

- 1) Turbulent velocity profiles in a boundary layer can be approximated by

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7}$$

Assuming the shear stress at the wall is given as

$$\tau_w = 0.01\rho U_\infty^2 \left(\frac{\nu}{\delta U_\infty}\right)^{1/6}$$

Starting from integral conservation laws, derive an expression for boundary layer thickness in the form

$$\frac{\delta}{x} = f(Re_x)$$

- 2) For a rotationally symmetric steady solution of Incompressible Navier-Stokes equations, two velocity components in cylindrical coordinates are given as follows:

$$v_\theta = \frac{\Omega c^2}{r} \left(1 - e^{-\frac{r^2}{c^2}}\right), \quad v_z = \frac{4\nu z}{c^2}$$

Find the radial velocity v_r and pressure p as a function of r and z . The pressure should be written in terms of the following function F :

$$F(\xi) = \int_\xi^\infty \frac{1}{x^2} (1 - e^{-x})^2 dx$$

What should be the units of Ω and c ?