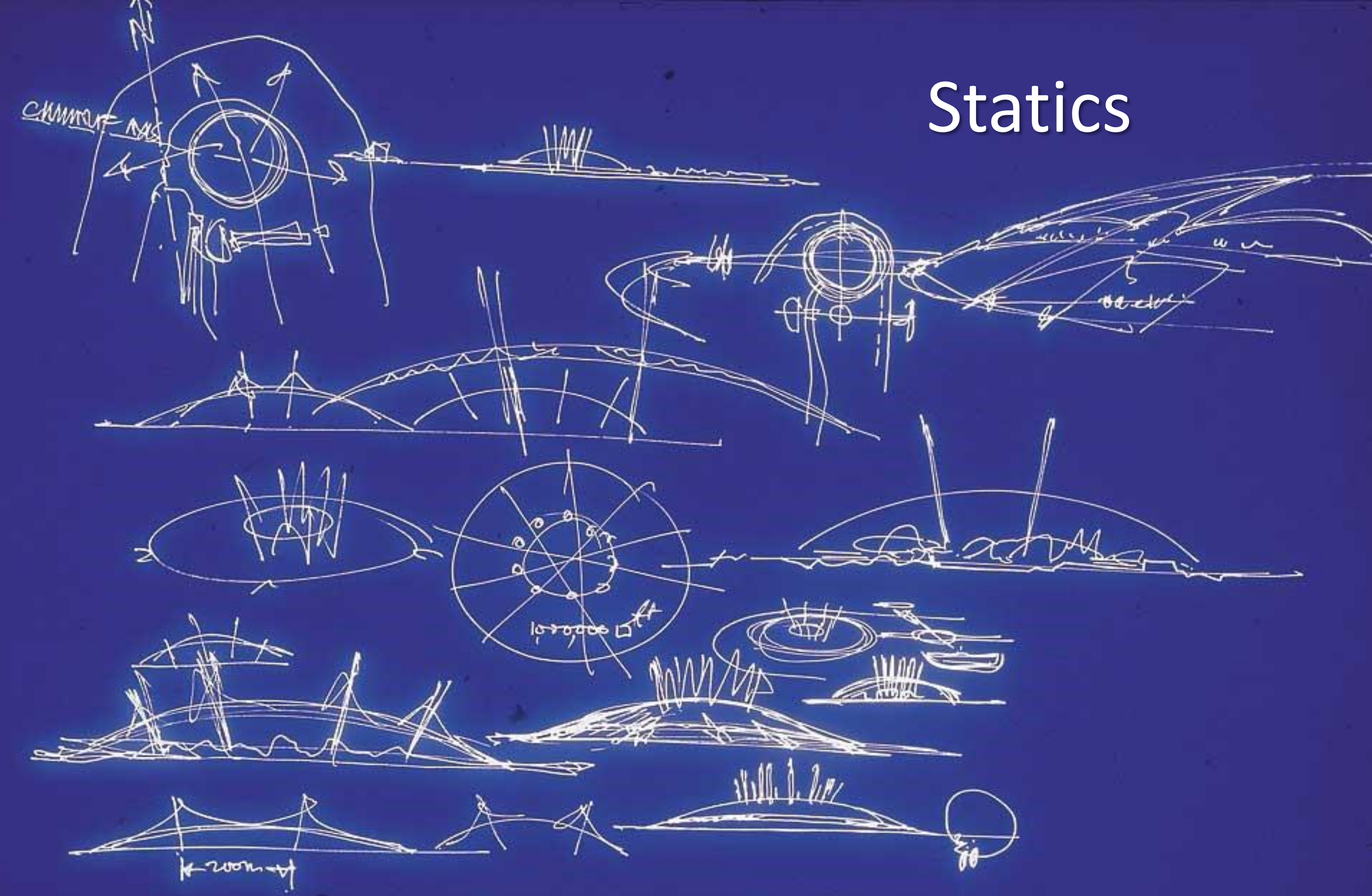
Statics



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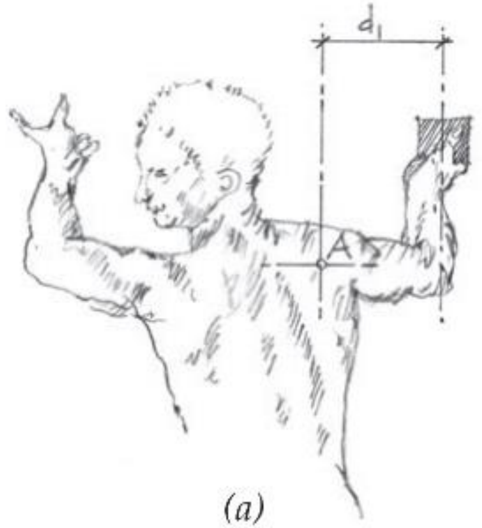
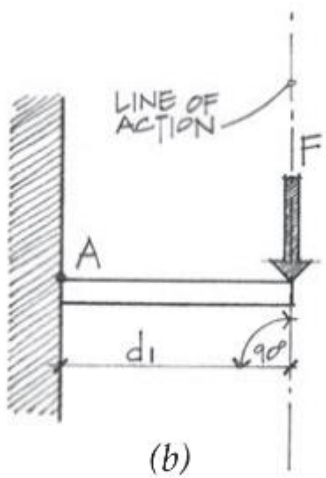


(a)



(b)

Figure 2.29 Moment of a force.

Moment of a Force

The tendency of a force to produce rotation of a body about some reference axis or point is called the *moment of a force* (see Figures 2.29a and 2.29b). Quantitatively, the moment M of a force F about a point A is defined as the product of the magnitude of the force F and perpendicular distance d from A to the line of action of F. In equation form,

$$M_A = F \times d$$

The subscript A denotes the point about which the moment is taken.

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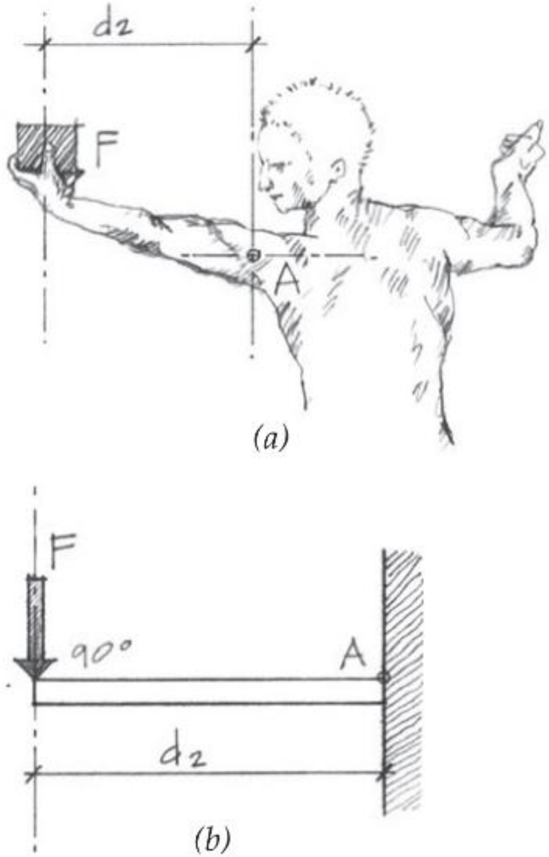


Figure 2.30 Moment of a force with an increased moment arm.

If the person now extends his arm so that the weight is at distance d_2 from point A, as shown in Figure 2.30(b), the amount of physical energy needed to carry the weight is increased. One reason for this is the increased moment about point A due to the increased distance d_2. The moment is now equal to that shown in Figure 2.30(a):

$$M_A = F \times d_2$$

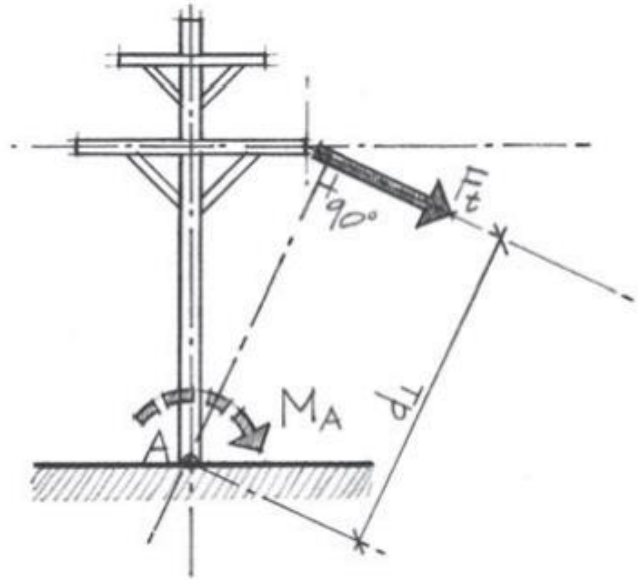


Figure 2.31 Perpendicular moment arm.

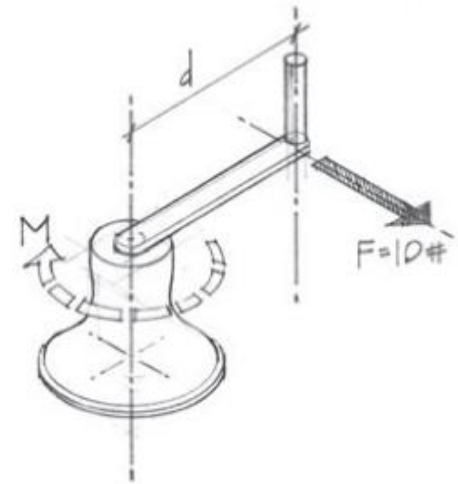
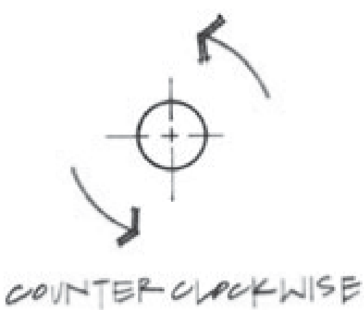
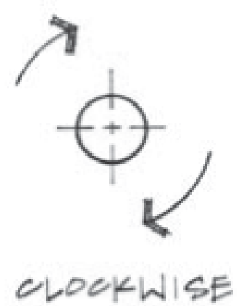


Figure 2.32 Sailboat winch-rotation about an axis.





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Varignon's Theorem

The French mathematician Pierre Varignon developed a very important theorem of statics. It states that the moment of a force about a point (axis) is equal to the algebraic sum of the moments of its components about the same point (axis). This may be best illustrated by an example (Figure 2.35).

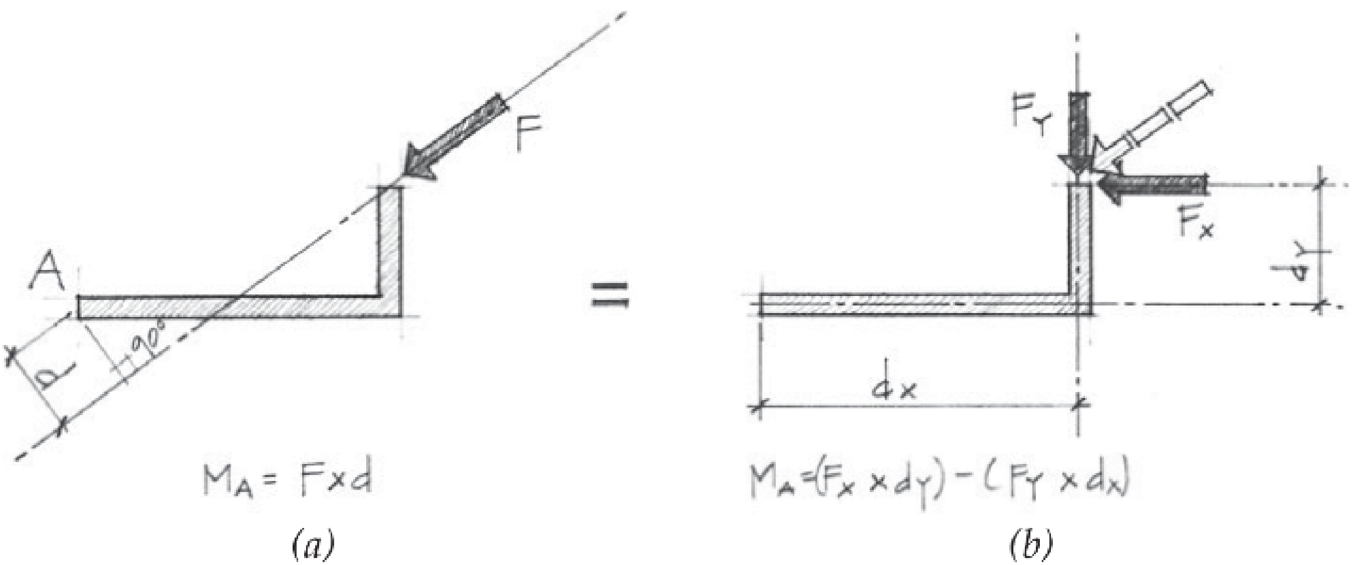

Figure 2.35 Moment of a force—Varignon's theorem.

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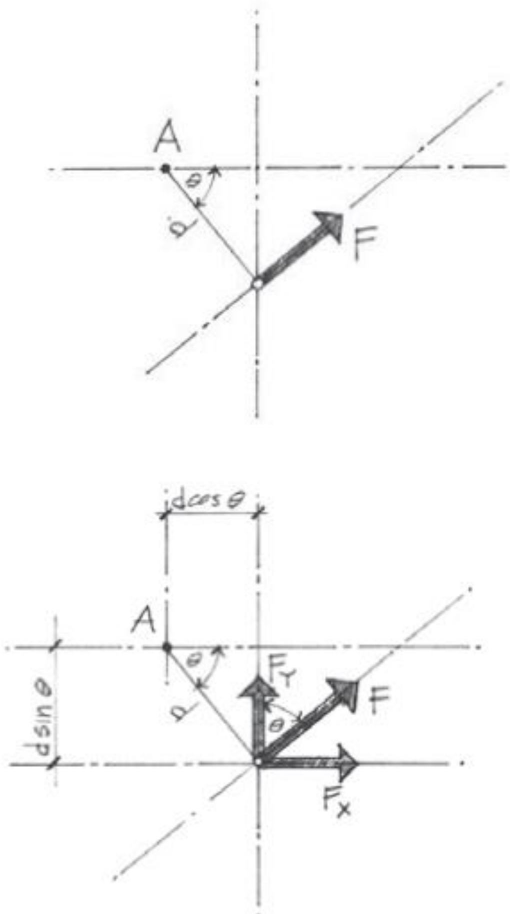

Figure 2.36 Varignon's theorem.

A proof of Varignon's theorem may be illustrated as in Figure 2.36:

$$F(d) = F_y(d\cos\theta) + F_x(d\sin\theta)$$

Substituting for F_x and F_y,

$$F(d) = F\cos\theta(d\cos\theta) + F\sin\theta(d\sin\theta)$$
$$F(d) = Fd\cos^2\theta + Fd\sin^2\theta = Fd(\cos^2\theta + \sin^2\theta)$$

But from the known trigonometric identity,

$$\sin^2\theta + \cos^2\theta = 1$$
$$F(d) = F(d) \qquad \therefore \text{ CHECKS}$$

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Couple and Moment of a Couple

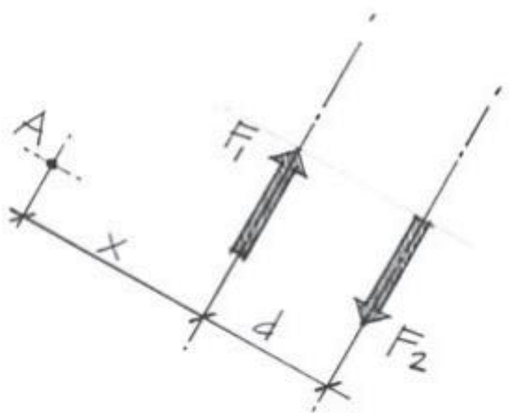

Figure 2.37 Force couple system.

Assume a point A on the rigid body about which the moment will be calculated. Distance x represents the perpendicular measurement from reference point A to the applied force F, and d is the perpendicular distance between the lines of action of F_1 and F_2.

$$M_A = +F_1(x) - F_2(x + d)$$

where

$$F_1 = F_2$$

and because F_1 and F_2 form a couple system,

$$M_A = +Fx - Fx - Fd$$

$$\therefore M_A = -Fd$$

Couple and Moment of a Couple

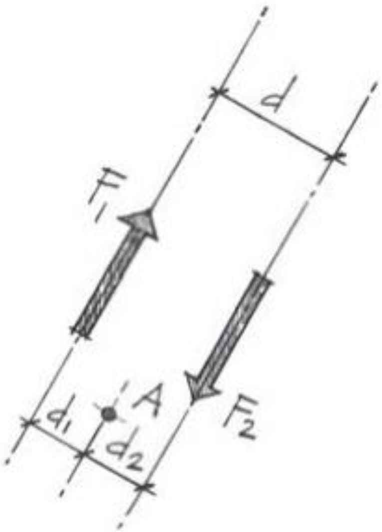

Figure 2.38 Moment of a couple about A.

The final moment M is called the *moment of the couple*. Note that M is independent of the location of the reference point A. M will have the same magnitude and the same rotational sense regardless of the location of A (Figure 2.38).

$$M_A = -F(d_1) - F(d_2) = -F(d_1 + d_2)$$

but

$$d = d_1 + d_2$$

$$M_A = -Fd$$

It can be concluded, therefore, that the moment M of a couple is constant. Its magnitude is equal to the product $(F) \times (d)$ of either F (F_1 or F_2) and the perpendicular distance d between their lines of action. The sense of M (clockwise or counterclockwise) is determined by direct observation.

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Resolution of a Force into a Force and Couple Acting at Another Point

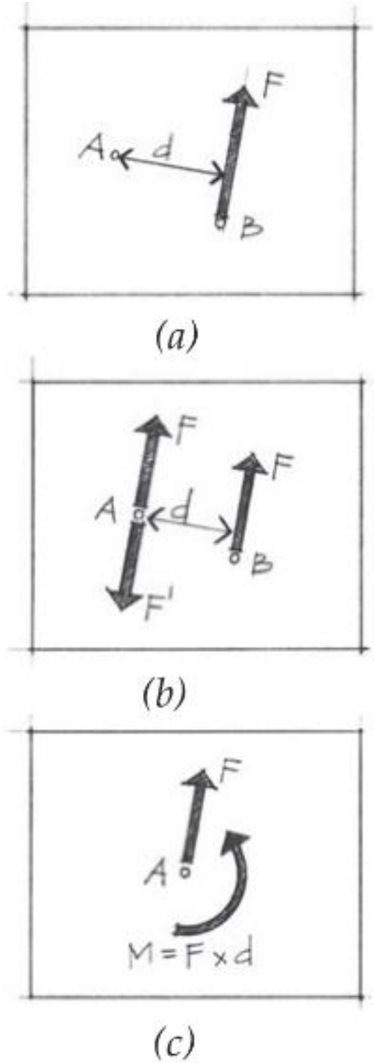
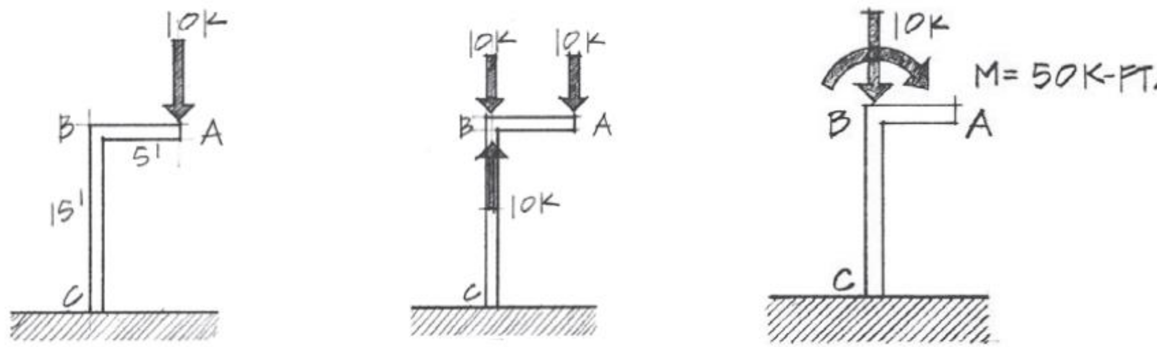


Figure 2.41 Moving a force to another parallel line of action.



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Resultant of two parallel Forces

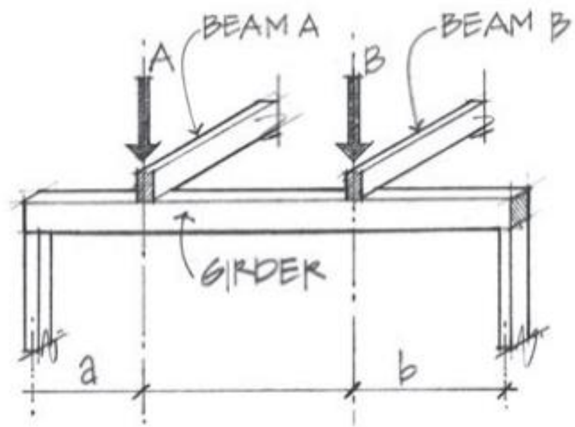

Figure 2.42(a) *Two parallel forces acting on a girder.*

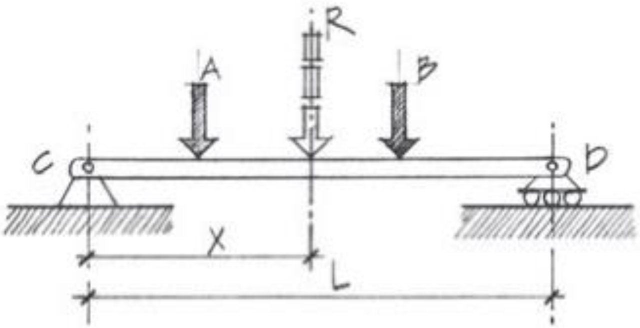

Figure 2.42(b) *Equivalent resultant force R for A and B.*

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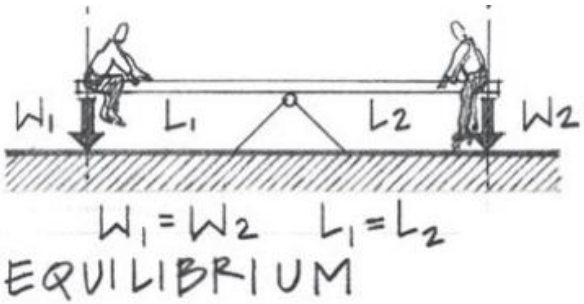
Equilibrium Equations: Two Dimensional

$$R_x = \Sigma F_x = 0$$
$$R_y = \Sigma F_y = 0$$
$$M_i = \Sigma M = 0; \quad \text{where } i = \text{any point}$$



ıre 2.44 *Example of equilibrium.*

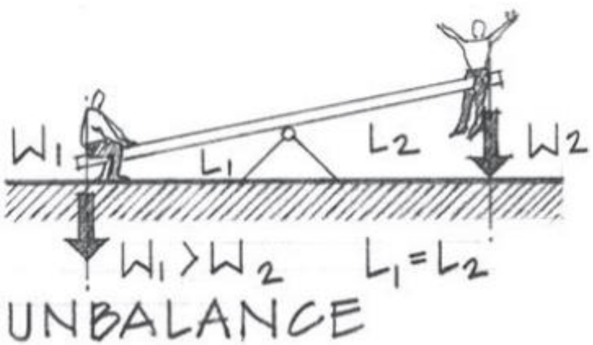


Figure 2.45 Example of nonequilibrium or unbalance.

Collinear Force System

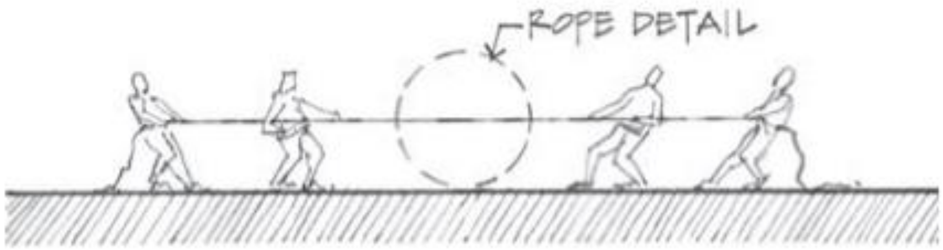


Figure 2.47(a) Tug-of-war (deadlocked).

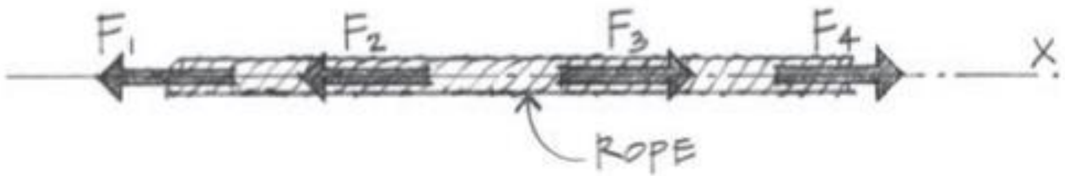


Figure 2.47(b) Detail of rope—collinear forces.

$$\Sigma F_x = 0$$

$$-F_1 - F_2 + F_3 + F_4 = 0$$

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Concurrent Force System

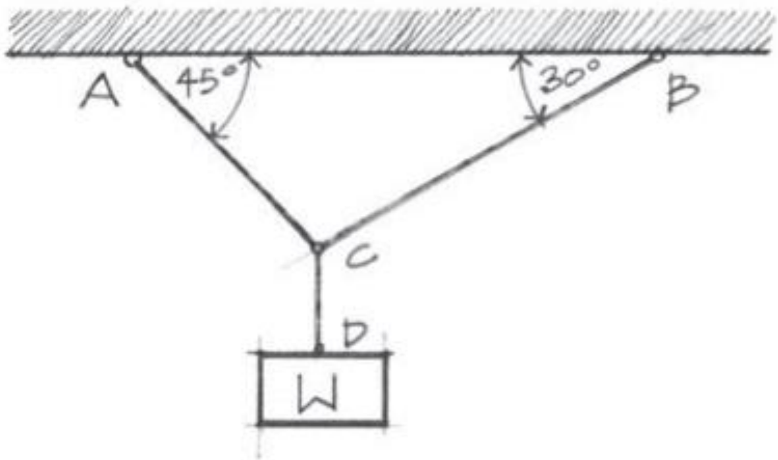

Figure 2.48 Concurrent force system at C.

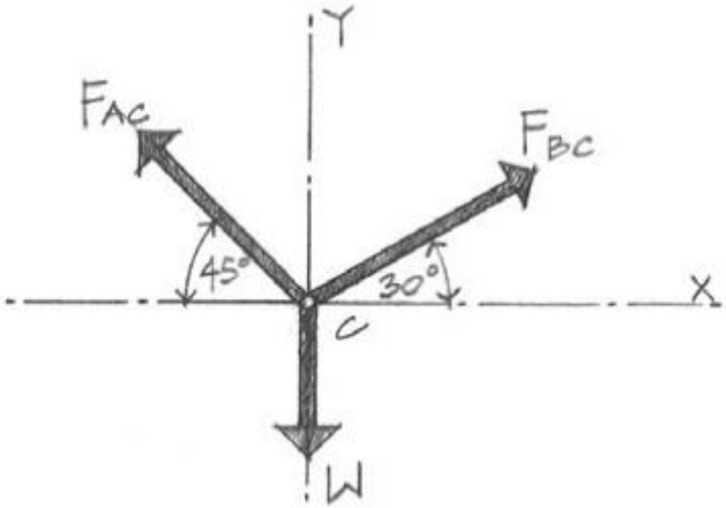

Figure 2.49 Force diagram of concurrent point C.

$$R_x = \Sigma F_x = 0$$
$$R_y = \Sigma F_y = 0$$

These two conditions must be satisfied before equilibrium is established. No translation in either the x or y direction is permitted.

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Figure 2.51 An example of nonequilibrium—Tacoma Narrows bridge before collapse.

$$\Sigma F_x = 0; \quad \Sigma M_i = 0; \quad \Sigma M_j = 0$$

or

$$\Sigma F_y = 0; \quad \Sigma M_i = 0; \quad \Sigma M_j = 0$$

or

$$\Sigma M_i = 0; \quad \Sigma M_j = 0; \quad \Sigma M_k = 0$$