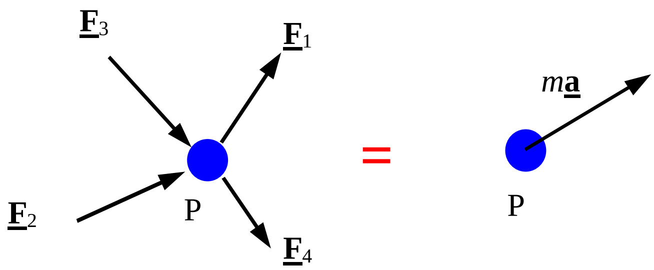
**MÜHENDİSLİK MEKANİĞİ**

## 10. HAFTA: KİNETİK
## Newton'un 2. kanunu

**Newton'un 2. kanunu:** Bir parçacığa etki eden kuvvetlerin bileşkesi parçacığın ivmesiyle orantılıdır.




$$\boxed{\sum \underline{\mathbf{F}} = m\underline{\mathbf{a}}}$$

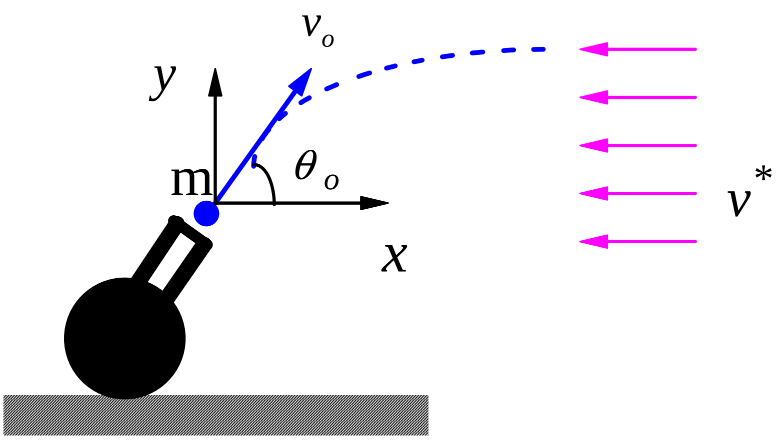
**Kartezyen koordinatlarda hareket denklemleri**:

$$\begin{array}{|l|}\hline \sum F_x = ma_x \\ \sum F_y = ma_y \\ \sum F_z = ma_z \\ \hline\end{array}$$

**Teğetsel ve normal koordinatlarda haraket denklemleri**:

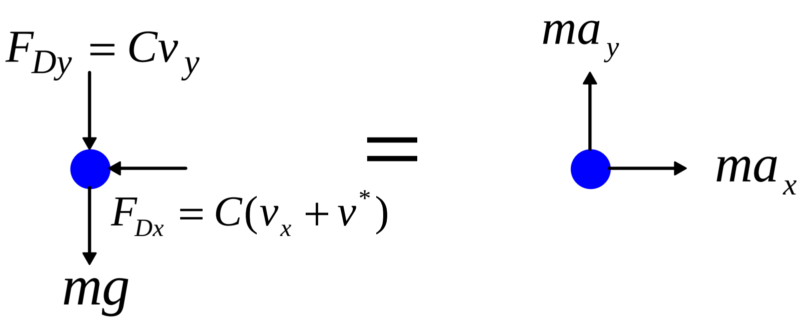
$$\begin{array}{|l|}\hline \begin{matrix} \sum F_t = ma_t \\ \sum F_n = ma_n \\ \sum F_b = ma_b \end{matrix} \quad where \quad \begin{cases} a_t = \dot{v} \\ a_n = \dfrac{v^2}{\rho} = \rho\,\dot{\beta}^2 \\ a_b = 0 \end{cases} \\ \hline\end{array}$$

**ÖRNEK1: Newton'un 2. kanunu**




A particle is ejected into a moving fluid of velocity $v^*$. The drag force applied by the fluid onto the particle is proportion to the velocity of the particle relative to the fluid. Calculate the equation of the motion of the projectile.

Çözüm:




$$\sum F_x = ma_x \Rightarrow$$

$$m\frac{dv_x}{dt} = -C(v_x + v^*)$$

İntegrasyon ile

$$\int \frac{dv_x}{(v_x + v^*)} = -\int \frac{C}{m} dt$$

$$\Rightarrow \ln(v_x + v^*) = -\frac{C}{m}t + D_1$$

$$t = 0, v_x = v_o \cos\theta, \Rightarrow D_1 = \ln(v_o \cos\theta + v^*)$$

$$\Rightarrow v_x = \frac{dx}{dt} = (v_o \cos\theta + v^*)e^{-\frac{C}{m}t} - v^*$$

Integrasyon ile

$$x=-\frac{m}{C}(v_o\cos\theta+v^*)e^{-\frac{C}{m}t}-v^*t+D_2$$

$$t=0, x=0 \Rightarrow D_2=\frac{m}{C}(v_o\cos\theta+v^*)$$

$$\Rightarrow x=\frac{m}{C}(v_o\cos\theta+v^*)(1-e^{-\frac{C}{m}t})-v^*t$$

$$\sum F_y=ma_y \Rightarrow$$

$$-Cv_y-mg=m\frac{dv_y}{dt}$$

$$\frac{dv_y}{v_v+\frac{m}{C}g}=-\frac{C}{m}dt$$

Integrasyonla

$$\ln(v_y+\frac{m}{C}g)=-\frac{C}{m}t+D_1$$

$$v_y=\frac{dy}{dt}=e^{-\frac{C}{m}t+D_1}-\frac{m}{C}g$$

$$t=0, v_y=\sin\theta_o \Rightarrow D_1=\ln(v_o\sin\theta+\frac{m}{C}g)$$

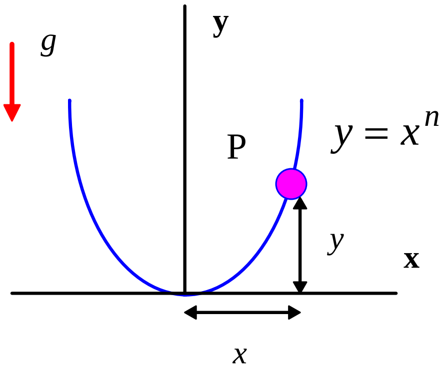
Integrasyon ile

$$y=-\frac{m}{C}e^{-\frac{C}{m}t+D_1}-\frac{m}{C}gt+D_2$$

$$t=0,\ y=0 \Rightarrow D_2=e^{D_1}$$

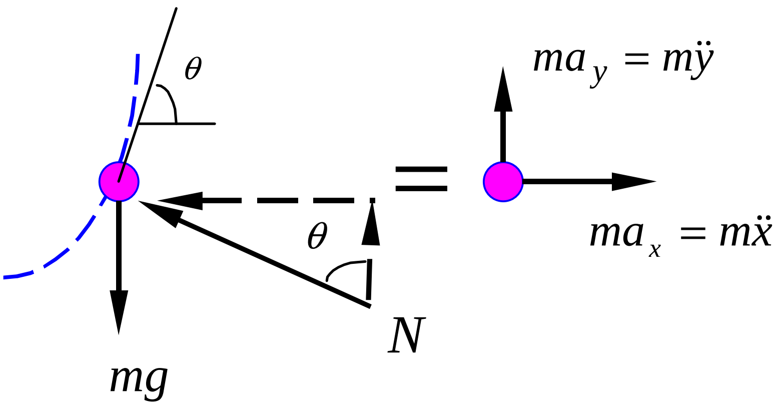
$$\Rightarrow y=(v_o\sin\theta+\frac{m}{C}g)(1-\frac{m}{C}e^{-\frac{C}{m}t})-\frac{m}{C}gt$$

**ÖRNEK2: Newton’un 2. kanunu**




A bead of mass *m* moves on a bar. The bar is shaped as shown in the figure so that $y = x^n$. If the bar is smooth and the particle is released from rest at the position $x = x_o$, calculate the equation of motion of the bead and the force applied by the bar on the bead at $x = x_o$.

Çözüm:




Kinematik ilişki:

$$\tan\theta = \frac{dy}{dx} = nx^{n-1}$$

$$\sin\theta = \frac{\frac{dy}{dx}}{\sqrt{1+(\frac{dy}{dx})^2}} = \frac{nx^{n-1}}{\sqrt{1+n^2x^{2(n-1)}}}$$

$$\cos\theta = \frac{1}{\sqrt{1+(\frac{dy}{dx})^2}} = \frac{1}{\sqrt{1+n^2x^{2(n-1)}}}$$

$$a_x = \ddot{x}$$

$$a_y = \ddot{y} = \frac{d}{dt}(\frac{dy}{dt}) = \frac{d}{dt}(\frac{dy}{dx}\frac{dx}{dt}) = \frac{d}{dt}(nx^{n-1}\dot{x})$$

$$\Rightarrow a_y = n(n-1)x^{n-2}\dot{x}^2 + nx^{n-1}\ddot{x}$$

Kinetik:

$$\begin{cases} \sum F_x = ma_x \Rightarrow -N\sin\theta = m\ddot{x} \quad (1) \\ \sum F_y = ma_y \Rightarrow -mg + N\cos\theta = m\ddot{y} \quad (2) \end{cases}$$

$$\Rightarrow -mg - \frac{m\ddot{x}}{\tan\theta} = m\ddot{y}$$

Hareket denklemi:

$$\Rightarrow -g - \frac{\ddot{x}}{nx^{n-1}} = n(n-1)x^{n-2}\dot{x}^2 + nx^{n-1}\ddot{x}$$

$$At\ x = x_o,\ \dot{x} = 0 \Rightarrow \ddot{y}_o = nx_o^{n-1}\ddot{x}_o \Rightarrow$$

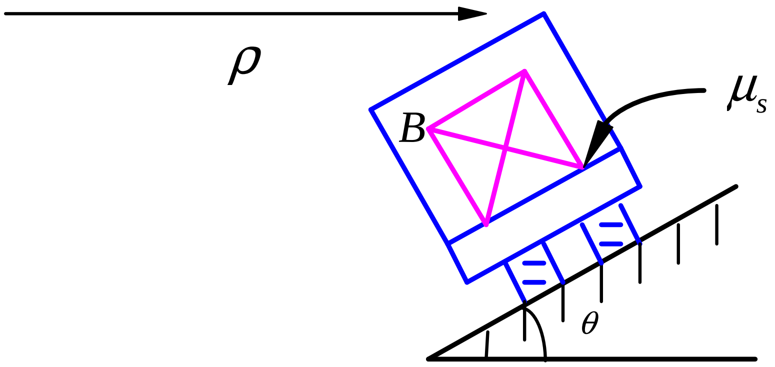
$$\begin{cases} (1) \to -N\sin\theta_o\ nx_o^{n-1} = mnx_o^{n-1}\ddot{x} \\ (2) \to -mg + N\cos\theta_o = mnx_o^{n-1}\ddot{x} \end{cases}$$

$$-N(\sin\theta_o\ nx_o^{n-1} + \cos\theta_o) = -mg$$

$$N = mg / (\sin\theta_o\ nx_o^{n-1} + \cos\theta_o)$$

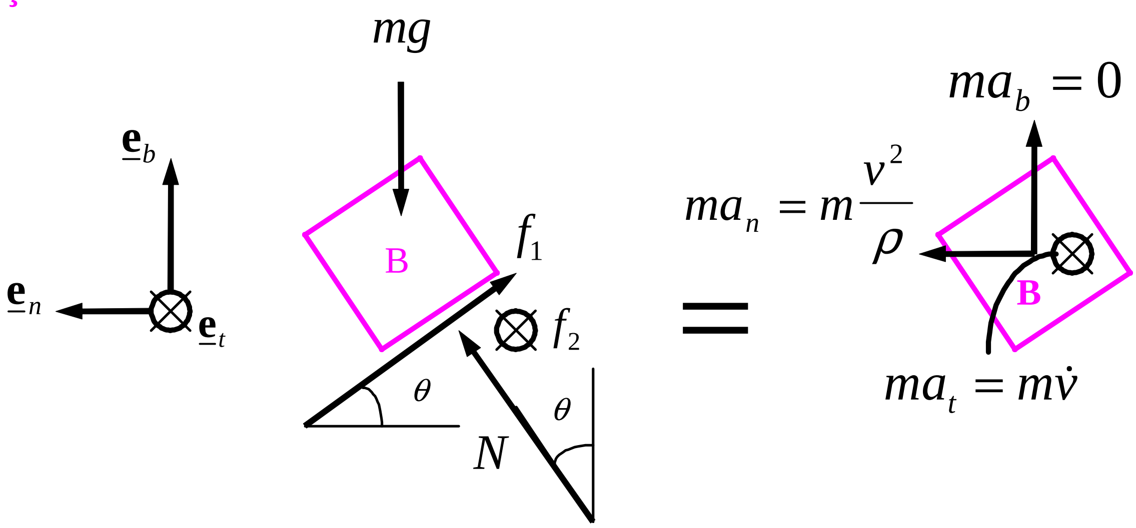
$$N = \frac{mg}{\sqrt{1+n^2 x_o^{2(n-1)}}}$$

**ÖRNEK3: Newton'un 2. kanunu**



The box B sits on the bed of a truck which is going around a turn of radius $\rho$ . At this instant the truck has a speed $v = v_o$. If the coefficient of friction between B and the bed is $\mu_s$, calculate the maximum rate at which the truck's speed can increase without making the box slip.

**Çözüm:**




$$\sum F_n = ma_n \Rightarrow -f_1 \cos\theta + N \sin\theta = m\frac{v_o^2}{\rho} \quad (1)$$

$$\sum F_t = ma_t \Rightarrow f_2 = m\dot{v} \quad (2)$$

$$\sum F_b = ma_b \Rightarrow -mg + f_1 \sin\theta + N\cos\theta = 0 \quad (3)$$

Kayma şartı,

$$\Rightarrow f = \sqrt{f_1^2 + f_2^2} = \mu_s N \quad (4)$$

(2) denklemi (4), te yerine konursa

$$N = \frac{\sqrt{f_1^2 + (m\dot{v})^2}}{\mu_s} \quad (5)$$

(5) denklemi (1) ve (3) te yerine konursa

$$-f_1\cos\theta+\frac{\sqrt{f_1^2+(m\dot{v})^2}}{\mu_s}\sin\theta=m\frac{v_o^2}{\rho} \quad (6)$$

$$-mg+f_1\sin\theta+\frac{\sqrt{f_1^2+(m\dot{v})^2}}{\mu_s}\cos\theta=0 \quad (7)$$

$$(6)\times\cos\theta-(7)\times\sin\theta\Rightarrow$$

$$f_1=mg\sin\theta-m\frac{v_o^2}{\rho} \quad (8)$$

(8) denklemi (7), te yerine konursa

$$-mg+(mg\sin\theta-m\frac{v_o^2}{\rho})\sin\theta$$

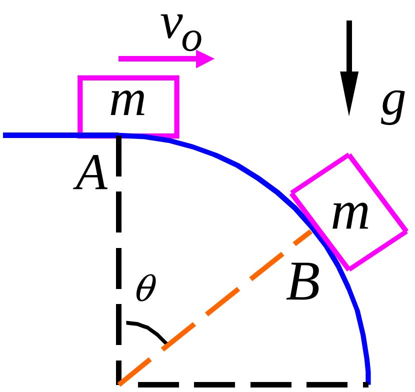
$$+\frac{\sqrt{(mg\sin\theta-m\frac{v_0^2}{\rho})^2+(m\dot{v})^2}}{\mu_s}\cos\theta=0 \quad (9)$$

$$\Rightarrow\dot{v}=\sqrt{\left\{\frac{[(g\sin\theta-\frac{v_o^2}{\rho})\sin\theta-g]\mu_s}{\cos\theta}\right\}^2-(g\sin\theta-\frac{v_o^2}{\rho})^2}$$

**Silindirik koordinat sisteminde hareket denklemi**:

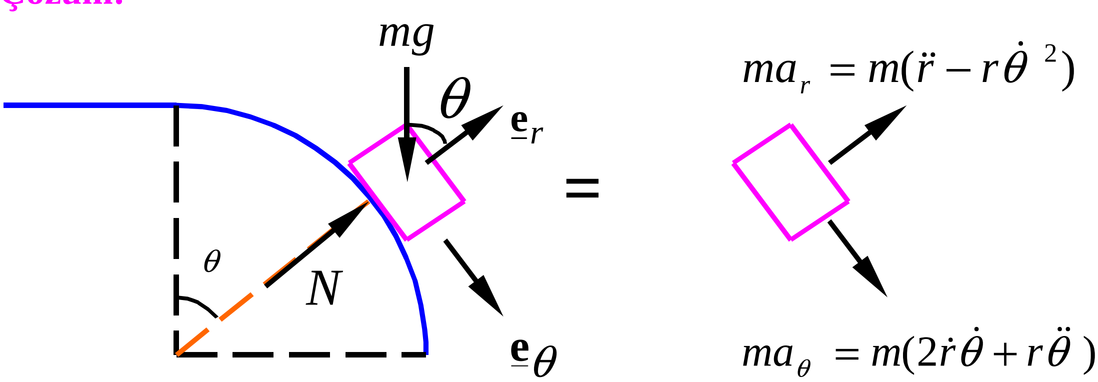
$$\sum F_r=ma_r \qquad \sum F_\theta=ma_\theta \qquad \sum F_z=ma_z \quad \text{where} \quad \begin{cases} a_r=\ddot{r}-r\dot{\theta}^2 \\ a_\theta=2\dot{r}\dot{\theta}+r\ddot{\theta}=\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) \\ a_b=\ddot{z} \end{cases}$$

**ÖRNEK 4**: **Newton'un 2. kanunu**




A particle starts at A with an initial velocity of $v_o$, and moves down the smooth circular ramp up to point B when it separates. Obtain the equation of motion of the particle and the angle $\theta_B$ at which it separates.

**Çözüm:**




Kinematik:

r = constant

$$\Rightarrow a_r = \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2$$

$$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = r\ddot{\theta}$$

Kinetik:

$$\sum F_r = ma_r \Rightarrow N - mg\cos\theta = -mr\dot{\theta}^2 \quad (1)$$

$$\sum F_\theta = ma_\theta \Rightarrow mg\sin\theta = mr\ddot{\theta} \quad (2)$$

(2)

N=0

$$(1) \Rightarrow -g\cos\theta_B = -r\dot{\theta}_B^{\,2} \quad (3)$$

Integrasyon

$$\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$$

$$mr\ddot{\theta} = mr\dot{\theta}\frac{d\dot{\theta}}{d\theta} = mg\sin\theta$$

$$\Rightarrow r\dot{\theta}\,d\dot{\theta} = g\sin\theta\,d\theta$$

$$\Rightarrow r\frac{\dot{\theta}^{\,2}}{2} = -g\cos\theta + C \quad (4)$$

A daki başlangıç koşullarından

$$v_r = \dot{r} = 0, v_\theta = r\dot{\theta} = v_0 \Rightarrow at \quad \theta = 0, \dot{\theta} = \frac{v_o}{r}$$

$$\Rightarrow C = g + \frac{1}{2}\frac{v_o^{\,2}}{r}$$

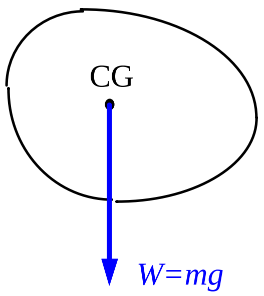
C (4) te yerine konursa

$$\Rightarrow r\frac{\dot{\theta}_B^{\,2}}{2} = -g\cos\theta_B + g + \frac{v_o^2}{2r} \quad (5)$$

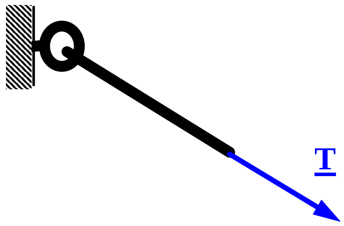
$$\Rightarrow \theta_B = \cos^{-1}\left(\frac{g + \frac{v_0^2}{2r}}{3g}\right)$$

**Free-Body diagram:** A diagram showing the particle under consideration and all the forces acting on the particle. Each force in this diagram must be labeled.

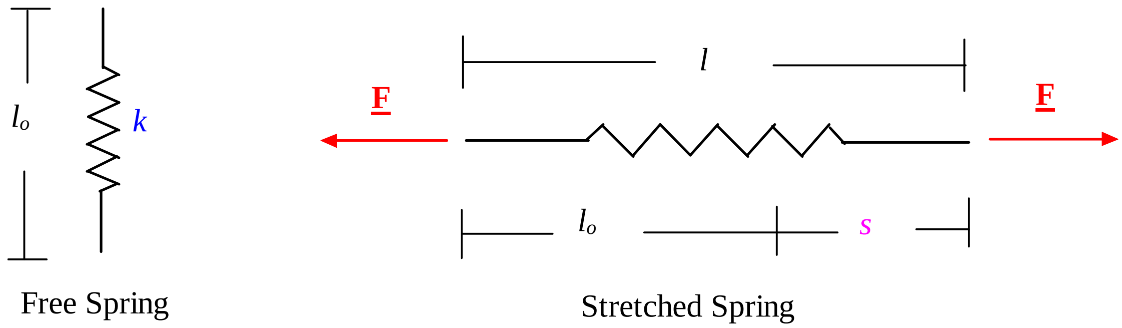
**Gravity:** The force of gravity is equal to the mass times the acceleration of gravity and is applied at the center of gravity (CG) of the body.



**String or cable:** A mechanical device that can only transmit a tensile force along itself.



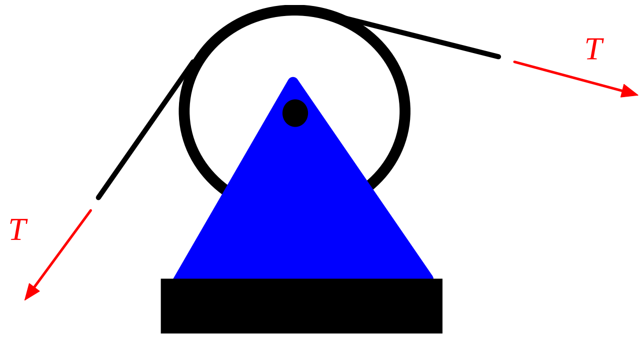

**Linear spring:** A mechanical device which exerts a force proportional to its extension along its line of action.



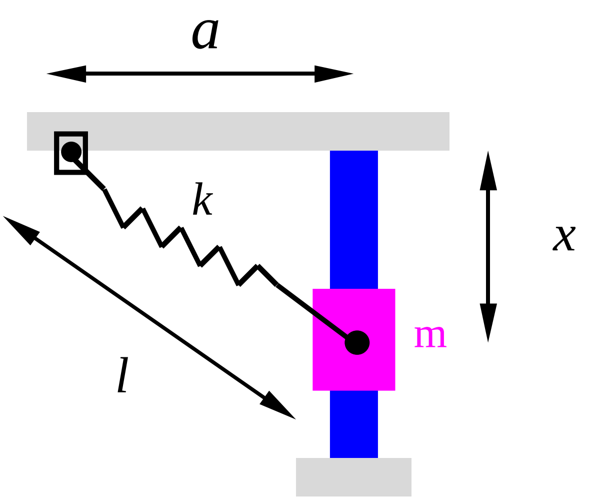

$$\boxed{F = ks}$$

Example 5:

**Frictionless and massless pulleys:** For a frictionless and massless pulley, the tension in the cable is the same on both sides of the pulley.

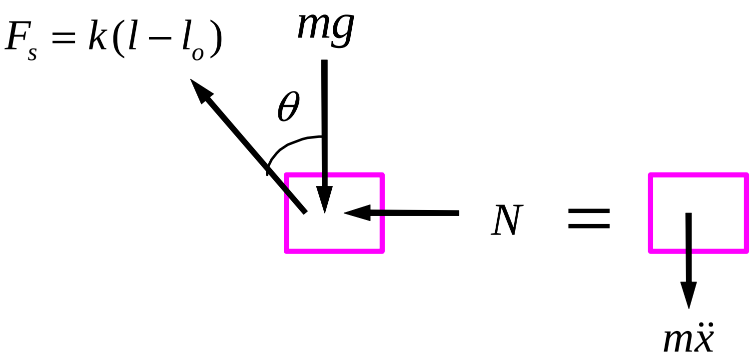


**EXAMPLE 5: Newton's 2nd law**




The particle of mass *m* moves on the smooth rod. The spring has an initial length of $l_0$ and a stiff of *k*. Obtain the equation of motion of the particle.

**Solution:**




Kinematics:

$$l = \sqrt{x^2 + a^2}, \cos\theta = \frac{x}{l}$$

Kinetics:

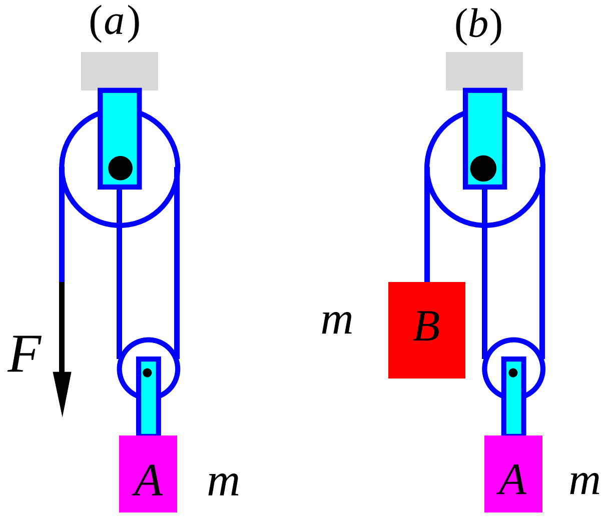
$$\sum F_x = ma_x \Rightarrow mg - F_s \cos\theta = m\ddot{x}$$

$$\Rightarrow mg - k(l - l_o)\frac{x}{l} = m\ddot{x}$$

$$\Rightarrow mg - k(1 - \frac{l_o}{\sqrt{x^2 + a^2}})x = m\ddot{x}$$

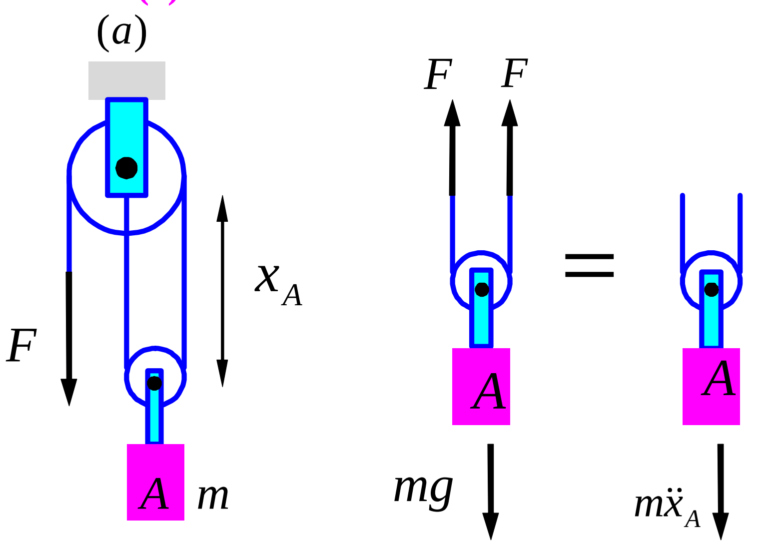
The above is the equation of motion.

## Example 6: **Newton's 2nd law**




Calculate the acceleration of A for each of the two systems. Set $F=mg$ and compare the acceleration of A in each system.

**Solution (a):**




$$\sum F = ma$$

$$\Rightarrow -2F + mg = m\ddot{x}_A$$

$$\Rightarrow \ddot{x}_A = g - \frac{2F}{m}$$

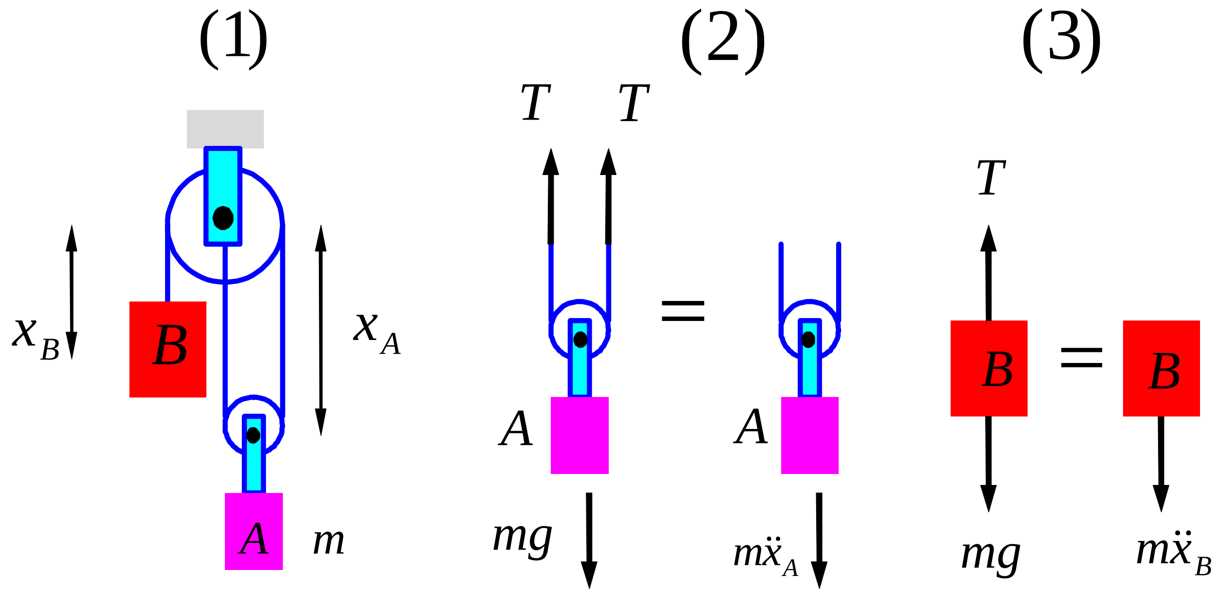
$$F = mg \Rightarrow$$

$$\ddot{x}_A = g - \frac{2mg}{m} = -g$$

(i.e. moves up)

**Solution (b):**

Kinematics:




Kinematics:

$$2x_A + x_B = \text{constant}$$

$$\Rightarrow 2\ddot{x}_A + \ddot{x}_B = 0$$

$$\Rightarrow -2\ddot{x}_A = \ddot{x}_B \quad (1)$$

$\sum F = ma$ for A

$$\Rightarrow -2T + mg = m\ddot{x}_A \quad (2)$$

$\sum F = ma$ for B

$$\Rightarrow -T + mg = m\ddot{x}_B \quad (3)$$

Solve (1), (2), (3) to get

$$\ddot{x}_A = -\frac{g}{5}$$