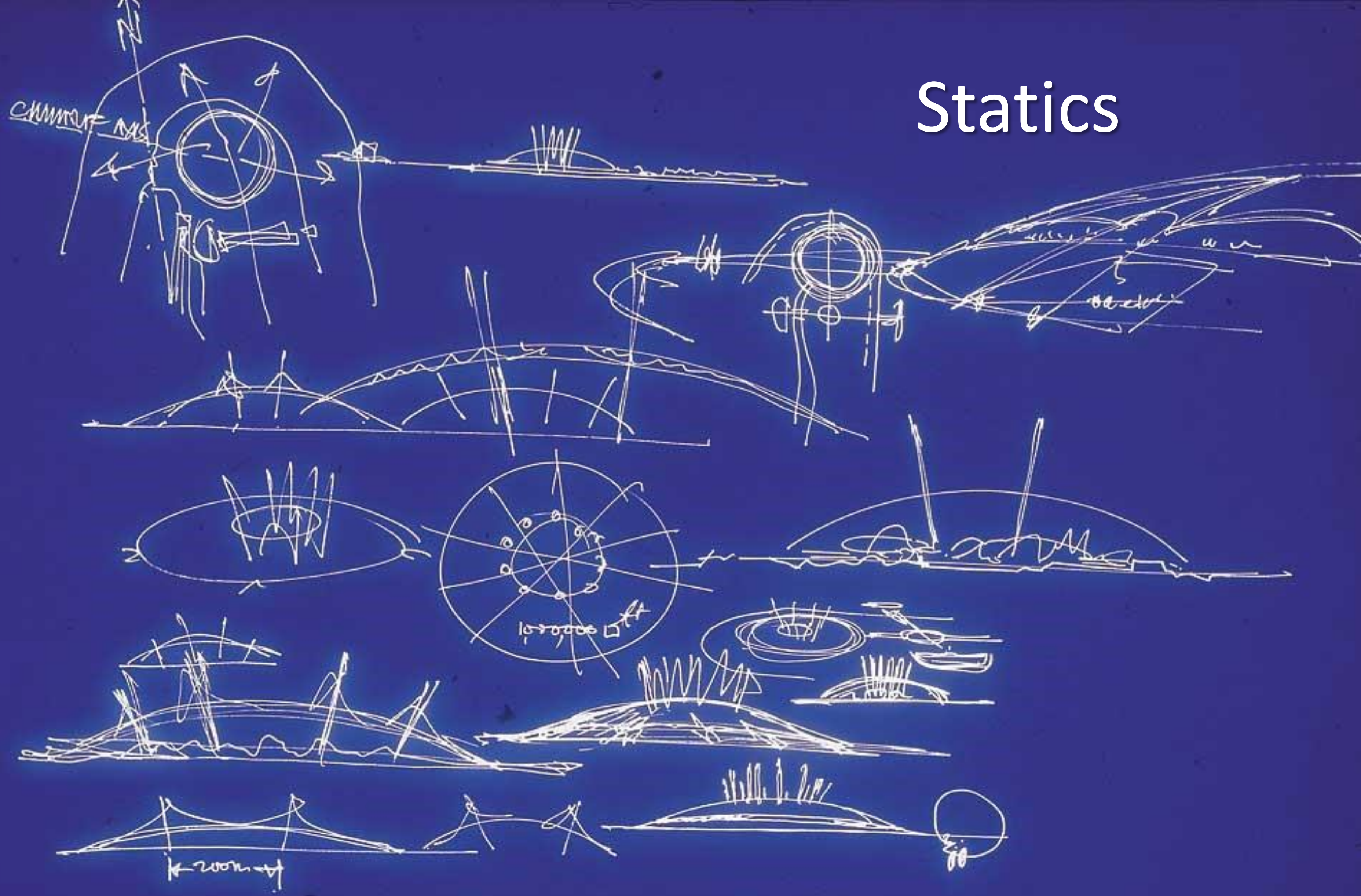
Statics

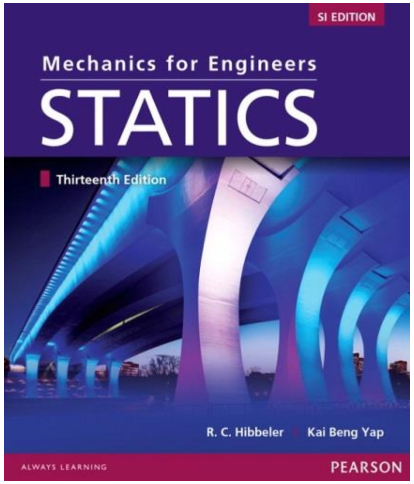


Dr. Haluk Sesigür
I.T.U. Faculty of Architecture
Structural and Earthquake Engineering WG

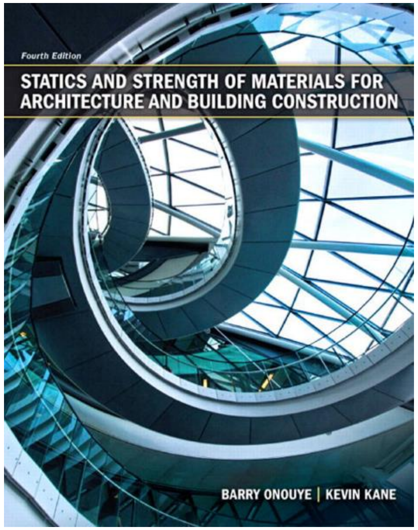
INTRODUCTION

Reference Book

Mechanics For Engineers: Statics, SI Editon, 13/E
Russell C. Hibbeler
Kai Beng Yap
©2013 • Prentice Hall • Paper, 646 pp

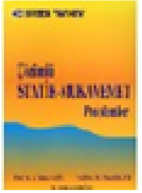


Statics and Strength of Materials for Architecture And Building Construction
Barry Onouye & Kevin Kane
©2012 • Prentice Hall • Paper, 600 pp



INTRODUCTION

Other References:

Beer, F.P., Johnston, R., Dewolf J.T., Mazurek D.	Statics and Mechanics of Materials	2010	McGraw-Hill	
Karataş, H., İşler, Ö.	Mühendislik Mekaniğinde Statik Problemleri	2003	Çağlayan	
Aköz, Y., Eratlı, N.	Çözümlü Statik-Mukavemet Problemleri	2005	Birsen	
Karataş, H	Mukavemet	1986	Çağlayan	

INTRODUCTION

Course treatment & Assessment Criteria :

Theory and Application
1 Midterm (%40) + Final Exam (%60)

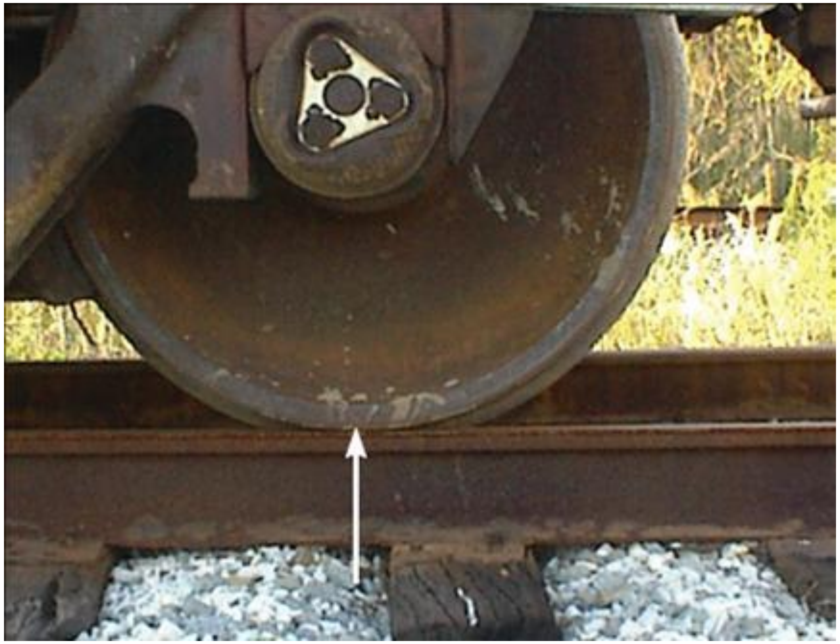
Course syllabus

- General, Concept of Force,
- Concurrent Forces in a Plane,
- Parallel Forces in a Plane,
- Moment of a Force and Couple,
- General Case of Forces in a Plane,
- Centroids,
- Supports and Reactions, Loads,
- Friction,
- Plane Trusses,
- Cables,
- Moments of Intertia.

WHAT IS MECHANICS?

Study of what happens to a “thing” (the technical name is “BODY”) when FORCES are applied to it.

Either the body or the forces can be large or small.



Mechanics

- Mechanics can be divided into 3 branches:

  - Rigid-body Mechanics

  - Deformable-body Mechanics

  - Fluid Mechanics

- Rigid-body Mechanics deals with

  - Statics

  - Dynamics

Mechanics

- Statics – Equilibrium of bodies
  - At rest
  - Move with constant velocity

- Dynamics – Accelerated motion of bodies

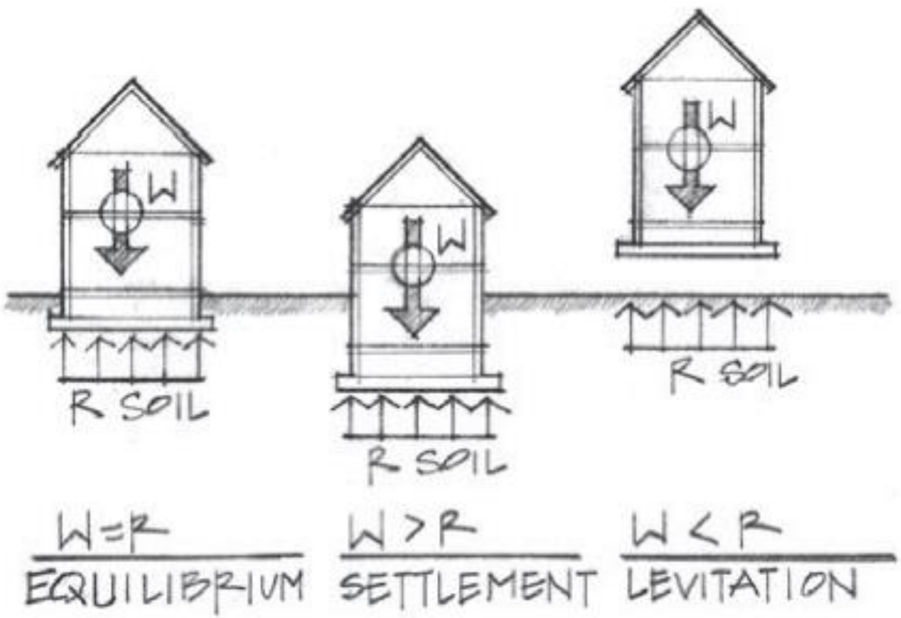
Force

What is force? Force may be defined as the action of one body on another that affects the state of motion or rest of the body. In the late 17th century, Sir Isaac Newton (Figure 2.1) summarized the effects of force in three basic laws:

- **First Law:** Any body at rest will remain at rest, and any body in motion will move uniformly in a straight line, unless acted upon by a force. (Equilibrium)
- **Second Law:** The time rate of change of momentum is equal to the force producing it, and the change takes place in the direction in which the force is acting. ($F = m \times a$)

Figure 2.1 Sir Isaac Newton (1642–1727).

- **Third Law:** For every force of action, there is a re-action that is equal in magnitude, opposite in direction, and has the same line of action. (Basic concept of force.)



Ground resistance on a building.

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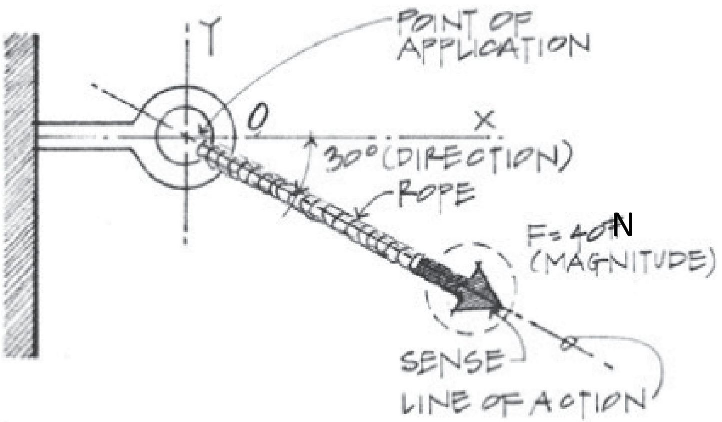

Figure 2.3 Rope pulling on an eyebolt.

Characteristics of a Force

A force is characterized by its (a) point of application, (b) magnitude, and (c) direction.

The *point of application* defines the point where the force is applied. In statics, the point of application does not imply the exact molecule on which a force is applied but a location that, in general, describes the origin of a force (Figure 2.3).

In the study of forces and force systems, the word *particle* will be used, and it should be considered as the location or point where the forces are acting.

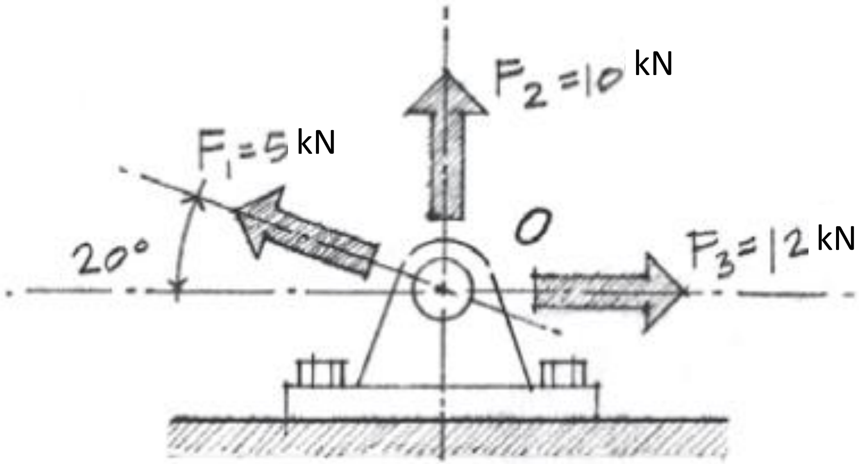

Figure 2.4(a) An anchor device with three applied forces.

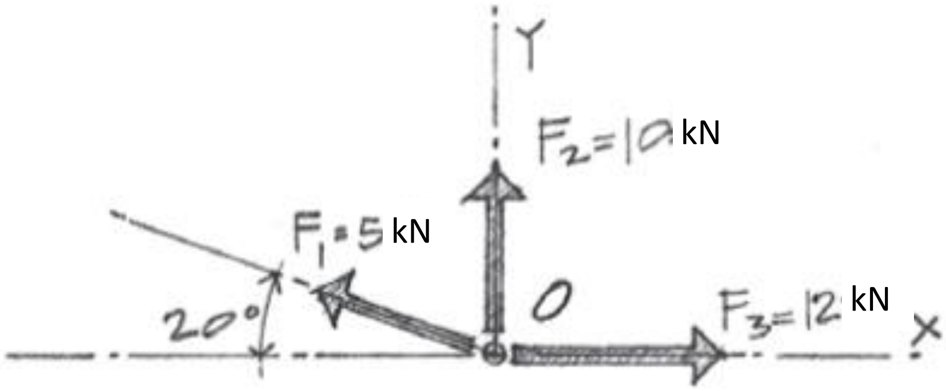

Figure 2.4(b) Force diagram of the anchor.

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The sense of the force is indicated by an arrowhead. For example, in Figure 2.6, the arrowhead gives the indication that a pulling force (tension) is being applied to the bracket at point *O*.

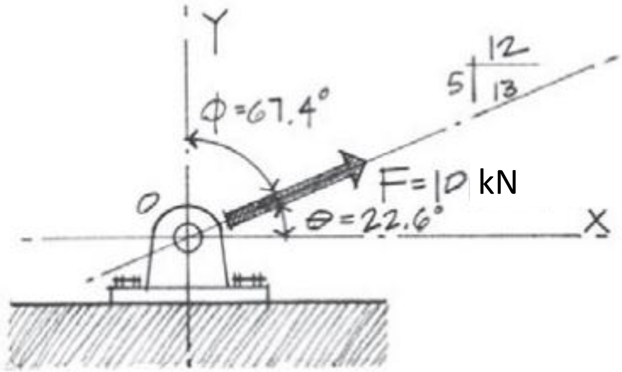

Figure 2.6 Three ways of indicating direction for an angular tension force.

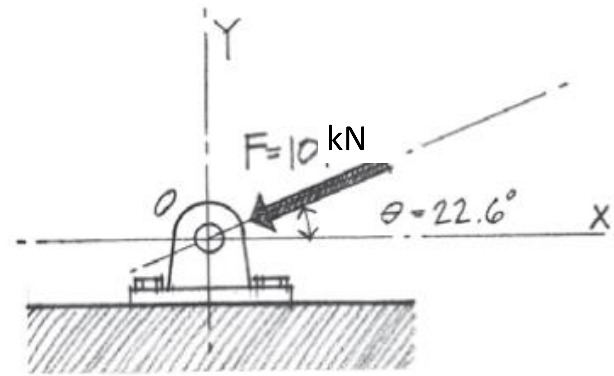

Figure 2.7 Force in compression.

By reversing only the arrowhead (Figure 2.7), we would have a pushing force (compression) applied on the bracket with the same magnitude ($F = 10$ k), point of application (point *O*), and line of action ($\theta = 22.6°$ from the horizontal).

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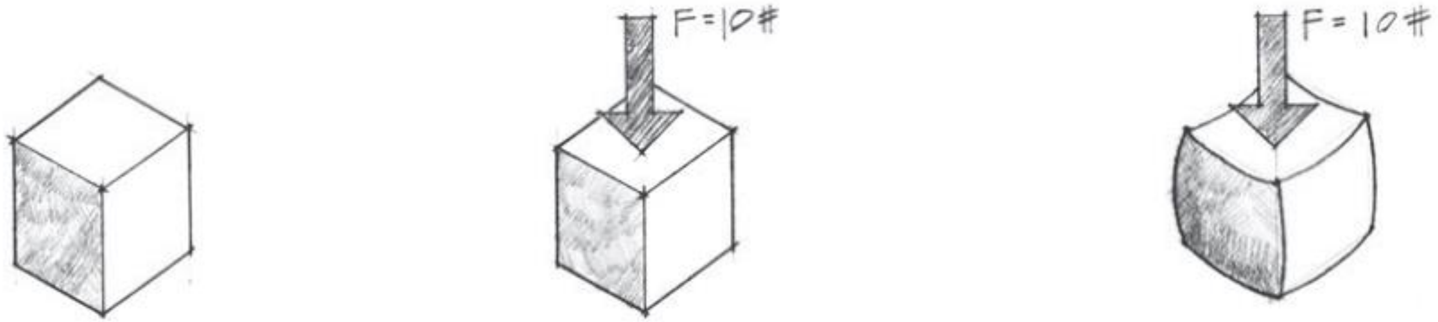
Rigid Bodies

When a force of F = 10# is applied to a box, as shown in Figure 2.8, some degree of deformation will result. The deformed box is referred to as a *deformable body*, whereas in Figure 2.8(b) we see an undeformed box called the *rigid body*. Again, you must remember that the rigid body is a purely theoretical phenomenon but necessary in the study of statics.



(a) Original, unloaded box. *(b) Rigid body (example: stone).* *(c) Deformable body (example: foam).*

Figure 2.8 Rigid body/deformable body.

Principle of Transmissibility

An important principle that applies to rigid bodies in particular is the *principle of transmissibility*. This principle states that the external effects on a body (cart) remain unchanged when a force F_1 acting at point A is replaced by a force F_2 of equal magnitude at point B, provided that both forces have the same sense and line of action (Figure 2.9).

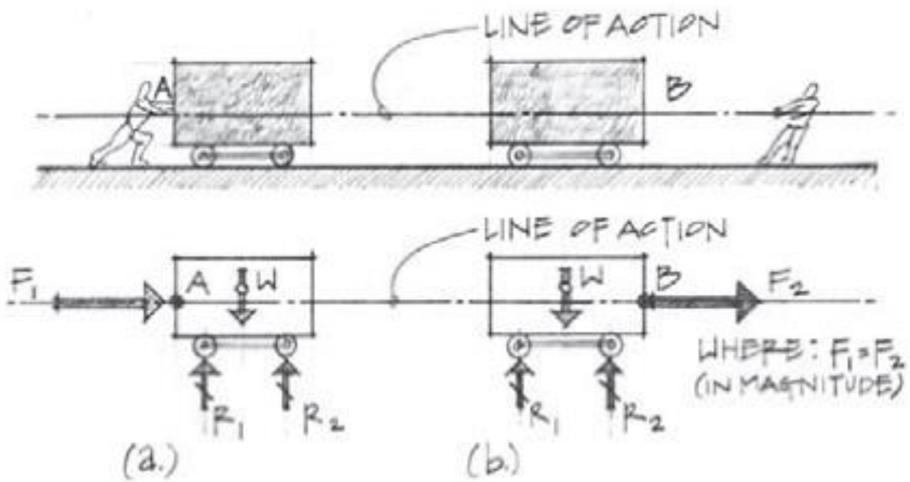


Figure 2.9 An example of the principle of transmissibility.

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External and Internal Forces

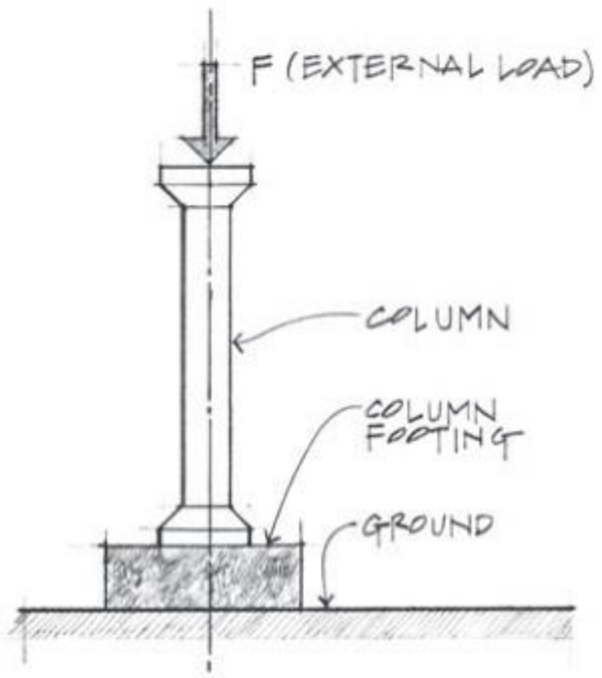

Figure 2.14 A column supporting an external load.

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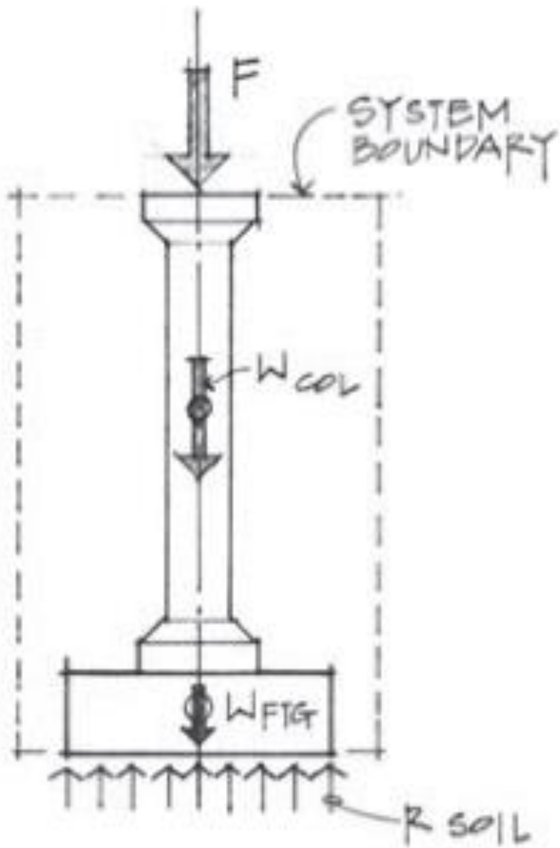
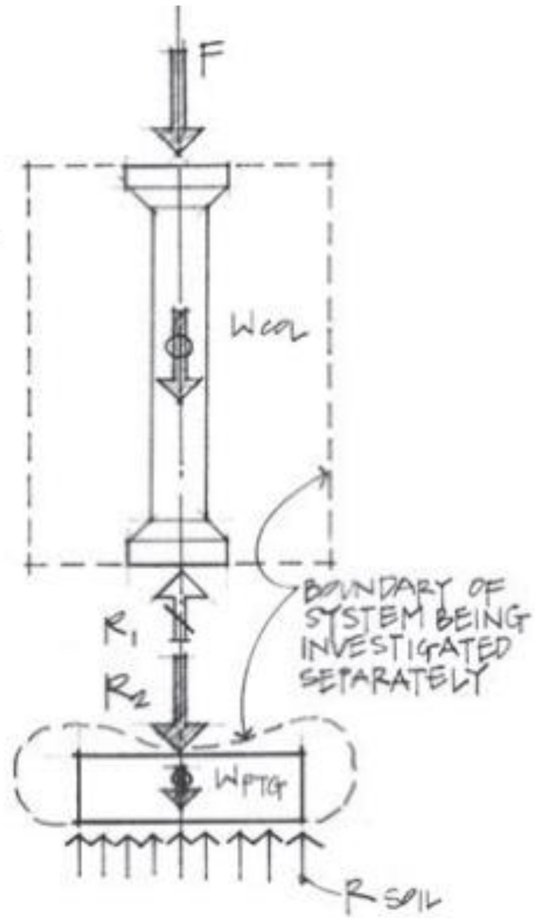
External and Internal Forces



(a) Column and footing.

(b) Column.



(c) Footing.

Figure 2.15 Different system groupings.

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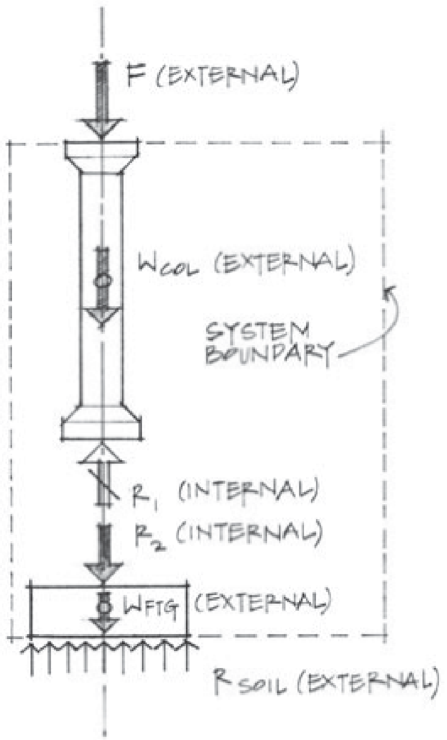


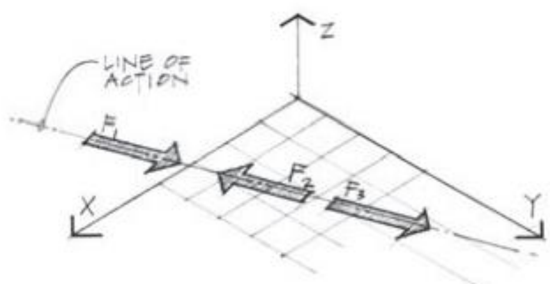
(a) Relationship of forces between the column and footing.

Now let's examine the internal forces that are present in each of the three cases examined above (Figure 2.16).

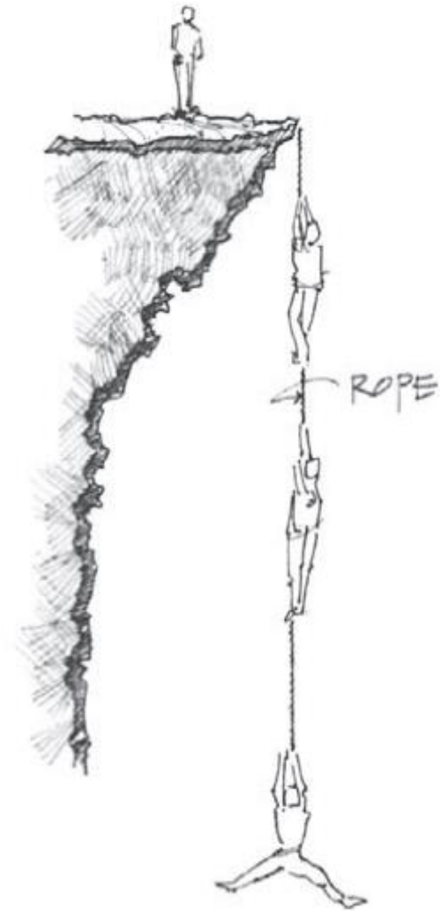
Examination of Figure 2.16(a) shows forces R_1 and R_2 occurring between the column and footing. The boundary of the system is still maintained around the column and footing, but by examining the interaction that takes place between members within a system, we infer internal forces. Force R_1 is the reaction of the footing on the column, while R_2 is the action of the column on the footing. From Newton's third law, we can then say that R_1 and R_2 are equal and opposite forces.

Types of Force Systems

Force systems are often identified by the type or types of systems on which they act. These forces may be *collinear*, *coplanar*, or *space force systems*. When forces act along a straight line, they are called *collinear*; when they are randomly distributed in space, they are called *space forces*. Force systems that intersect at a common point are called *concurrent*, while parallel forces are called *parallel*. If the forces are neither concurrent nor parallel, they fall under the classification of *general force systems*. Concurrent force systems can act on a particle (point) or a rigid body, whereas parallel and general force systems can act only on a rigid body or a system of rigid bodies. (See Figure 2.17 for a diagrammatic representation of the various force system arrangements.)

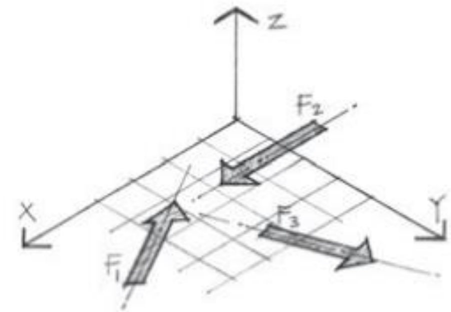


Collinear—*All forces acting along the same straight line.*

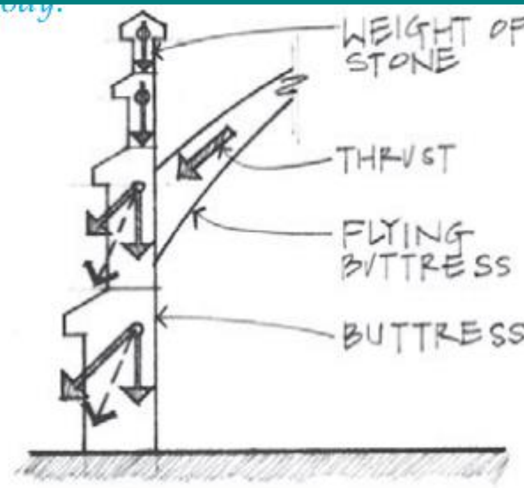


One intelligent hiker observing three other hikers dangling from a rope.

Figure 2.17(a) Particle or rigid body.

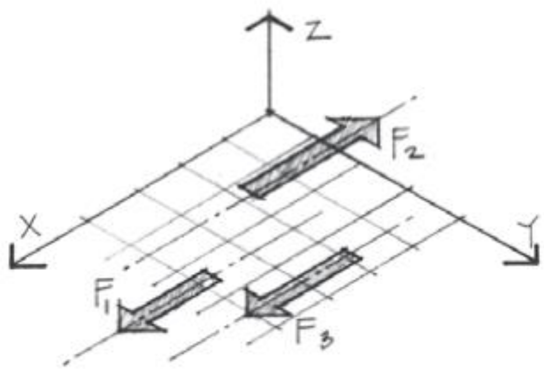


***Coplanar**—All forces acting in the same plane.*

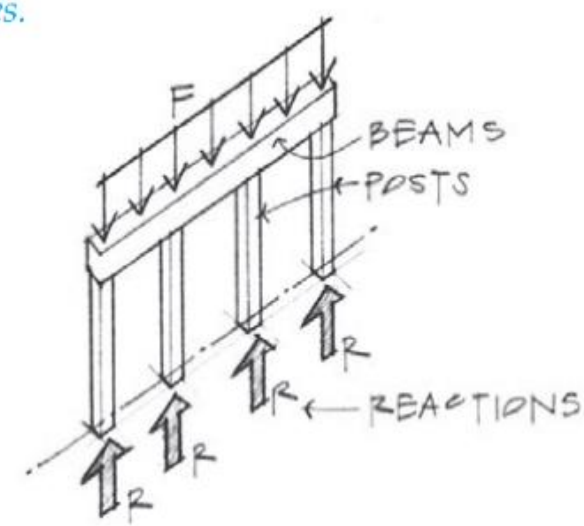


Forces in a buttress system.

Figure 2.17(b) Rigid bodies.



***Coplanar, parallel**—All forces are parallel and act in the same plane.*



A beam supported by a series of columns.

Figure 2.17(c) Rigid bodies.

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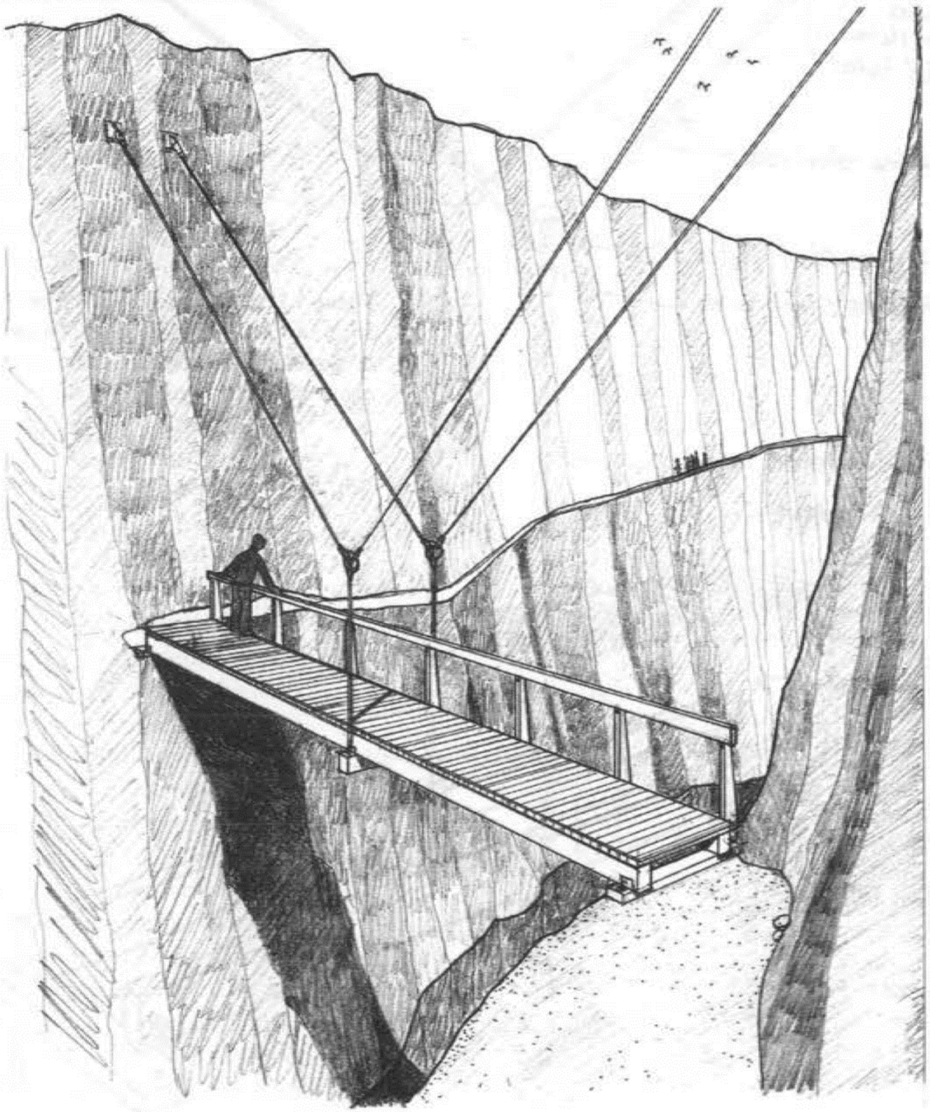


Figure 1.1 This suspension span of 40 ft is the first bridge we will consider in this chapter.

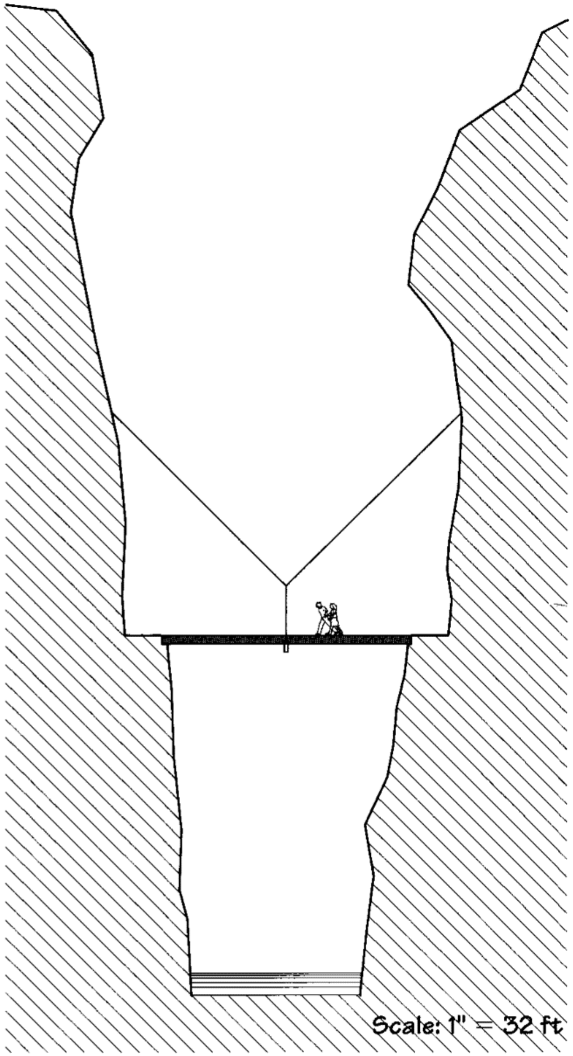


Figure 1.6 Bridge #1 utilizes two 20-ft lengths of deck beams to span 40 ft.

Form and Forces, E.Allen, W. Zalewski

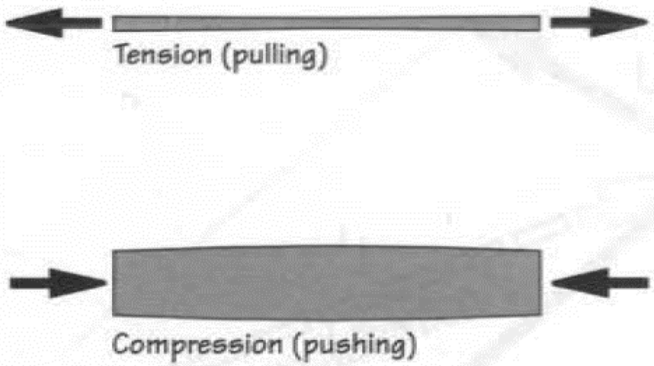


Figure 1.11 Tension and compression represented as bodies acted on by vectors.

"The remarkable, inherent simplicity of nature allows the structure to perform its task through two elementary actions only: pulling and pushing."

—MARIO SALVADORI

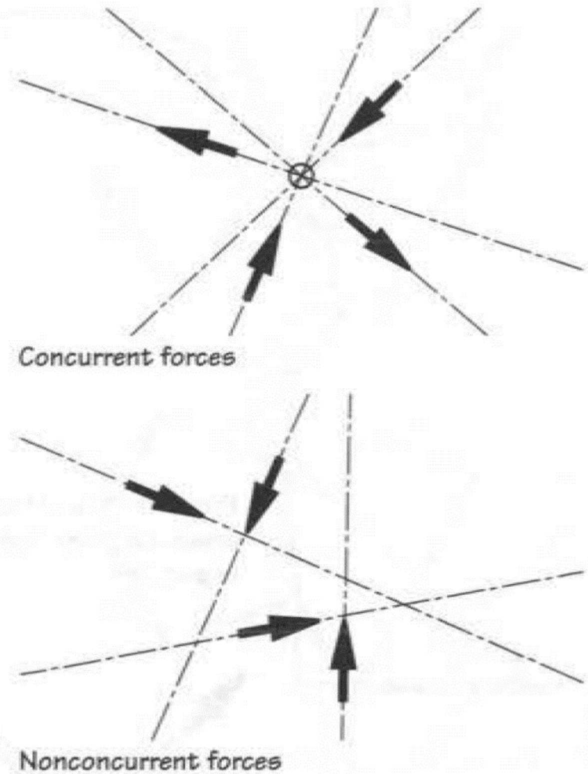


Figure 1.13 Concurrent and nonconcurrent forces. There are pairs of concurrent forces in the lower diagram, but there is no concurrence of more than two forces.

Form and Forces, E.Allen, W. Zalewski

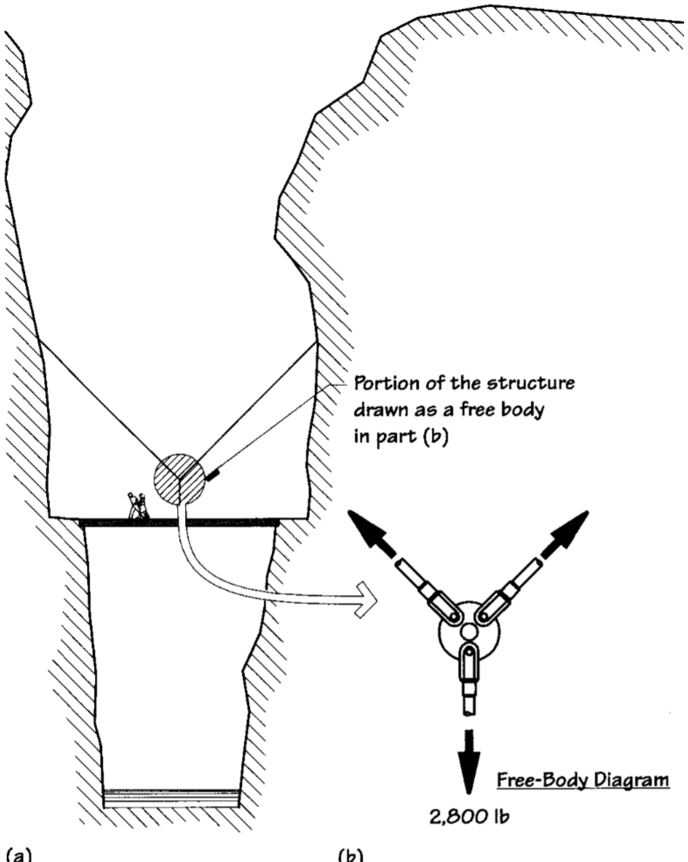


Figure 1.14 In part (a), a free-body diagram has been constructed of the rod intersection or node in Bridge #1. The intersection is cut imaginarily from the larger structure (a) and represented as a free body, isolated from the rest of the structure. Vectors are added to indicate all the external forces that act upon the free body. We know the magnitude and direction of the downward force on the free body (2,800 lb) but we know only the directions of the other two forces.

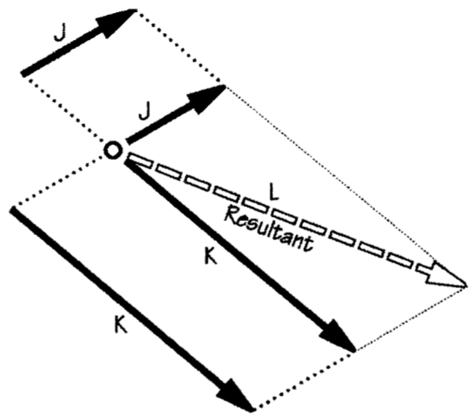


Figure 1.15 The parallelogram law. The resultant of vectors *J* and *K* is found by bringing them together as the adjacent sides of a parallelogram. The diagonal of the parallelogram, *L*, is their resultant, a single force that has the same effect as *J* and *K* acting together.

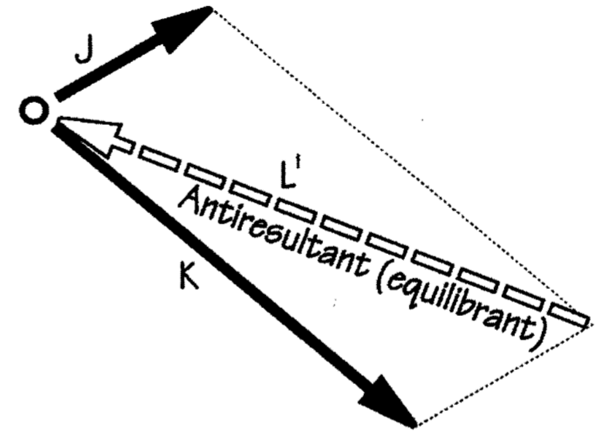


Figure 1.16 If the direction of the resultant in Figure 1.15 is reversed, it becomes an antiresultant or equilibrant, and the three forces are in static equilibrium.

Form and Forces, E.Allen, W. Zalewski

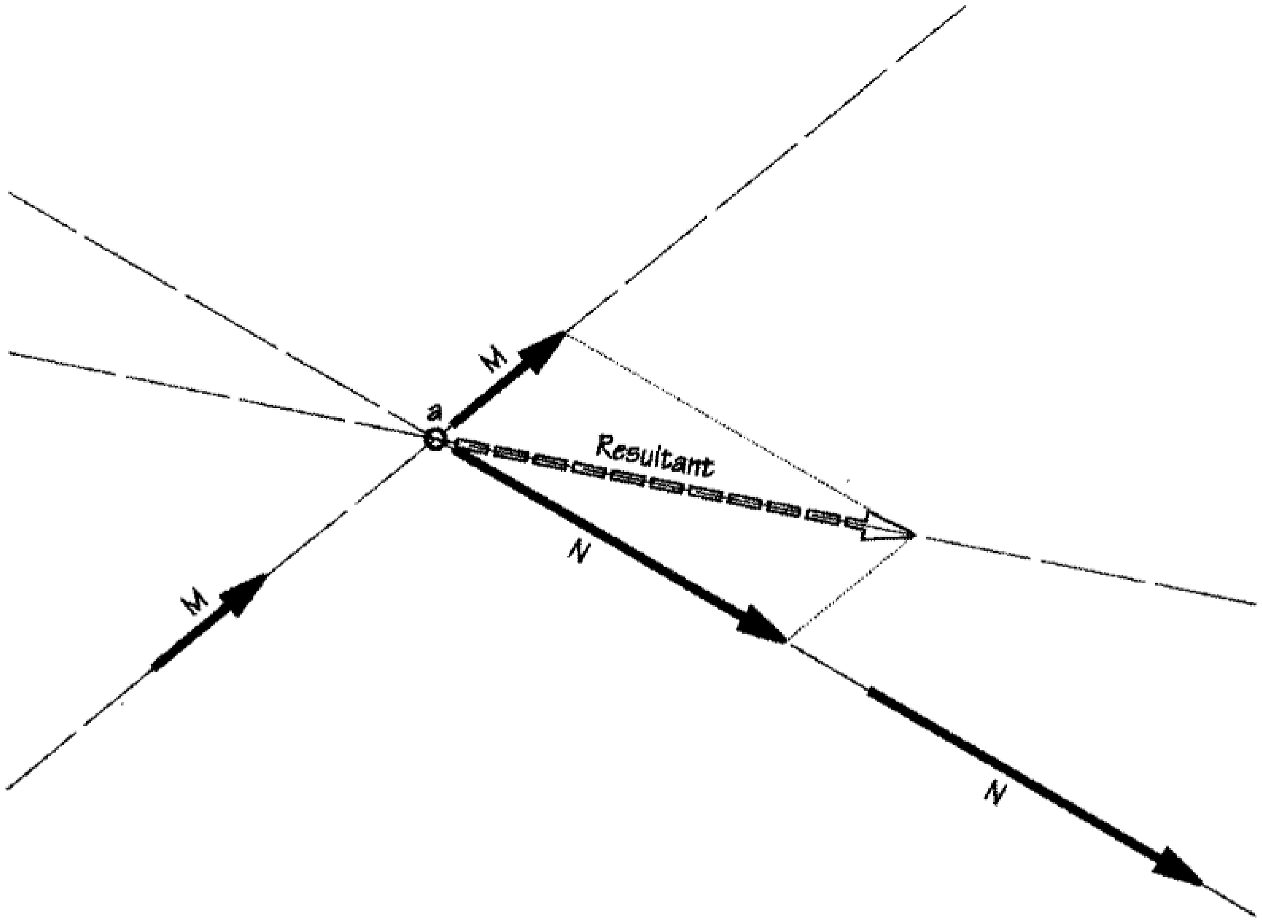
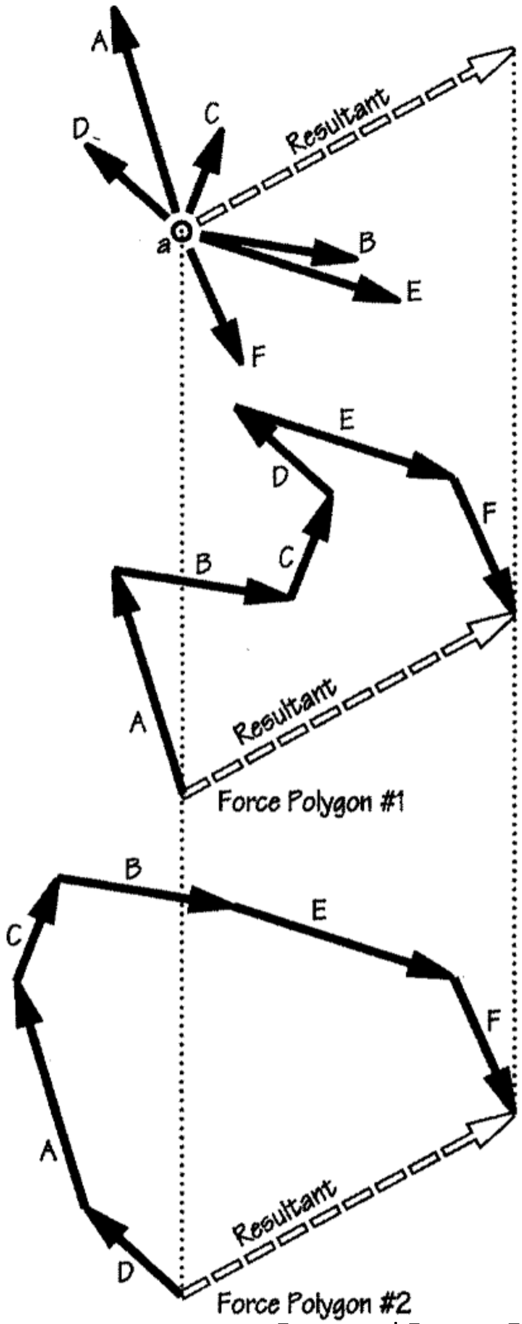


Figure 1.17 The resultant of two forces may also be found by connecting their vectors tip to tail and drawing a vector from the tail of the first force vector to the tip of the second. The line of action of the resultant passes through the intersection of the lines of action of the two original forces.

Form and Forces, E.Allen, W. Zalewski

Figure 1.18 Tip-to-tail addition can be applied to any number of forces. The forces in this example have been connected in two different orders to show that the resultant is the same regardless of the order of connection. The line of action of the resultant passes through the point of concurrence of the vectors in their original locations.



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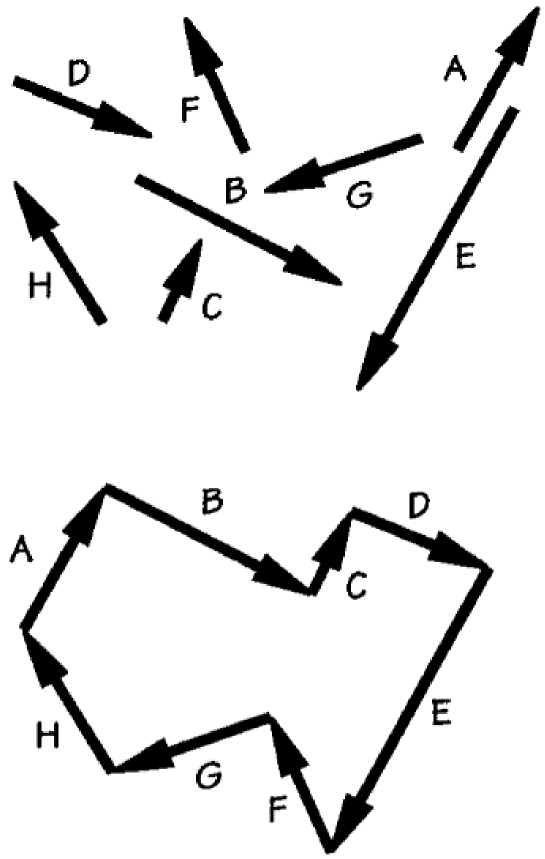

Figure 1.19 If the tip-to-tail polygon of forces closes upon itself, the resultant is zero. If all the forces are concurrent, the closure of the force polygon means that they are in static equilibrium.

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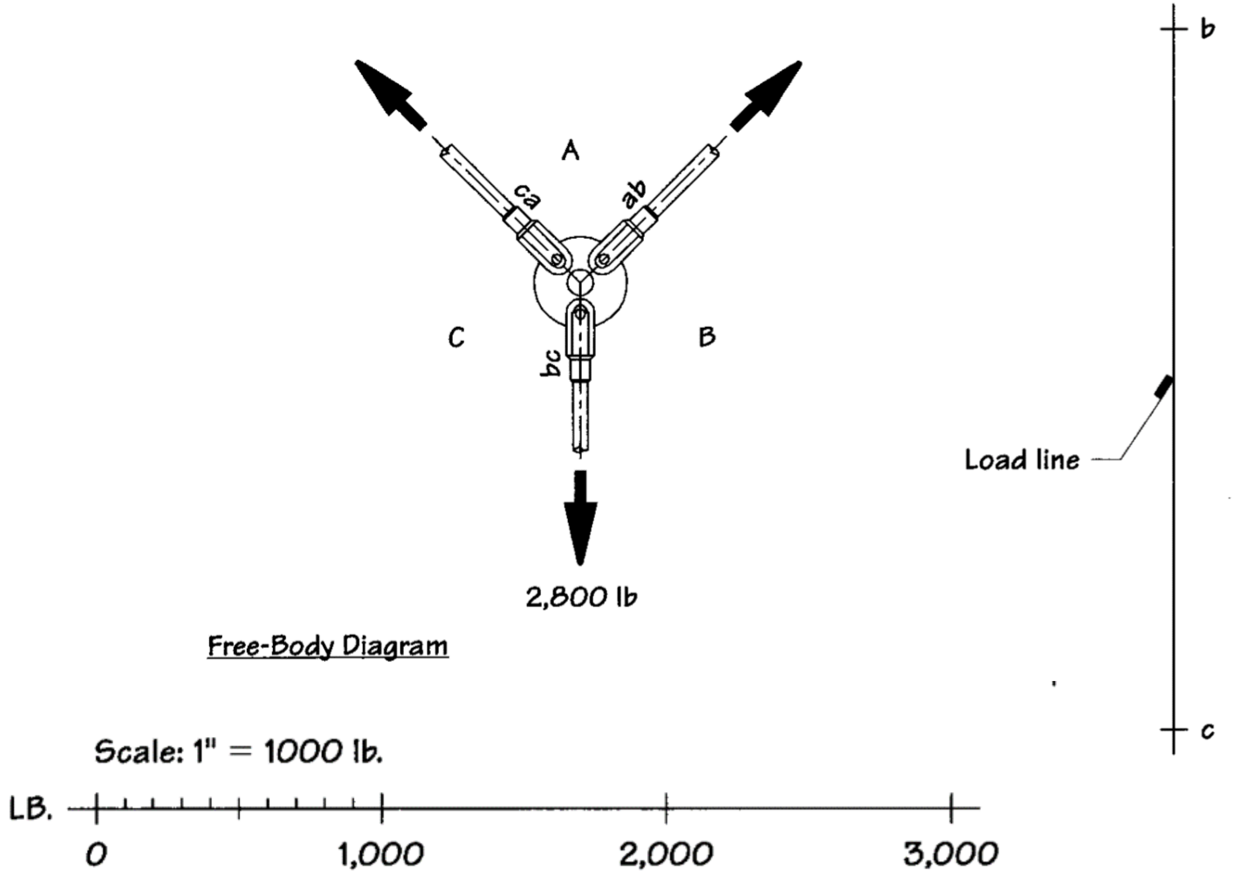

Figure 1.21 The first step in finding the magnitudes of the unknown rod forces is to represent the known force, *bc*, as a line segment whose length is equal to the magnitude of the force and whose direction is parallel to the force. Line *bc* is a force vector, but is generally drawn without an arrowhead. Any convenient scale of length to force may be used in this construction. There is no relationship between the scale of the free-body diagram and the scale of the force polygon.

Form and Forces, E.Allen, W. Zalewski

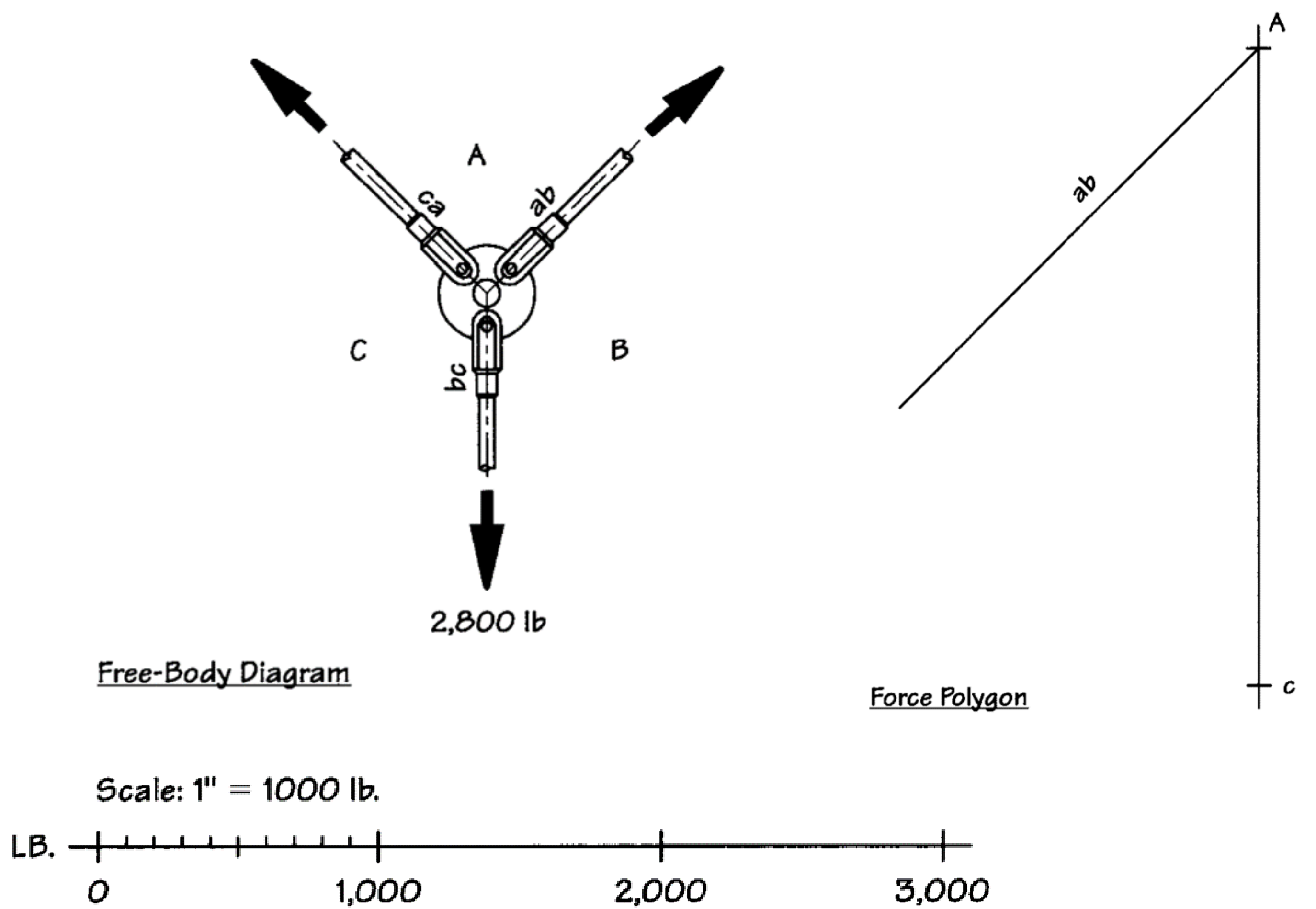

Figure 1.22 The second step in finding the magnitudes of the unknown rod forces is to draw a line on the force polygon to represent the direction of one of the forces. Bow's notation helps us to know where to connect this line to the line that represents the known force: The unknown force is *ab* and there is no letter *a* on the force polygon at the moment; therefore the new line must connect at *b*.

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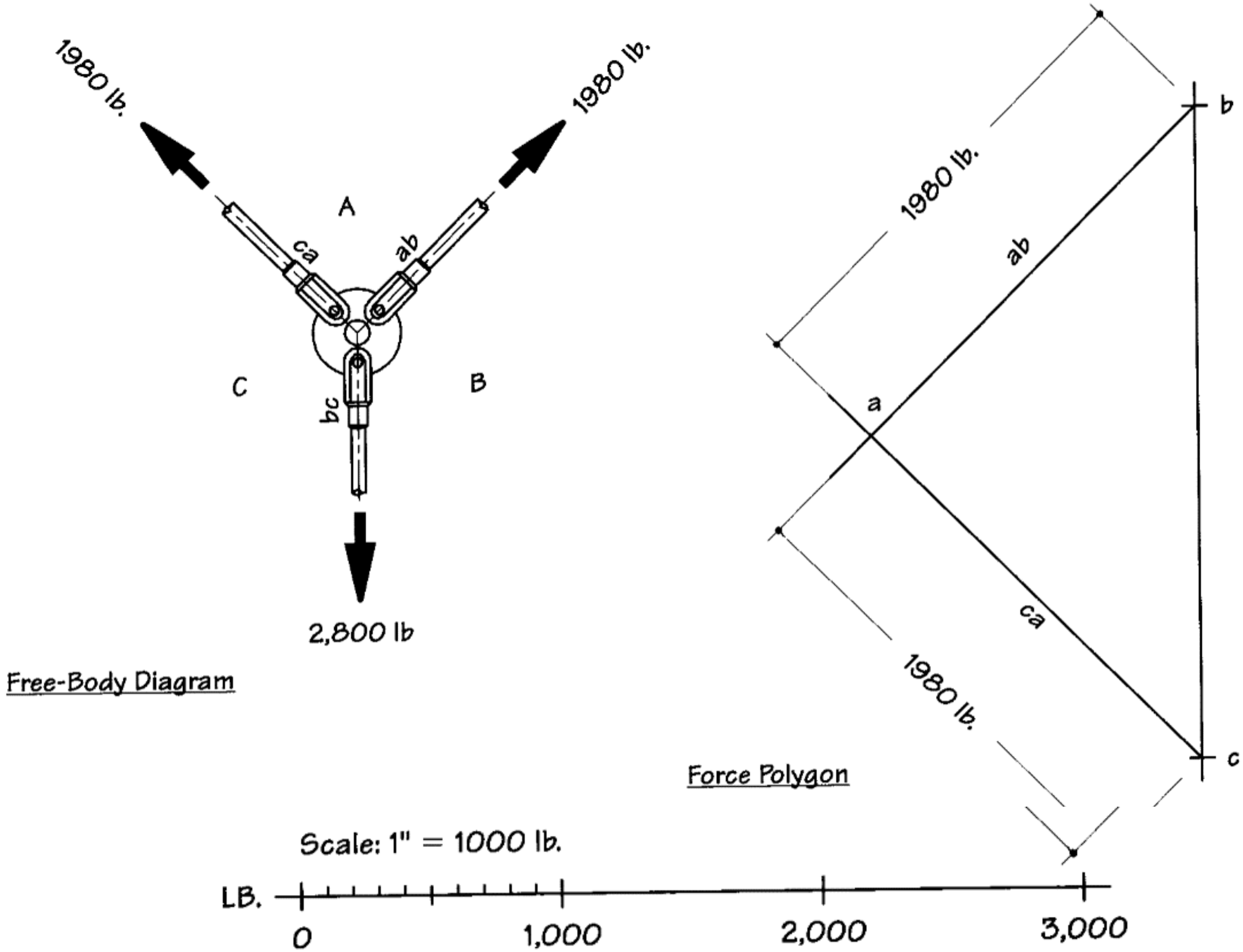
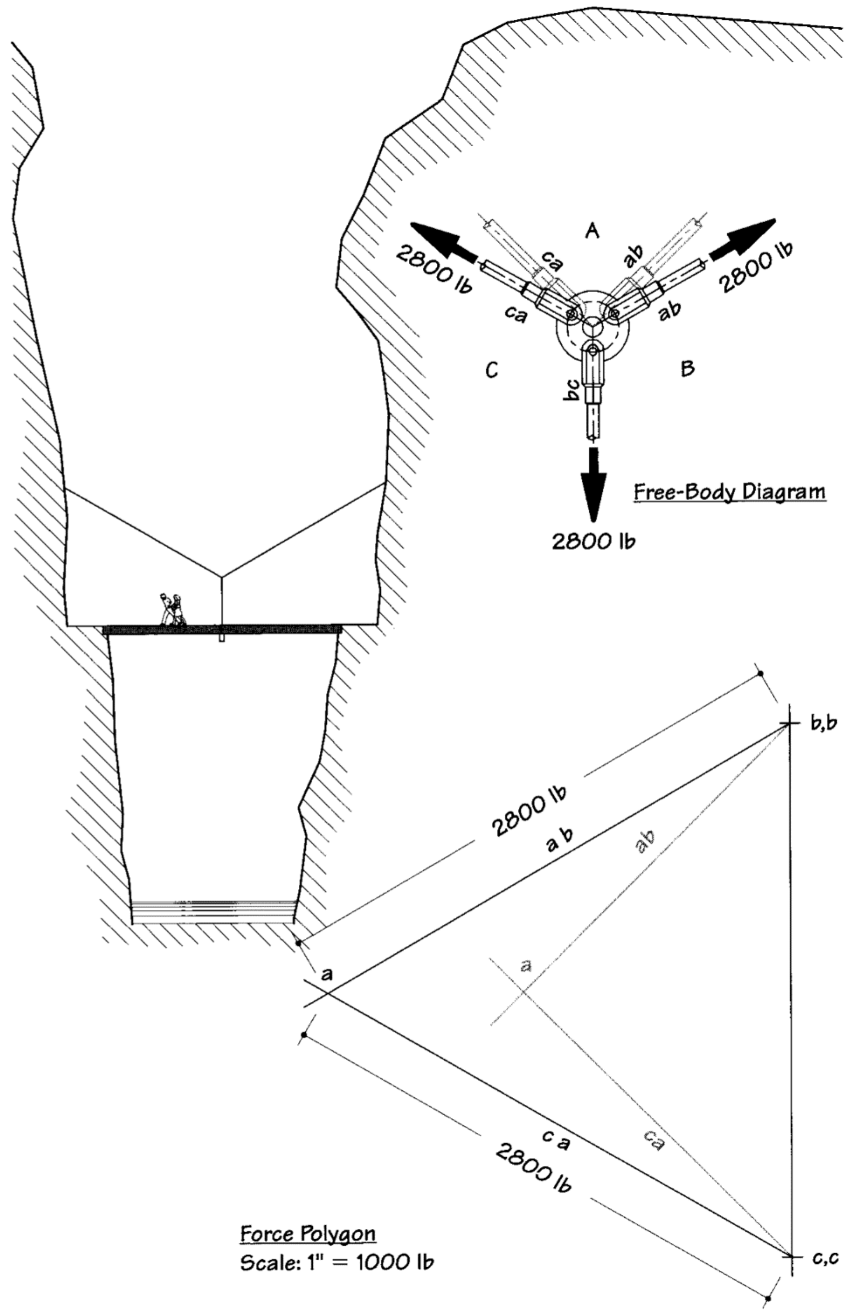


Figure 1.23 The force polygon is completed by repeating the previous step for the remaining unknown force. *a* is the intersection of the two lines that represent the unknown forces. Lines *ab* and *ca* are vectors that may be measured at the given scale to determine the magnitudes of the unknown forces.

Form and Forces, E.Allen, W. Zalewski



Form and Forces, E.Allen, W. Zalewski

Figure 1.27 In Bridge #2, the slopes of the inclined rods are decreased, which causes the forces in the rods to rise. The lighter lines are the vectors for the original slopes; the solid lines represent the new slopes.

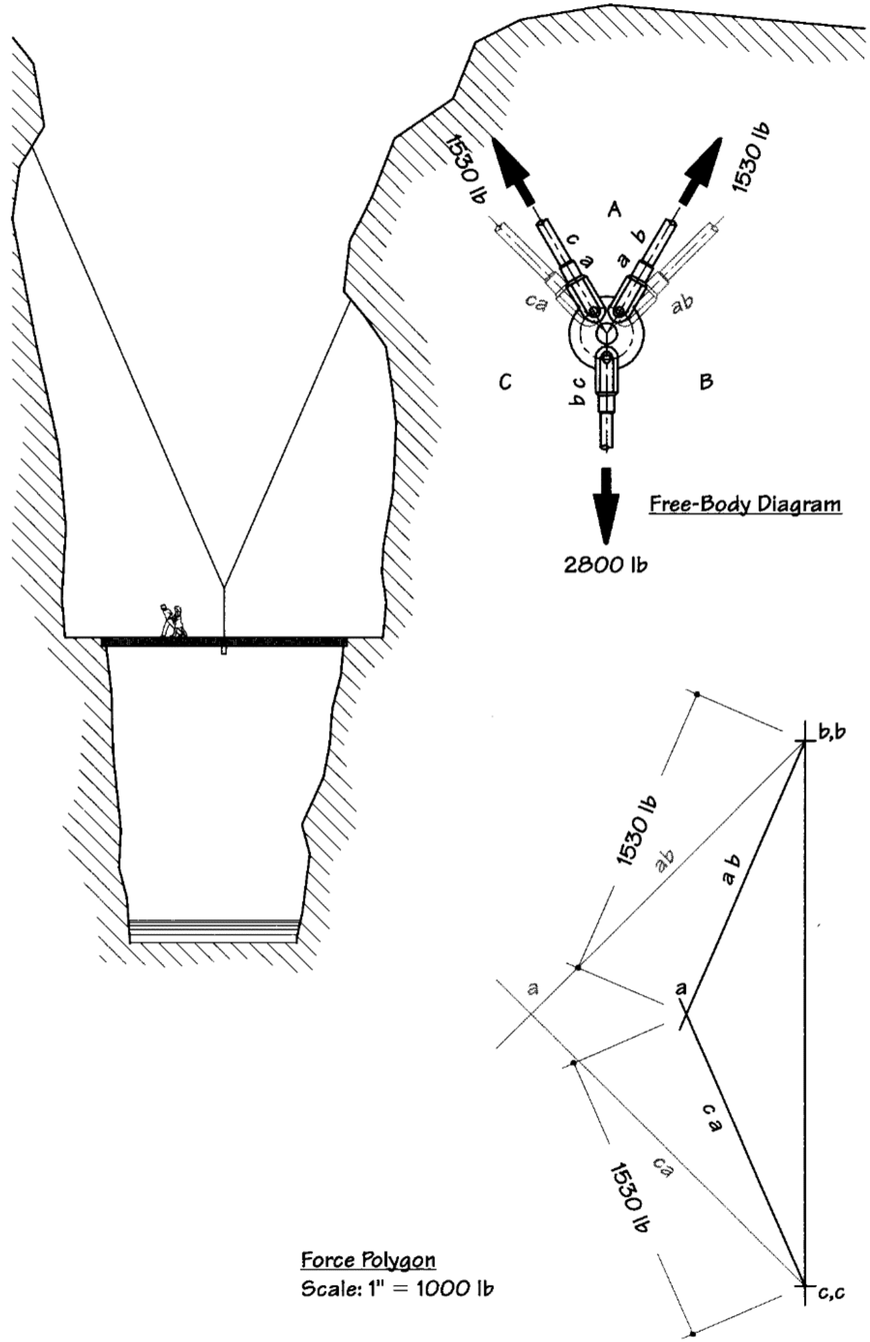


Figure 1.28 If the rods are sloped more steeply, as they are in Bridge #3, the forces in them decrease.

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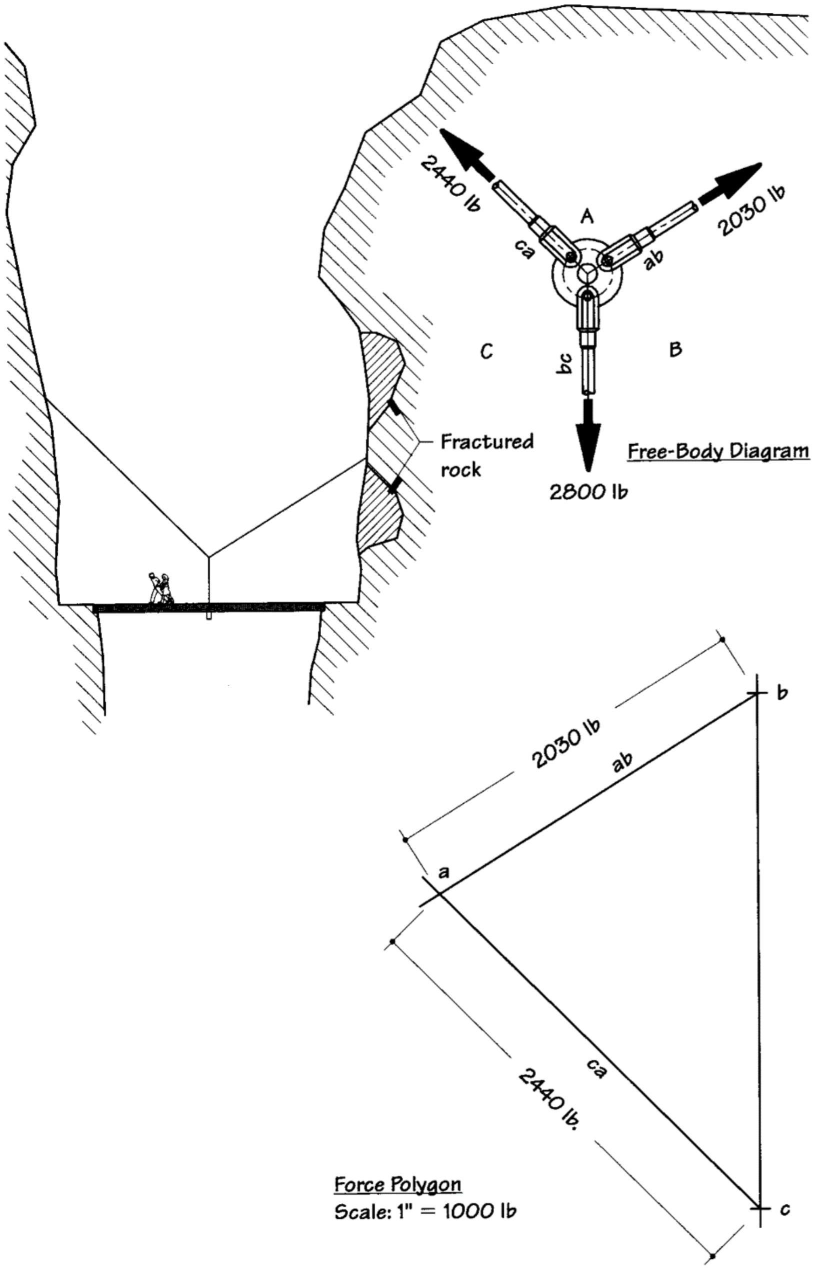

Figure 1.29 The outer end of one of the inclined rods in Bridge #4 is lowered to avoid areas of fractured rock. The magnitudes of the forces in the rods rise, but the bridge remains in static equilibrium.

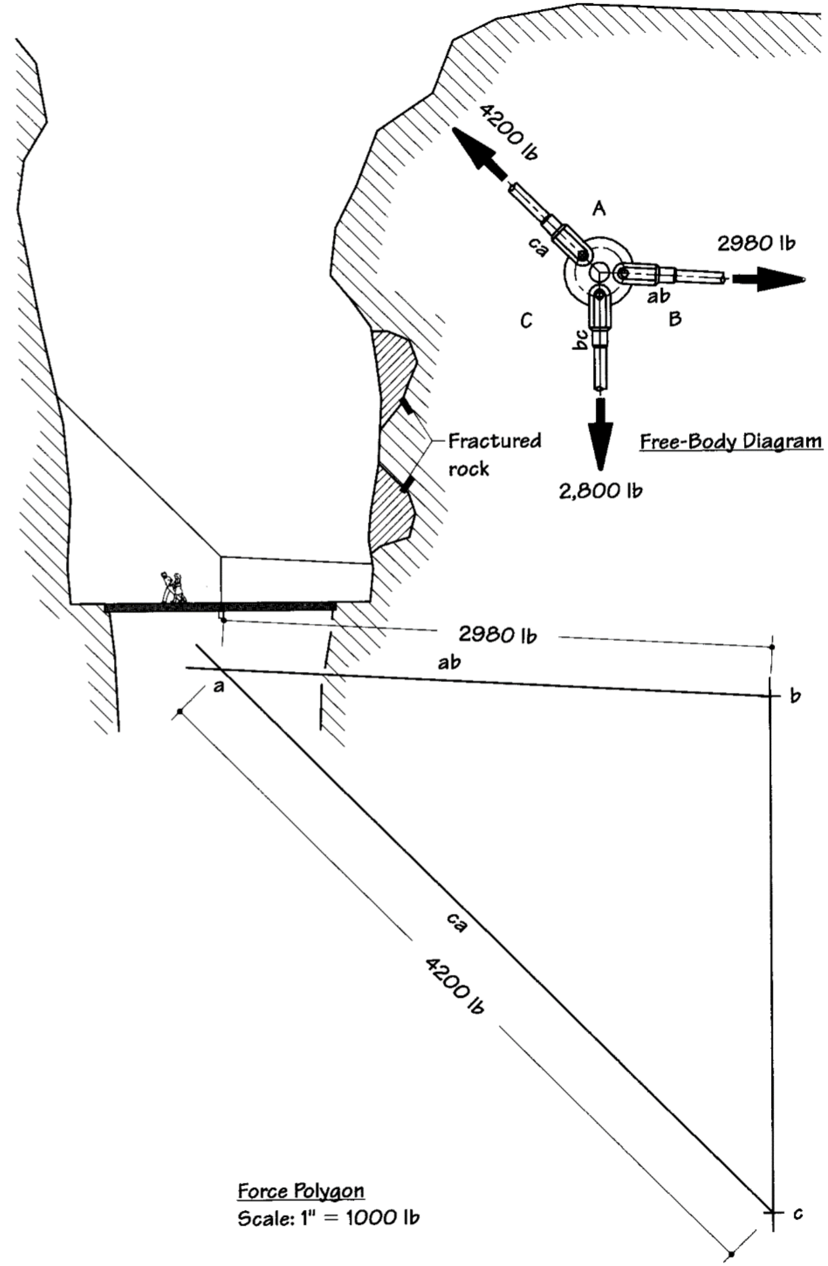

Figure 1.30 Static equilibrium is possible even with the outer end of one rod lower than its inner end.

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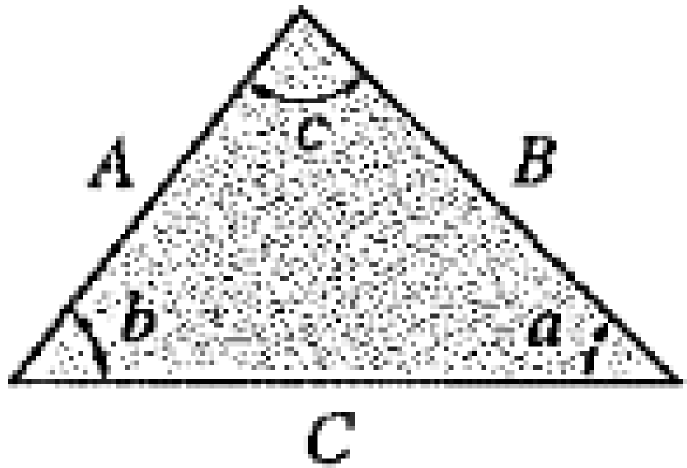


Sine law:

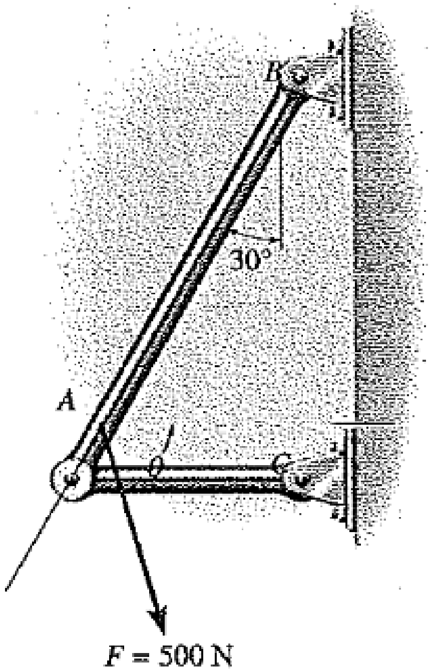
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

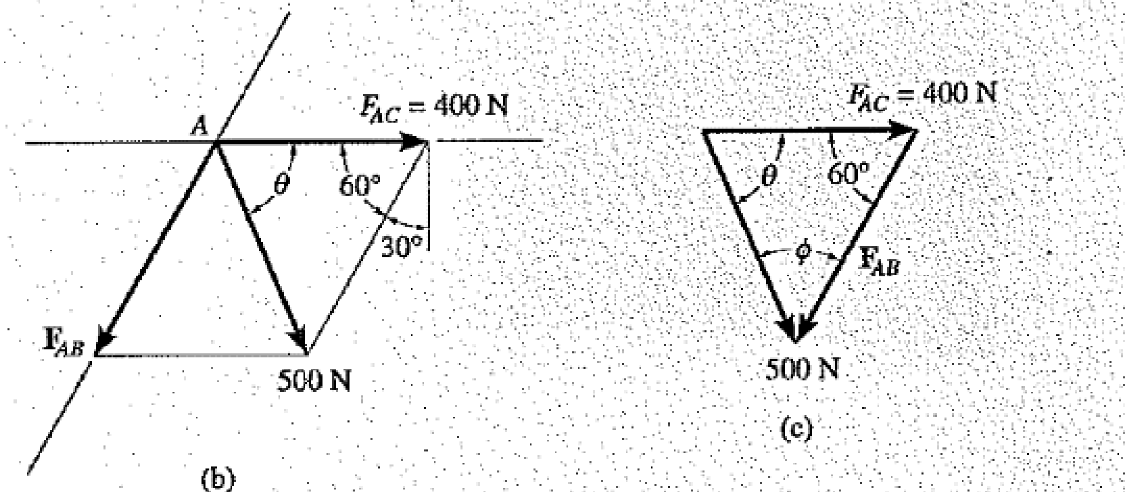
Statics and Strength of Materials, R.C. Hibbeler

The force **F** acting on the frame shown in Fig. 2–12*a* has a magnitude of 500 N and is to be resolved into two components acting along members *AB* and *AC*. Determine the angle θ, measured *below* the horizontal, so that the component $\mathbf{F}_{AC}$ is directed from *A* toward *C* and has a magnitude of 400 N.



(a)

Fig. 2–12



Solution

By using the parallelogram law, the vector addition of the two components yielding the resultant is shown in Fig. 2–12*b*. Note carefully how the resultant force is resolved into the two components

(a)

Fig. 2–12

Solution

By using the parallelogram law, the vector addition of the two components yielding the resultant is shown in Fig. 2–12*b*. Note carefully how the resultant force is resolved into the two components $\mathbf{F}_{AB}$ and $\mathbf{F}_{AC}$, which have specified lines of action. The corresponding vector triangle is shown in Fig. 2–12*c*.

The angle ϕ can be determined by using the law of sines:

$$\frac{400\text{ N}}{\sin\phi} = \frac{500\text{ N}}{\sin 60^\circ}$$

$$\sin\phi = \left(\frac{400\text{ N}}{500\text{ N}}\right)\sin 60^\circ = 0.6928$$

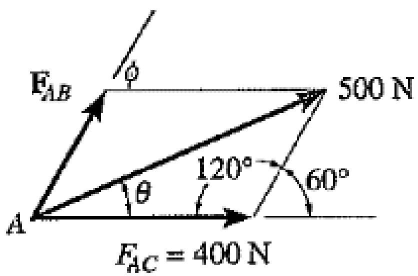
$$\phi = 43.9^\circ$$

Hence,

$$\theta = 180^\circ - 60^\circ - 43.9^\circ = 76.1^\circ \measuredangle_\theta \qquad \textit{Ans.}$$

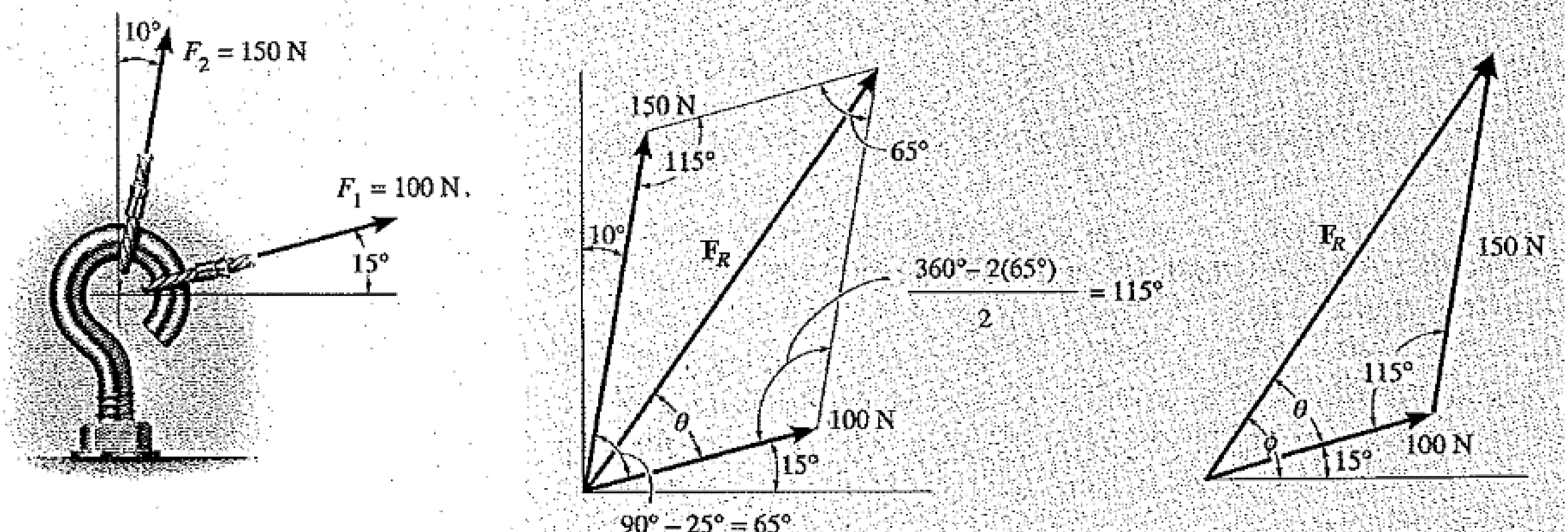
Using this value for θ, apply the law of cosines or the law of sines and show that $\mathbf{F}_{AB}$ has a magnitude of 561 N.

Notice that $\mathbf{F}$ can also be directed at an angle θ *above* the horizontal, as shown in Fig. 2–12*d*, and still produce the required component $\mathbf{F}_{AC}$. Show that in this case $\theta = 16.1^\circ$ and $F_{AB} = 161$ N.



(d)

The screw eye in Fig. 2–10*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



(a) (b) (c)

Fig. 2-10

Solution

Parallelogram Law. The parallelogram law of addition is shown in Fig. 2–10*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From Fig. 2–10*b*, the vector triangle, Fig. 2–10*c*, is constructed. F_R is determined by using the law of cosines:

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115°}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$

The angle θ is determined by applying the law of sines, using the computed value of F_R.

$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115°}$$

$$\sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(0.9063)$$

$$\theta = 39.8°$$

Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8° + 15.0° = 54.8° \measuredangle^{\phi} \qquad \textit{Ans.}$$

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The ring shown in Fig. 2–13*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. If it is required that the resultant force have a magnitude of 1 kN and be directed vertically downward, determine (a) the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ provided $\theta = 30°$, and (b) the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ if F_2 is to be a minimum.

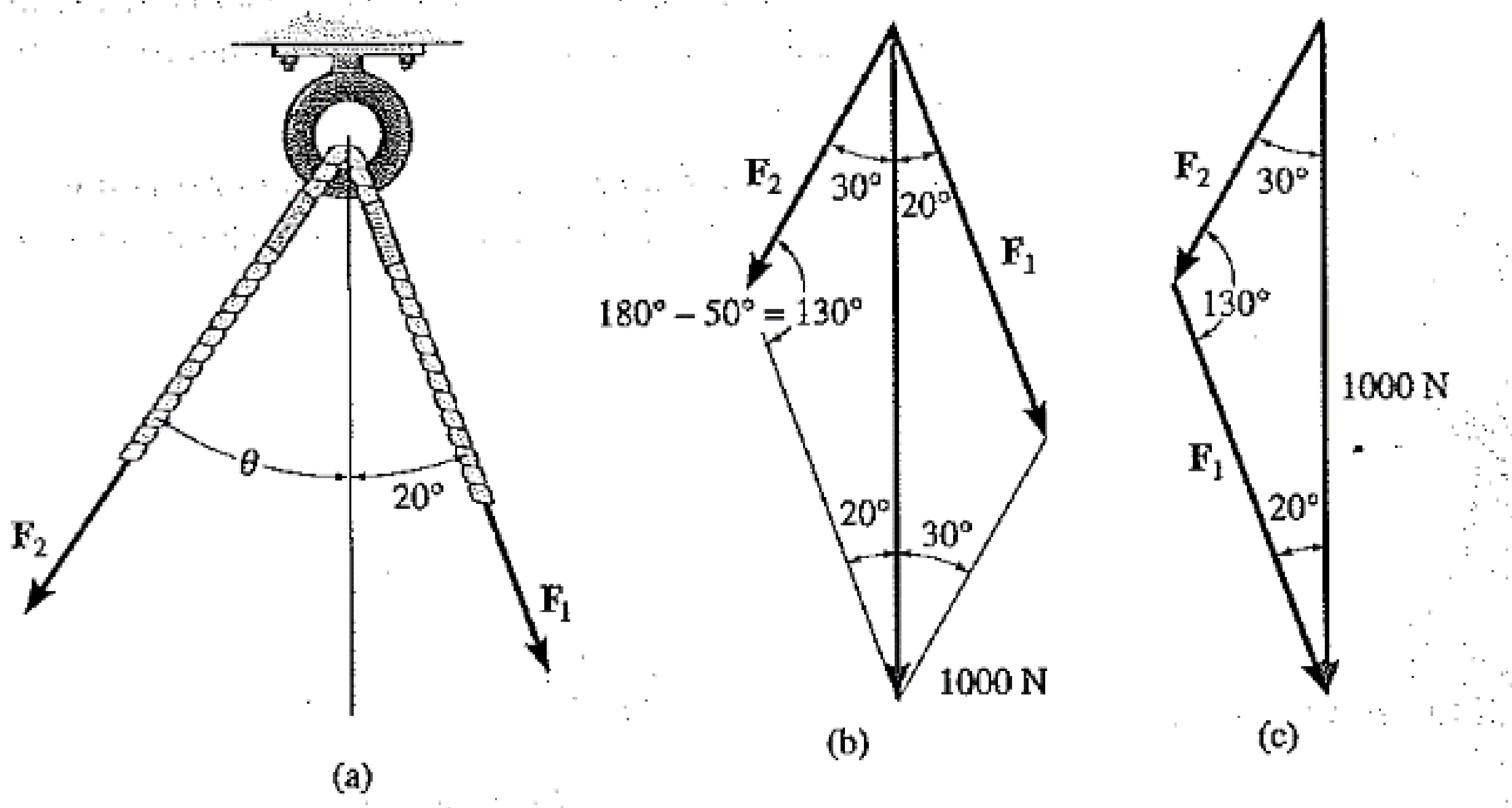

Fig. 2–13

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Solution

Part (a). A sketch of the vector addition according to the parallelogram law is shown in Fig. 2–13*b*. From the vector triangle constructed in Fig. 2–13*c*, the unknown magnitudes F_1 and F_2 are determined by using the law of sines:

$$\frac{F_1}{\sin 30^\circ} = \frac{1000\text{ N}}{\sin 130^\circ}$$

$$F_1 = 653\text{ N} \qquad \textit{Ans.}$$

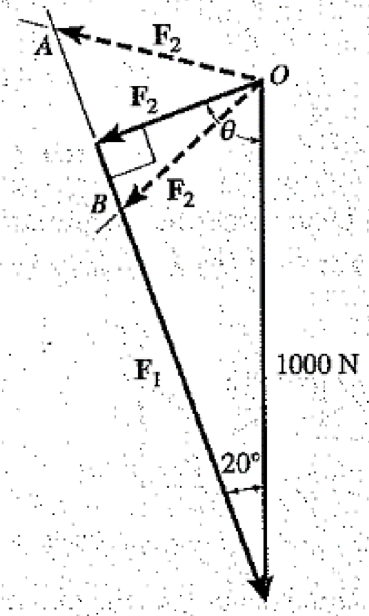
$$\frac{F_2}{\sin 20^\circ} = \frac{1000\text{ N}}{\sin 130^\circ}$$

$$F_2 = 446\text{ N} \qquad \textit{Ans.}$$

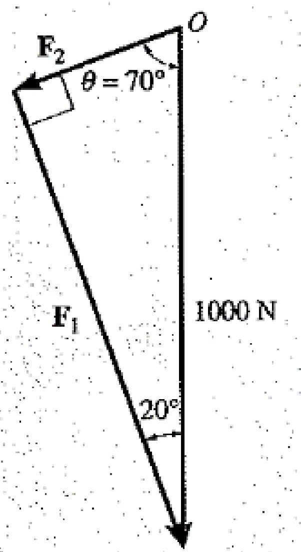
Part (b). If θ is not specified, then by the vector triangle, Fig. 2–13*d*, $\mathbf{F}_2$ may be added to $\mathbf{F}_1$ in various ways to yield the resultant 1000-N force. In particular, the *minimum* length or magnitude of $\mathbf{F}_2$ will occur when its line of action is *perpendicular to* $\mathbf{F}_1$. Any other direction, such as *OA* or *OB*, yields a larger value for F_2. Hence, when $\theta = 90^\circ - 20^\circ = 70^\circ$, F_2 is minimum. From the triangle shown in Fig. 2–13*e*, it is seen that

$$F_1 = 1000 \sin 70^\circ\text{N} = 940\text{ N} \qquad \textit{Ans.}$$

$$F_2 = 1000 \cos 70^\circ\text{N} = 342\text{ N} \qquad \textit{Ans.}$$



(d)



(e)

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The link in Fig. 2–18*a* is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and orientation of the resultant force.

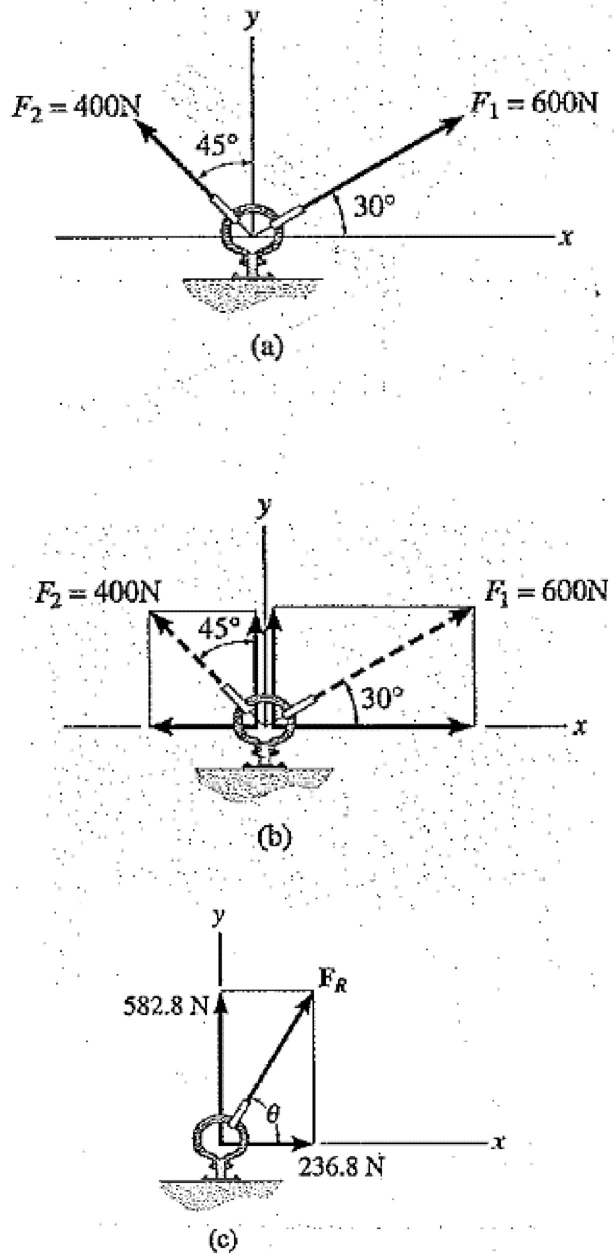

Fig. 2–18

Solution I

Scalar Notation. This problem can be solved by using the parallelogram law; however, here we will resolve each force into its *x* and *y* components, Fig. 2–18*b*, and sum these components algebraically. Indicating the "positive" sense of the *x* and *y* force components alongside each equation, we have

$$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N}$$
$$= 236.8 \text{ N} \rightarrow$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N}$$
$$= 582.8 \text{ N} \uparrow$$

The resultant force, shown in Fig. 2–18*c*, has a *magnitude* of

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2}$$
$$= 629 \text{ N} \qquad \textit{Ans.}$$

From the vector addition, Fig. 2–18*c*, the direction angle θ is

$$\theta = \tan^{-1}\left(\frac{582.8\text{N}}{236.8\text{N}}\right) = 67.9^\circ \qquad \textit{Ans.}$$