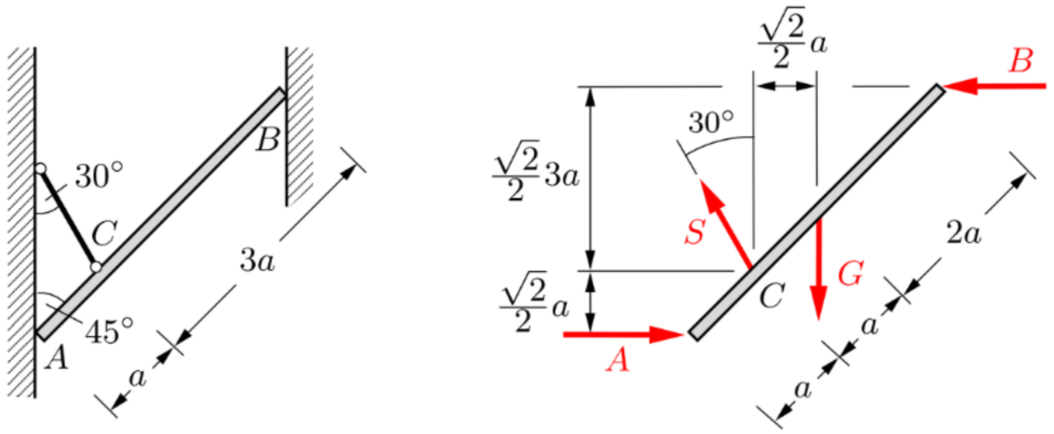
Example 3.4 A homogeneous beam (length $4\,a$, weight G) is suspended at C by a rope. The beam touches the smooth vertical walls at A and B (Fig. 3.16a).

Find the force in the rope and the contact forces at A and B.



a b

Fig. 3.16

Solution We isolate the beam by cutting the rope and removing the two walls. The free-body diagram (Fig. 3.16b) shows the contact forces A and B acting perpendicularly to the planes of contact (smooth walls), the internal force S in the rope and the weight G (acting at the center of the beam, compare Chapter 4). To formulate the sum of the moments, point C is chosen as reference point; the lever arms of the forces follow from simple geometrical relations. The equilibrium conditions then are given by

$$\uparrow: \quad S\cos 30^\circ - G = 0\,,$$

$$\rightarrow: \quad A - B - S\sin 30^\circ = 0\,,$$

$$\overset{\curvearrowleft}{C}: \quad \frac{\sqrt{2}}{2}\,a\,A - \frac{\sqrt{2}}{2}\,a\,G + \frac{\sqrt{2}}{2}\,3\,a\,B = 0\,.$$

With $\cos 30^\circ = \sqrt{3}/2$, $\sin 30^\circ = 1/2$ the three unknown forces are obtained as

$$\underline{\underline{S = \frac{2\sqrt{3}}{3}\,G}}\,, \quad \underline{\underline{A = \frac{1+\sqrt{3}}{4}\,G}}\,, \quad \underline{\underline{B = \frac{3-\sqrt{3}}{12}\,G}}\,.$$

Example 3.6 A lever (length l) that is subjected to a vertical force F (Fig. 3.18a) exerts a contact force on a circular cylinder (radius r, weight G). The weight of the lever may be neglected. All surfaces are smooth.

Determine the contact force between the cylinder and the floor if the height h of the step is equal to the radius r of the cylinder.

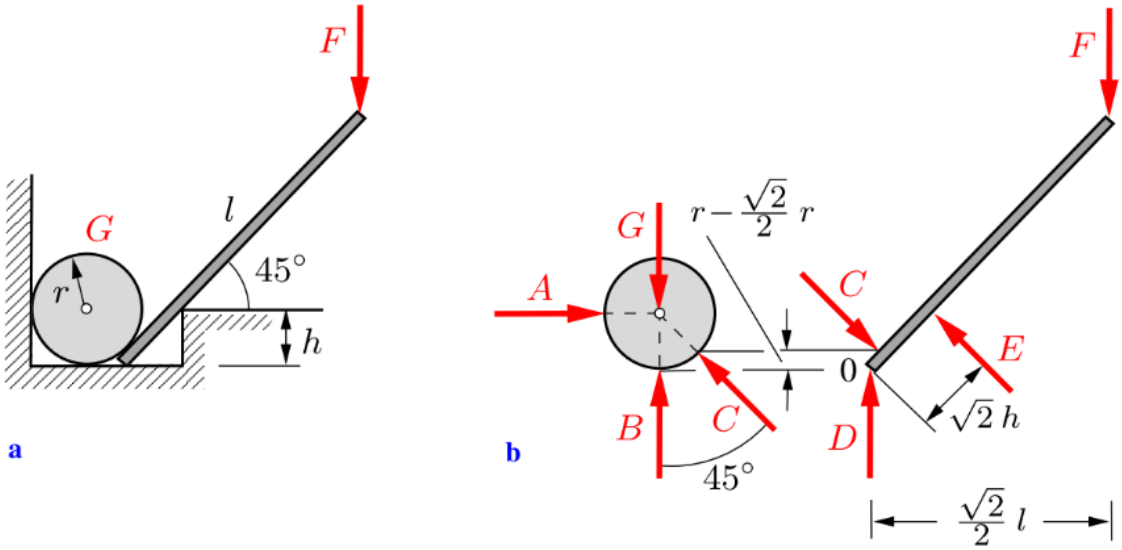


Fig. 3.18

Solution The cylinder and the lever are isolated and the contact forces A to E, which are perpendicular to the planes of contact at the respective points of contact, are introduced (Fig. 3.18b). Note that the floor and the lever represent the planes of contact at D and E, respectively. The equilibrium conditions for the lever are given by

$$\rightarrow: \quad \frac{\sqrt{2}}{2}\,C - \frac{\sqrt{2}}{2}\,E = 0\,,$$

$$\uparrow: \quad D - \frac{\sqrt{2}}{2}\,C + \frac{\sqrt{2}}{2}\,E - F = 0\,,$$

$$\overset{\curvearrowleft}{0}: \quad \sqrt{2}\,r\Big(1 - \frac{\sqrt{2}}{2}\Big)C - \sqrt{2}\,h\,E + \frac{\sqrt{2}}{2}\,l\,F = 0$$

and equilibrium at the cylinder (concurrent forces) requires

$$\rightarrow: \quad A - \frac{\sqrt{2}}{2}\,C = 0\,,$$

$$\uparrow: \quad B + \frac{\sqrt{2}}{2}\,C - G = 0\,.$$

These are five equations for the five unknown forces A to E. With $h = r$, they yield the contact force at point B:

$$\underline{\underline{B = G - \frac{l}{2\,r}\,F}}\,.$$

In the case of $F = (2\,r/l)G$, contact force B vanishes. For larger values of F there is no equilibrium: the cylinder will be lifted.

Example 3.1 A disc is subjected to four forces as shown in Fig. 3.11a. The forces have the given magnitudes F or $2\,F$, respectively.

Determine the magnitude and direction of the resultant and the location of its line of action.

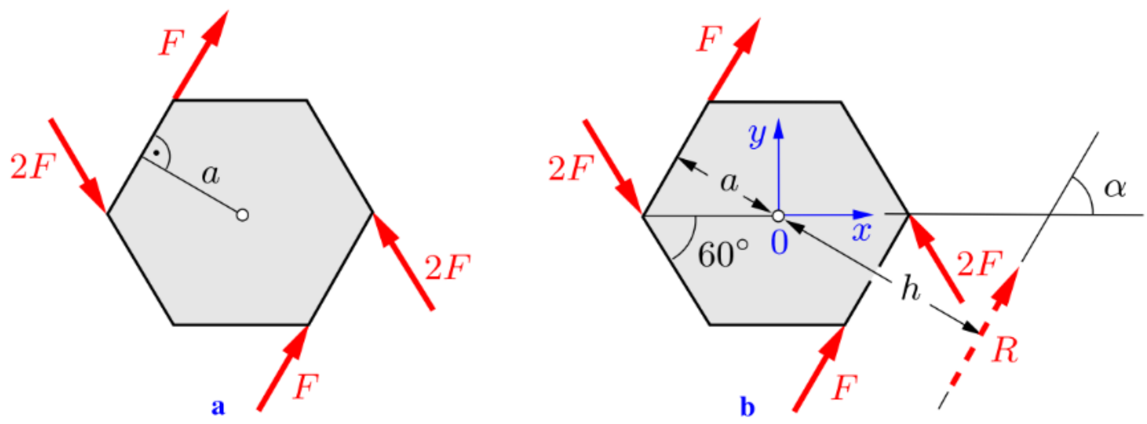


Fig. 3.11

Solution We choose a coordinate system x, y (Fig. 3.11b), and its origin 0 is taken as the point of reference. According to the sign convention, positive moments tend to rotate the disk counterclockwise ($\curvearrowleft$). Thus, from (3.9) we obtain

$$\begin{aligned}
R_x &= \sum F_{ix} = 2\,F\cos 60^\circ + F\cos 60^\circ \\
&\qquad + F\cos 60^\circ - 2\,F\cos 60^\circ = F\,, \\
R_y &= \sum F_{iy} = -\,2\,F\sin 60^\circ + F\sin 60^\circ \\
&\qquad + F\sin 60^\circ + 2\,F\sin 60^\circ = \sqrt{3}\,F\,, \\
M_R^{(0)} &= \sum M_i^{(0)} = 2\,a\,F + a\,F + 2\,a\,F - a\,F = 4\,a\,F\,,
\end{aligned}$$

which yield (see (3.10))

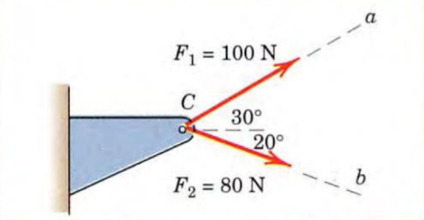
$$\underline{\underline{R}} = \sqrt{R_x^2 + R_y^2} = \underline{\underline{2\,F}}\,, \quad \tan\alpha = \frac{R_y}{R_x} = \sqrt{3} \quad \to \quad \underline{\underline{\alpha = 60^\circ}}\,.$$

The perpendicular distance of the resultant from point 0 follows from (3.11):

$$\underline{\underline{h}} = \frac{M_R^{(0)}}{R} = \frac{4\,a\,F}{2\,F} = \underline{\underline{2\,a}}\,.$$

Sample Problem 2/4

Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bracket as shown. Determine the projection F_b of their resultant $\mathbf{R}$ onto the b-axis.

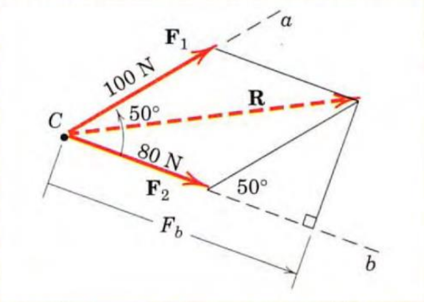


Solution. The parallelogram addition of $\mathbf{F}_1$ and $\mathbf{F}_2$ is shown in the figure. Using the law of cosines gives us

$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130^\circ \qquad R = 163.4 \text{ N}$$

The figure also shows the orthogonal projection $\mathbf{F}_b$ of $\mathbf{R}$ onto the b-axis. Its length is

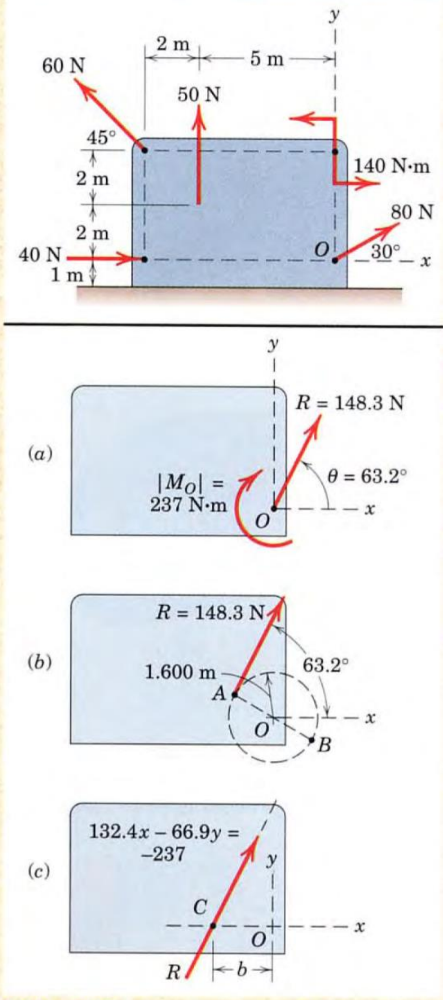
$$F_b = 80 + 100 \cos 50^\circ = 144.3 \text{ N} \qquad \textit{Ans.}$$



Note that the components of a vector are in general not equal to the projections of the vector onto the same axes. If the a-axis had been perpendicular to the b-axis, then the projections and components of $\mathbf{R}$ would have been equal.

Sample Problem 2/8

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution. Point O is selected as a convenient reference point for the force–couple system that is to represent the given system.

$[R_x = \Sigma F_x]$ $\qquad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9 \text{ N}$

$[R_y = \Sigma F_y]$ $\qquad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4 \text{ N}$

$[R = \sqrt{R_x^2 + R_y^2}]$ $\qquad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$ *Ans.*

$\left[\theta = \tan^{-1}\dfrac{R_y}{R_x}\right]$ $\qquad \theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$ *Ans.*

① $[M_O = \Sigma(Fd)]$ $\qquad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$
$\qquad = -237 \text{ N·m}$

The force–couple system consisting of **R** and M_O is shown in Fig. *a*.

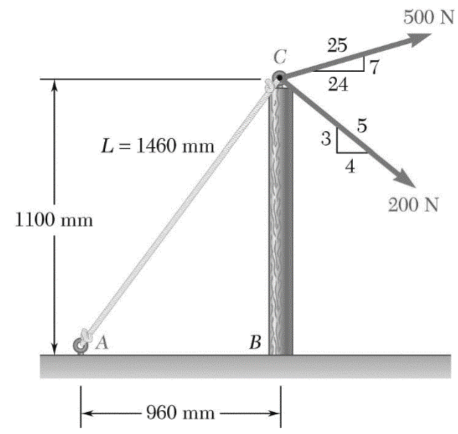
We now determine the final line of action of **R** such that **R** alone represents the original system.

$[Rd = |M_O|]$ $\qquad 148.3d = 237 \qquad d = 1.600 \text{ m}$ *Ans.*

Hence, the resultant **R** may be applied at any point on the line which makes a 63.2° angle with the x-axis and is tangent at point A to a circle of 1.6-m radius with center O, as shown in part b of the figure. We apply the equation $Rd = M_O$ in an absolute-value sense (ignoring any sign of M_O) and let the physics of the situation, as depicted in Fig. a, dictate the final placement of **R**. Had M_O been counterclockwise, the correct line of action of **R** would have been the tangent at point B.

The resultant **R** may also be located by determining its intercept distance b to point C on the x-axis, Fig. c. With R_x and R_y acting through point C, only R_y exerts a moment about O so that

$$R_y b = |M_O| \qquad \text{and} \qquad b = \frac{237}{132.4} = 1.792 \text{ m}$$

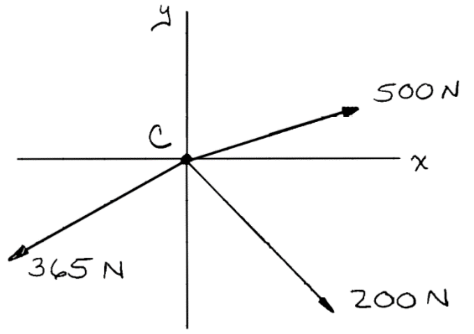


PROBLEM 2.36

Knowing that the tension in rope *AC* is 365 N, determine the resultant of the three forces exerted at point *C* of post *BC*.

SOLUTION

Determine force components:



Cable force *AC*: $F_x = -(365\text{ N})\dfrac{960}{1460} = -240\text{ N}$

$F_y = -(365\text{ N})\dfrac{1100}{1460} = -275\text{ N}$

500-N Force: $F_x = (500\text{ N})\dfrac{24}{25} = 480\text{ N}$

$F_y = (500\text{ N})\dfrac{7}{25} = 140\text{ N}$

200-N Force: $F_x = (200\text{ N})\dfrac{4}{5} = 160\text{ N}$

$F_y = -(200\text{ N})\dfrac{3}{5} = -120\text{ N}$

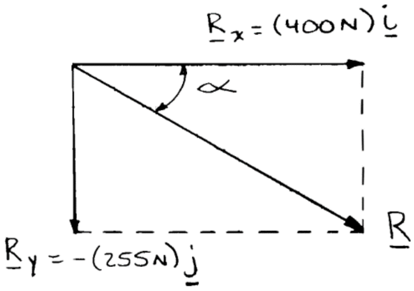
and $R_x = \Sigma F_x = -240\text{ N} + 480\text{ N} + 160\text{ N} = 400\text{ N}$

$R_y = \Sigma F_y = -275\text{ N} + 140\text{ N} - 120\text{ N} = -255\text{ N}$

$R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{(400\text{ N})^2 + (-255\text{ N})^2}$

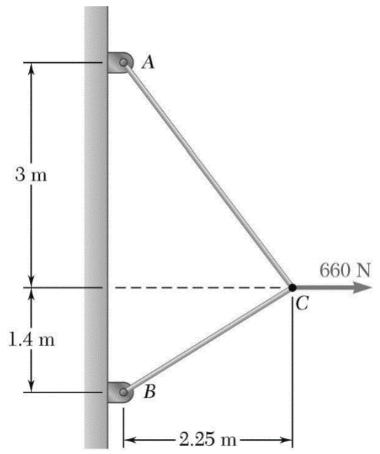
$= 474.37\text{ N}$



Further: $\tan\alpha = \dfrac{255}{400}$

$\alpha = 32.5°$

R = 474 N ⦪ 32.5° ◀



PROBLEM 2.44

Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

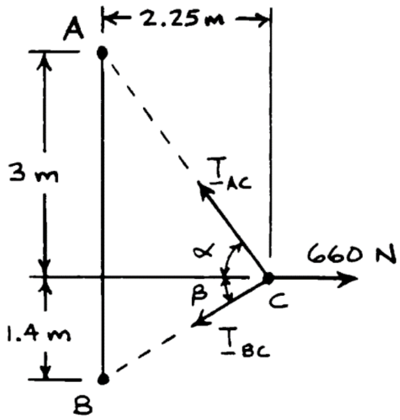
$$\tan\alpha = \frac{3}{2.25}$$

$$\alpha = 53.130°$$

$$\tan\beta = \frac{1.4}{2.25}$$

$$\beta = 31.891°$$

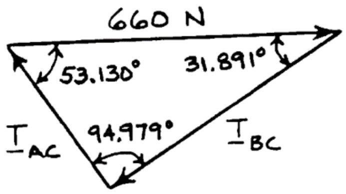
Free-Body Diagram



Law of sines:

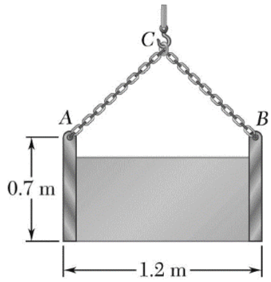
$$\frac{T_{AC}}{\sin 31.891°} = \frac{T_{BC}}{\sin 53.130°} = \frac{660\text{ N}}{\sin 94.979°}$$

Force-Triangle



(a) $$T_{AC} = \frac{660\text{ N}}{\sin 94.979°}(\sin 31.891°)$$ $T_{AC} = 350$ N ◀

(b) $$T_{BC} = \frac{660\text{ N}}{\sin 94.979°}(\sin 53.130°)$$ $T_{BC} = 530$ N ◀

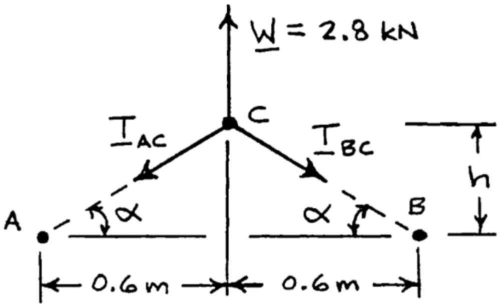


PROBLEM 2.62

A movable bin and its contents have a combined weight of 2.8 kN. Determine the shortest chain sling *ACB* that can be used to lift the loaded bin if the tension in the chain is not to exceed 5 kN.

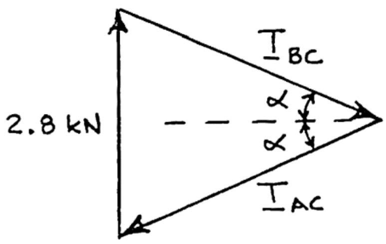
SOLUTION

Free-Body Diagram



$$\tan\alpha = \frac{h}{0.6\text{ m}} \qquad (1)$$

Isosceles Force Triangle



Law of sines: $\sin\alpha = \frac{\frac{1}{2}(2.8\text{ kN})}{T_{AC}}$

$$T_{AC} = 5\text{ kN}$$

$$\sin\alpha = \frac{\frac{1}{2}(2.8\text{ kN})}{5\text{ kN}}$$

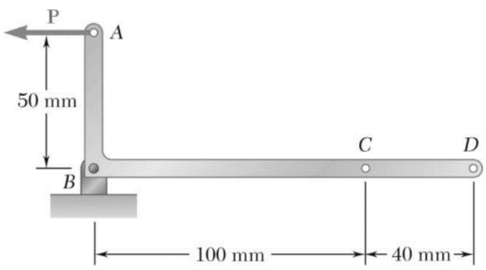
$$\alpha = 16.2602°$$

From Eq. (1): $\tan 16.2602° = \frac{h}{0.6\text{ m}}$ $\quad\therefore\quad h = 0.175000\text{ m}$

Half length of chain $= AC = \sqrt{(0.6\text{ m})^2 + (0.175\text{ m})^2}$

$$= 0.625\text{ m}$$

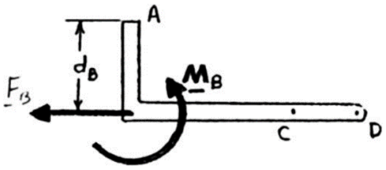
Total length: $= 2\times 0.625\text{ m}$ 1.250 m ◀



PROBLEM 3.85

The 80-N horizontal force **P** acts on a bell crank as shown. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two vertical forces at *C* and *D* that are equivalent to the couple found in part *a*.

SOLUTION



(*a*) Based on ΣF: $F_B = F = 80\text{ N}$ or $\mathbf{F}_B = 80.0\text{ N} \leftarrow$ ◀

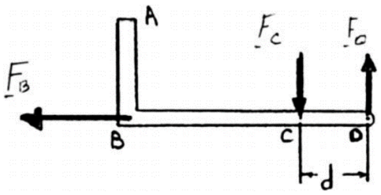
ΣM: $M_B = Fd_B$

$= 80\text{ N }(0.05\text{ m})$

$= 4.0000\text{ N}\cdot\text{m}$

or $\mathbf{M}_B = 4.00\text{ N}\cdot\text{m}$ ↻ ◀

(*b*) If the two vertical forces are to be equivalent to $\mathbf{M}_B$, they must be a couple. Further, the sense of the moment of this couple must be counterclockwise.



Then with F_C and F_D acting as shown,

ΣM: $M_D = F_C d$

$4.0000\text{ N}\cdot\text{m} = F_C(0.04\text{ m})$

$F_C = 100.000\text{ N}$ or $\mathbf{F}_C = 100.0\text{ N}\downarrow$ ◀

ΣF_y: $0 = F_D - F_C$

$F_D = 100.000\text{ N}$ or $\mathbf{F}_D = 100.0\text{ N}\uparrow$ ◀