WORKSHEET # III

1. Find the asymtotes of the curve $f(x) = x + \sqrt{x^2 - 1}$.

2. Let $f(x) = \begin{cases} x+3 & -3 \le x < -1\ , \\ -1 & x = -1\ , \\ -x+1 & -1 < x \le 1\ , \\ \dfrac{1}{x-1} & 1 < x \le 2\ , \\ x & x > 2\ . \end{cases}$

   a) Graph $f(x)$.

   b) Find the points, if any, at which the function $f(x)$ is discontinuous and classify the types of the discontinuities.

3. For the following functions find the discontinuity points, if any, and classify the types of the discontinuities.

   a) $f(x) = \begin{cases} \sin^{-1} \dfrac{x}{3} & 0 < x < 3 \\ \dfrac{\pi}{2} & x = 0 \text{ and } x = 4 \\ 2^{\frac{1}{x-4}} & 3 \le x < 4 \text{ and } x > 4 \end{cases}$

   b) $f(x) = \begin{cases} \dfrac{1 - \cos x}{x^2} & x \neq 0 \\ 1 & x = 0 \end{cases}$

4. Let $f(x) = x^3 - 2x + 2$. Show that f must have a zero between -2 and 0.

5. Show that the following functions have at least one root.

   (a) $f(x) = \sqrt[3]{x} + x - 2$

   (b) $f(x) = \cos x + \sin x - x$

6. Suppose that f and g are continuous on $[a, b]$ and that $f(a) < g(a)$ but $f(b) > g(b)$. Prove that $f(c) = g(c)$ for some $c \in [a, b]$.

7. Consider the function $f(x) = x^2 - 3x$. Use the limit definition to find the slope of the tangent line at $x = 2$. Write the equation for the tangent and normal line.

8. Find the equation for the tangent line to the function $f(x) = \dfrac{1}{x-1}$ with the slope -1.

9. Does any tangent to the curve $y = \sqrt{x-1}$ cross the $x-$axsis at $x = -3$? If so, find an equation for the line and the point of tangency.

10. Find the points at which the function $f(x) = x^2 + 4x - 1$ has the horizontal tangent.

11. Find the values of a and b that make $f(x)$ differentiable at $x = 0$?

$$f(x) = \begin{cases} ax + b & x < 0 \\ 2\sin x + 3\cos x & x \ge 0 \end{cases}$$

12. Consider the function

$$f(x) = \begin{cases} 3x, & , \quad x < 0 \\ -(2-x)^2, & 0 \le x \le 2 \\ x^2 - 4 & , \quad x > 2 \end{cases}.$$

   (a) Analyze the continuity of $f(x)$ at $x = 0$ and $x = 2$?

   (b) Analyze the differentiability of $f(x)$ at $x = 0$ and $x = 2$?

   Give reasons for your answers.