STATİK - MUKAVEMET

10. HAFTA ŞEKİL DEĞİŞTİRME
HOOKE KANUNLARI

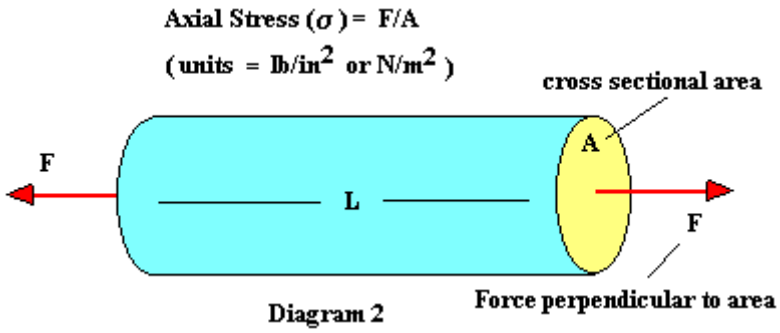


Diagram 2

Düzlem Şekil değiştirme hali

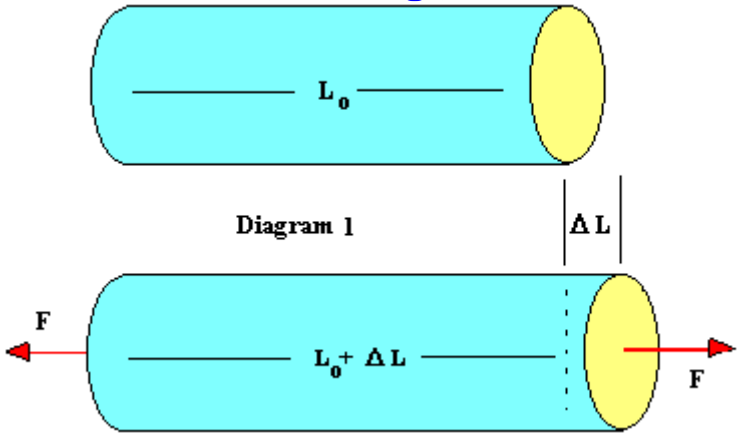
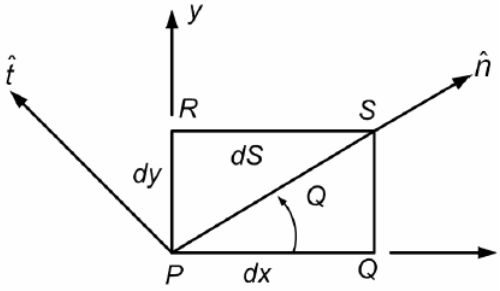


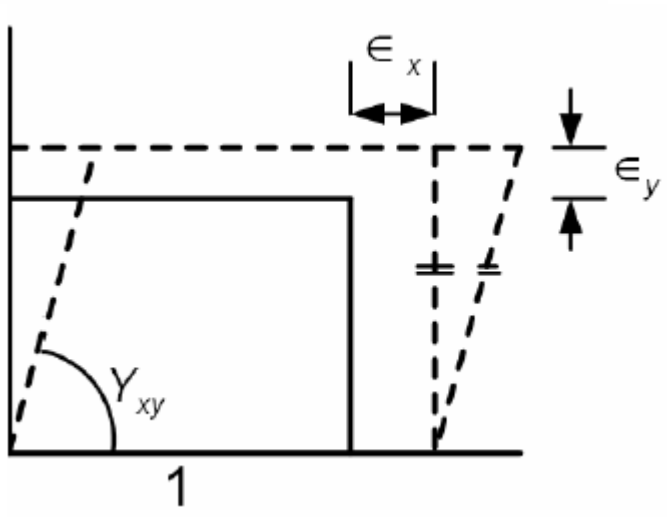
Diagram 1

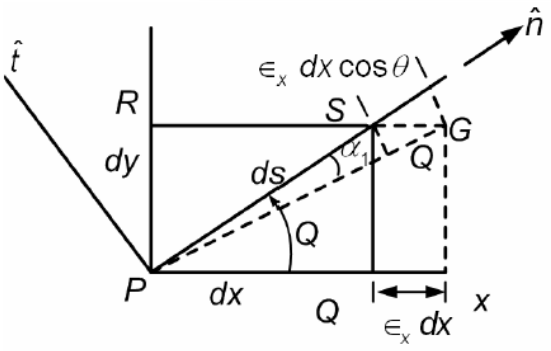
$$\begin{bmatrix} \epsilon_x & \gamma_{xy} \\ \gamma_{xy} & \epsilon_y \end{bmatrix}$$

$$\epsilon_x = \epsilon_x(x,y)$$
$$\epsilon_y = \epsilon_y(x,y)$$
$$\gamma_{xy} = \gamma_{xy}(x,y)$$



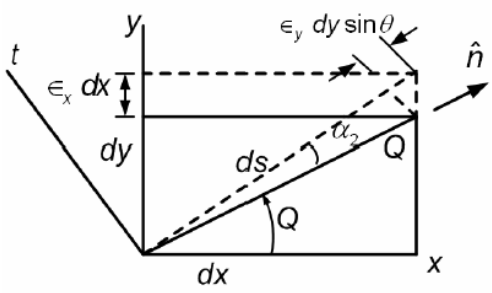
θ açısı yapan düzlemdeki şekil değiştirmeler
x- yönü





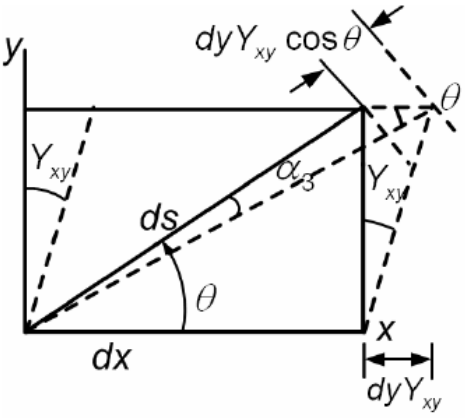
$$\alpha_1 = \frac{\in_x dx Sin\theta}{ds}$$
$$= \in_x sin\theta \frac{dx}{ds}$$
$$= \in_x sin\theta cos\theta$$

Y yönüdeki şekikl değiştirme



$$\alpha_2 = \in_y cos\theta \frac{dy}{ds} = \in_y cos\theta sin\theta$$

Dönme şekil değiştirme



$$\alpha_3 = Y_{xy}\sin\theta\frac{dy}{ds}$$
$$= Y_{xy}\sin\theta\sin\theta$$

$$dL = \in_x dx\cos\theta + \in_y dy\sin\theta + Y_{xy}dy\cos\theta$$

$$\frac{dL}{dS} = \in_n = \in_x \frac{dx}{ds}\cos\theta + \in_y \frac{dy}{ds}\sin\theta + Y_{xy}\frac{dy}{ds}\cos\theta$$

$$= \in_x \cos\theta\cos\theta + \in_y \sin^2\theta + Y_{xy}\sin\theta\cos\theta$$

n-n eksenindeki şekil değiştirme

$$\in_n = \frac{\in_x + \in_y}{2} + \frac{\in_x - \in_y}{2}\cos 2\theta + \frac{Y_{xy}}{2}\sin 2\theta$$

$$\alpha = -\in_x \sin\theta\cos\theta + \in_y \sin\theta\cos\theta - Y_{xy}\sin^2\theta$$

$$\beta = -\in_x \cos\theta(-\sin\theta) + \in_y \cos\theta(-\sin\theta) - Y_{xy}\cos^2\theta$$

$$= \in_x \cos\theta\sin\theta - \in_y \cos\theta\sin\theta - Y_{xy}\cos^2\theta$$

n-t eksenindeki birim dönme açısı

$$\frac{Y_{nt}}{2} = -\frac{(\in_x - \in_y)}{2}\sin 2\theta + \frac{Y_{xy}}{2}\cos 2\theta$$

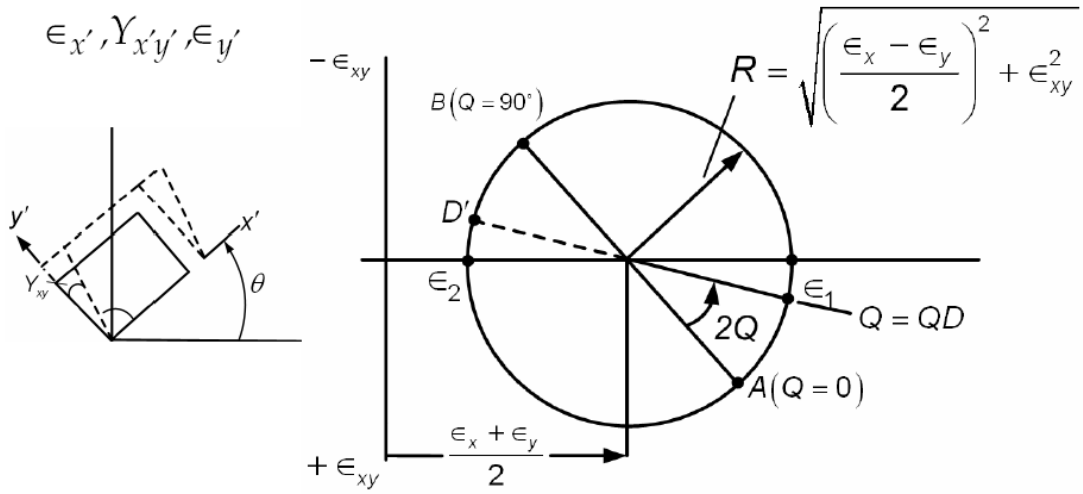
$$\epsilon_{xy} = \frac{\gamma_{xy}}{2}$$

$$\sigma_x \rightarrow \epsilon_x$$

$$\sigma_y \rightarrow \epsilon_y$$

$$\tau_{xy} \rightarrow \epsilon_{xy} = \frac{\gamma_{xy}}{2}$$

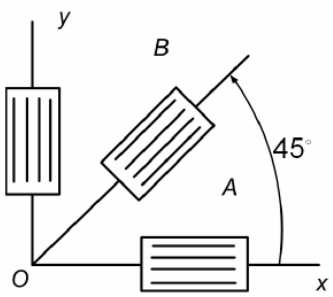
Mohr diyagramı



Problem

A, B ve C noktalarındaki şekil değiştirmeler sırasıyla, A noktasında 520x10 -6 , B noktasında 360x10 -6 ve C noktasında -80x10 -6
Olduğuna göre asal uzama oranlarını bulunuz.

Solution:



$$\epsilon_x = 520\times10^{-6}$$

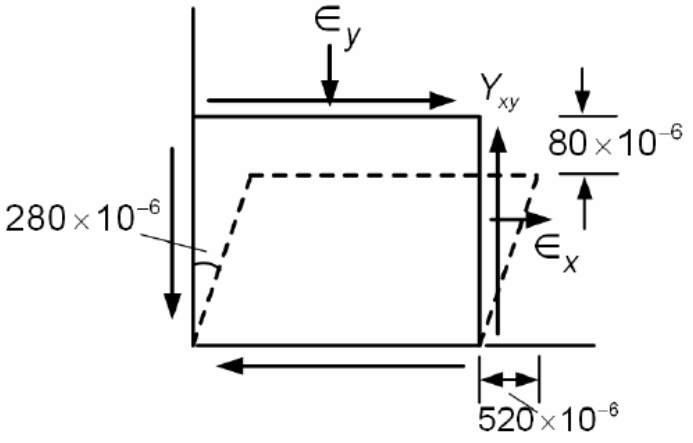
$$\epsilon_{OB} = 360\times10^{-6}$$

$$\epsilon_y = -80\times10^{-6}$$

$$\gamma_{xy} = 2\epsilon_{OB} - (\epsilon_x + \epsilon_y)$$

$$= 2\times360\times10^{-6} - (520\times10^{-6} - 80\times10^{-6})$$

$$= 280\times10^{-6} \quad rad$$



(1) $$\frac{\in_x + \in_y}{2} = \frac{520\times10^{-6} - 80\times10^{-6}}{2} = 220\times10^{-6}$$

$$\frac{\in_x - \in_y}{2} = \frac{520\times10^{-6} + 80\times10^{-6}}{2} = 300\times10^{-6}$$

$$tan\,2\theta_p = \frac{2\in_{xy}}{\in_x - \in_y} = \frac{e\times140\times10^{-6}}{300\times10^{-6}}$$

$$\in_{xy} = \frac{\gamma_{xy}}{2} = \frac{280\times10^{-6}}{2} = 140\times10^{-6}$$

$$\therefore \quad 2\theta_p = 25.02^\circ$$

$$\theta_p = 12.51 \qquad \theta_p = 102.51^\circ$$

$$\in_1 \; or \; \in_2 = \frac{\in_x + \in_y}{2} \pm \sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \in_{xy}^2}$$

$$= 220\times10^{-6} \pm \sqrt{\left(300\times10^{-6}\right)^2 + \left(140\times10^{-6}\right)^2}$$

$$= 220\times10^{-6} \pm 331.06\times10^{-6}$$

$$\therefore \qquad \in_1 = 551.06\times10^{-6}$$

$$\in_2 = -111.06\times10^{-6}$$

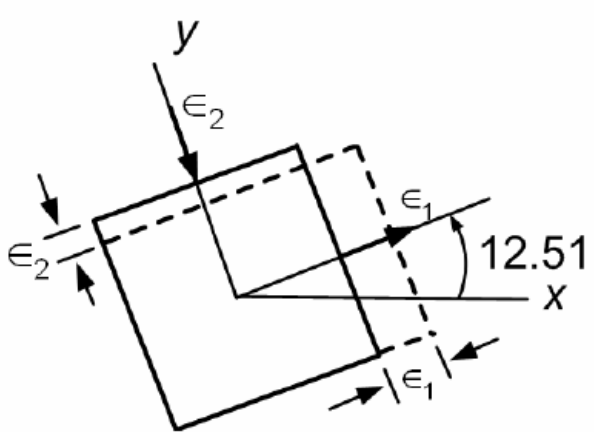
$$\in_{x'}\Big|_{\theta=12.51^\circ}$$

$$= \frac{\in_x + \in_y}{2} + \frac{\in_x - \in_y}{2} Cos2\theta + \in_{xy} Sin2\theta$$

$$= 220\times10^{-6} + 300\times10^{-6} cos(2\times12.51) + 140\times10^{-6} Sin(2\times12.51)$$

$$= 551.06\times10^{-6}$$

$$\theta_{p_1} = 12.51 \quad \text{and} \quad \theta_{p_2} = 102.51$$



$$\in_{max}^{xy} \text{ or } \in_{min}^{xy} = \pm\sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \in_{xy}^2}$$

$$= \pm331.06\times10^{-6}$$

$$\left(\in_{xy}\right)_{max} = 331.06\times10^{-6}$$

$$\left(\in_{xy}\right)_{min} = -331.06\times10^{-6}$$

$$tan\,2Q_s = -\frac{\left(\epsilon_x - \epsilon_y\right)}{2.\epsilon_{xy}} = \frac{-300\times10^{-6}}{140\times10^{-6}}$$

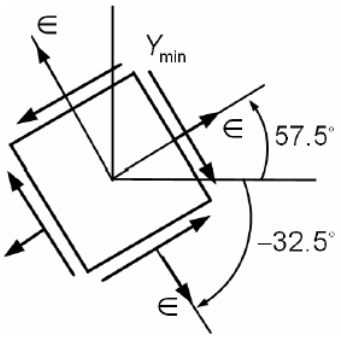
$$2Q_s = 64.98$$

$$Q_s = -32.5^\circ \qquad Q_s = 57.5^\circ$$

$$\epsilon_{x'y'}\Big|_{Q=57.5^\circ} = -\frac{\left(\epsilon_x - \epsilon_y\right)}{2}Sin2(57.5) + \epsilon_{xy}\,Cos2(57.5)$$

$$= -271.89\times10^{-6} - 59.17\times10^{-6} = 331.06\times10^{-6}$$

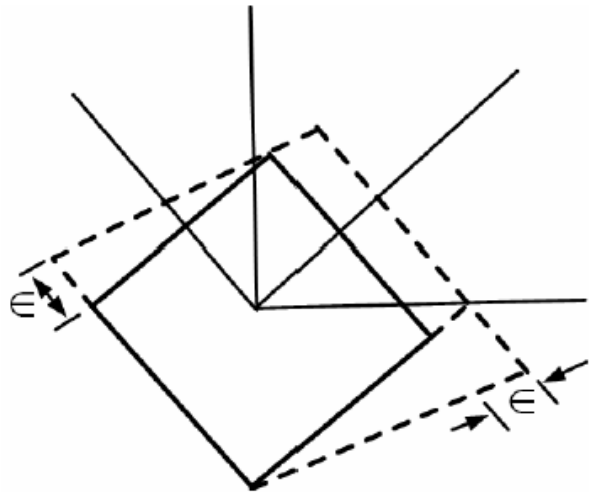
$\theta_{s_1} = -32.5^\circ$ and $\theta_{s_2} = -32.5^\circ$



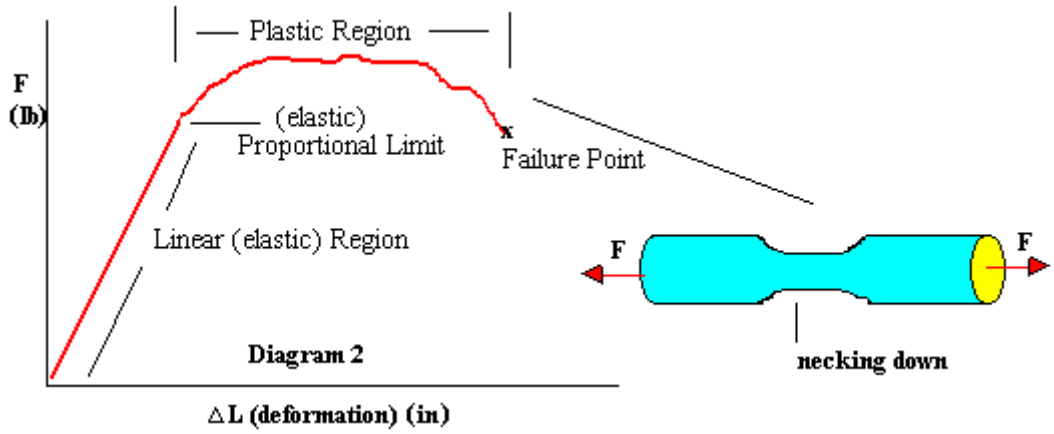
$$Y_{min} = -662.11\times10^{-6}$$

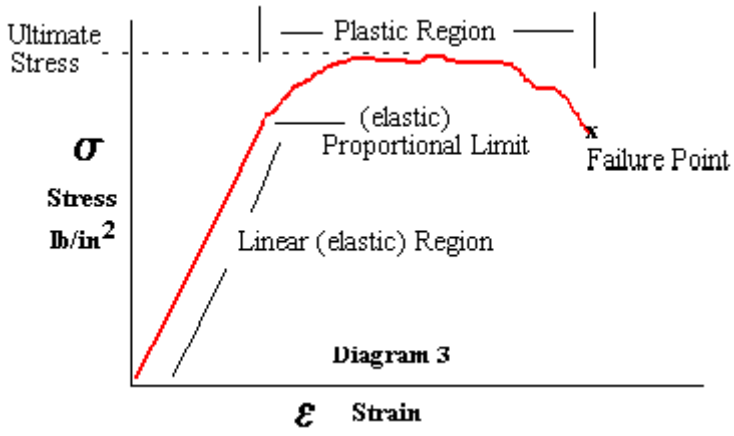
$$Y_{max} = 662.11\times10^{-6}$$

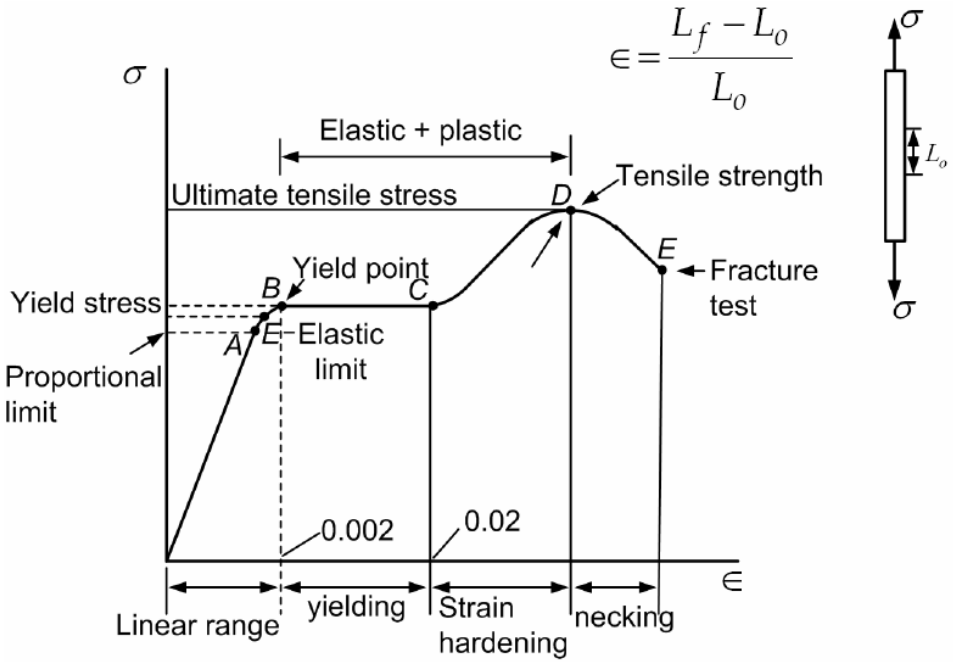
$$\epsilon = \frac{\epsilon_x + \epsilon_y}{2} = 220\times10^{-6}$$



GERİLME – ŞEKİL DEĞİŞTİRME BAĞINTILARI (HOOKE YASALARI)

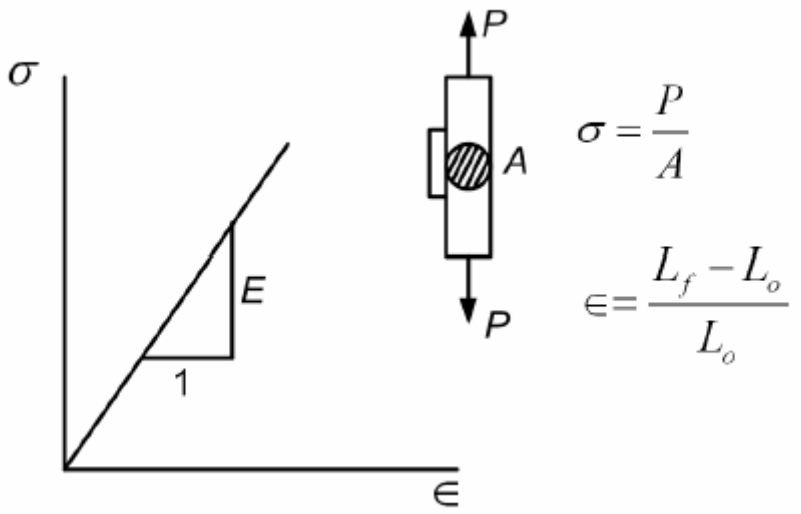






E:Malzemenin elastisite modülü
G:Malzemenin kayma modülü
v:Malzemenin Poisson oranı

elastik bölgede gerilme şekil değiştirme diyagramı



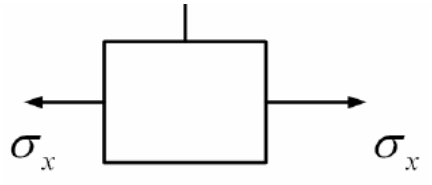
$$\sigma_x = E \in_x \quad \text{or} \quad \sigma_y = E \in_y$$

$$\tau = G\gamma$$

$$\in_x = \frac{\sigma_x}{E}$$

$$\in_y = -v\frac{\sigma_x}{E}$$

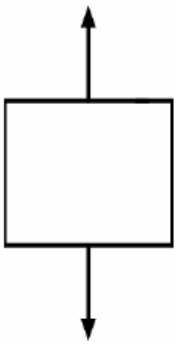
$$\in_z = -v\frac{\sigma_x}{E}$$



$$\in_y = \frac{\sigma_y}{E}$$

$$\in_x = -v\frac{\sigma_y}{E}$$

$$\in_z = -v\frac{\sigma_y}{E}$$

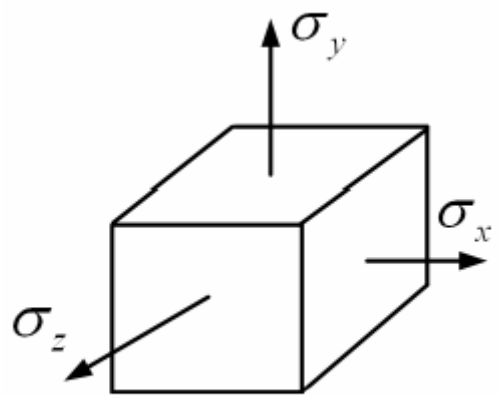


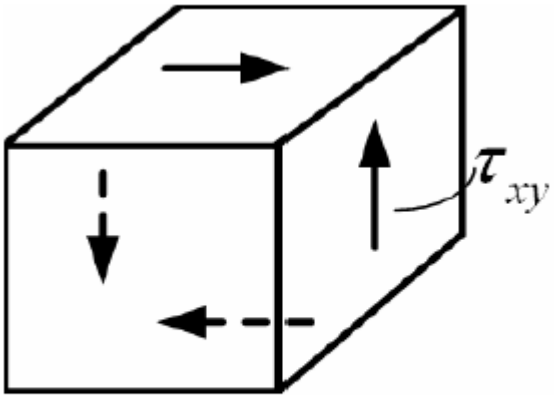
$$\in_x = \frac{\sigma_x}{E} - \frac{v\sigma_y}{E} - \frac{v\sigma_3}{E}$$

$$\in_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right]$$

$$\in_y = \frac{1}{E}\left[\sigma_y - v\left(\sigma_x + \sigma_z\right)\right]$$

$$\in_x = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right]$$





$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

DÜZLEM GERİLME

$$\sigma_x = \sigma_x(x,y)$$
$$\sigma_y = \in_y(x,y)$$
$$\tau_{xy} = \tau_{xy}(x,y)$$
$$\sigma_z = \tau_{yz} = \tau_{zx} = 0$$

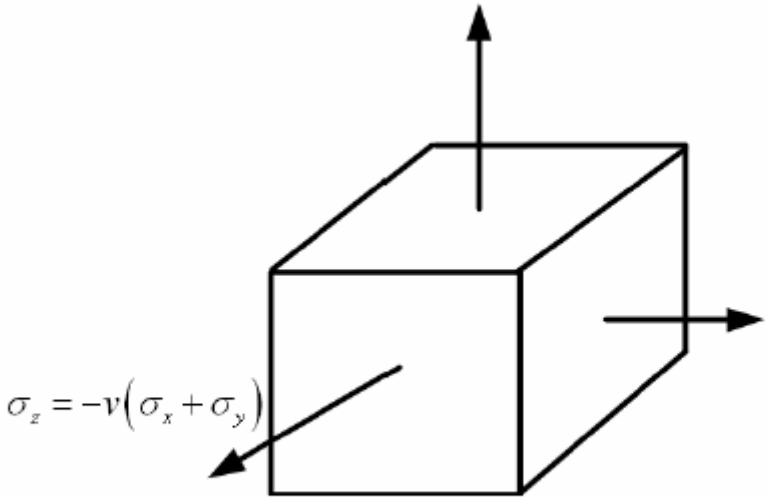
$$\in_x = \frac{1}{E}\left(\sigma_x - v\sigma_y\right)$$
$$\in_y = \frac{1}{E}\left(\sigma_y - v\sigma_x\right)$$
$$\in_z = -\frac{v}{E}\left(\sigma_x + \sigma_y\right) = \frac{-v}{1-v}\left(\in_x + \in_y\right)$$
$$Y_{xy} = \frac{\tau_{xy}}{G}$$
$$Y_{yz} = Y_{xz} = 0$$

DÜZLEM ŞEKİL DEĞİŞTİRMRE

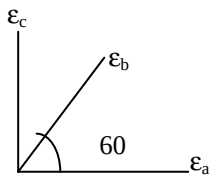
$$\in_x = \in_x(x,y)$$
$$\in_y = \in_y(x,y)$$
$$Y_{xy} = Y_{xy}(x,y)$$
$$\in_3 = Y_{xz} = Y_{yz} = 0$$
$$e = \in_x + \in_y$$

$$\sigma_x = e\lambda + \mu \in_x = \sigma_x(x,y)$$
$$\sigma_y = e\lambda + \mu \in_y = \sigma_y(x,y)$$
$$\sigma_z = -v\left(\sigma_x + \sigma_y\right) = \sigma_z(x,y)$$
$$= -v(2e\lambda + \mu e)$$
$$= -ve(2\lambda + \mu)$$
$$= -v(2\lambda + \mu)\left(\in_x + \in_y\right)$$

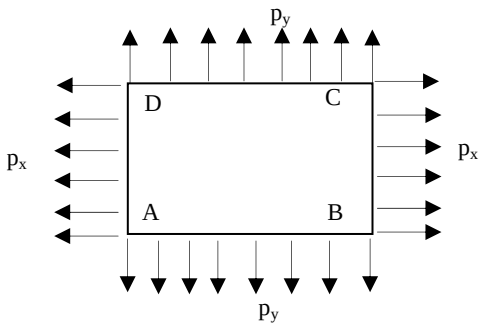
$$\tau_{xy} = GY_{xy}$$
$$\tau_{xz} = \tau_{yz} = 0$$



Şekilde a, b, c doğrultularında $\varepsilon_a=10^{-4}$, $\varepsilon_b=2*10^{-4}$, $\varepsilon_c=4*10^{-4}$ birim boy uzamaları verilmiştir. Asal şekil değiştirme, asal gerilmeleri ve doğrultularını bulunuz. E= $2x10^4$ N/cm^2, ν=0.2



AB kenarı 3m, BC kenarı 2m ve kalınlığı 0.5 cm olan bir levha şekilde görüldüğü gibi p_x, p_y düzgün yayılı yükler ile yüklenmiştir. Yüklemeden sonra AB kenarının 0.195 cm, BC kenarının 0.08 cm uzadığı ölçülmüştür. Bu duruma göre p_x ve p_y nin şiddetleri ile levha kalınlığının ne kadar değiştiğini bulunuz. E=$2*10^6$ N/cm^2, G=$0.8*10^6$ N/cm^2



$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$G = \frac{E}{2(1+\nu)}$$

GERİLME – ŞEKİL BAĞINTILARI

$$\varepsilon_\xi = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{(\frac{\varepsilon_x - \varepsilon_y}{2})^2 + (\frac{\gamma_{xy}}{2})^2}$$

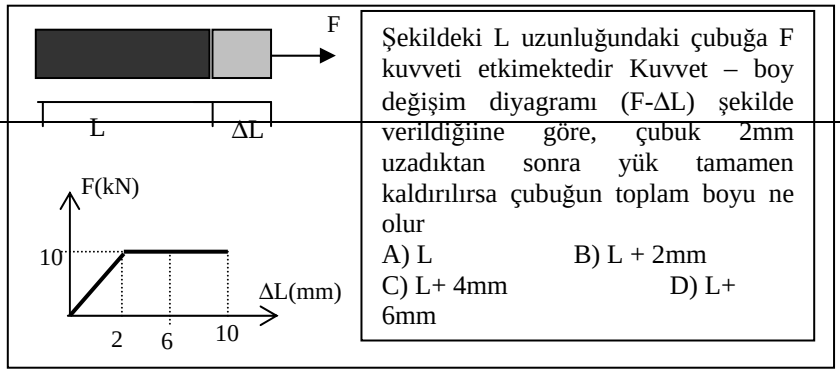
$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right], \qquad \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \qquad \varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_y + \sigma_x)\right]$$

$$\gamma_{\xi\eta} = -2(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \gamma_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\tan 2\theta_0 = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\sigma_{1,2} = \frac{\sigma_X + \sigma_Y}{2} \pm \sqrt{\left(\frac{\sigma_X - \sigma_Y}{2}\right)^2 + \tau_{XY}^2}\ , \quad \sigma = \sigma_X \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

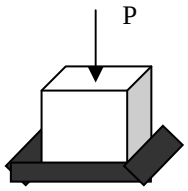
$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$



Şekildeki L uzunluğundaki çubuğa F kuvveti etkimektedir Kuvvet – boy değişim diyagramı (F-ΔL) şekilde verildiğiine göre, çubuk 2mm uzadıktan sonra yük tamamen kaldırılırsa çubuğun toplam boyu ne olur

A) L B) L + 2mm

C) L+ 4mm D) L+ 6mm



Boyutları a ve elastisite modülü E olan küp şeklindeki bir cisim, rijit bir tabana oturmuştur. Cismin üzerine etkiyen P tekil yükü etkisinde düşey yönde uzama oranı ε_y nedir.

A) P/(aE) B) P/(a^2E)

C) P/a^2 D) P/E