

- 1) Show that divergence of vorticity is zero.

Vorticity w_i and its divergence is given as follows in tensor notation

$$w_i = \varepsilon_{ijk} \frac{\partial u_k}{\partial x_j}, \quad \frac{\partial w_i}{\partial x_i} = 0$$

- 2) Write the following identity in tensor notation and prove it

$$(\vec{V} \cdot \vec{\nabla})\vec{V} = \nabla \left(\frac{V^2}{2} \right) - \vec{V} \times \vec{\nabla} \times \vec{V}$$

Hint: use the identity $\varepsilon_{ijk}\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$

- 3) Use the following form of the momentum equation to derive an equation for kinetic energy per volume: $\kappa = \rho \mathbf{u}_i \mathbf{u}_i / 2$

$$\rho \frac{D\mathbf{u}_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ji}}{\partial x_j}$$

Result should be in the following form

$$\frac{\partial \kappa}{\partial t} + \frac{\partial (\mathbf{u}_i \kappa)}{\partial x_i} = ?$$