WORKSHEET # VI

1. Evaluate the following limit (Do not use L'Hôpital Rule !)

$$\lim_{x\to 0} \frac{1 - \frac{x^2}{8} - \cos x}{x^2}$$

2. For the following functions find the discontinuity points, if any, and classify the types of the discontinuities.

$$f(x) = \begin{cases} \tan^{-1} \dfrac{x}{x-2} & 0 \le x < 2, \\ \dfrac{\pi}{2} & x = 2, \\ \sin^{-1} \dfrac{2}{x} & 2 < x \le 4, \\ \dfrac{1}{x-4} & 4 < x \le 5. \end{cases}$$

3. Show that the function $f(x) = |x - 5|$ is continuous everywhere but not differentiable at $x = 5$. (Sketching graph of the function may provide insight, but will not considered as a complete solution)

4. Compute $f'(x)$ if $f(x) = \dfrac{\sec(3x)}{(x^2+1)\sin(2x)}$

5. Find $\dfrac{d^2y}{dx^2}$ at given point if y is a differentiable function of x satisfying the equation

$$2\sin^2(x+y) = 3\sin x + \sin y \qquad \left(\frac{\pi}{2}, \frac{\pi}{6}\right)$$

6. Does the following function satisfy the conditions of the Mean Value Theorem? If so, find the admissible value of c on given interval .

$$f(x) = 3\sqrt{x} + 4x \qquad [1, 4]$$

7. Graph the following functions in details.

a) $y = x - 3x^{2/3}$

b) $y = \dfrac{x^2 - x + 1}{x - 1}$

c) $y = \dfrac{(x-1)^2}{x+2}$

d) $y = \dfrac{1}{4 - x^2}$

e) $y = \dfrac{x^3 - 4x}{x^2 - 1}$

f) $y = \dfrac{x^3}{x^2 - 9}$

Steps:

i) Find the domain and, if any, the intercepts of $f(x)$.

ii) If any, find the asymptotes of $f(x)$.

iii) Find the intervals on which the function is increasing and decreasing, and identify the function's local extreme values, if any. stating where they are taken on.

iv) Determine the concavity and, if any, find the the points of inflection.