WORKSHEET # V

1. Prove that the functions $f(x) = \dfrac{x}{x^4+1}$ and $g(x) = \dfrac{x}{x^3+1}$ satisfy the equation $f'(x) = g'(x)$ at least one x in the interval $(0,1)$

2. Does the function $f(x) = \sqrt{-2x^2+11x-12}$ satisfy the hypothesis Rolle's Theorem on the interval $[\frac{3}{2}, 4]$? If so, find the admissible value of $c \in (\frac{3}{2}, 4)$

3. Show that $2x^3 + x + 4 = 0$ has exactly one zero.

4. Does the function $f(x) = \sqrt{x - x^2}$ satisfy the hypothesis of Mean Value Theorem on the interval $[0,1]$? If so, find the admissible value of $c \in (0,1)$.

5. For what values of a and b does the following function

$$f(x) = \begin{cases} ax + 4\pi & , \quad -\pi \le x \le 0 \\ b\cos(2x) + 2x, & 0 \le x < \pi \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval $[-\pi, \pi]$?

6. Show that for any numbers a and b, the following inequality is true.

$$|\sin b - \sin a| \le |b - a|$$

7. Find the critical point and classify the extreme values of the function

$$f(x) = 2\cos^3 x + 3\cos x \qquad , \quad [0, \pi]$$

8. Let $f(x) = x^{\frac{2}{3}}(x^2 - 4)$.

   a) Identify the function's local extreme values, if any, saying where they are taken on. Which, if any of the extreme values are absolute.

   b) Find the absolute extreme of this function on the interval $[-2, 2]$