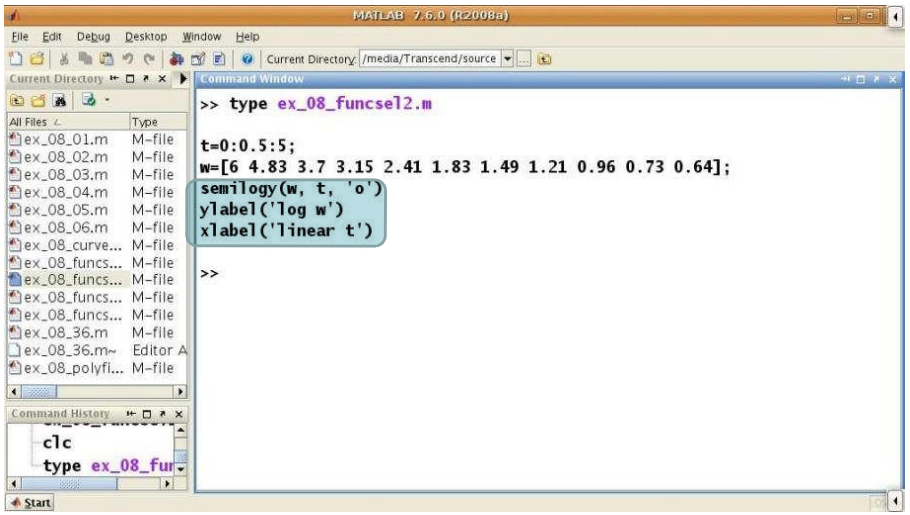
Choose the function for the given data. (in revers order)



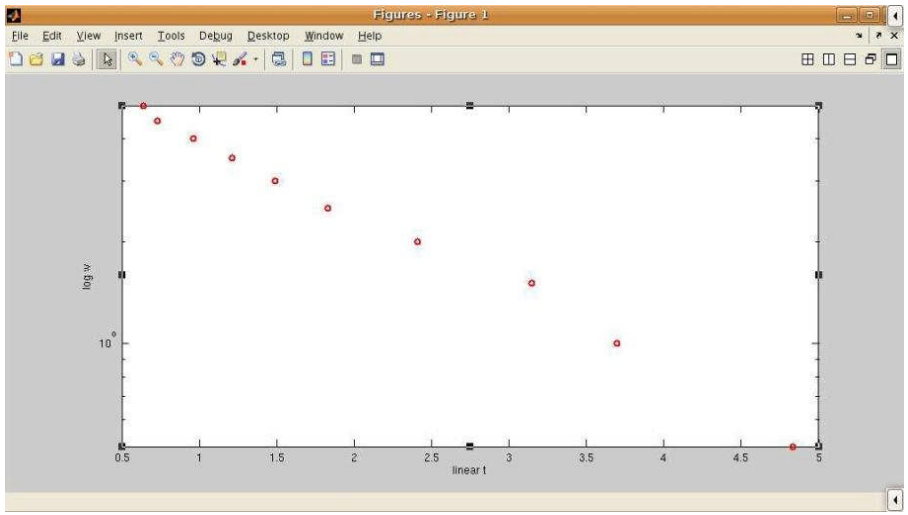
CURVE FITTING



CURVE FITTING



CURVE FITTING



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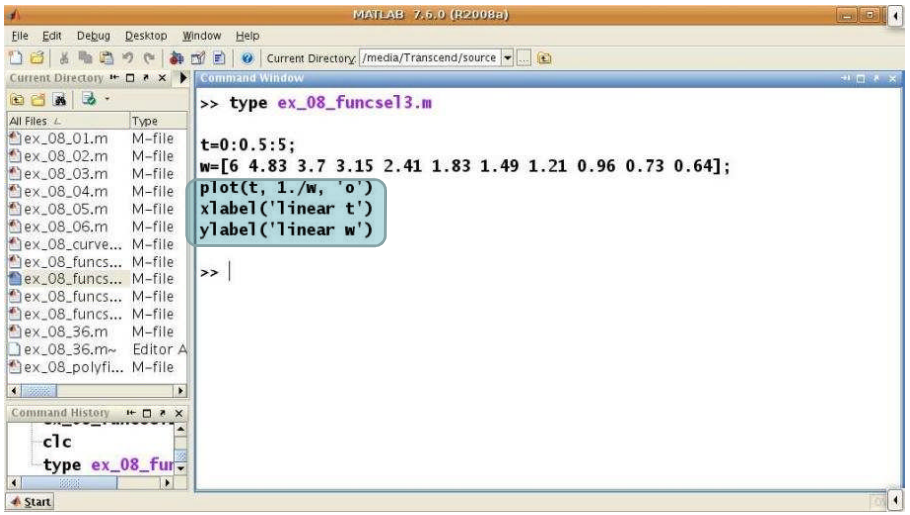
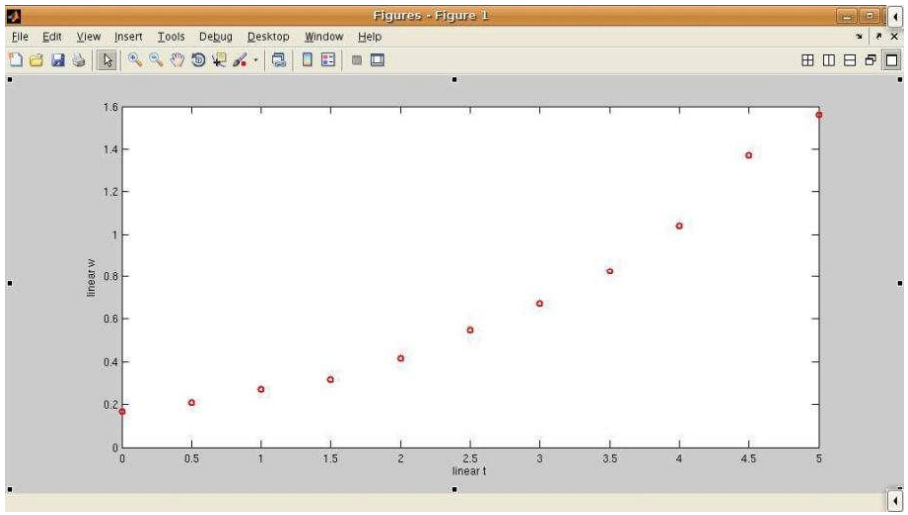


CURVE FITTING



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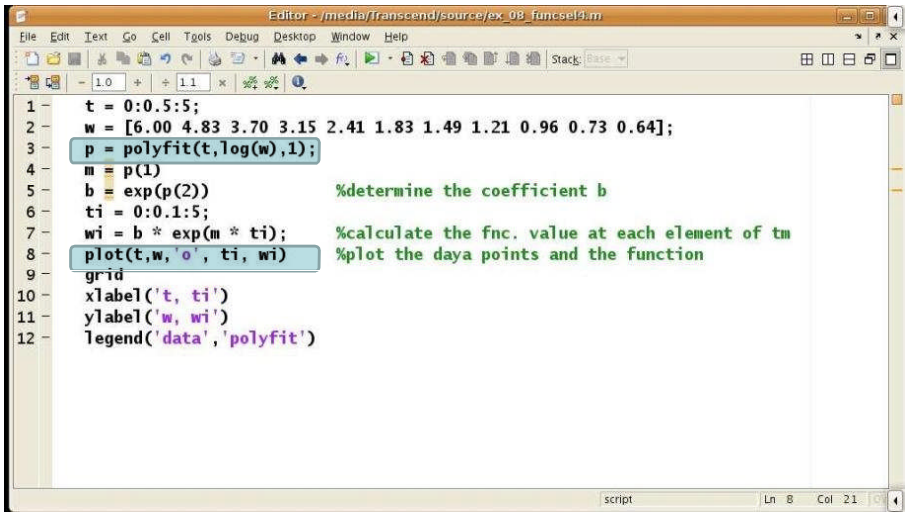
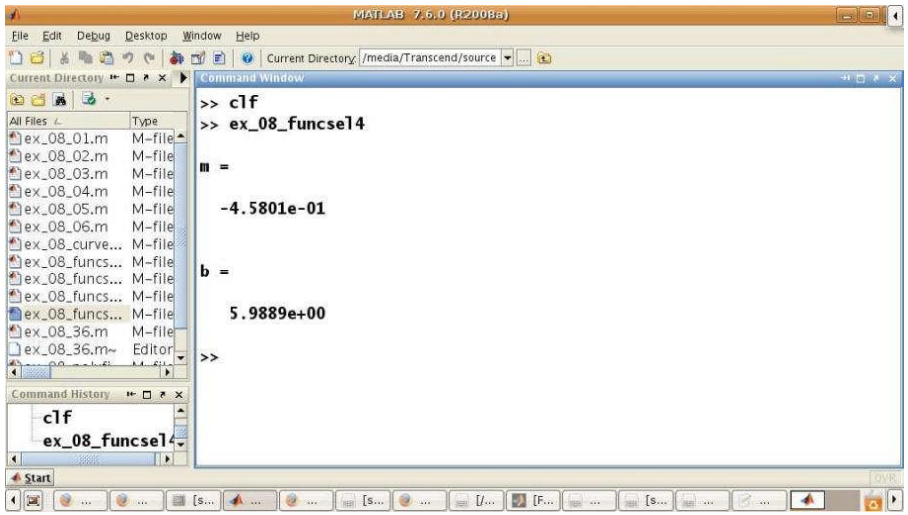
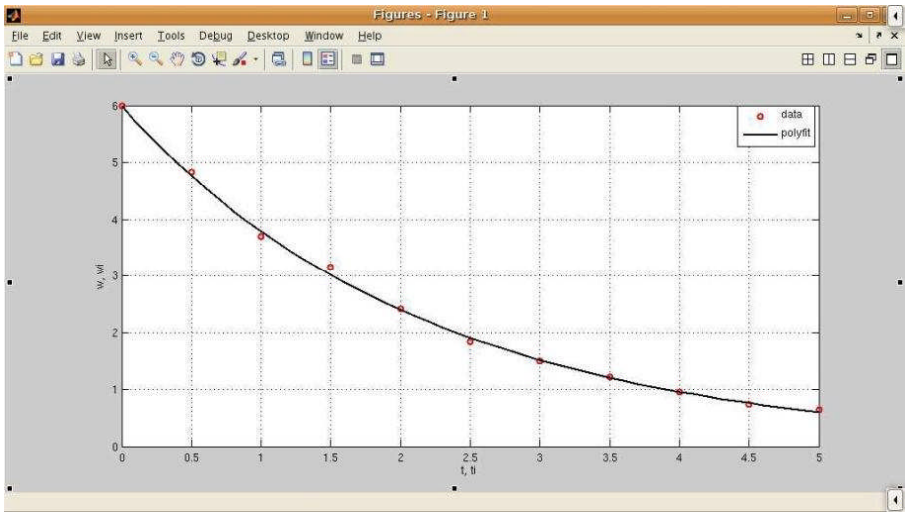


CURVE FITTING



CURVE FITTING



CURVE FITTING

EXAMPLE:

- The price (in euros) of a magazine has changed as follows:

Nov.87	*Dec.88*	*Nov.90*	*Jan.93*	*Jan.95*	*Jan.96*	*Nov.96*	*Nov.00*
4.5	5.0	6.0	6.5	7.0	7.5	8.0	8.0

Estimate the price in November 2002 by extrapolating these data.

CURVE FITTING

EXAMPLE:

	δ_K			
Latitude	$K = 0.67$	$K = 1.5$	$K = 2.0$	$K = 3.0$
65	-3.1	3.52	6.05	9.3
55	-3.22	3.62	6.02	9.3
45	-3.3	3.65	5.92	9.17
35	-3.32	3.52	5.7	8.82
25	-3.17	3.47	5.3	8.1
15	-3.07	3.25	5.02	7.52
5	-3.02	3.15	4.95	7.3
-5	-3.02	3.15	4.97	7.35
-15	-3.12	3.2	5.07	7.62
-25	-3.2	3.27	5.35	8.22
-35	-3.35	3.52	5.62	8.8
-45	-3.37	3.7	5.95	9.25
-55	-3.25	3.7	6.1	9.5

- Variation of the average yearly temperature on the Earth for four different values of the concentration K of carbon acid at different latitudes.
  - Compute the least-squares polynomial of degree 4 that approximates the values of K reported in the Table given above.

References for Week 8

1 Alfio Quarteroni, Fausto Saleri, Scientific Computing with Matlab and Octave, Springer, 2006.

2 Brian Hahn, Daniel T.Valentine, Essential Matlab for Engineers and Scientists, Elsevier, 2010.

3 http://www.mathworks.com/help/techdoc/ref/ f16-5872.html#f16-6325

Have a nice Week End