Impedance of a cylindrical coil over an infinite metallic half-space with shallow surface features

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An analytical solution is derived for the electric and magnetic fields produced by an air-core cylindrical coil driven by a constant ac current source and placed over an infinite metallic half-space with small surface perturbations. The depth of the features on the half-space is small compared to the mean radius of the coil. The perturbation technique is used to solve the Maxwell equations in the quasistatic approximation. Formulas are given for the electric field and the impedance. Real and imaginary parts of the impedance are computed and plotted for several test cases. © *1999 American Institute of Physics.* [S0021-8979(99)02016-2]

I. INTRODUCTION

Eddy-current air-core coils are widely used to detect surface flaws and to measure properties of the surface layers. The technique consists of measuring the impedance of the coil over a homogeneous half-space and then over a half-space with the flaw or the surface layer at various frequencies and for positions of the coil, and then inferring the properties of the flaw or the surface layer from the difference in the impedance. A crucial part of this process is the ability to solve the forward problem, i.e., computing the impedance when the properties of the flaw or the layer are given.

Layered-surface problems have been widely addressed before since the necessary forward solutions are known.[1–4] Analytical solutions for the more general problem of a surface flaw are scarce. Nair and Rose[5] presented an analytical solution for the fields in a half-space in the presence of inhomogeneities, but in their study the fields were excited by space-sinusoidal current sheets rather than cylindrical coils. They also gave an analytical inversion formula. Kahn, Spal, and Feldman[6] treated a long crack in a uniform incident field in the thin-skin regime, which was extended later by Harfield and Bowler[7–9] using a variety of methods. Harfield and Bowler[10] also used a perturbation approach at low frequency. Lewis[11] treated a long crack in a material of arbitrary permeability with a cylindrical coil in the thin-skin regime. Burke[12] treated a long crack with a cylindrical coil in the thin-skin regime and also considered inversion. Other studies aiming to solve the forward problem for a cylindrical coil placed over an inhomogeneous half-space use numerical methods; see, for example, Palanisamy, Ida, and Lord.[13]

In this study we would like to address the problem of a cylindrical air-core coil placed over a metallic half-space with voids or cracks on the surface. Because of the complicated geometry of the surface of the half-space a full analytical solution is not possible. Here, we adopt the perturbation theory approach and assume that the disturbance of the surface from a flat surface is small compared to the skin depth of the fields into the metal at the given frequency. This allows us to expand the jump conditions on the surface into a Taylor series and keep only a few terms. The governing equations are also expanded into a Taylor series. Each of the resulting series of problems is amenable to analytical treatment. The leading order problem is just the coil over a flat half-space. The next order problem gives a first correction to this due to the geometry of the half-space's surface. It also provides us with the leading order impedance change due to the surface feature. The higher order terms in the series give the corrections to this impedance change. We will only derive the leading order impedance change. Then, this formula will be used to compute the impedance difference for several types of surface features.

In Sec. II, we setup the problem and present its solution by the perturbation technique. In Sec. III, we derive the impedance formula and compare it with a known solution in a simple case in Sec. IV. Finally, we conclude with some results and discussion in Sec. V.

II. SOLUTION BY THE PERTURBATION TECHNIQUE

In this section, we will setup the basic problem in subsection A and solve it by the perturbation technique in subsection B.

A. Problem formulation

Consider a cylindrical coil with n turns of wire driven by the constant time-harmonic current $Ie^{i\varpi t}$ and placed over a half-space with a small surface perturbation (Fig. 1). Coil's axis is perpendicular to the unperturbed surface of the half-space. The equation of the surface is

$$z=\epsilon f(x,y), \tag{2.1}$$

where ϵ is a small parameter that will be used in the Taylor series expansions. The material's conductivity, σ, is constant, and the magnetic permeability is that of free-space everywhere. The time dependence of electric and magnetic fields is of the form

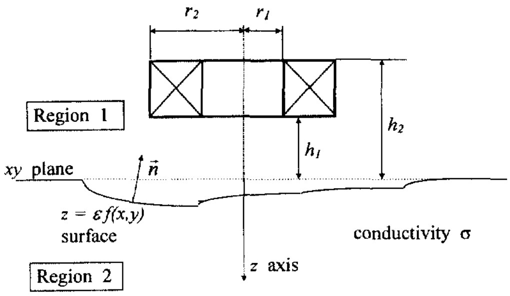

FIG. 1. Cylindrical coil over the flat surface with a void expressed by the equation $z=\epsilon f(x,y)$.

$$\mathbf{E}e^{i\varpi t} \quad \text{and} \quad \mathbf{H}e^{i\varpi t},$$

where **E** and **H** are time-independent complex vectors. In the quasistatic approximation, i.e., ignoring the displacement current term, the Maxwell equations can be simplified to give, for the electric field

$$\nabla^2\mathbf{E}-i\varpi\mu\sigma\mathbf{E}=i\varpi\mu\mathbf{J}. \tag{2.2}$$

Here J is the driving current density. We assume that the current is uniformly distributed over the cross section of the coil, thus

$$\mathbf{J}=j\mathbf{e}_\phi, \tag{2.3a}$$

where

$$j=\frac{nI}{(r_2-r_1)(h_2-h_1)} \quad \text{if } -h_2<z<-h_1$$

$$\text{and } r_1<r<r_2 \tag{2.3b}$$

$$j=0 \quad \text{otherwise.} \tag{2.3c}$$

(R,ϕ,z) are the cylindrical coordinates and $\mathbf{e}_\phi$ is the unit vector in the ϕ direction. It is understood that the term in Eq. (2.2) containing σ vanishes in free-space. The magnetic field is given by

$$-i\omega\mu\mathbf{H}=\nabla\times\mathbf{E}. \tag{2.4}$$

We will solve Eq. (2.2), therefore the jump conditions are all expressed in terms of the electric field. For a nonmagnetic material, these become

$$\mathbf{n}\times\Delta\mathbf{E}=0 \quad \text{on} \quad z=\epsilon f(x,y), \tag{2.5a}$$

$$\mathbf{n}\times(\nabla\times\Delta\mathbf{E})=0 \quad \text{on} \quad z=\epsilon f(x,y), \tag{2.5b}$$

$$\mathbf{n}\times(\nabla\times\Delta\mathbf{E})=0 \quad \text{on} \quad z=\epsilon f(x,y), \tag{2.5c}$$

where **n** is the normal to the surface as shown in Fig. 1. The first two equations express the continuity of the tangential electric field and perpendicular magnetic intensity vector. The last equation expresses the fact that there are no surface currents on the surface of the metal. We use the notation

$$\Delta\mathbf{E}=\mathbf{E}^{(1)}-\mathbf{E}^{(2)}, \tag{2.6}$$

where $\mathbf{E}^{(1)}$ and $\mathbf{E}^{(2)}$ represent the electric fields in regions 1 and 2. Region 1 is $z<\epsilon f(x,y)$ and region 2 is $z>\epsilon f(x,y)$ as shown in Fig. 1. Note that Eqs. (2.5b) and (2.5c) simplify to

$$\nabla\times\Delta\mathbf{E}=0 \quad \text{on} \quad z=\epsilon f(x,y). \tag{2.7}$$

In addition, we must impose the condition

$$\mathbf{E}\to 0 \quad \text{as} \quad x^2+y^2+z^2\to\infty. \tag{2.8}$$

We assume that ϵ is small in some sense and write the vector **n** using Taylor expansion as

$$\mathbf{n}=-\mathbf{e}_z+\epsilon\left(\frac{\partial f}{\partial x}\mathbf{e}_x+\frac{\partial f}{\partial y}\mathbf{e}_y\right)+O(\epsilon^2). \tag{2.9}$$

The second and higher order terms will be neglected. This expansion is valid if the slope of the surface feature is smaller everywhere than the order $1/\epsilon$. The electric field is also expanded as

$$\mathbf{E}=\mathbf{E}_0+\epsilon\mathbf{E}_1+O(\epsilon^2). \tag{2.10}$$

Substituting Eq. (2.10) in Eq. (2.2) and ordering in terms of ϵ gives

$$\nabla^2\mathbf{E}_0-i\varpi\mu\sigma\mathbf{E}_0=i\varpi\mu\mathbf{J}, \tag{2.11a}$$

$$\nabla^2\mathbf{E}_1-i\varpi\mu\sigma\mathbf{E}_1=0. \tag{2.11b}$$

In the jump conditions, we use the Taylor series expansion

$$[\Phi]_{z=\epsilon f(x,y)}=[\Phi]_{z=0}+\left[\frac{\partial\Phi}{\partial z}\right]_{z=0}\cdot\epsilon f(x,y)+O(\epsilon^2) \tag{2.12}$$

for any quantity $\Phi(x,y,z)$. This gives, for example,

$$[\Delta\mathbf{E}]_{z=\epsilon f(x,y)}=[\Delta\mathbf{E}_0]_{z=0}+\epsilon\left[f(x,y)\Delta\left(\frac{\partial\mathbf{E}_0}{\partial z}\right)+\Delta\mathbf{E}_1\right]_{z=0}+O(\epsilon^2). \tag{2.13}$$

The expansion procedure allows us to write the jump conditions on $z=0$ rather than on the surface $z=\epsilon f(x,y)$. Finally, the leading and the first order jump conditions become, by substituting Eqs. (2.9) and (2.10) into Eqs. (2.5a) and (2.7)

$$\mathbf{e}_z\times\Delta\mathbf{E}_0=0 \quad \text{on} \quad z=0, \tag{2.14a}$$

$$\nabla\times\Delta\mathbf{E}_0=0 \quad \text{on} \quad z=0, \tag{2.14b}$$

and

$$\mathbf{e}_z\times\Delta\mathbf{E}_1=\left(\frac{\partial f}{\partial x}\mathbf{e}_x+\frac{\partial f}{\partial y}\mathbf{e}_y\right)\times\Delta\mathbf{E}_0-f\cdot\mathbf{e}_z\times\Delta\left(\frac{\partial\mathbf{E}_0}{\partial z}\right) \quad \text{on} \quad z=0, \tag{2.15a}$$

$$\nabla\times\Delta\mathbf{E}_1=-f\cdot\nabla\times\Delta\left(\frac{\partial\mathbf{E}_0}{\partial z}\right) \quad \text{on} \quad z=0. \tag{2.15b}$$

B. Solution

The solution of the problem for E_0 is readily available,[1,2]

$$\mathbf{E}_0=E_0(R,z)\mathbf{e}_\phi, \tag{2.16a}$$

where, for $z<0$

$$E_0=-\frac{1}{2}i\varpi\mu j\int_0^{\infty}\left(e^{-\alpha z}+\frac{\alpha-\alpha_0}{\alpha+\alpha_0}e^{\alpha z}\right)\times\frac{P(\alpha)}{\alpha}(e^{-\alpha h_1}-e^{-\alpha h_2})J_1(\alpha R)d\alpha \tag{2.16b}$$

and, for $z>0$

$$E_0=-i\varpi\mu j\int_0^{\infty}\frac{e^{-\alpha_0 z}}{\alpha+\alpha_0}\times P(\alpha)(e^{-\alpha h_1}-e^{-\alpha h_2})J_1(\alpha R)d\alpha. \tag{2.16c}$$

α_0 and $P(\alpha)$ are defined as

$$\alpha_0=\sqrt{\alpha^2+i\varpi\mu\sigma}, \tag{2.16d}$$

$$P(\alpha)=\int_{r_1}^{r_2}xJ_1(\alpha x)dx, \tag{2.16e}$$

and (R,ϕ) are the polar coordinates in the xy plane. To solve the E_1 problem, we express Cartesian coordinates as

$$\mathbf{E}_1=U\mathbf{e}_x+V\mathbf{e}_y+W\mathbf{e}_z. \tag{2.17}$$

Let

$$U=U^{(1)},\quad \text{for } z<0, \tag{2.18a}$$

$$U=U^{(2)},\quad \text{for } z>0, \tag{2.18b}$$

and similarly for V and W. Then $U^{(1)}$ and $U^{(2)}$ satisfy

$$\nabla^2U^{(1)}=0, \tag{2.19a}$$

$$\nabla^2U^{(2)}-i\varpi\mu\sigma U^{(2)}=0, \tag{2.19b}$$

and V and W satisfy similar equations. We also need to use $\nabla\cdot\mathbf{E}_1=0$, i.e.,

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0 \tag{2.20}$$

in both regions. The jump conditions become

$$\Delta U=0\quad \text{on}\quad z=0, \tag{2.21a}$$

$$\Delta V=0\quad \text{on}\quad z=0, \tag{2.21b}$$

$$\Delta\left(\frac{\partial U}{\partial z}\right)-\Delta\left(\frac{\partial W}{\partial x}\right)=yf(x,y)\psi(R)\quad \text{on}\quad z=0, \tag{2.21c}$$

$$\Delta\left(\frac{\partial W}{\partial y}\right)-\Delta\left(\frac{\partial V}{\partial z}\right)=xf(x,y)\psi(R)\quad \text{on}\quad z=0, \tag{2.21d}$$

where

$$\psi(R)=\frac{i\varpi\mu nj}{R}\int_0^{\infty}(\alpha_0-\alpha)P(\alpha)(e^{-\alpha h_1}-e^{-\alpha h_2})\times J_1(\alpha R)d\alpha. \tag{2.22}$$

We should also add the condition that all of these components vanish as $x^2+y^2+z^2\to\infty$.

In order to solve Eqs. (2.19)–(2.21) we apply the two-dimensional Fourier transform; for any function $u(x,y)$, its Fourier transform is defined by

$$\tilde{u}(\xi,\eta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u(x,y)e^{i(\xi x+\eta y)}dxdy \tag{2.23a}$$

and the inversion formula is

$$u(x,y)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{u}(\xi,\eta)e^{-i(\xi x+\eta y)}d\xi d\eta. \tag{2.23b}$$

Applying the Fourier transform to the governing equations and noting that the fields vanish as $x^2+y^2+z^2\to\infty$, we find

$$\tilde{U}_1=a_1e^{rz},\quad \tilde{U}_2=a_2e^{-\rho z}, \tag{2.24a}$$

$$\tilde{V}_1=b_1e^{rz},\quad \tilde{V}_2=b_2e^{-\rho z}, \tag{2.24b}$$

$$\tilde{W}_1=c_1e^{rz},\quad \tilde{W}_2=c_2e^{-\rho z}. \tag{2.24c}$$

Note that in the perturbed problem, the shape of the surface feature only appears as a source term in Eqs. (2.21c) and (2.21d). The solution domain is bound by the $z=0$ plane. Therefore, the solutions in the Fourier space are of the form of Eq. (2.24). Equation (2.20), after applying Fourier transform, gives

$$-i\xi a_1-i\eta b_1+rc_1=0, \tag{2.25a}$$

$$-i\xi a_2-i\eta b_2-\rho c_2=0, \tag{2.25b}$$

where

$$r^2=\xi^2+\eta^2, \tag{2.26a}$$

$$\rho^2=r^2+i\varpi\mu\sigma. \tag{2.26b}$$

Applying the jump conditions in Fourier-transformed form, we find four more equations

$$a_1-a_2=0, \tag{2.27a}$$

$$b_1-b_2=0, \tag{2.27b}$$

$$ra_1+\rho a_2+i\xi(c_1-c_2)=\tilde{X}, \tag{2.27c}$$

$$-i\eta(c_1-c_2)-(rb_1+\rho b_2)=\tilde{Y}, \tag{2.27d}$$

where

$$X=yf(x,y)\psi(R), \tag{2.28a}$$

$$Y=xf(x,y)\psi(R). \tag{2.28b}$$

Solving Eqs. (2.25) and (2.27), we find

$$a_1=a_2=\frac{1}{i\varpi\mu\sigma}\left[\left(\rho-\frac{\eta^2}{r}\right)\tilde{X}-\frac{\xi\eta}{r}\tilde{Y}\right], \tag{2.29a}$$

$$b_1=b_2=\frac{1}{i\varpi\mu\sigma}\left[\frac{\xi\eta}{r}\tilde{X}-\left(\rho-\frac{\xi^2}{r}\right)\tilde{Y}\right], \tag{2.29b}$$

$$c_1=\frac{1}{\varpi\mu\sigma}\frac{\rho}{r}(\xi\tilde{X}-\eta\tilde{Y}), \tag{2.29c}$$

$$c_2=-\frac{1}{\varpi\mu\sigma}(\xi\tilde{X}-\eta\tilde{Y}). \tag{2.29d}$$

The perturbation in the electric field can be found by substituting Eq. (2.29) in Eq. (2.24) and then taking the inverse Fourier transform.

III. THE IMPEDANCE FORMULA

The formula for the electric field developed in the previous section is quite complicated since it involves two-dimensional Fourier and inverse Fourier transforms. In this section we will write a formula for the impedance difference due to the surface features. It turns out that the formula can be simplified considerably. In general, the voltage induced in the n-turn coil is given by

$$V=-\frac{n}{(h_2-h_1)(r_2-r_1)}\int\int_{\text{cross section}}\left\{\oint_{(r_0,h)}\mathbf{E}\cdot d\mathbf{l}\right\}dr_0dh. \tag{3.1}$$

Here the outer double integral is taken over the cross section of the coil, the inner curvilinear integral is along a current filament located at (r_0,h), and the factor in front is the turns per unit area of the coil cross section. We are interested in the impedance over an unflawed half-space minus the impedance over the flawed surface:

$$\Delta Z=Z_{\text{unflawed}}-Z_{\text{flaw}}. \tag{3.2}$$

The impedance difference due to the surface perturbation can thus be written as

$$\Delta Z=-\frac{\epsilon}{I}\frac{n}{(h_2-h_1)(r_2-r_1)}\times\int\int_{\text{cross section}}\left\{\oint_{(r_0,h)}(U^{(1)}dx+V^{(1)}dy)\right\}dr_0dh. \tag{3.3}$$

In the innermost integral in Eq. (3.3), we substitute $U^{(1)}$ and $V^{(1)}$ using Eq. (2.23b), and also make the change of variables

$$\xi=r\cos\theta,\quad \eta=r\sin\theta. \tag{3.4}$$

The result for the innermost integral is

$$-\frac{r_0}{i\varpi\mu\sigma}\int_0^\infty\int_0^\infty F_0(R)e^{-rh}(\rho-r)\times J_1(r_0r)rRJ_1(rR)\psi(R)RdRdr. \tag{3.5}$$

Here we used the identity

$$2\pi J_1(x)=ix\int_0^{2\pi}e^{-ix\cos z}\sin z dz \tag{3.6}$$

and J_1 is the Bessel function of order 1. $F_0(R)$ is defined as

$$F_0(R)=\int_0^{2\pi}f(R,\phi)d\phi \tag{3.7}$$

and $f(R,\phi)$ is $f(x,y)$ expressed in polar coordinates. In Eq. (3.3) we perform the integration on the cross section first; the end result is

$$\Delta Z=\frac{\epsilon}{\sigma}\frac{n^2}{(h_2-h_1)^2(r_2-r_1)^2}\int_0^\infty R[G(R)]^2F_0(R)dR, \tag{3.8}$$

where

$$G(R)=\int_0^\infty(\alpha_0-\alpha)P(\alpha)(e^{-\alpha h_1}-e^{-\alpha h_2})J_1(\alpha R)d\alpha. \tag{3.9}$$

If we nondimensionalize all lengths by the mean coil radius $r_0=(r_1+r_2)/2$, then Eq. (3.8) becomes

$$\Delta Z=\frac{\epsilon}{\sigma r_0^2}\frac{n^2}{(h_2-h_1)^2(r_2-r_1)^2}\int_0^\infty R[G(R)]^2F_0(R)dR. \tag{3.10}$$

Equation (3.9) remains in the same form but the definition of α_0 should be changed to

$$\alpha_0=\sqrt{\alpha^2+i\varpi\mu\sigma r_0^2}. \tag{3.11}$$

Assuming that the integral in Eq. (3.10) is well behaved, the impedance change scales as $1/\sigma r_0$ and its size essentially depends on ϵ/r_0. For the perturbation theory to be valid, ϵ/r_0 should be much less than unity.

The impedance difference seems to depend on the circular average of the surface feature through Eq. (3.7). Therefore, different flaws may give the same impedance change, at the linear order in ϵ.

IV. VERIFICATION OF THE IMPEDANCE FORMULA FOR $f(x,y)=1$

In this section we will compare the impedance formula with a known solution in a simple case. For simplicity, we take a single turn coil of radius r_0 and lift-off distance h. The impedance change due to a void becomes, in this case,

$$\Delta Z=\frac{\epsilon r_0^2}{\sigma}\int_0^\infty R[G(R)]^2F_0(R)dR, \tag{4.1}$$

where $F_0(R)$ is defined as before and

$$G(R)=\int_0^\infty\alpha(\alpha_0-\alpha)e^{-\alpha h}J_1(\alpha r_0)J_1(\alpha R)d\alpha. \tag{4.2}$$

If $f(x,y)=1$, this means that the coil is on an unflawed half-space, the lift-off is changed from h to $h+\epsilon$ and the difference is taken. The impedance of a single turn coil due to the presence of a half-space is[1,2]

$$Z_{hs}=\pi r_0^2 i\varpi\mu\int_0^\infty\frac{\alpha-\alpha_0}{\alpha+\alpha_0}e^{-2\alpha h}J_1^2(\alpha r_0)d\alpha. \tag{4.3}$$

For our case

$$\Delta Z_{hs}=Z_{hs}(\text{with } h+\epsilon)-Z_{hs}(\text{with } h)\quad\text{and } \epsilon\alpha\ll 1. \tag{4.4}$$

The restriction in Eq. (4.4) means that ϵ is much smaller than the inverse of the important range of integration in Eq. (4.3). If we expand Eq. (4.3) in a Taylor series and take up to linear terms in ϵ we find

$$\Delta Z=\frac{2\epsilon\pi r_0^2}{\sigma}\int_0^\infty\alpha(\alpha-\alpha_0)^2e^{-2\alpha h}J_1^2(\alpha r_0)d\alpha. \tag{4.5}$$

This should be compared with Eq. (4.1) for $f(x,y)=1$. In this case $F_0(R)=2\pi$ and using the orthogonality of the Bessel functions

$$\int_0^\infty RJ_1(\alpha R)J_1(\beta R)dR=\frac{1}{\alpha}\delta(\beta-\alpha), \tag{4.6}$$

we arrive at the same formula as (4.5).

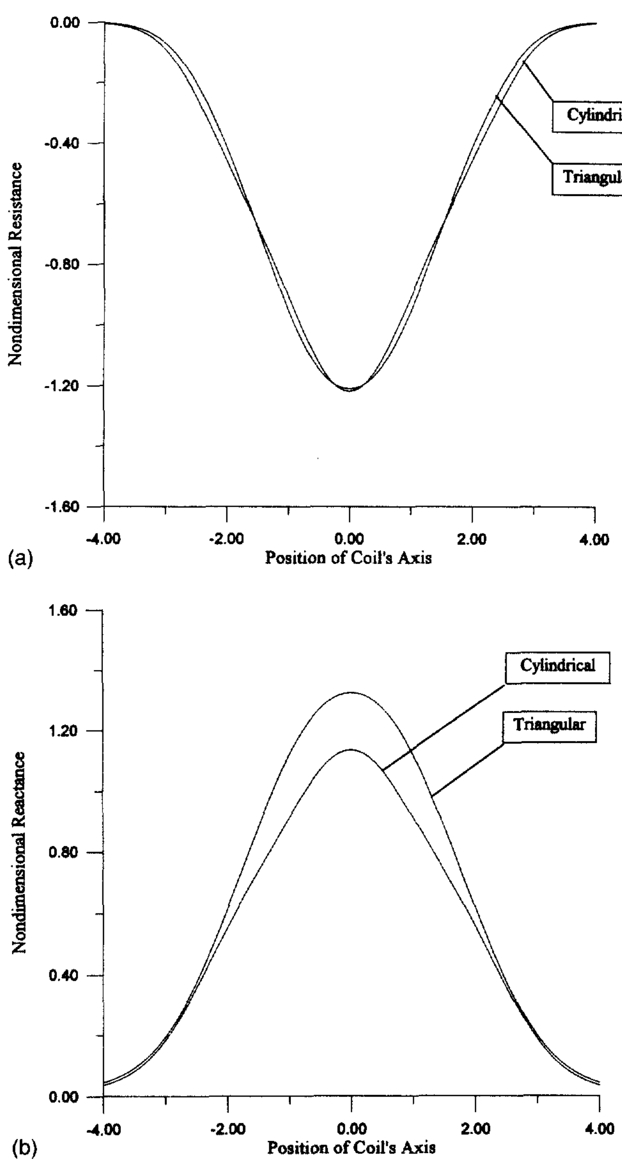

FIG. 2. (a) Real part of the impedance difference as the coil traverses over a cylindrical and a triangular groove (both grooves have an opening width of 4). (b) Imaginary part of the impedance difference as the coil traverses over a cylindrical and a triangular groove (both grooves have an opening width of 4).

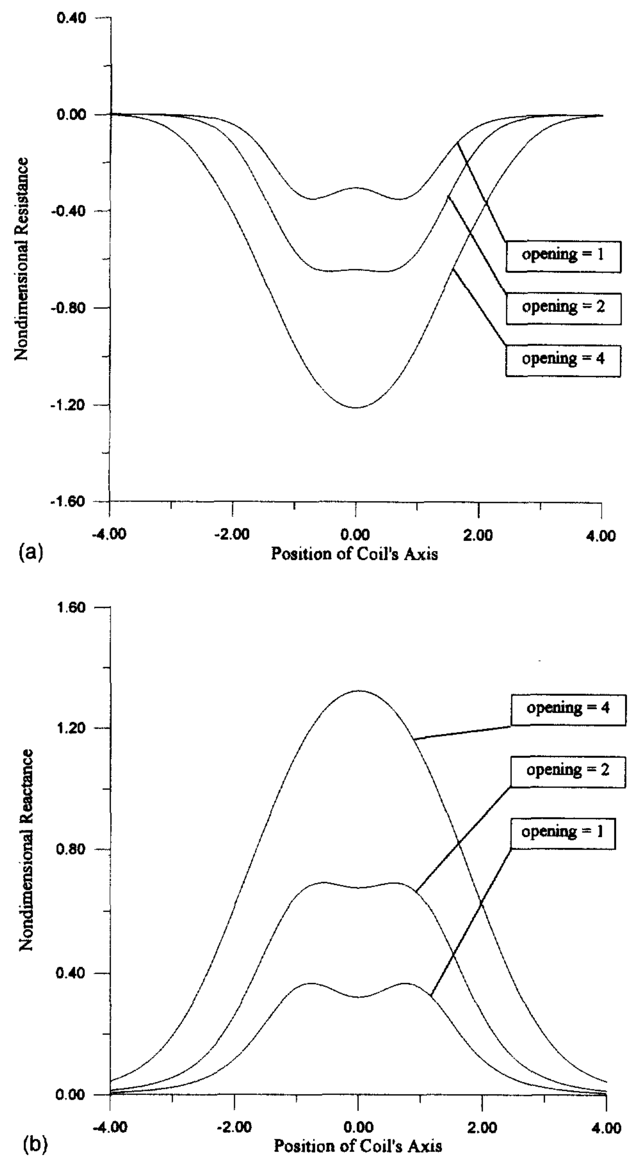

FIG. 3. (a) Real part of the impedance difference as the coil traverses over three different triangular grooves with openings 4, 2, and 1 at the surface. (b) Imaginary part of the impedance difference as the coil traverses over three different triangular grooves with openings 4, 2, and 1 at the surface.

V. RESULTS AND DISCUSSION

In this section, we will use Eq. (3.10) to compute the impedance difference due to several surface features. In all the computations we use the coil with the nondimensional parameters $r_1=0.5$, $r_2=1.5$, $h_1=0.1$, $h_2=3.5$ (see Fig. 1); also, we fix the constant $\varpi\mu\sigma r_0^2=5$. The shape of the surface feature is defined in a $x'y'z$ coordinate system as

$$z=\epsilon f(x',y').$$

The coordinates of the coil's axis in the $x'y'$ system are (h_x,h_y). Thus the relation to the xy system fixed to the coil's axis is

$$x'=x+h_x,\quad y'=y+h_y.$$

As a first example, we consider a void in the shape of a groove extending to infinity in the y direction. The groove opening occupies $-2<x<2$ (nondimensional) and its depth is 1 (i.e., ϵ in dimensional coordinates). Two different grooves are considered:

Triangular:

$$f(x',y')=1+\tfrac{1}{2}x',\quad \text{for } -2<x<0,$$

$$1-\tfrac{1}{2}x',\quad \text{for } 0<x<2,$$

$$0,\quad \text{elsewhere,}$$

and cylindrical

$$f(x',y')=-1.5+\sqrt{6.25-x'^2},\quad \text{for } -2<x<2$$

$$0,\quad \text{elsewhere.}$$

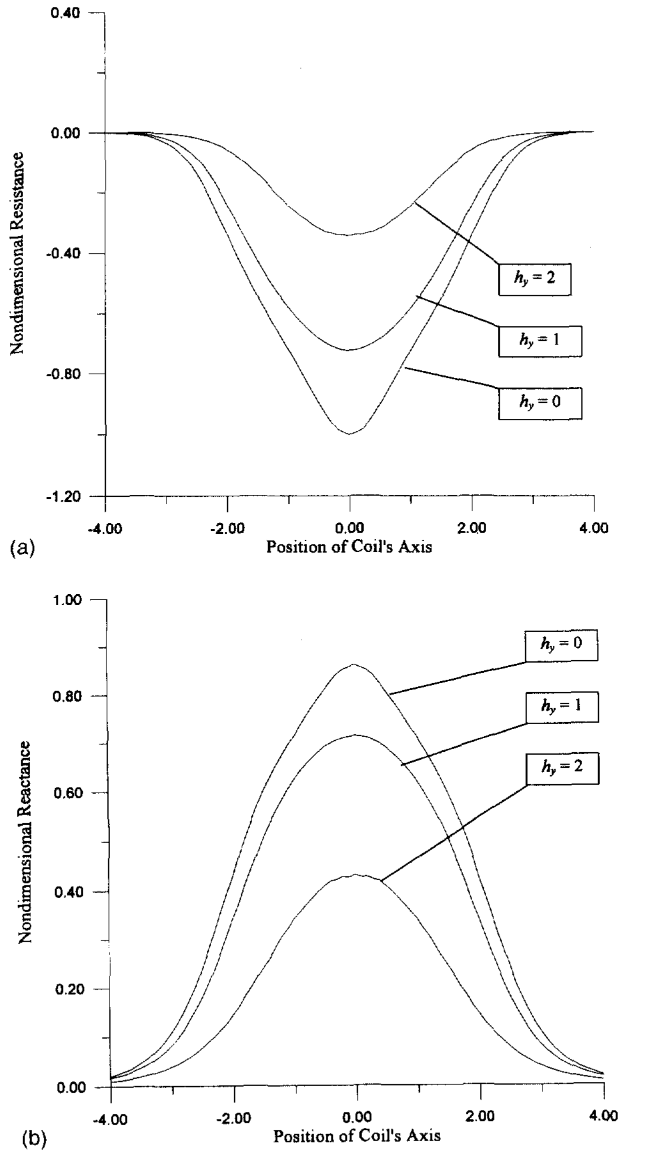

FIG. 4. (a) Real part of the impedance difference as the coil traverses through three different positions: $h_y=0$, 1, and 2, over a spherical pit with an opening radius of 2. (b) Imaginary part of the impedance difference as the coil traverses through three different positions: $h_y=0$, 1, and 2, over a spherical pit with an opening radius of 2.

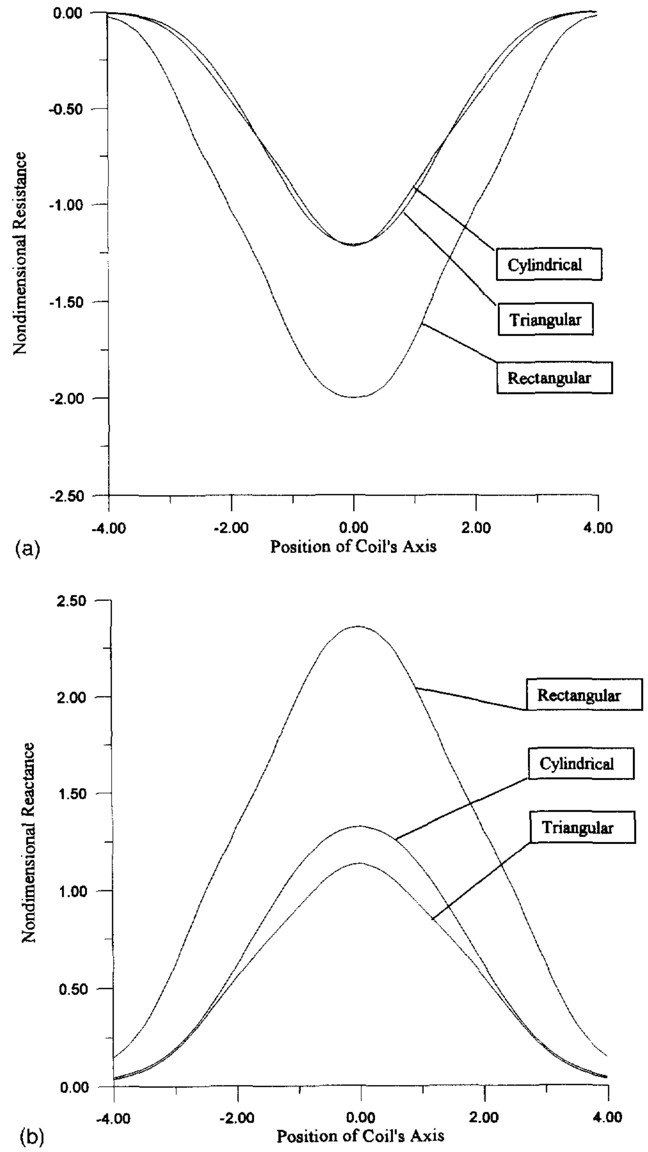

FIG. 5. (a) Real part of the impedance difference for the coil traversing over a rectangular groove compared to triangular and cylindrical grooves (opening width of all the grooves is equal to 4). (b) Imaginary part of the impedance difference for the coil traversing over a rectangular groove compared to triangular and cylindrical grooves (opening width of all the grooves is equal to 4).

The axis of the cylinder is at $z=-1.5$ so that the slopes of the groove at the sides $x'=\pm 2$ are finite. The impedance is independent of h_y and only changes with h_x which shows the coil's position relative to the groove. Figures 2(a) and 2(b) show the real and imaginary parts of the impedance difference as h_x changes from -4 to $+4$. Note that only the integral in Eq. (3.10) (a nondimensional impedance) is plotted. The real part of the impedance difference is relatively insensitive to the shape of the groove compared to the imaginary part.

In order to see the effect of the crack opening on the impedance change, we take the triangular groove, fix its depth at 1, and take three different opening widths, 4, 2, and 1. The resulting impedance changes are shown in Figs. 3(a) and 3(b). As the opening gets smaller, the signal levels drop but the width of the signals decrease relatively less. Also noteworthy is the oscillatory behavior observed for tighter grooves both in the real and imaginary parts of the signal.

As a three dimensional example, we consider a spherical pit given as

$$f(x',y')=-1.5+\sqrt{6.25-x'^2-y'^2}, \quad \text{for } x'^2+y'^2<4.$$

Again, the center of the sphere is at $z=-1.5$ so that the slope condition is not violated at the edges. In this case, we fix h_y at three different locations, $h_y=0$, 1, and 2, and change h_x from -4 to $+4$. The resulting impedance changes are shown in Figs. 4(a) and 4(b). The signal is weaker than the case of the cylindrical groove.

Finally, we consider a rectangular groove. The theory breaks down at the points $x'=\pm 2$ in this case. However, it may be expected that the singularities at these points are localized and the solution for the fields is valid away from them. The effect of the singularities on the impedance change remains to be investigated. For comparison, we show the impedance change for the rectangular groove together with triangular and cylindrical grooves in Figs. 5(a) and 5(b).

Any void on the surface seems to decrease the resistance and increase the reactance from that of an unflawed half-space. But this change is not monotonic; for tighter voids there are oscillations in the impedance signal (both in real and imaginary parts). Oscillations get higher as the void gets tighter.

We have presented an asymptotic-analytic solution for the impedance of an air core coil on a half-space with a surface breaking flaw. This solution can be used to infer the general properties of the expected signal from a class of flaws. If surface scan measurements of the flaw signal at various frequencies are provided, the solution can be used in the inversion studies and its real value can be assessed.

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