



INTRODUCTION TO SCIENTIFIC & ENG.
COMPUTING
BIL 108E, CRN 44448



Week.4_ Algorithms and Logic Operators

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04-03-2011



LECTURE # 4

- 1 NUMERICAL DATA TYPES IN MATLAB
- 2 ERRORS
- 3 LOOPS
- 4 ALGORITHMS
- 5 DATA ANALYSIS

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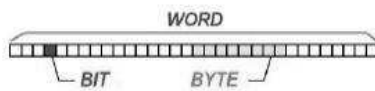


BITS, BYTES AND WORDS

VARIABLES ARE REPRESENTED BY WORDS,
COMPOSED OF BYTES,
COMPOSED OF BITS

- BIT = elemental circuit, ON (1) / OFF (0)
- BYTE = string of 8 BITS
- WORD = string of N BYTES

(partially controllable by the programmer)



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BITS, BYTES AND WORDS

| base 10 | conversion | base 2 |
|---------|--|-----------|
| 1 | $1 = 2^0$ | 0000 0001 |
| 2 | $2 = 2^1$ | 0000 0010 |
| 4 | $4 = 2^2$ | 0000 0100 |
| 8 | $8 = 2^3$ | 0000 1000 |
| 9 | $8 + 1 = 2^3 + 2^0$ | 0000 1001 |
| 10 | $8 + 2 = 2^3 + 2^1$ | 0000 1010 |
| 27 | $16 + 8 + 2 + 1 = 2^4 + 2^3 + 2^1 + 2^0$ | 0001 1011 |

one byte = 8 bits

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NUMERICAL DATA TYPES IN MATLAB

NUMERICAL DATA TYPES IN MATLAB

- `int8(-128,127)`, `int16(-32768, 32767)`,
`int32(- 2 147 483 648, 2 147 483 647)`,
`int64(-9 223 372 036 854 775 808,`
`9 223 372 036 854 775 807)`,
`uint8(0, 255)`, `uint16(0, 65535)`, `uint32(0, 4 294 967 295)`,
`uint64(0, 18 446 744 073 709 551 615)`
- `single(2-126, 3.4 × 1038)`
- `double(2.2251 × 10-308, 1.7977 × 10308)`

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NUMERICAL DATA TYPES IN MATLAB

REAL NUMBERS,
FLOATING-POINT NUMBERS

Significant digits × base^{exponent}

- Real numbers, \mathbb{R} .
- Floating-point numbers, \mathbb{F} .

Only a subset \mathbb{F} of finite dimension \mathbb{R} can be represented.

Any real number x is truncated by the machine as $fl(x)$.

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NUMERICAL DATA TYPES IN MATLAB

FLOATING-POINT NUMBERS (FPU) Kayan noktalı sayılar

- Numeric values with non-zero fractional parts are stored as floating point numbers.
- All floating point values are represented with a normalized scientific notation.



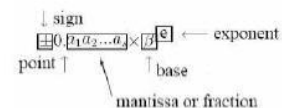
NUMERICAL DATA TYPES IN MATLAB

FLOATING-POINT NUMBERS

Floating-point number representation of a real number

$$x = (-1)^s \times (0.a_1a_2a_3 \dots a_t) \times \beta^e$$

$a_1 \neq 0$



EXAMPLES:

$$12.7887 = 0.127887 \times 10^2 \text{ (base 10)}$$

$$-0.099 = -0.99 \times 10^{-1} \text{ (base 10)}$$

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DIGITAL STORAGE OF INTEGERS

- Integers can be exactly represented by base 2
- Typical size is 16 bits
- 32 bit and larger integers are available

Note: All standard mathematical calculations in Matlab use floating point numbers.



- Floating point values have fixed number of bits allocated for storage of the mantissa and fixed number of bits allocated for storage of the exponent.
- Two common precisions are provided in numerical computing: single precision and double precision.
- Fixed number of bits are allocated to each number: single precision uses 32 bits per floating point number and double precision uses 64 bits per floating point number



Total number of bits are split into separate storage for both the mantissa and the exponent.

(Bir sayının logaritmasının ondalık bölümü)

- single precision: 1 sign bit, 8 bit exponent, 23 bit mantissa



- double precision: 1 sign bit, 11 bit exponent, 52 bit mantissa



- Limiting the number of bits allocated for storage of the exponent means that there are upper and lower limits on the magnitude of floating point numbers
- Limiting the number of bits allocated for storage of the mantissa means that there is a limit to the precision (number of significant digits) for any floating point number.



- 1 PP Physical Problem
- 2 MP Mathematical Problem
- 3 NP Numerical Problem

Each of these steps involve errors.



```

MATLAB 7.6.0 (R2008a)
File Edit Debug Desktop Window Help
Current Directory /media/Transcend/source
Command Window
New to MATLAB? Watch this Video, see Demos, or read Getting Started.
>> type ex_04_01
x = 1.e-15;
((1 + x) - 1) / x
>> ex_04_01
ans =
    1.1102
>> |
Command History
ex_04_01
clc
type ex_04_01
ex_04_01
Start: Click and drag to move Command Window.
    
```



EXAMPLE:

Computational Errors

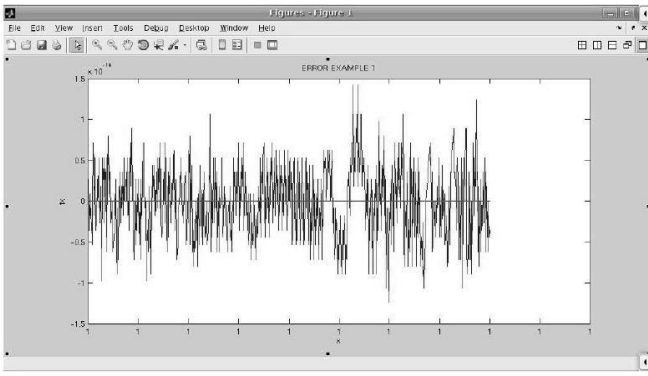
$$f(x) = (x - 1)^7$$

$$f(x) = x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1$$



```

MATLAB 7.6.0 (R2008a)
File Edit Debug Desktop Window Help
Current Directory /media/Transcend/source
Command Window
New to MATLAB? Watch this Video, see Demos, or read Getting Started.
>> type ex_04_02
x = linspace(1-2e-8, 1+2e-8, 801);
% x_min = 0.999999998
% x_max = 1.000000002
fx = x.^7 - 7*x.^6 + 21*x.^5 - 35*x.^4 + 35*x.^3 - 21*x.^2 + 7*x - 1;
plot(x, fx), title('ERROR EXAMPLE 1')
xlabel('x')
ylabel('fx')
>> |
Command History
type ex_04_02
ex_04_02
clc
type ex_04_02
Start: Click and drag to move Command Window.
    
```



EXAMPLE:

Calculation of π

$$z_2 = 2, z_{n+1} = 2^{n-1/2} \sqrt{1 - \sqrt{1 - 4^{1-n} z_n^2}}$$

$$n = 2, 3, \dots$$

EXAMPLE:

Calculation of π

$$z_2 = 2, z_{n+1} = 2^{n-1/2} \sqrt{1 - \sqrt{1 - 4^{1-n} z_n^2}}$$

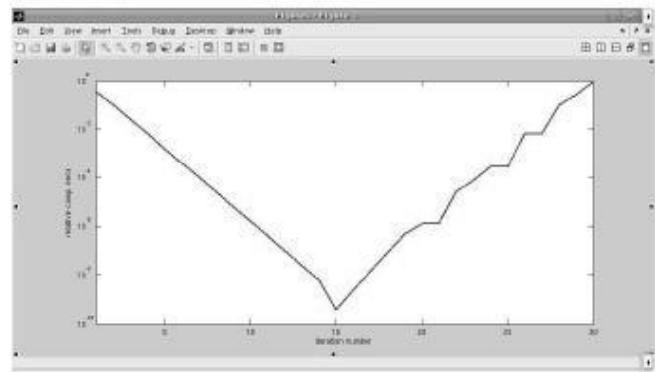
$$n = 2, 3, \dots$$

```

>> type ex_04_10

z=zeros(1,30);
z(1)=2;
for n = 2:1:30
    z(n)=2^(n-0.5)*sqrt(1-sqrt(1-4^(1-n)*z(n-1)^2));
end
semilogy([1:30], abs(z-pi)/pi)
ylabel('relative comp. error')
xlabel('iteration number')
axis([1, 30, 1e-10, 1])
    
```

EXAMPLE:



COMPUTATIONAL ERRORS

e_c : computational Error

x : exact solution of mathematical model

\hat{x} : numerical solution of mathematical model

Absolute Computational Error

$$e_c^{abs} = |x - \hat{x}|$$

Relative Computational Error

$$e_c^{rel} = |x - \hat{x}|/|x|$$

ERRORS RESULTING FROM PROBLEMS

- SYNTAX ERRORS
- LOGIC ERRORS
- ROUND OFF ERRORS

ERRORS RESULTING FROM PROBLEMS

- SYNTAX ERRORS
- LOGIC ERRORS
- ROUND OFF ERRORS

SYNTAX ERRORS

- Typo errors.
- Incompatible vector sizes.
- Name hiding (try "help command").

LOGIC ERRORS

- Try to run the program for some special cases where you know the answer.
- If you don't know any exact answer, use your insight to check whether the answer seems to be of the right order of magnitude.
- Try working through the program by hand to see if you can spot where things start going wrong.

ROUNDING ERRORS

- Finite-precision leads round-off in individual calculations (Yuvarlama)
- Effects of round-off accumulate slowly
- The round-off errors are inevitable, solution is to create better algorithms
- Subtracting nearly equal may lead to severe loss of precision

MACHINE PRECISION

The magnitude of roundoff errors is quantified by machine precision ϵ_M

There is a number, ϵ_M such that

$$1 + \delta = 1$$

whenever $\delta < \epsilon_M$

In exact arithmetics, ϵ_M is identically 0.

$\text{eps} = 2.2204 \times 10^{-16}$ in Matlab

FLOATING-POINT NUMBERS

Roundoff-Error

$$\frac{|x - f(x)|}{|x|} \leq \frac{1}{2}\epsilon_M$$

Ref: Standard for Floating Point Arithmetic P754, IEEE.
 $\epsilon_M = \beta^{1-t}$, here t is the distance between 1 and its closest floating-point number greater than 1.

In Matlab ϵ_M is obtained through the command `eps`.
 Number 0 does not belong to \mathbb{F}

FLOATING-POINT NUMBERS

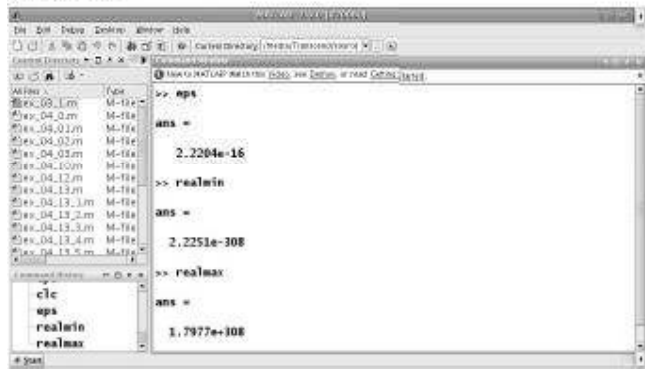
`realmin`, and `realmax`

If x is less than x_{min} is treated as 0, UNDERFLOW

If x is greater than x_{max} Inf OVERFLOW

The elements in \mathbb{F} are more dense near x_{min} , and less dense while approaching x_{max} .

EXAMPLE:



TRUNCATION ERROR

Example:

Consider the series for $\sin x$

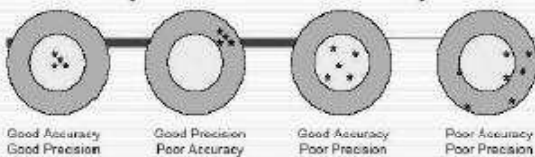
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

For small x , only a few terms are needed to get an accurate approximation to $\sin x$. The higher order terms are truncated.

$$f_{true} = f_{sum} + \text{truncation error}$$

The size of truncation error depends on x and the number of terms included in f_{sum}

Numbers: precision and accuracy



Numbers: precision and accuracy

- Low precision: $\pi = 3.14$
- High precision: $\pi = 3.140101011$
- Low accuracy: $\pi = 3.10212$
- High accuracy: $\pi = 3.14159$
- High accuracy & precision: $\pi = 3.141592653$

- 1 FOR statements
- 2 IF statements
- 3 SWITCH statements
- 4 WHILE statements

FLOW CONTROL

- 1 FOR statements
- 2 IF statements
- 3 SWITCH statements
- 4 WHILE statements

REPETITIVE TASKS

Loops are used for repetitive tasks.

Basic for Construct

The most common form of the loop is;

```
for index = j : k
    statements
end
```

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LOOPS

- $j : k$ is a vector with elements $j, j + 1, j + 2, \dots, k$.
- $j : m : k$ is a vector with elements $j, j + m, j + 2m, \dots$ such that the last element can not exceed k .
- $index$ must be a variable. Each time through the loop it will contain the next element of the vector $j : k$ or $j : m : k$.

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LOOPS

- $j : k$ is a vector with elements $j, j + 1, j + 2, \dots, k$.
- $j : m : k$ is a vector with elements $j, j + m, j + 2m, \dots$ such that the last element can not exceed k .
- $index$ must be a variable. Each time through the loop it will contain the next element of the vector $j : k$ or $j : m : k$.

$index = first : increment : last$

The number of times that the loop is executed is defined as iteration:

$iteration = \text{floor}(\frac{last - first}{increment}) + 1$ Here $\text{floor}(x)$ is a function, that rounds x down toward $-\infty$

This value is called iteration or trip count

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LOOPS

- On completion of the for loop the index contains the last value used.
- If the vector ($j : k$) or ($j : m : k$) is empty, statements are not executed and control passes to the statement following end.
- If the index does appear explicitly in statements, the for can often be vectorized. It runs faster.
- It is good programming style to indent (tabulate) the statements inside a for loop.

for in a single line

for index = j : k, statements, end

or

for index = j : m : k, statements, end

- Don't forget the commas.

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LOOPS

More general form of the for is

```
for index = v
```

Here v is any vector.

The index moves through each element of the vector.

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LOOPS

More general form of the for is

```
for index = v
```

Here v is any vector.

The index moves through each element of the vector.

EXAMPLES:

```
% display vector elements from 1 to i
x = [1 : 7];
for i = 1:7
    disp(x(1:i))
end
```

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LOOPS

EXAMPLE:

```
>> type ex_04_0
% display vector elements from 1 to i
x = [1 : 7];
for i = 1:7
    disp(x(1:i))
end
>>
```

```
type ex_04_0
ex_04_0
clc
type ex_04_0
```

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LOOPS

EXAMPLE:

```
>> ex_04_0
1
1 2
1 2 3
1 2 3 4
1 2 3 4 5
1 2 3 4 5 6
1 2 3 4 5 6 7
```

```
realmin
realmax
clc
ex_04_0
>>
```

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EXAMPLE:

```

function f = fib(n)
% This program calculates
% Fibonacci number for a given number.
% 25.02.2010
%
num = input('calculate fibonacci number for: ');
f(1) = 1;
f(2) = 2;
for k = 3:num
    f(k) = f(k-1) + f(k-2);
end
    
```

```

>> type ex_04_01a
ex_04_01a
c1c
type ex_04_01a
    
```

```

>> ex_04_01a
ans =
     12
    
```

EXAMPLE:

```

for k = 3:num
    f(k) = f(k-1) + f(k-2);
end
    
```

```

>> ex_04_01a
calculate fibonacci number for: 12
>> f
    
```

| Columns 1 through 11 |
|------------------------------|
| 1 2 3 5 8 13 21 34 55 89 144 |
| Column 12 |
| 233 |

```

>>
type ex_04_01a
ex_04_01a
12
f
    
```

EXAMPLES:

Vectorize if possible
 Evaluate the expression given below without the formula for the sum.

$$\sum_{n=1}^{100000} n$$

- clock function
 returns a six element vector.
- etime function
 returns the time in seconds between its two arguments.

EXAMPLE:

```

>> help cputime
CPUTIME CPU time in seconds.
CPUTIME returns the CPU time in seconds that has been used
by the MATLAB process since MATLAB started.

For example:

t=cputime; your_operation; cputime-t

returns the cpu time used to run your_operation.

The return value may overflow the internal representation
and wrap around.

See also etime, tic, toc, clock

Reference page in Help browser
doc cputime
    
```

EXAMPLE:

```

>> help etime
ETIME Elapsed time.
ETIME(T1,T0) returns the time in seconds that has elapsed between
vectors T1 and T0. The two vectors must be six elements long, in
the format returned by CLOCK:

T = [Year Month Day Hour Minute Second]

Time differences over many orders of magnitude are computed accura
The result can be thousands of seconds if T1 and T0 differ in their
first five components, or small fractions of seconds if the first
components are equal.

t0 = clock;
operation
etime(clock, t0)

See also tic, toc, clock, cputime, determ
    
```

```

>> help tic
>> help toc
>> help cputime
>> help etime
    
```

EXAMPLES

with for loop

```

t0 = clock;
s = 0;
for n = 1 : 100000
    s = s + n;
end
etime(clock, t0)
    
```

EXAMPLE:

```

>> type ex_04_2
t0 = cLock;
s = 0;
for n = 1 : 100000
    s = s + n;
end
etime(clock, t0)
>> ex_04_2
ans =
    0.0057
    
```

```

>> |
    
```

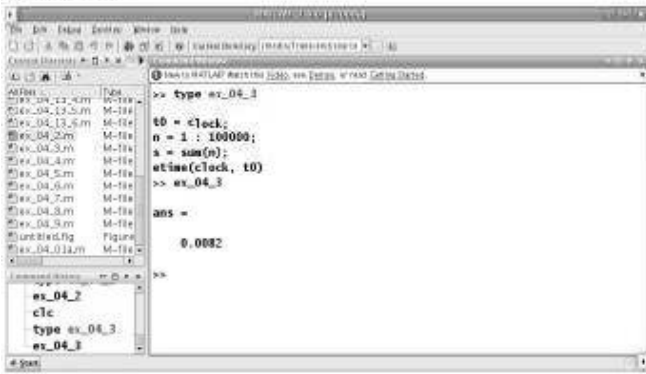
EXAMPLES

with vectorization

```

t0 = clock;
n = 1 : 100000;
s = sum(n);
etime(clock, t0)
    
```

EXAMPLE:

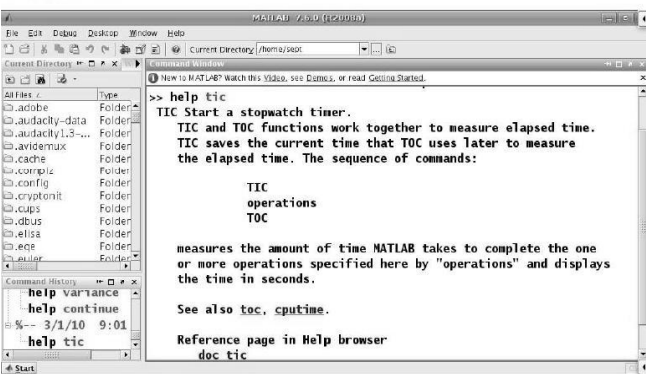


EXAMPLES

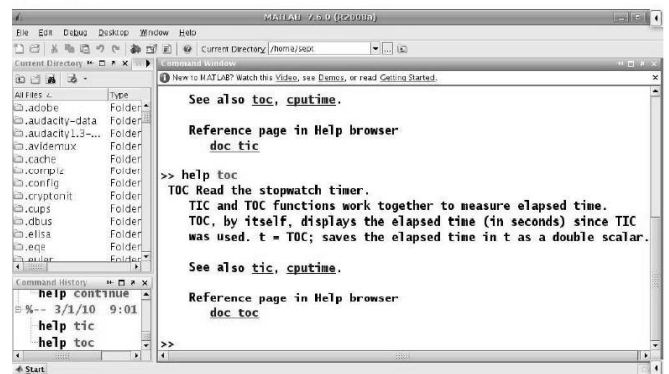
tic and toc functions
monitor the time to interpret MATLAB statements
Evaluate:

$$\sum_{n=1}^{100000} \frac{1}{n^2}$$

EXAMPLE:



EXAMPLE:



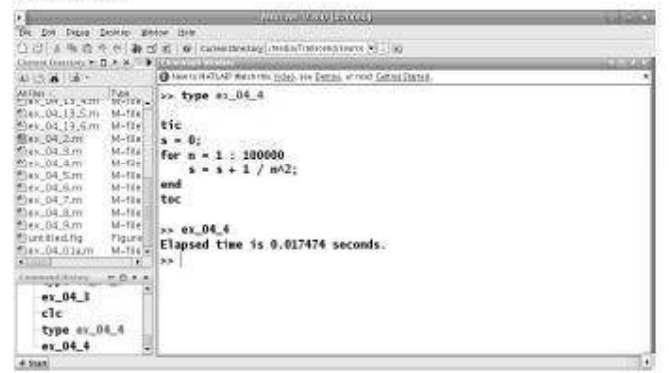
EXAMPLES

with for loop

```

tic
s = 0;
for n = 1 : 100000
    s = s + 1 / n^2;
end
toc
    
```

EXAMPLE:



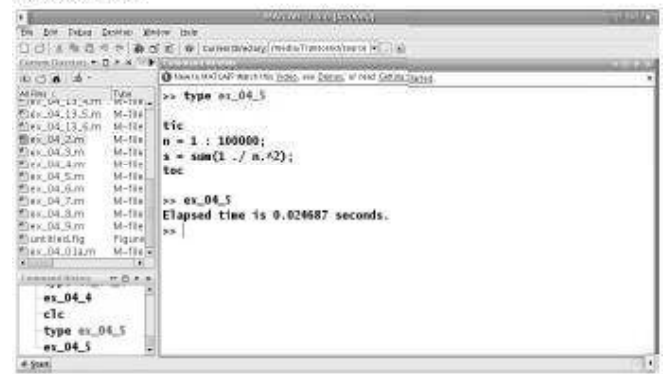
EXAMPLES

with vectorization

```

tic
n = 1 : 100000;
s = sum(1 ./ n.^2);
toc
    
```

EXAMPLE:





ERRORS

if statements

- Relational Operators

Usage:

if condition statement, end

- condition is usually a logical expression
- if condition is true statement is executed but if condition is false, nothing happens.
- Condition may be a vector or a matrix, in which case it is true only if all of its elements are nonzero. A single zero element in a vector or matrix renders it false.



CONDITIONAL STATEMENTS

LOGICAL EXPRESSIONS

Logical operators are used to combine logical expressions (with "and" or "or"), or to change a logical value with "not"

Operators:

& AND, | OR, ~ NOT

| INPUT | | OUTPUT | | | |
|-------|-------|--------|-------|-------|-------|
| A | B | A&B | A B | ~A | ~B |
| false | false | false | false | true | true |
| false | true | false | true | true | false |
| true | false | false | true | false | true |
| true | true | true | true | false | false |



CONDITIONAL STATEMENTS

EXAMPLE

```
a = rand
if a > 0.5 disp('greater 0.5'), end

% if logical expression is TRUE ---> 1
% if logical expression is FALSE --> 0
```



CONDITIONAL STATEMENTS

EXAMPLE:



CONDITIONAL STATEMENTS

IF-ELSE

```
if condition
    blockofstatementsA
else
    blockofstatementsB
end
```

- blockofstatementsA or blockofstatementsB represents one or more statements.
- If condition is true blockofstatementsA is executed and if false blockofstatementsB is executed.
- else is optional.



CONDITIONAL STATEMENTS

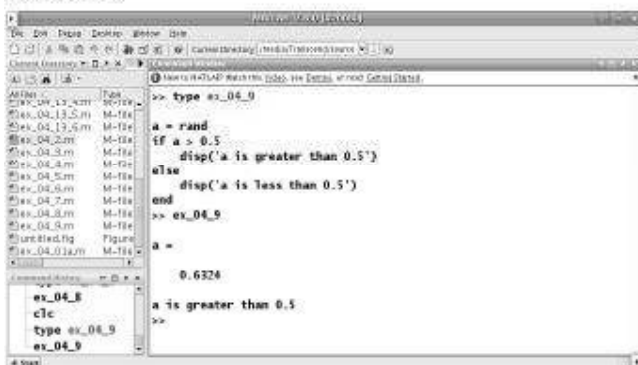
EXAMPLE

```
a = rand
if a > 0.5
    disp('a is greater than 0.5')
else
    disp('a is less than 0.5')
end
```



ERRORS

EXAMPLE:



CONDITIONAL STATEMENTS

```
elseif
if condition1
    statementA
elseif condition2
    statementB
elseif condition3
    statementC
...
else
    statementE
end
```

This is called elseif ladder.



CONDITIONAL STATEMENTS

`elseif`

- 1 `condition1` is tested. If it is true, `statementA` are executed; MATLAB then moves to the next statement after `end`.
- 2 If `condition1` is false, MATLAB checks `condition2`. If it is true, `statementB` are executed, followed by the statement after `end`.
- 3 In this way, all conditions are tested until a true one is found. As soon as a true condition is found, no further `elseif`s are examined and MATLAB jumps off the ladder.
- 4 If none of the conditions is true, `statementE` after `else` are executed.
- 5 Arrange the logic so that not more than one of the conditions is true.
- 6 There can be any number of `elseif`s, but at most one `else`.
- 7 `elseif` must be written as one word.
- 8 It is good programming style to indent each group of statements as shown.

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CONDITIONAL STATEMENTS

NESTED `ifs`

(İççe kullanım)

An `if` construct can contain further `ifs`.

This is called NESTING.

`else` belongs to the most recent `ifs`.

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CONDITIONAL STATEMENTS

`switch` STATEMENT

```

switch value
  case val1
    statement1
  case val2
    statement2
  case [val3 val4 val5]
    statement3
  ...
  otherwise
    statementN
end

```

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CONDITIONAL STATEMENTS

EXAMPLE:

```

val = 3;

switch val
  case 1
    disp('one')
  case 2
    disp('two')
  case 3
    disp('three')
  otherwise
    disp('not a number between 1-3')
end

```

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CONDITIONAL STATEMENTS

EXAMPLE:

```

>> type ex_04_04
val = 3;
switch val
  case 1
    disp('one')
  case 2
    disp('two')
  case 3
    disp('three')
  otherwise
    disp('not a number between 1-3')
end
>>

```

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CONDITIONAL STATEMENTS

EXAMPLE:

```

>> type ex_04_04
val = 3;
switch val
  case 1
    disp('one')
  case 2
    disp('two')
  case 3
    disp('three')
  otherwise
    disp('not a number between 1-3')
end
>> ex_04_04
three
>>

```

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LOOPS

WHILE LOOP

While loops are most often used when an iteration is repeated until some termination criterion is met.

Usage;

```

while expression
  block of statements
end

```

The block of statements is executed as long as expression is true.

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LOOPS

WHILE LOOP

To execute a `while`-`end` loop properly;

- The conditional expression in the `while` command must include at least one variable;
- The variables in the conditional expression must have been assigned when MATLAB executes the `while` command for the first time;
- At least one of the variables in the conditional execution must be assigned a new value in the commands that are between the `while` and the `end`. Otherwise once the looping starts it will never stop since the conditional expression will remain true.

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LOOPS

EXAMPLE:

```

h = 0.001;
x = [0:h:2];
y = 0*x;
y(1) = 1;
i = 1;

while(i<max(size(x)))
    y(i+1) = y(i) + h*(x(i)-abs(y(i)));
    i = i + 1;
end
plot(x,y)

```

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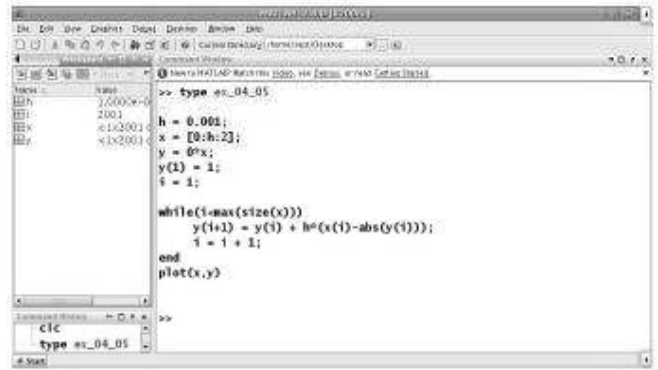
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LOOPS

EXAMPLE:



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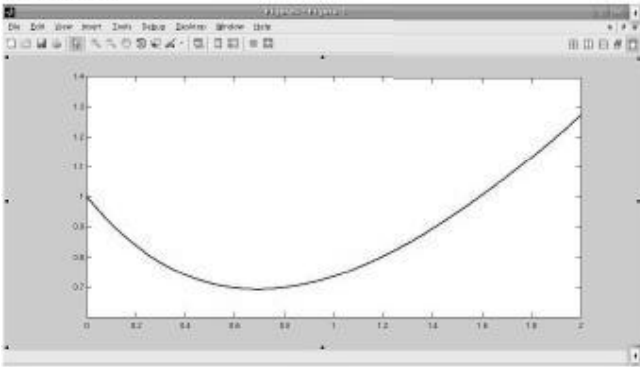
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LOOPS

EXAMPLE:



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LOOPS

INTERRUPTING THE INFINITE LOOP

In case of an Infinite Loop

```

% ***
i=100;
while i == 100
    disp(i)
end
% ***

```

Use CTRL+C or CTRL+BREAK to stop the program.

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LOOPS

EXAMPLE:



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LOOPS

EXAMPLE:



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LOOPS

BREAK, CONTINUE AND RETURN

- The break and return statements provide an alternative way to exit from the flow of the program.
- Continue passes control to the next iteration of for or while loop and skips any remaining statements in the body of the loop.
- Break is used to escape from an enclosing while or for loop. Execution continues at the end of the enclosing loop construct.
- Return is used to force an exit from a function. This can have the effect of escaping from a loop. Any statement following the loop that are in the function body are skipped (Next week "Functions").

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LOOPS

EXAMPLE:

```

i=0
sum = 0;
while (i <= 100)
    if(i == 72)
        disp(i);
        disp('loop stopped');
        break;
    end
    if(i == 12)
        disp(i);
        disp('loop continued without ending');
        i = i+1
    end
    i = i + 1;
    sum = sum + i;
end

```

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EXAMPLE:

```

i=0
sum = 0;
while (i <= 100)
    if(i == 72)
        disp(i);
        disp('loop stopped');
        break;
    end
    if(i == 12)
        disp(i);
        disp('loop continued without ending');
        i = i+1
    end
    i = i + 1;
    sum = sum + i;
end
disp(i);
    
```

EXAMPLE:

```

i = 0
sum = 0;
while (i <= 100)
    if(i == 72)
        disp(i);
        disp('loop stopped');
        break;
    end
    if(i == 12)
        disp(i);
        disp('loop continued without ending');
        i = i+1
    end
    i = i + 1;
    sum = sum + i;
end
disp(i);
    
```

- Design Process
- Structure Plan

To design a successful program you need to understand a problem thoroughly and break it down into its most fundamental logical stages.

In other words, you have to develop a systematic procedure or an algorithm for solving it.

- The program must be readable and hence clearly understandable.
- It is useful to decompose the main program into subprograms that do specific parts of it.
- Add comments and references so that you know exactly what was done and for what purpose.

- 1 Problem analysis.
- 2 Problem statement. Develop a detailed statement of the mathematical problem to be solved with a computer program.
- 3 Processing scheme. Define the inputs required and the outputs to be produced by the program.
- 4 Algorithm. Design the step-by-step procedure in a top-down process that decomposes the overall problem into subordinate problems.

- 5 Program algorithm. Translate or convert the algorithm into a computer language.
- 6 Evaluation. Test all of the options and conduct a validation study of the program. For example, compare results with other programs.
- 7 Application. Solve the problems, the program was designed to solve. If the program is well designed and useful, it can be saved in your working directory

EXAMPLE

A function M-file is a script file designed to handle a particular task that may be activated (invoked) whenever needed.

EXAMPLE TRAJECTORY

A ball is thrown with an initial angle of θ and initial velocity of v_0 .

Given

- velocity and theta angle

Find

- Projectile Flight Path
- Projectile speed vs. angle



DESIGN PROCESS

EXAMPLE

- 1 $v_{0,x} = v_0 \times \cos(\theta_0)$
- 2 $v_{0,y} = v_0 \times \sin(\theta_0)$
- 3 $x(t) = v_{0,x} \times t$ (horizontal distance from origin as a function of t)
- 4 $y(t) = v_{0,y} \times t - 0.5 \times gt^2$ (vertical distance from origin as a function of t)
- 5 $y(x) = \frac{v_{0,y}}{v_{0,x}} \times x - 0.5 \times g \frac{x^2}{v_{0,x}^2}$ (with using # 3 and # 4 vertical location of the point is a function of x horizontal distance)



DESIGN PROCESS

EXAMPLE

```
%
% The projectile problem with zero air resistance
% in a gravitational field with constant g.
%
% Written by ##### 01.03.2010

% Written by D. T. Valentine ..... September 2006
% Revised by D. T. Valentine ..... November 2008
% An eight-step structure plan applied in MATLAB:
%
```



DESIGN PROCESS

EXAMPLE cont'd.

```
% 1. Definition of the input variables.
%
% Gravity in m/s**2
g = 9.81;
disp('*** INPUT DATA FOR PROJECTILE PROBLEM ***');
vo = input('Launch speed in m/s: ');
theta = input('Launch angle in degrees: ');
% Convert degrees to radians
theta = pi*theta/180;
```



DESIGN PROCESS

EXAMPLE cont'd.

```
% 2. Calculate the range and duration of the flight.
%
txmax = (2*v0/g) * sin(theta);
xmax = txmax * v0 * cos(theta);
```



DESIGN PROCESS

EXAMPLE cont'd.

```
% 3. Calculate the sequence of time
% steps to compute trajectory.
%
dt = txmax/100;
t = 0:dt:txmax;
%
% 4. Compute the trajectory.
%
x = (v0 * cos(theta)) .* t;
y = (v0 * sin(theta)) .* t - (g/2) .* t.^2;
```



DESIGN PROCESS

EXAMPLE cont'd.

```
% 5. Compute the speed and angular
% direction of the projectile.
% Note that vx = dx/dt, vy = dy/dt.
%
vx = v0 * cos(theta);
vy = v0 * sin(theta) - g .* t;
v = sqrt(vx.*vx + vy.*vy);
th = (180/pi) .* atan2(vy,vx);
%
% 6. Compute the time, horizontal
% distance at maximum altitude.
%
tymax = (v0/g) * sin(theta);
xymax = xmax/2;
ymax = (v0/2) * tymax * sin(theta);
```



DESIGN PROCESS

EXAMPLE cont'd

```
% 7. Display output.
%
disp([' Range in m = ', num2str(xmax), ...
' Duration in s = ', num2str(txmax)])
disp(' ')
disp([' Maximum altitude in m = ', num2str(ymax), ...
' Arrival in s = ', num2str(tymax)])
plot(x,y,'k',xmax,y(size(t)), 'o',xmax/2,ymax,'o')
title([' Projectile flight path, vo = ', num2str(vo), ...
' th = ', num2str(180*th/pi)])
xlabel(' x '), ylabel(' y ') % Plot of Figure 1.
figure % Creates a new figure.
plot(v,th,'r')
title(' Projectile speed vs. angle ')
xlabel(' V '), ylabel(' \theta ') % Plot of Figure 2.
%
% 8. Stop.
%
```



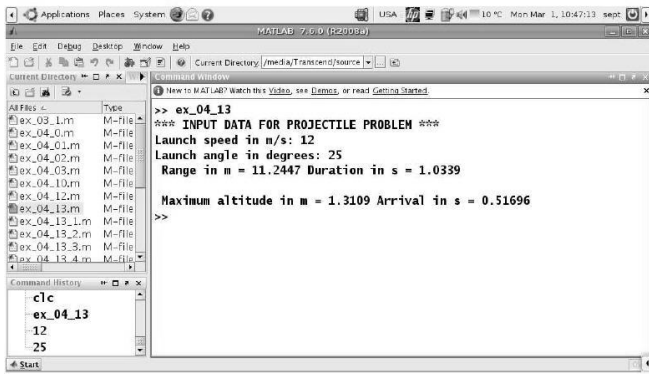
DESIGN PROCESS

EXAMPLE:

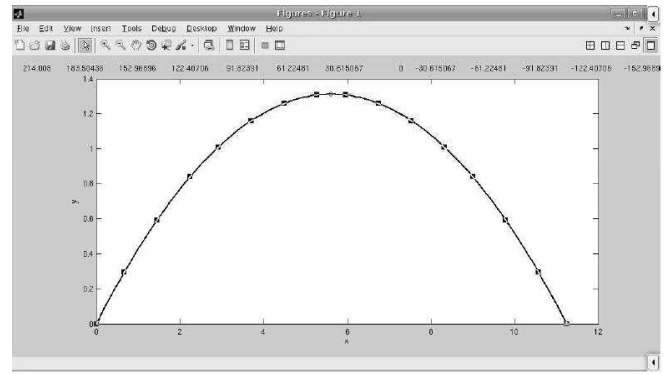
```
>> ex_04_13
*** INPUT DATA FOR PROJECTILE PROBLEM ***
Launch speed in m/s: 12
Launch angle in degrees: 25
```



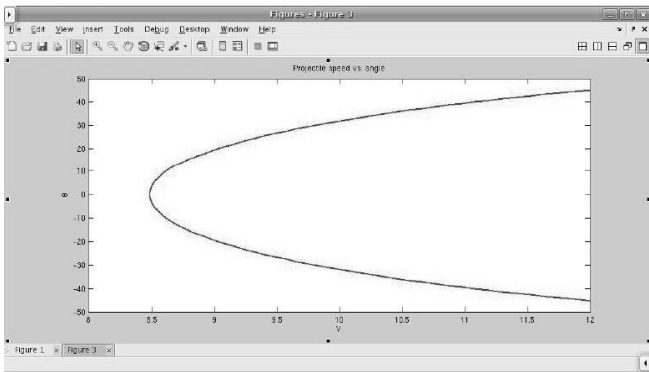
EXAMPLE:



EXAMPLE:



EXAMPLE:



DATA ANALYSIS FUNCTIONS

- **max(x)** Determines the largest value in x.
- **min(x)** Determines the smallest value in x.
- **sum(x)** Determines the sum of the elements in x.
- **prod(x)** Determines the product of the elements in x.
- **sort(x)** Returns a vector with the values of x in ascending order.



MEAN AND MEDIAN

- **mean(x)** Computes the mean(average value) of the elements of the vector x.

$$\bar{x} = \frac{\sum_{k=1}^N x_k}{N}$$

where $\sum_{k=1}^N x_k = x_1 + x_2 + \dots + x_N$

- **median(x)** Determines the median value of the elements in the vector x.



VARIANCE AND STANDARD DEVIATION

- **var(x)** Computes the variation of the values in x.
- **std(x)** Computes the standard deviation of the values in x.
- The standard deviation is defined as the square root of the variance.

$$\sigma^2 = \frac{\sum_{k=1}^N (x_k - \bar{x})^2}{(N-1)}$$



References - Week 4

- 1 Alfio Quarteroni, Fausto Saleri, Scientific Computing with Matlab and Octave, Wissenschaftliches Rechnen mit Matlab, Springer Verlag, 2006.
- 2 Cleve Moler, Numerical Computing with Matlab, Mathworks, 2008.
- 3 Brian Hahn, Daniel T.Valentine, Essential Matlab for Engineers and Scientists, Elsevier, 2010.

Have a nice Week End