

## MAT104 Problem Saati

Araş. Gör. İsmail GÜZEL

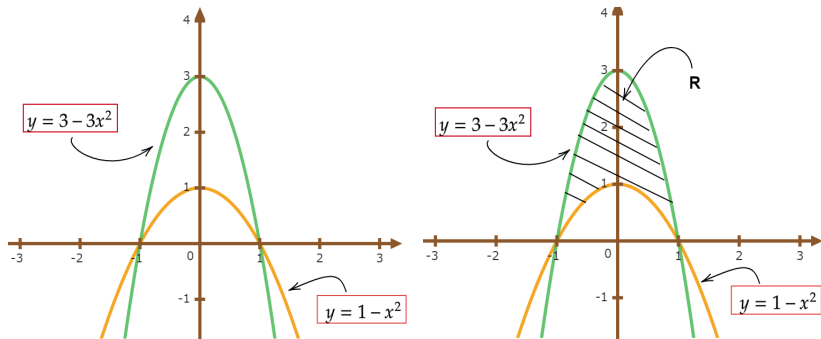
*iguzel@itu.edu.tr*

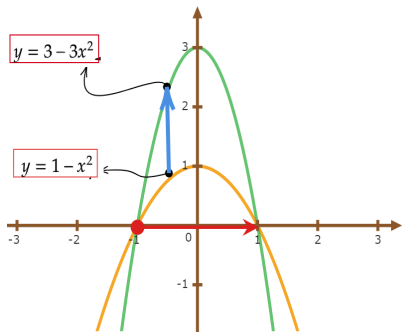
<https://web.itu.edu.tr/iguzel/>

İstanbul Teknik Üniversitesi

### Soru 1.

$R$ ,  $y = 1 - x^2$ ,  $y = 3 - 3x^2$  eğrileri ile sınırlanmış bölge olmak üzere  $\iint_R (x + 2y) \, dA$  integralini hesaplayınız.

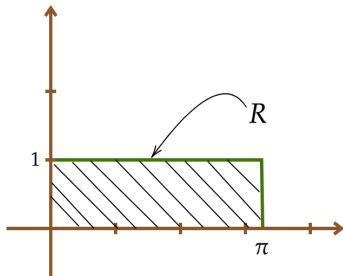




$$\begin{aligned}
 \iint_R (x+2y) dA &= \int_{-1}^1 \int_{1-x^2}^{3-3x^2} (x+2y) dy dx \\
 &= \int_{-1}^1 \left[ xy + y^2 \right]_{y=1-x^2}^{y=3-3x^2} dx \\
 &= \int_{-1}^1 \left[ x(3-3x^2) + (3-3x^2)^2 \right. \\
 &\quad \left. - x(1-x^2) + (1-x^2)^2 \right] dx \\
 &= \int_{-1}^1 (8x^4 - 2x^3 - 16x^2 + 2x + 8) dx \\
 &= \left[ \frac{8x^5}{5} - \frac{2x^4}{4} - \frac{16x^3}{3} + x^2 + 8x \right]_{x=-1}^{x=1} \\
 &= \frac{128}{5} //
 \end{aligned}$$

## Soru 2.

Eğer  $R = [0, \pi] \times [0, 1]$ ,  $\iint_R y \cos xy \, dA = ?$

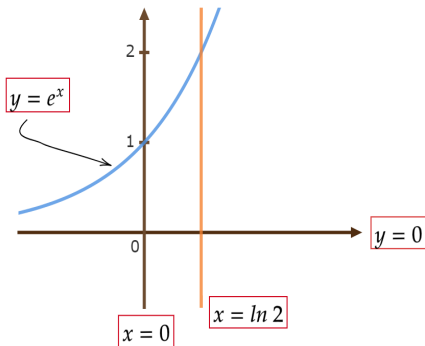


Fubini Teoreminde,

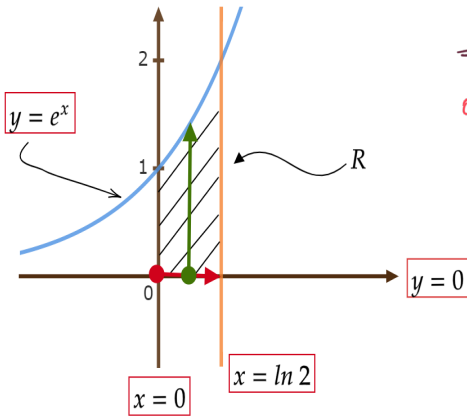
$$\int_0^{\pi} \int_0^1 y \cos(xy) \, dy \, dx = \int_0^1 \int_0^{\pi} y \cos(xy) \, dx \, dy$$
$$= \int_0^1 \left[ \sin(xy) \right]_{x=0}^{x=\pi} \, dy$$
$$= \int_0^1 \sin(\pi y) \, dy$$
$$= - \left. \frac{\cos(\pi y)}{\pi} \right|_{y=0}^{y=1} = 2/\pi$$

### Soru 3.

İki katlı integral kullanarak  $y = e^x$ ,  $y = 0$ ,  $x = 0$ ,  $x = \ln 2$  eğrileri arasında kalan bölgenin alanını hesaplayınız.



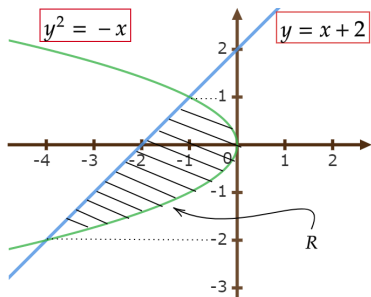
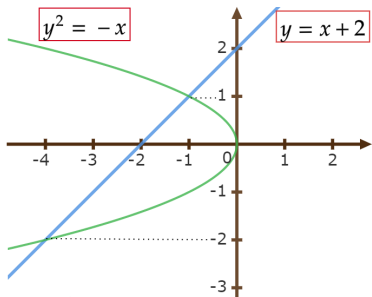
$$\text{Alan} = \iint_R dA$$



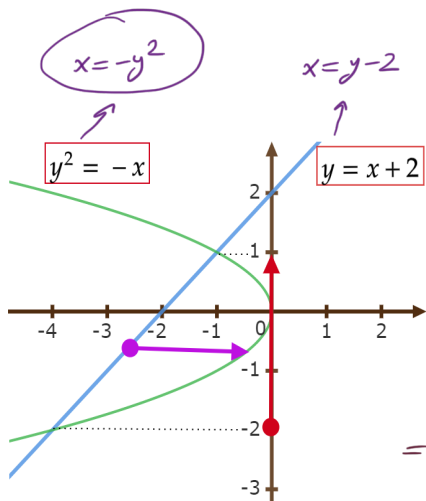
$$\begin{aligned}
 A_{\text{lon}} &= \iint_R dA \\
 &= \int_0^{\ln 2} \int_0^{e^x} 1 \, dy \, dx \\
 &= \int_0^{\ln 2} e^x \, dx \\
 &= e^x \Big|_{x=0}^{x=\ln 2} \\
 &= e^{\ln 2} - e^0 = 1,
 \end{aligned}$$

#### Soru 4.

İki katlı integral kullanarak  $y^2 = -x$  ve  $y = x + 2$  eğrileri arasında kalan bölgenin alanını hesaplayınız.



$$\left. \begin{array}{l} y = x + 2 \\ y^2 = -x \end{array} \right\} \Rightarrow y^2 = 2 - y \Rightarrow y^2 + y - 2 = 0 \Rightarrow \underline{\underline{y = -2}} \text{ ve } \underline{\underline{y = 1}}$$



$$A_{\text{kon}} = \iint_R dA$$

$$= \int_{-2}^1 \int_{-y^2}^{y-2} dx dy$$

$$= \int_{-2}^1 (-y^2 - y + 2) dy$$

$$= \left. -\frac{y^3}{3} - \frac{y^2}{2} + 2y \right|_{y=-2}^{y=1}$$

$$= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$$

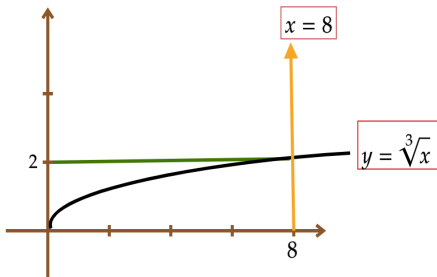
$$= \frac{9}{2}$$



### Soru 5.

$$\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{1+y^4} dy dx \text{ integralini hesaplayınız.}$$

$R$  :  $x = 0$  ve  $x = 8$  doğruları arasında  $y = \sqrt[3]{x}$  eğrisi ve  $y = 2$  doğrularının sınırladığı bölge



Fubini teoreminden,

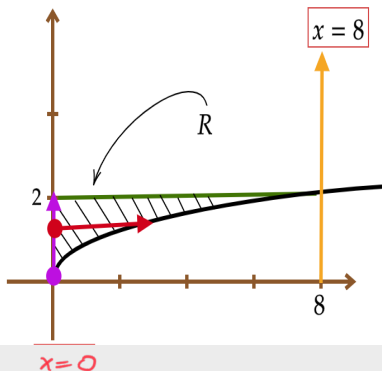
$$\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{1+y^4} dy dx = \int_0^2 \int_0^{y^3} \frac{1}{1+y^4} dx dy$$

$$= \int_0^2 \left( \frac{x}{1+y^4} \right)_{x=0}^{x=y^3} dy$$

$$= \int_0^2 \frac{y^3}{1+y^4} dy$$

$$= \frac{\ln|1+y^4|}{4} \Big|_{y=0}^{y=2}$$

$$= \frac{\ln 17}{4} \cdot 11$$



$$y = \sqrt[3]{x}$$

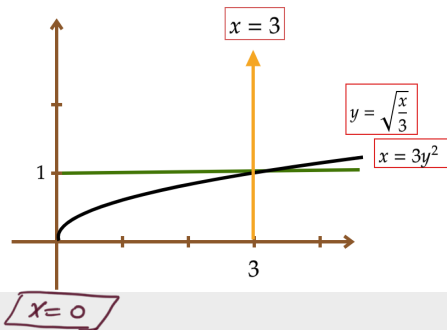
$$x = y^3$$

$$x=0$$

### Soru 6.

$$\int_0^3 \int_{\sqrt{\frac{x}{3}}}^1 e^{y^3} dy dx \text{ integralini hesaplayınız.}$$

$R$  :  $x = 0$  ve  $x = 3$  doğruları arasında  $y = \sqrt{\frac{x}{3}}$  eğrisi ve  $y = 1$  doğrularının sınırladığı bölge



Fubini teoreminden,

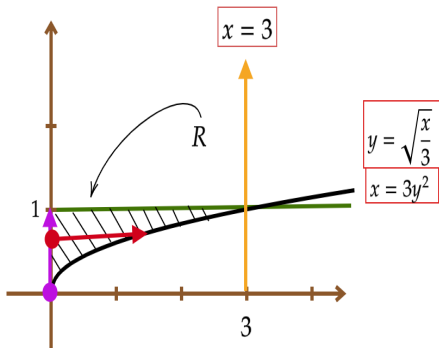
$$\int_0^3 \int_{\sqrt{\frac{x}{3}}}^1 e^{y^3} dy dx = \int_0^1 \int_0^{3y^2} e^{y^3} dx dy$$

$$= \int_0^1 \left[ e^{y^3} x \right]_{x=0}^{x=3y^2} dy$$

$$= \int_0^1 3y^2 e^{y^3} dy$$

$$= e^{y^3} \Big|_{y=0}^{y=1}$$

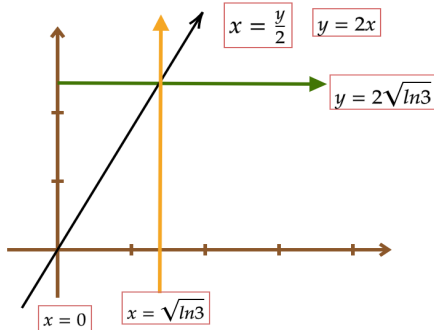
$$= e - 1 //$$



### Soru 7.

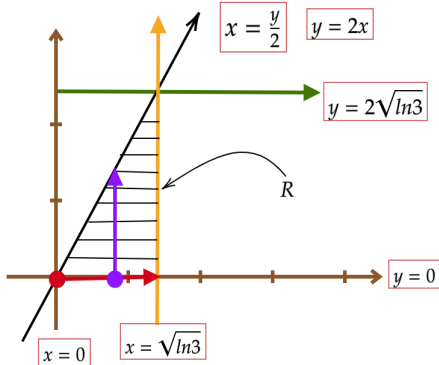
$$\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2} dx dy \text{ integralini hesaplayınız.}$$

$R : y = 0$  ve  $y = 2\sqrt{\ln 3}$  doğruları arasında  $x = \frac{y}{2}$  eğrisi ve  $x = \sqrt{\ln 3}$  doğrusunun sınırladığı bölge



Fubini teoreminden,

$$\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2} dx dy = \int_0^{\sqrt{\ln 3}} \int_0^{2x} e^{x^2} dy dx$$



$$= \int_0^{\sqrt{\ln 3}} \left[ y e^{x^2} \right]_{y=0}^{y=2x} dx$$

$$= \int_0^{\sqrt{\ln 3}} 2x e^{x^2} dx$$

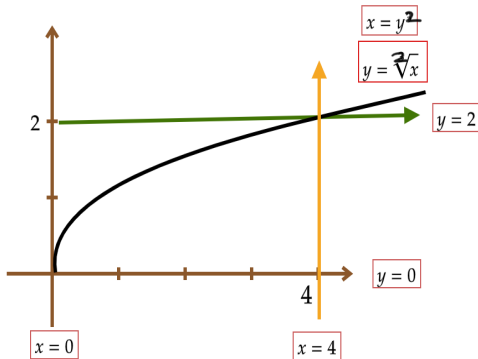
$$= \left. e^{x^2} \right|_{x=0}^{x=\sqrt{\ln 3}}$$

$$= 3 - 1 = 2 //$$

### Soru 8.

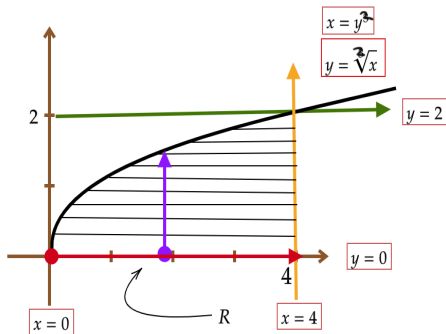
$\int_0^2 \int_{y^2}^4 \frac{3}{2} e^{y/\sqrt{x}} dx dy$  integralini hesaplayınız ?

$R$  :  $y = 0$  ve  $y = 2$  doğruları arasında  $x = y^2$  eğrisi ve  $x = 4$  doğrusunun sınırladığı bölge



Fubini teoreminden,

$$\int_0^2 \int_{y^2}^4 \frac{3}{2} e^{y/\sqrt{x}} dx dy = \int_0^4 \int_0^{\sqrt{x}} \frac{3}{2} e^{y/\sqrt{x}} dy dx$$



$$= \int_0^4 \frac{3}{2} \cdot \frac{1}{\sqrt{x}} \cdot e \Big|_{y=0}^{y=\sqrt{x}} dx$$

$$= \int_0^4 \left( \frac{3}{2} e\sqrt{x} - \frac{3}{2} \sqrt{x} \right) dx$$

$$= x^{3/2} (e-1) \Big|_{x=0}^{x=4}$$

$$= 8(e-1) //$$