A DYNAMICAL MODEL OF A COGNITIVE FUNCTION: ACTION SELECTION

Özkan Karabacak, N. Serap Şengör

Faculty of Electrical and Electronics Engineering, İstanbul Technical University, İstanbul, Turkey

What was our expectancy?

To find out how "action selection" function arises in our brain, so we modelled cortex-basal ganglia-thalamus (C-BG-TH-C) loop.

How did we approach?

Neural substrates are considered as simple cells and when necessary the number of cells are increased. As the number of cells are increased, more than one parallel C-BG-TH-C loops arised each corresponding to a different action.

What is C-BG-TH-C loop?

- Basal Ganglia once → only motor control
  now → also cognitive processes
- Dysfunctions - Parkinson's disease - Schizophrenia
  - Huntington's disease - Tourette's syndrome

Who has dealt with these?

- Alexander'90 proposed five basal ganglia-thalamocortical loops.
- Mathematical models are proposed by Taylor'00, Gurney'01, Suri'01.

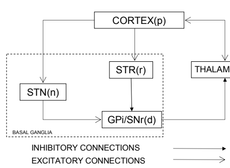


Figure 1: C-BG-TH-C loop

How did we model it?

$$\begin{bmatrix} r(k+1) \\ n(k+1) \\ d(k+1) \\ m(k+1) \\ p(k+1) \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&\lambda&0 \\ 0&0&0&0&\lambda \end{bmatrix} \begin{bmatrix} r(k) \\ n(k) \\ d(k) \\ m(k) \\ p(k) \end{bmatrix} + \begin{bmatrix} 0&0&0&0&1 \\ 0&0&0&0&1 \\ -a&b&0&0&0 \\ 0&0&-1&0&1 \\ 0&0&0&1&0 \end{bmatrix} \begin{bmatrix} g(r(k)) \\ f(n(k)) \\ f(d(k)) \\ f(m(k)) \\ f(p(k)) \end{bmatrix}$$

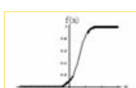


Figure 2: Activation function of a neuron and its saturation regions (painted in red)

$$\Sigma_1: \quad x(k+1) = \Gamma x(k) + \Lambda F(x(k))$$

$$g(x) = f(x-\theta)$$

What is the ability of this model?

- It gives bounded solutions for $\lambda < 1$

$$\|x(n)\| \le \|\Gamma\|^n \|x(0)\| + \left(\|\Gamma\|^{n-1} + \|\Gamma\|^{n-2} + \dots + \|I\|\right)\cdot \xi \,, \quad \xi \triangleq \max\{\|\Lambda F(x(0))\|, \|\Lambda F(x(1))\|, \dots, \|\Lambda F(x(n-1))\|\}$$

- In the saturation region of F(x), Σ_1 is a contraction mapping and has fixed points there: $\|\Gamma\| + \alpha\|\Lambda\| < 1$, $\alpha \triangleq \max_{x\in\Re}\left(\left\|\frac{df(x)}{dx}\right\|, \left\|\frac{dg(x)}{dx}\right\|\right)$
- As observed from simulation results the system ends up at two fixed points:

$[0.1 \quad 0.1 \quad 0.2 \quad -0.1 \quad 0.1]^T$ → passive point

$[0.9 \quad 0.9 \quad 0.2 \quad 1.8 \quad 1.8]^T$ → active point

- For different values of a and b the qualitative behaviour of Σ_1 change:

$b < 0.34\cdot a + 0.87$ (1) → ends up in active point

$b > 0.65\cdot a + 0.9$ (2) → ends up in passive point

$0.34\cdot a + 0.87 < b < 0.65\cdot a + 0.9$ (3) → either in passive or in active point according to initial conditions

How can we interpret these?

- From the five component(r,n,d,m,p) only the last one is considered, as it models the behaviour of cortex.
- active point → loop is activated → action is selected
- passive point → loop is inhibited → action is not selected

How to select one action among two?

By binding two loops…

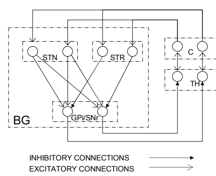


Figure 3: Connected C- BG-TH-C loops

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} \Lambda & \Pi \\ \Pi & \Lambda \end{bmatrix} \begin{bmatrix} F(x_1(k)) \\ F(x_2(k)) \end{bmatrix}$$

$$\Sigma_2: \quad x_\Pi(k+1) = \Gamma_\Pi x_\Pi(k) + \Lambda_\Pi F(x_\Pi(k))$$

The elements of $\Pi \in \Re^{5\times5}$ are zero, except the one on the third row second column which is denoted by c.

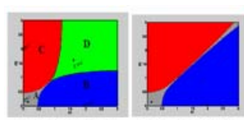
…without changing qualitative behaviour:

- solution is bounded. - contraction regions occur.

What about the fixed points?

Maximum number of fixed points increased from 2 to 4.

How can we interpret?



x*1 : loops are passive

x*2 : first loop is active

x*3 : second loop is active

x*4 : loops are active

The region D vanished. Why?

To understand why, consider the solutions of Σ_2 with $x_1(0)=x_2(0)$. As Γ_Π and Λ_Π is symmetric, $x_1(k)=x_2(k)$ ✂ k, and these solutions are in the subspace $x_1=x_2$ which lays in the regions A and D. These are also the solutions of:

$$\Sigma_2': \quad \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} \Lambda+\Pi & 0 \\ 0 & \Lambda+\Pi \end{bmatrix} \begin{bmatrix} F(x_1(k)) \\ F(x_2(k)) \end{bmatrix}$$

Σ'_2 consists of two subsytems as:

$$x(k+1) = \Gamma x(k) + \Lambda_2 F(x(k)), \qquad \Lambda_2 = \Lambda + \Pi$$

Using Ineq. (1), (2), and (3) and substituting b+c for b:

$b + c < 0.34\cdot a + 0.87$ (4) → A vanishes.

$b + c > 0.65\cdot a + 0.9$ (5) → D vanishes.(Fig. 5)

$0.34\cdot a + 0.87 < b + c < 0.65\cdot a + 0.9$ (6) → Both exist.(Fig. 4)

How to select ℓ actions among n?

By constructing a soft discreminator binding n loops...

$$\Sigma_n: \quad \begin{bmatrix} x_1(k+1) \\ \vdots \\ x_\ell(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} \Gamma & 0 & 0 & \cdots & 0 \\ 0 & \Gamma & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0 & \Gamma \end{bmatrix} \begin{bmatrix} x_1(k) \\ \vdots \\ x_\ell(k) \\ \vdots \\ x_n(k) \end{bmatrix} + \begin{bmatrix} \Lambda & \Pi & \Pi & \cdots & \Pi \\ \Pi & \Lambda & \ddots & \vdots & \vdots \\ \Pi & \ddots & \ddots & \ddots & \Pi \\ \vdots & \ddots & \ddots & \ddots & \Pi \\ \Pi & \cdots & \Pi & \Pi & \Lambda \end{bmatrix} \begin{bmatrix} F(x_1(k)) \\ \vdots \\ F(x_\ell(k)) \\ \vdots \\ F(x_n(k)) \end{bmatrix}$$

...without changing qualitative behaviour:

-solution is bounded. - contraction regions occur.

What about the fixed points?

Maximum number of fixed points increase from 2 to 2^n.

What are the conditions to select ℓ actions?

Restricting the system to the subspaces $x_{a_1} = x_{a_2} = \dots = x_{a_\ell}$, $x_{a_{\ell+1}} = x_{a_{\ell+2}} = \dots = x_{a_n} = 0$ we obtain the system below:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_\ell(k+1) \end{bmatrix} = \begin{bmatrix} \Gamma & 0 & \dots & 0 \\ 0 & \Gamma & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \Gamma \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_\ell(k) \end{bmatrix} + \begin{bmatrix} \Lambda+(\ell-1)\cdot\Pi & 0 & \dots & 0 \\ 0 & \Lambda+(\ell-1)\cdot\Pi & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \Lambda+(\ell-1)\cdot\Pi \end{bmatrix} \begin{bmatrix} F(x_1(k)) \\ F(x_2(k)) \\ \vdots \\ F(x_\ell(k)) \end{bmatrix}$$

It consists of ℓ disconnected subsystems as:

$$x(k+1) = \Gamma x(k) + \Lambda_\ell F(x(k)) \quad , \quad \Lambda_\ell = \Lambda + (\ell-1)\cdot\Pi$$

Using Ineq. (1) and (3) again by substituting b+(ℓ-1)c for b:

$$b + (\ell-1)\cdot c < 0.65\cdot a + 0.9 \qquad (7)$$

Ineq. (7) should be satisfied for selecting ℓ actions.

An example…

Consider Σ_5 for a=1.5, b=1, c=0.35. Ineq. (7) is satisfied for ℓ is equal to one, two, and three but not four. So the system can select up to three actions.

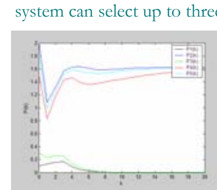


Figure 6

p1(0)=0.1, p2(0)=2, p3(0)=0.3, p4(0)=1.5, p5(0)=1.8

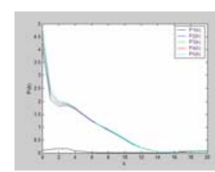


Figure 7

p1(0)=0.1, p2(0)=4, p3(0)=4.3, p4(0)=4.5, p5(0)=4.8

What is dopamine?

- One of the neurotransmitters in BG
- Dopamine ↓ → parkinson
- Dopamine ↑ → schizopherenia

Some comments on modelling the effect of dopamine...

- ❑ models dopamine discharge
- ❑ affects action selection

❑ = 0.3 → good discriminator

❑ = 0.2 → poor discriminator

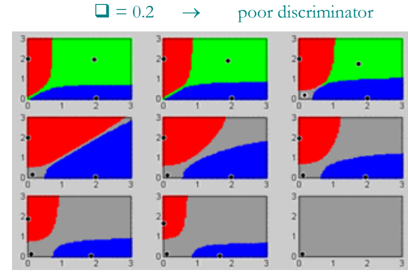


Figure 8:Domains of attraction for different values of θ

What did we get at the end?

- A cognitive process which is attributed to a neural circuit, namely, C-BG-TH-C loop can be generated by a mathematical model.
- The effect of dopamine, a neurotransmitter, can be evoked by a parameter θ and it has been observed that the interconnection weights a, b between BG substructures can be also used to establish this effect.
- We get the idea that the dopamine may be effective when we got stuck between more than one choice.