A New Method for Calculating of Electric Fields Around or Inside Any Arbitrary Shape Electrode Configuration

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Abstract: - In this paper a new method for calculating of electric fields intensity around or inside any arbitrary shape electrode systems or conductors with the same or different configuration have been presented. As we know that in design and construction of a high voltage equipment, analysis of electric field intensity around and or inside that equipment has very important role therefore, according to precision and ability of this new computer program or new computer code, we can use this code in research centers for design new equipments in electric industries. According to complexity of some electrodes or conductors configuration, solving of Laplace's or Poisson's equations by different numerical methods are very difficult in actual field. In this new method we transfer actual field (electrode or conductors and their outer boundary) to rectangular plane. In general this computer code solve two couple Poisson's or Laplace's equations in general curvilinear coordinate. For solving elliptic differential equations we have been used TTM (THOMPSON, THAMES & MASTIN) algorithm.

Key-Words: - Electric fields, Equipotential lines, Numerical method, Arbitrary shape electrode configuration, TTM algorithm, Conformal mapping, Curvilinear coordinate

1 Introduction

In this paper, we can obtain electric field intensity and also equipotential lines around or inside any arbitrary shape electrodes with TTM algorithm. With this computer code we can solve two couple Poisson or Laplace equations in general curvilinear coordinate. For finding electric field intensity and also equipotential lines around any electrode, we transfer actual field or actual coordinate system (surface of any arbitrary shape electrode) and also its outer boundary to rectangular calculating field or rectangular coordinate system. Solving of Poisson and also Laplace equations are easy in rectangular field than actual field [1].

As we know that, by using conformal mapping, we can solve electric fields and also equipotential lines around some definite electrodes [2]. In conformal mapping method we define any complex transfer function and then, by using this function, for every point in actual complex plane z with coordinates x and y we can find one or many points in an other complex plane w with coordinates u and v. The relationship between points in plane z and plane w have been determined by complex analytic function w = f(z). This complex analytic function is named transfer function also. Since definition of transfer function is not possible for all type electrode system therefore In general we can use conformal mapping technique under some special conditions [2].

In this computer code, similar to conformal mapping technique for calculating electric field and equipotential lines around or inside any arbitrary shape electrodes first of all we transfer actual field to rectangular calculating field and after solving Laplace or Poisson equations in rectangular coordinate system we again transfer the results to actual field also. The constraints of the conformal mapping method have been not seen in this code. In mathematical method of this computer program, we will explain transfer function briefly.

In the past several methods such as finite difference, finite element, boundary element and charge simulation have been used for solving electric field distribution [1-16]. According to some difficulties in the above mentioned different methods we have obtained a paper that is comparing finite difference, finite element and charge simulation methods with each other [4]. In finite difference method if the shape of the electrode is complicated therefore the results obtained by this method will have a large errors. In this new method we can analysis electric field and also equipotential lines around or inside any arbitrary shape electrode with minimum errors [17, 18].

2 Mathematical Discussion

Since calculation of electric field intensity in rectangular plane is quite simple therefore in this computer program we transfer actual computational field and its outer boundary to any computational rectangular plane. Henceforth, we show actual field with E and calculating field with E^T. Figure 1 shows actual field (body or electrode and its outer boundary) in complex z-plane with x and y coordinates.

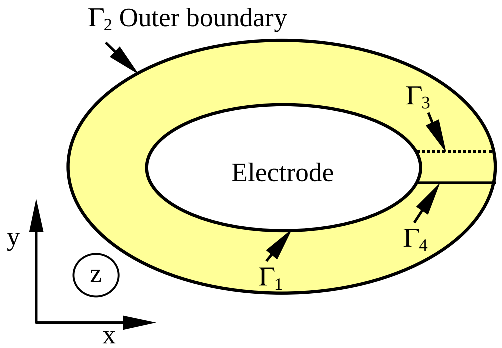

Figure 1. One electrode and its outer boundary (actual field)

Figure 2 shows calculating field in complex w-plane with coordinates u and v respectively. In general, electrode surface and outer boundary transfer to v constant lines (v_1 and v_2) that is shown in figure 2. In figure 1, Γ_1 shows body or electrode surface and Γ_2 shows outer boundary in the computational field, in this figure also we assume two cutting lines or curves Γ_3 and Γ_4 respectively these hypothesis lines or curves connect body or electrode to outer boundary in actual field.

These hypothesis lines or curves in rectangular computational field have been shown by lines Γ_3^T and Γ_4^T. Notice that Γ_3 and Γ_4 in actual field have been coincide to each other therefore coordinates x(u, v) and y(u, v) for Γ_3 and Γ_4 will be equal.

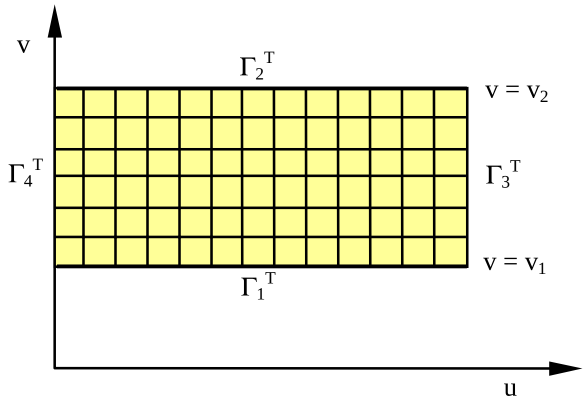

Figure 2. Computational field of one electrode and its outer boundary

In the above discussion we assumed only one electrode or body in actual field. In this code we can assume many electrodes in actual field therefore by this new method electric field intensity and equipotential lines around three phase transmission line can be investigated. Figure 3 shows actual field and figure 4 shows computational field of two electrodes configuration.

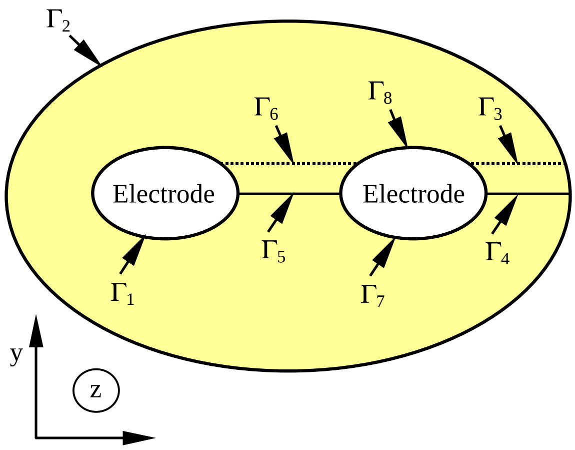

Figure 3. Two electrodes and their boundary (actual field)

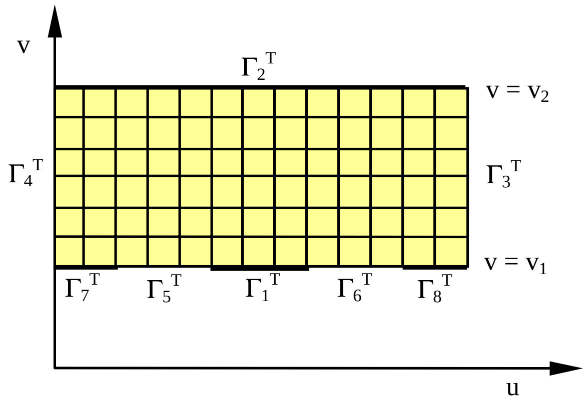

Figure 4. Computational field of two electrodes and their outer boundary

From the point of view of mathematics, transfer from actual field to computational field and vice versa, can be shown by the following equations.

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u(x, y) \\ v(x, y) \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(u, v) \\ y(u, v) \end{bmatrix} \tag{2}$$

The matrices of this transformation are as follows:

$$J_1 = \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix}, \quad J_2 = \begin{bmatrix} x_u & x_v \\ y_u & y_v \end{bmatrix} \qquad (3)$$

According to above equation we can say that $[J_1] = [J_2]^{-1}$. In general the elements of matrix J_1 are obtained as follows:

$$u_x = y_v / J, \qquad u_y = -x_v / J, \qquad (4a)$$

$$v_x = -y_u / J, \quad v_y = x_u / J \qquad (4b)$$

In the above relationships J, shows the determinant of matrix J_2, following equation shows the determinant of matrix J_2:

$$J = \det[J_2] = x_u y_v - x_v y_u \qquad (5)$$

The solved equations in this computer program are Laplace or Poisson. At first step we consider Laplace equations solution and at the second step we investigate Poissons equations solution. In general, in curvilinear coordinate system Laplace equations are shown as follows:

$$u_{xx} + u_{yy} = 0 \qquad (6)$$

$$v_{xx} + v_{yy} = 0 \qquad (7)$$

With respect to Dirichlet boundary conditions can be obtained following equations:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_1(x,y) \\ v_1 \end{bmatrix}, \qquad [x, y] \in \Gamma_1^T \qquad (8)$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_2(x,y) \\ v_2 \end{bmatrix}, \qquad [x, y] \in \Gamma_2^T \qquad (9)$$

In the above equations u_1 and u_2 are known functions that are obtained from curves Γ_1 and Γ_2 (electrode surface and outer boundary) separately. In computational field (rectangular coordinate), we use following equations:

$$\alpha x_{uu} - 2\beta x_{uv} + \gamma x_{vv} = 0 \qquad (10)$$

$$\alpha y_{uu} - 2\beta y_{uv} + \gamma y_{vv} = 0 \qquad (11)$$

In equations (10) and (11), constants α, β and γ can be obtained from following equations:

$$\alpha = x_v^2 + y_v^2 \qquad (12)$$

$$\beta = x_u x_v + y_u y_v \qquad (13)$$

$$\gamma = x_u^2 + y_u^2 \qquad (14)$$

Under transferred boundary conditions, we obtain following equations:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_1(u,v_1) \\ f_2(u,v_1) \end{bmatrix}, \qquad [u, v] \in \Gamma_1^T \qquad (15)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} g_1(u,v_2) \\ g_2(u,v_2) \end{bmatrix}, \qquad [u, v] \in \Gamma_2^T \qquad (16)$$

Functions f_1, f_2, g_1, g_2 in equations (15) and (16) are well defined, since these functions are the coordinates of region (Γ_1 and Γ_2) that we define in the actual field.

Notice that the systems of the above equations are quasi-linear and elliptic, therefore the solution of these functions are very complicated than Laplace equations in actual field coordinates, but in this code we solve these equations in rectangular coordinate system that make easy our calculations.

If we have two electrodes in actual field for second electrode also we can obtain following equations:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} h_1(u,v_1) \\ h_2(u,v_1) \end{bmatrix}, \qquad [u, v_1] \in \Gamma_7^T \qquad (17)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} o_1(u,v_1) \\ o_2(u,v_1) \end{bmatrix}, \qquad [u, v_1] \in \Gamma_8^T \qquad (18)$$

In general, in curvilinear coordinate system Poisson equations are shown as follows:

$$u_{xx} + u_{yy} = \varepsilon_1 \qquad (19)$$

$$v_{xx} + v_{yy} = \varepsilon_2 \qquad (20)$$

In the above equations ε_1 and ε_2 can be equal or different. According to above discussion, we obtain following equations in transformed field or plane:

$$\alpha x_{uu} - 2\beta x_{uv} + \gamma x_{vv} + J^2(\varepsilon_1 x_u + \varepsilon_2 x_v) = 0 \qquad (21)$$

$$\alpha y_{uu} - 2\beta y_{uv} + \gamma y_{vv} + J^2(\varepsilon_1 y_u + \varepsilon_2 y_v) = 0 \qquad (22)$$

3 Flowchart and Preparation of The Code

The figure 5 shows the flowchart of this computer code. As we have mentioned previously, by this computer code we can analysis electric field intensity and also equipotential lines around or inside any arbitrary shape electrode configuration.

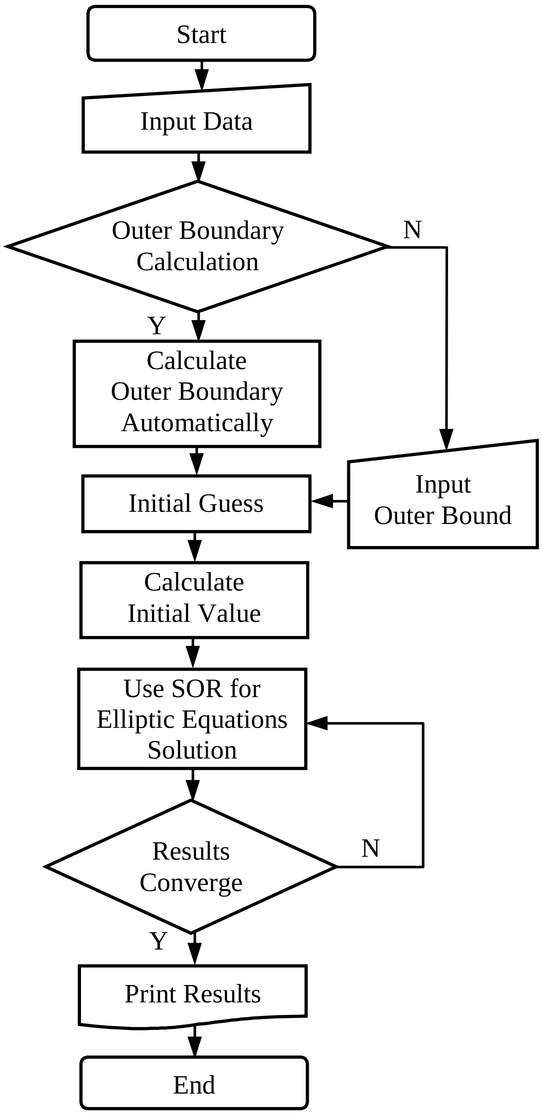

Figure 5. Flowchart

In this computer code, first of all we define input file (GEM2DIN) according to given electrode configuration. After execution of this computer code, we obtain two different output files (ZOUT, GEMTEC.DAT). ZOUT shows specification of input file with given initial boundary conditions, errors in each iteration and also final results that are obtained. we use GEMTEC.DAT file in TECPLOT software to plot electric field and equipotential lines inside or outside any arbitrary shape electrode configurations.

4 Comparison of The Results

Figure 6 and figure 7 shows the electric field intensity and also equipotential lines around or inside any arbitrary shape electrode configuration. In Figure 6 electric field intensities and also equipotential lines around a complicated shape electrode, and also in Figure 7 electric field and equipotential lines around three conductors with the same shape As like transmission lines, have been obtained by this new method. Comparison these results with analogue electrode systems electric field intensity and equipotential lines, that obtained with different methods in given references, shows that we can use this new algorithm for analyzing electric field intensity around any arbitrary shape electrode or conductors with the same or different configuration.

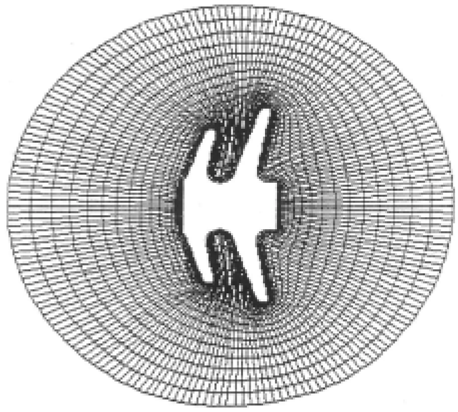

Figure 6. Electric field intensities and equipotential lines around arbitrary shape electrode

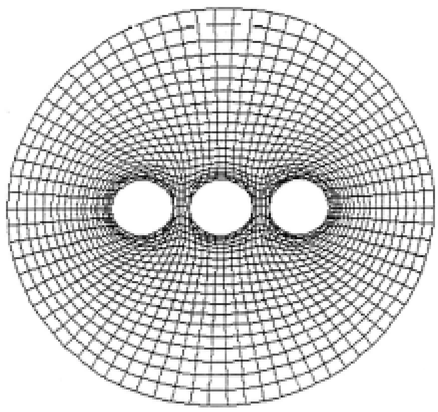

Figure 7. Electric field intensities and equipotential lines around three conductors

Figure 8 shows electric field intensities and also equipotential lines inside a system as like rod-plane electrode configuration and also figure 9 shows electric field intensities and equipotential lines inside a system as like sphere-plane electrode configuration with surface roughness on plane electrode. As we know that analysis of surface roughness on electrode has very important role in design and application of high voltage apparatus. In general if there is a symmetry inside any electrode system configuration then, in analysis of the electric field and equipotential lines inside the system we can assume only one half of the system as shown in figure 8 and 9 respectively.

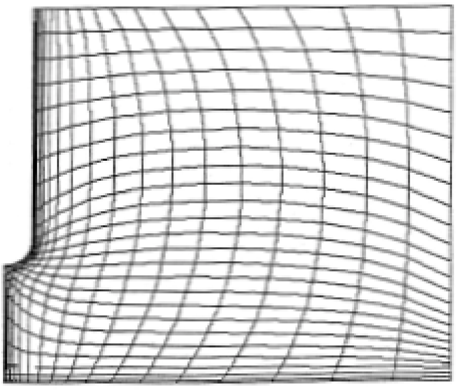

Figure 8. Electric field intensities and equipotential lines inside a system as like rod-plane electrode configuration

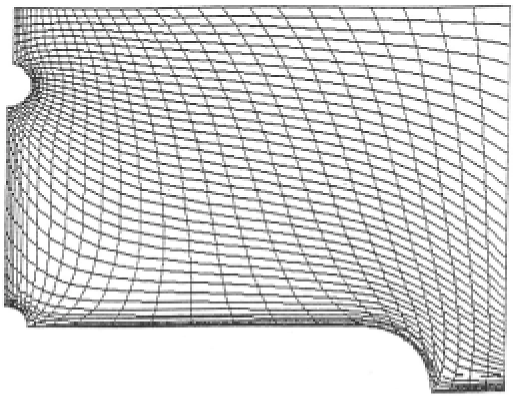

Figure 9. Electric field intensities and equipotential lines inside a system as like sphere-plane electrode configuration with electrode surface roughness on plane electrode

After comparison these results with the electric field and equipotential lines of the same electrode configuration that are obtained with different numerical methods as mentioned in different references, we can say that in general, we can use also this new computer code for analysis of electric field intensities inside any high voltage equipment.

5. Conclusion

We have described here a new computer program with the ability to solve Poisson's and Laplace's equations. With this new method we can calculate electric field intensity and also equipotential lines around or inside any arbitrary shape electrode or conductors with the same or different configurations. As we know that in high voltage engineering, analysis of electric field around or inside any equipment has very important role in the design of that equipment therefore, we can use this new computer code in the design and construction of any HV equipment that are used in electric industry. Concerning to precision and ability of this new method we can use this code also in research and educational activities.

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