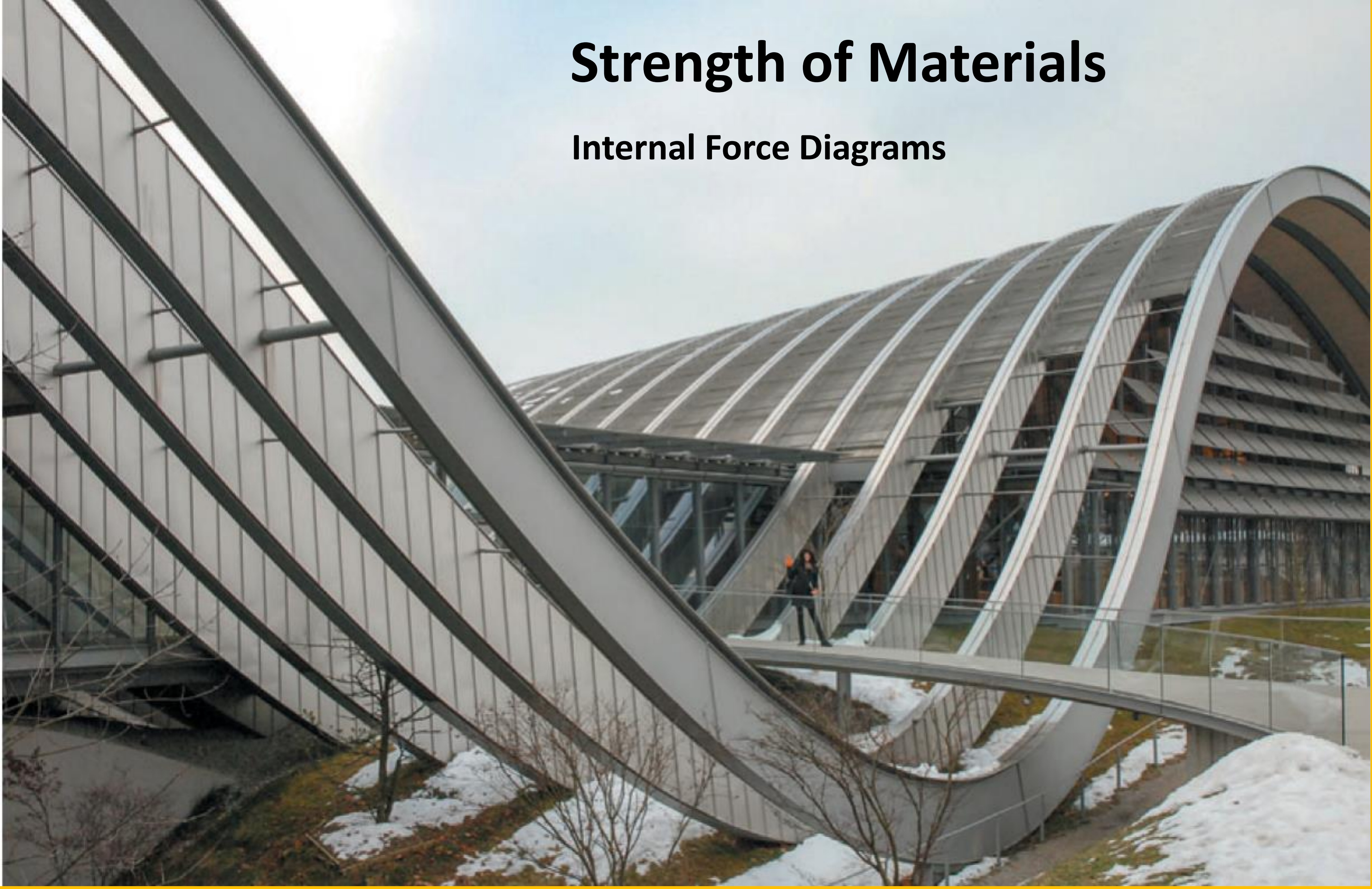
# Strength of Materials

## Internal Force Diagrams



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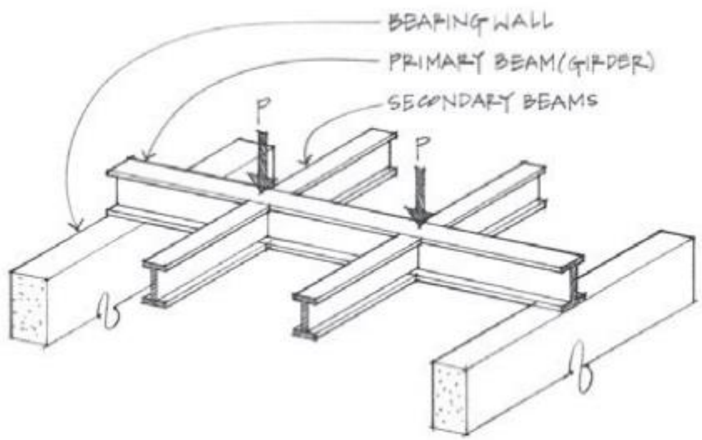
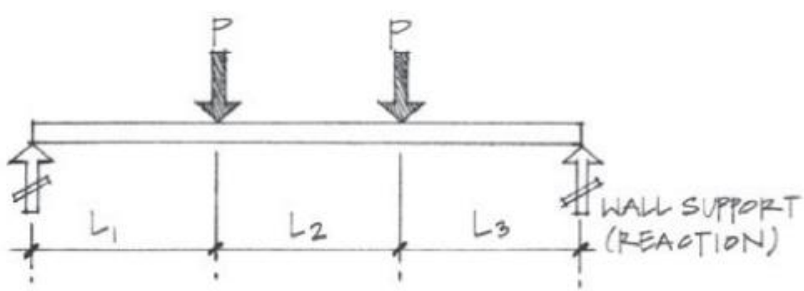
## CLASSIFICATION OF BEAMS AND LOADS

The design of a beam entails the determination of size, shape, and material based on the bending stress, shear stress, and deflection due to the applied loads (Figure 7.1).




*(a) Pictorial diagram of a loaded beam.*

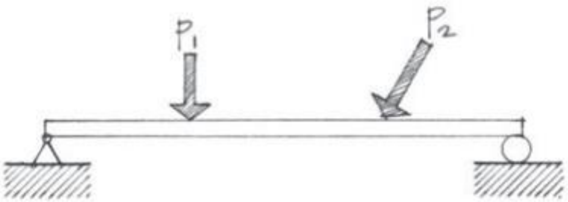



*(b) FBD of the beam.*

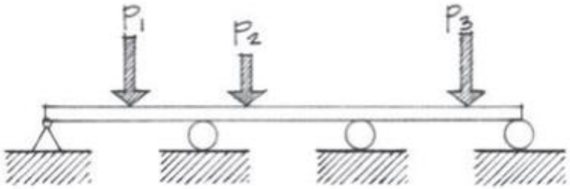
*Figure 7.1 Steel beam with loads and support reactions.*

# CLASSIFICATION OF BEAMS AND LOADS

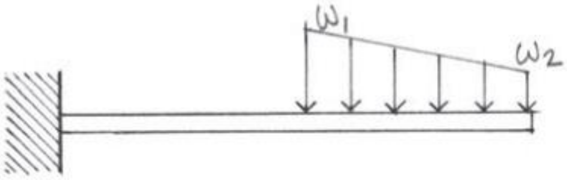
Beams are often classified according to their support conditions. Figure 7.2 illustrates six major beam classifications.




*(a) Simply supported: two supports.*



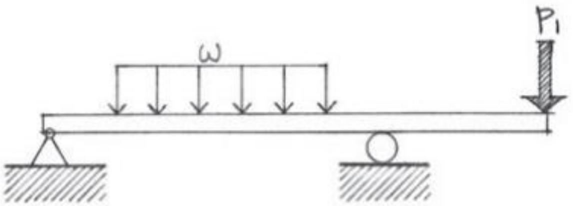

*(b) Continuous: three or more supports.*



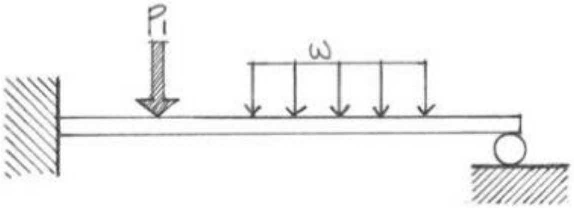

*(c) Cantilever: one end supported rigidly.*

# CLASSIFICATION OF BEAMS AND LOADS

*(d) Overhang: two supports—one or both supports not located at the end.*




*(e) Propped: two supports—one end is fixed.*




*(f) Restrained or fixed: both supports are fixed, allowing no rotation at the restrained ends.*

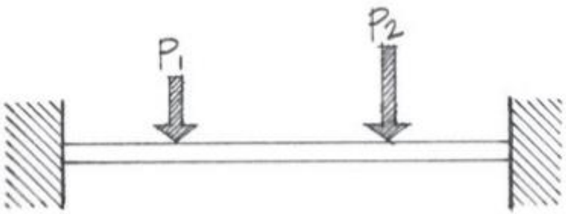
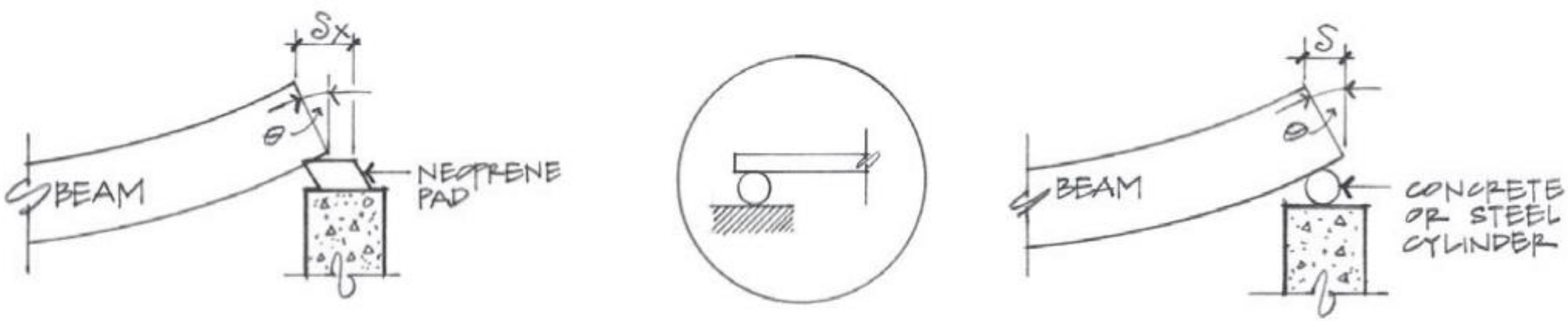



*Figure 7.2 Classification based on support conditions.*

# TYPES OF CONNECTIONS

Actual support and connection conditions for beams and columns are idealized as rollers, hinges (pins), or fixed. Figure 7.3 illustrates examples of common support/ connection conditions found in practice.



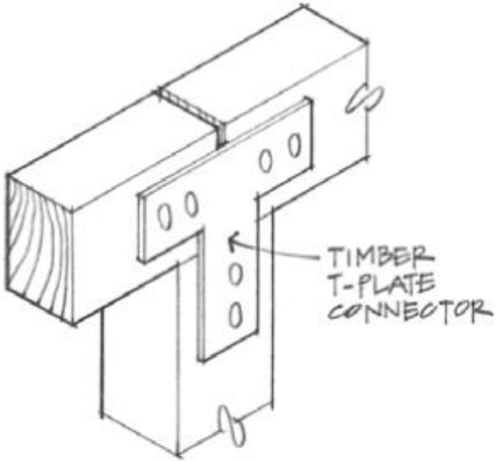

*(a) Beam supported by a neoprene pad.*

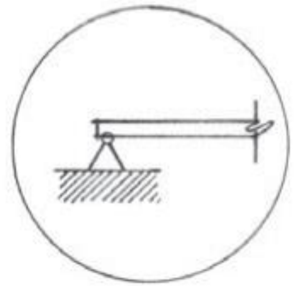
*(b) Beam supported by a concrete or steel cylinder.*

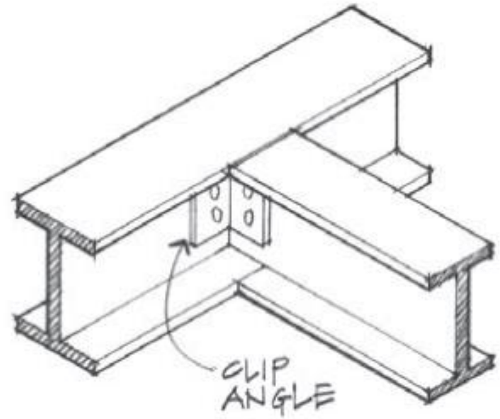
*Examples of roller supports (a, b). Horizontal displacement and rotation are permitted; may be due to loads or thermal conditions.*

*Examples of hinge or pin supports (c, d, e, f). Allows a certain amount of rotation at the connection.*

## TYPES OF CONNECTIONS

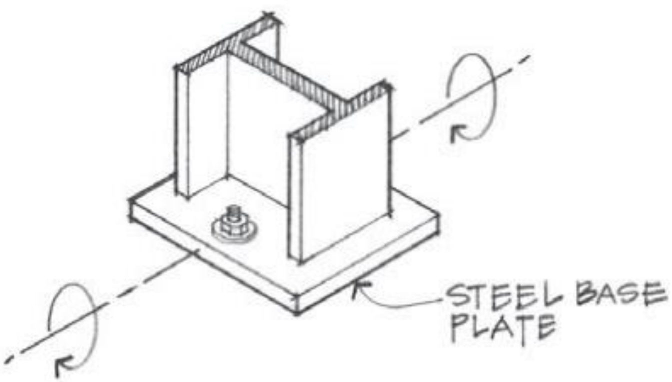


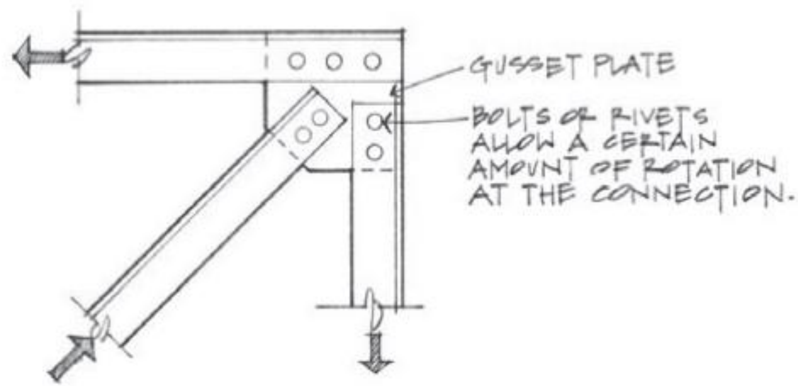






*(c) Timber beam-column connection with T-plate.*

*(d) Steel beam connected to a steel girder.*



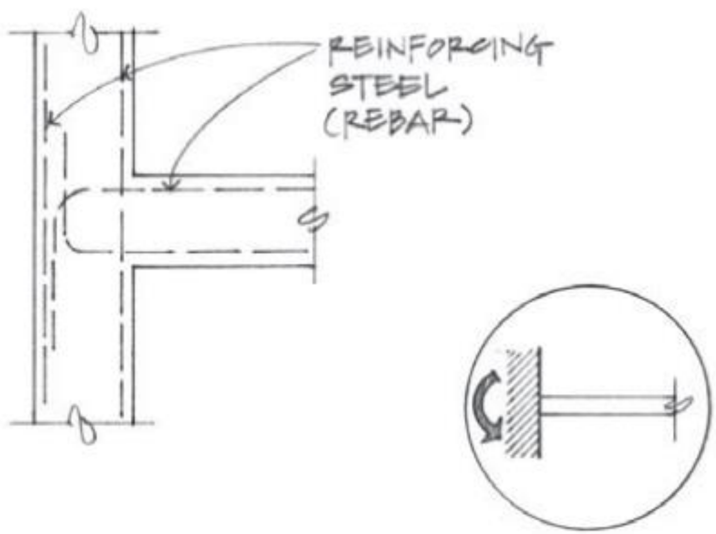




*(e) Typical pin-connected column base.*

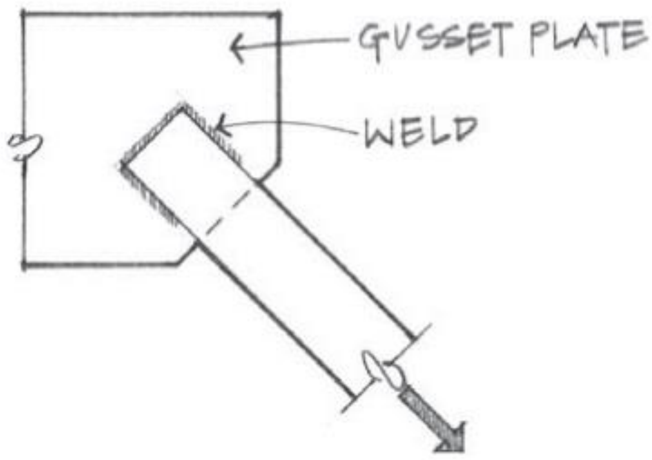
*(f) Truss joint—three steel angles with gusset plate.*

Figure 7.3 Classification based on connection types (continues on next page).

# TYPES OF CONNECTIONS

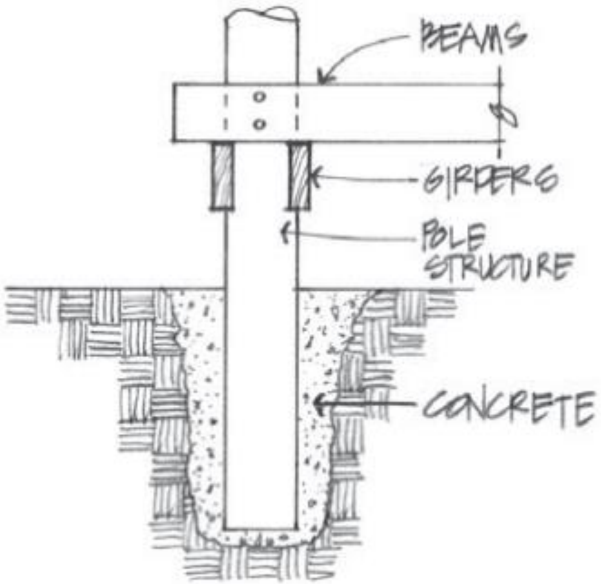



(g) *Reinforced concrete floor-wall connection.*

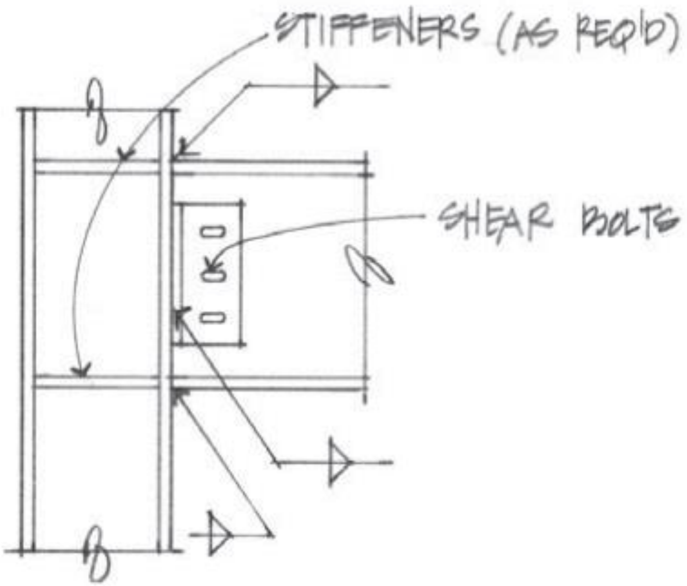



(h) *Steel strap welded to a gusset plate.*

# TYPES OF CONNECTIONS




*(i) Timber pole structure—embedded base.*

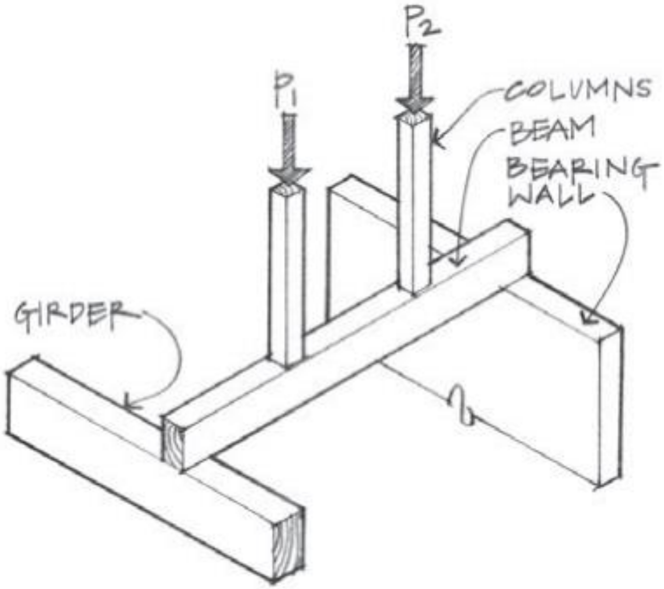


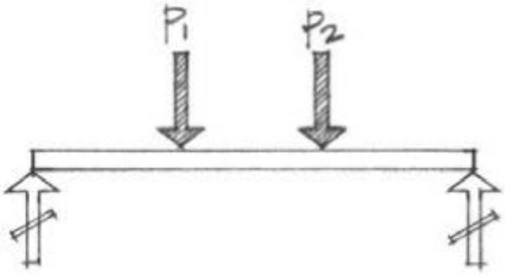

*(j) Beam-column moment connection.*

*Examples of fixed support (g, h, i, j). No rotation at the connection.*

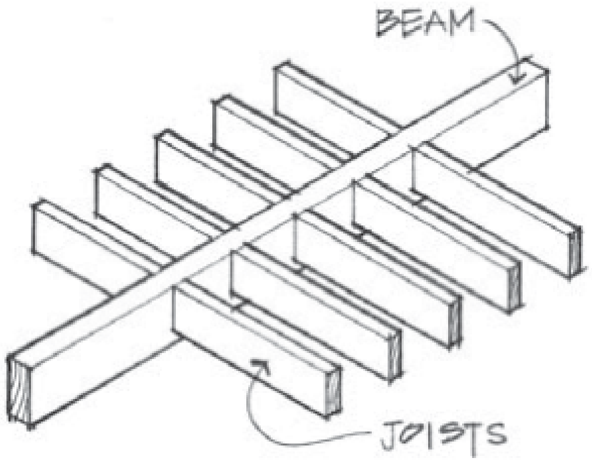
# TYPES OF LOADS

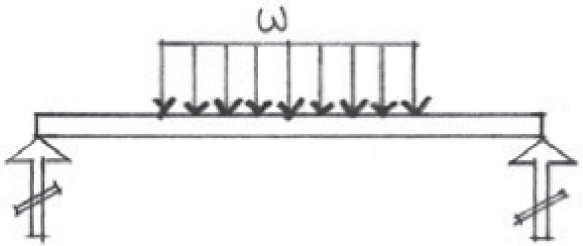
Figure 7.4 illustrates the four fundamental types of loads that can act on a beam.



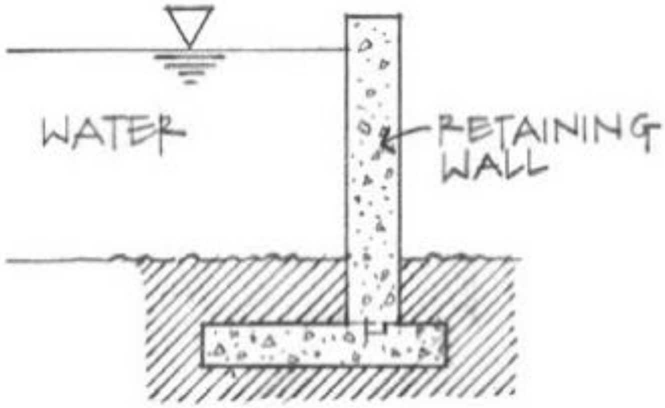

(a) Concentrated load.

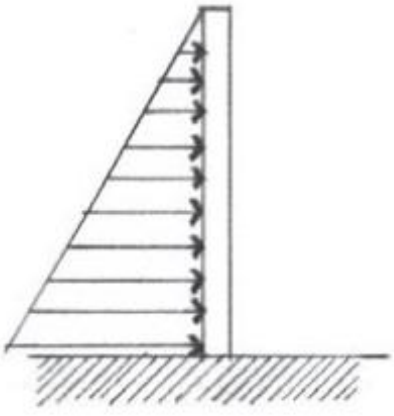


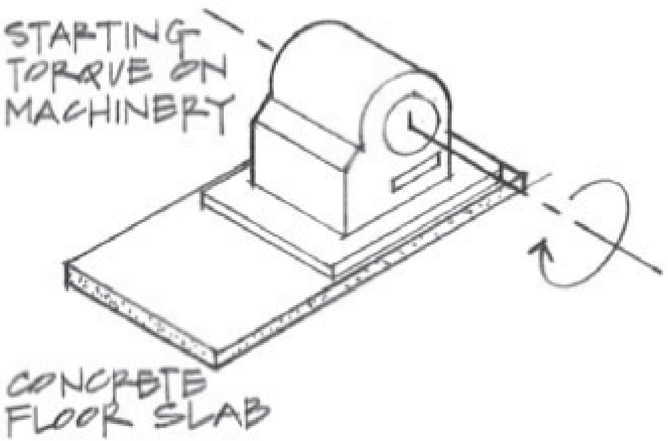

(b) Uniformly distributed load.

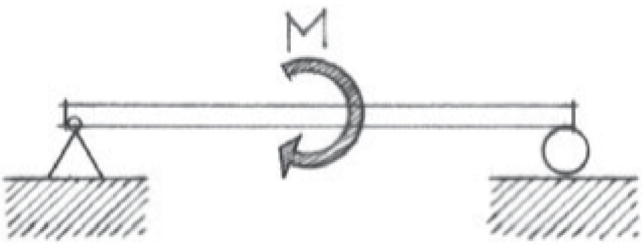
# TYPES OF LOADS



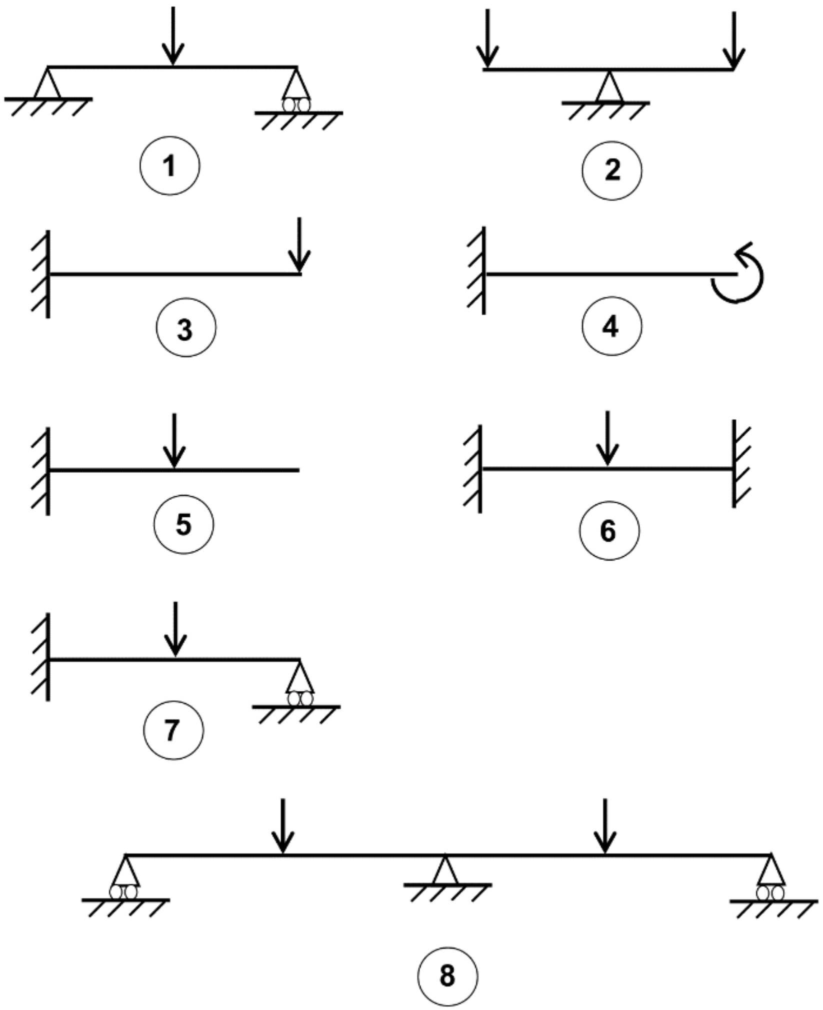
*(c) Nonuniformly distributed load.*



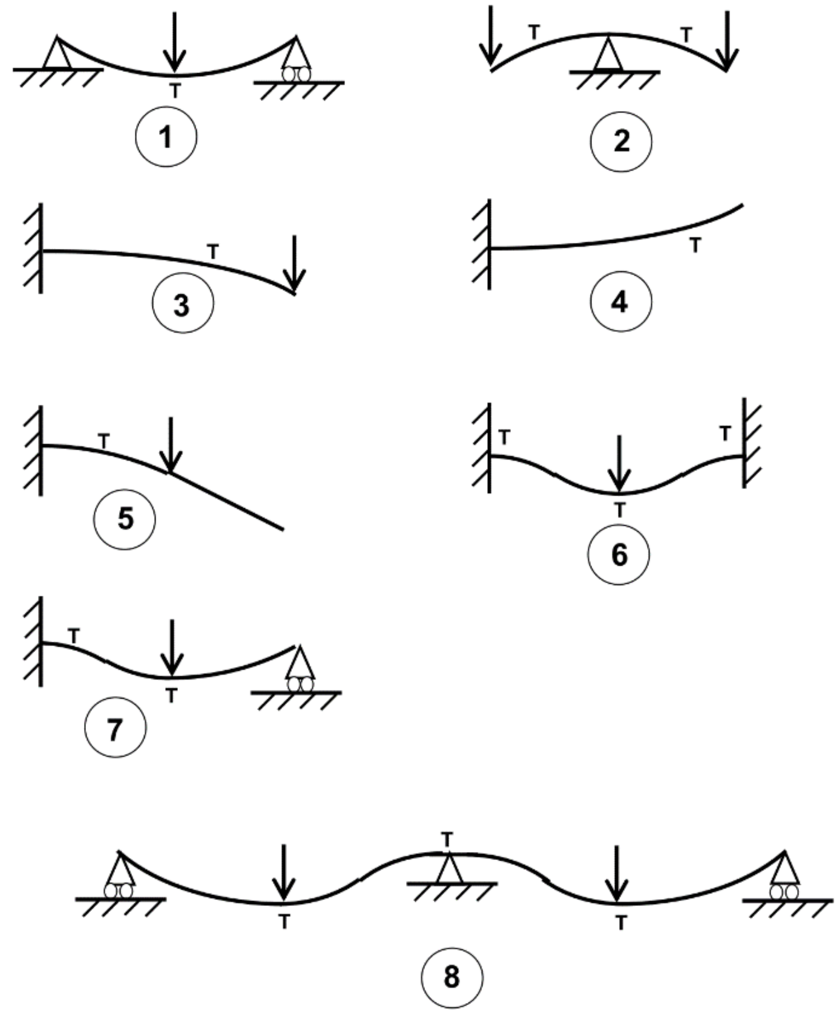




*(d) Pure moment.*

# DEFORMATION OF STRUCTURES

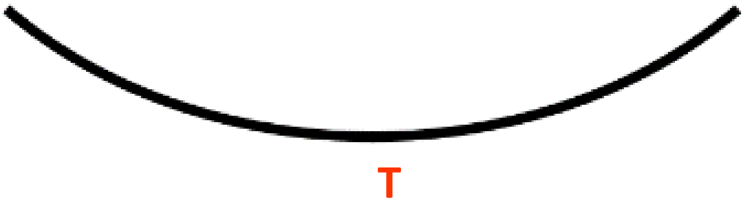



Point loaded beams



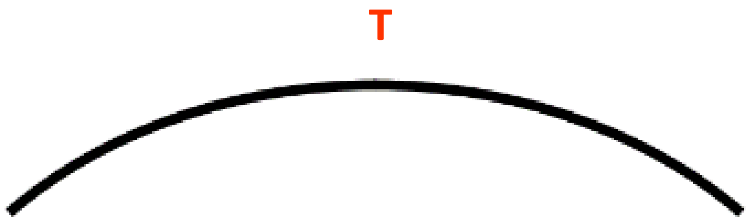

Deformations in beams due to point loads

## DEFORMATION OF STRUCTURES




(a) sagging

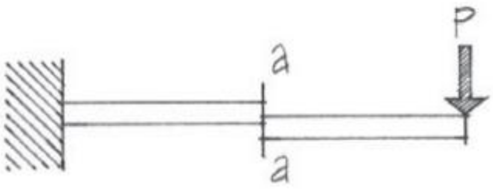
A downward deformation leads TENSION (T) stresses at the bottom of the beam element



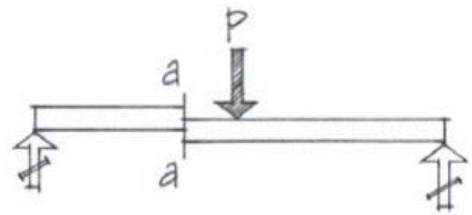

(b) hogging

An upward deformation leads TENSION (T) stresses at the top of the beam element

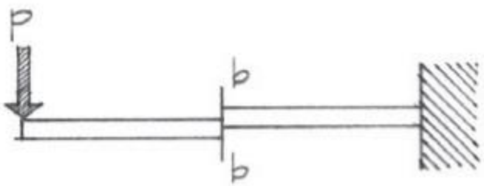
## SHEAR FORCE



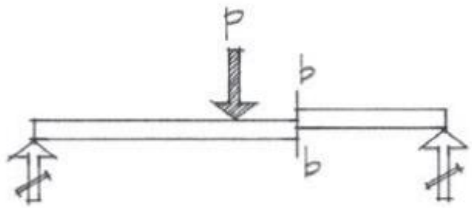

(+) Shear.




(+) Shear.




(–) Shear.




(–) Shear.

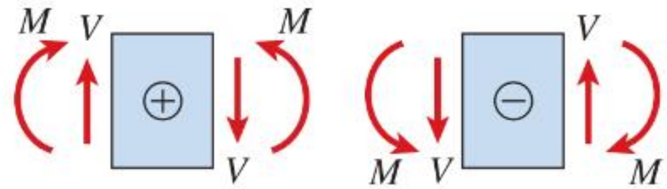



**FIG. 4-9** Sign conventions for shear force $V$ and bending moment $M$

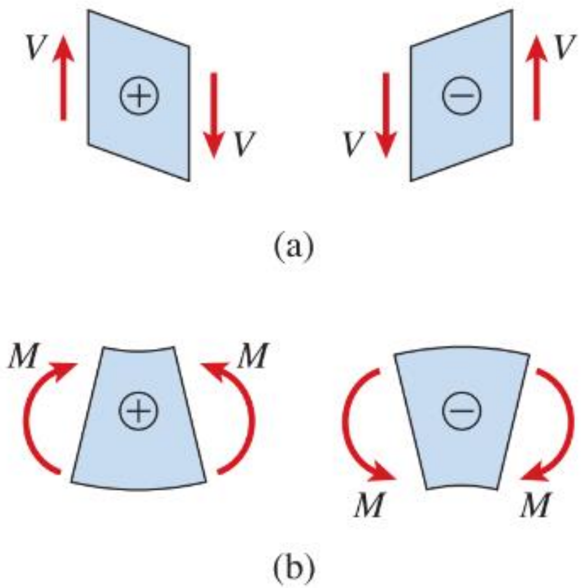
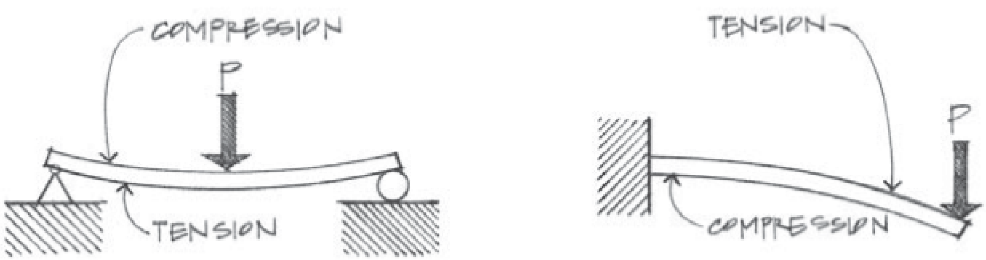



**FIG. 4-10** Deformations (highly exaggerated) of a beam element caused by (a) shear forces, and (b) bending moments

# BENDING MOMENT




(+) *Moment.* (–) *Moment.*


(+) *Moment.* (–) *Moment.*

*Figure 7.7 Sign convention for moment.*

*Positive moment* generates a curvature that tends to hold water (concave-upward curvature), whereas *negative moment* causes a curvature that sheds water (concave-downward curvature).

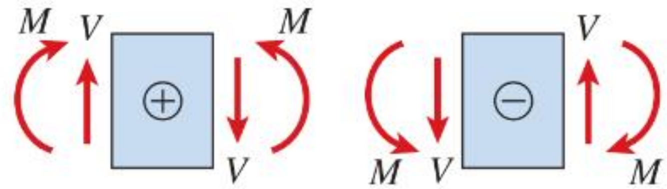
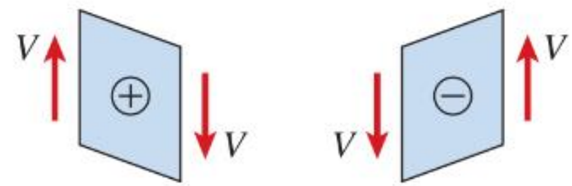
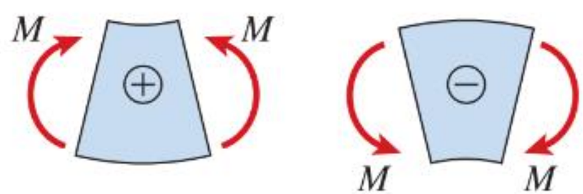



**FIG. 4-9** Sign conventions for shear force $V$ and bending moment $M$




(a)




(b)

**FIG. 4-10** Deformations (highly exaggerated) of a beam element caused by (a) shear forces, and (b) bending moments

## DEFORMATION DUE TO BENDING (T:Tension, C:Compression)

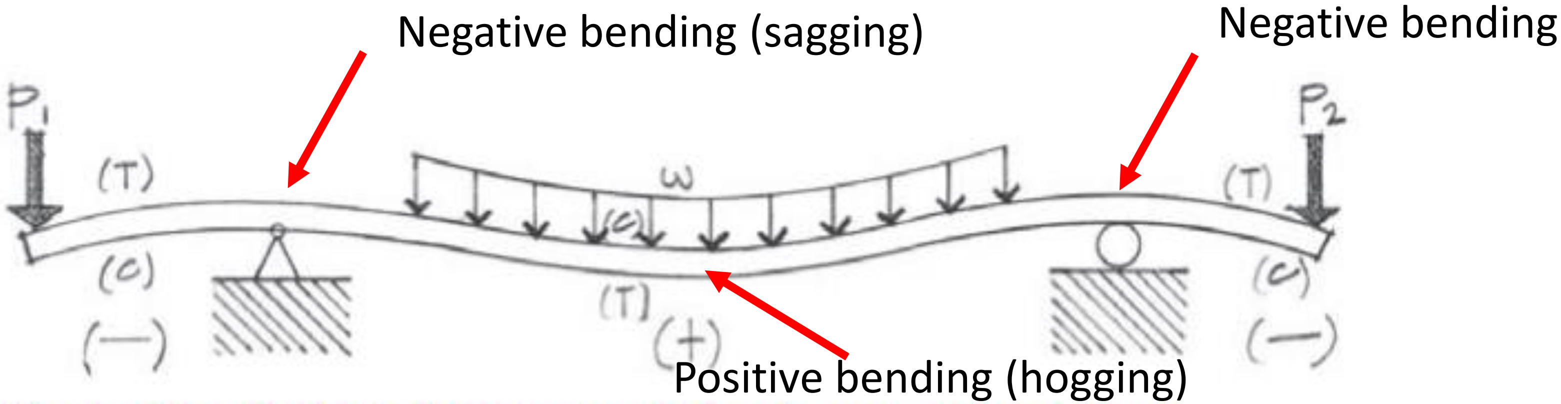
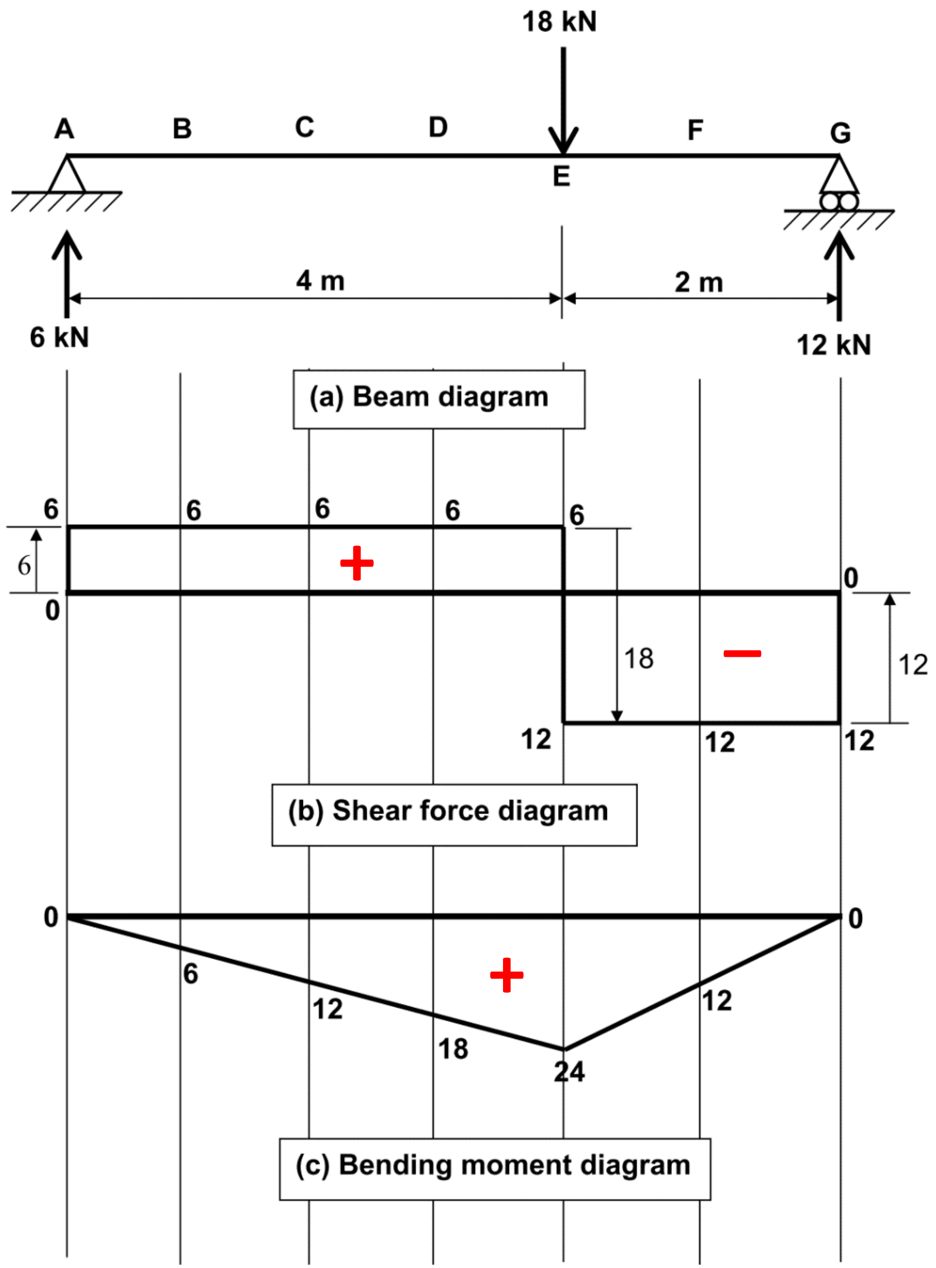



*Figure 7.8 Deflected shape due to loads on overhang beam.*

The overhang beam shown in Figure 7.8 exhibits a changing curvature that results in negative to positive to negative moments. The implication here is that the beam span includes one or more transverse sections where the bending moment is zero to accommodate the required sign change. Such a section, termed the *inflection point* or *point of inflection*, is almost always present in overhang and multiple-span beams.




( V )

( M )

**Fig. 16.8** Example 16.2: Shear force and bending moment diagrams.

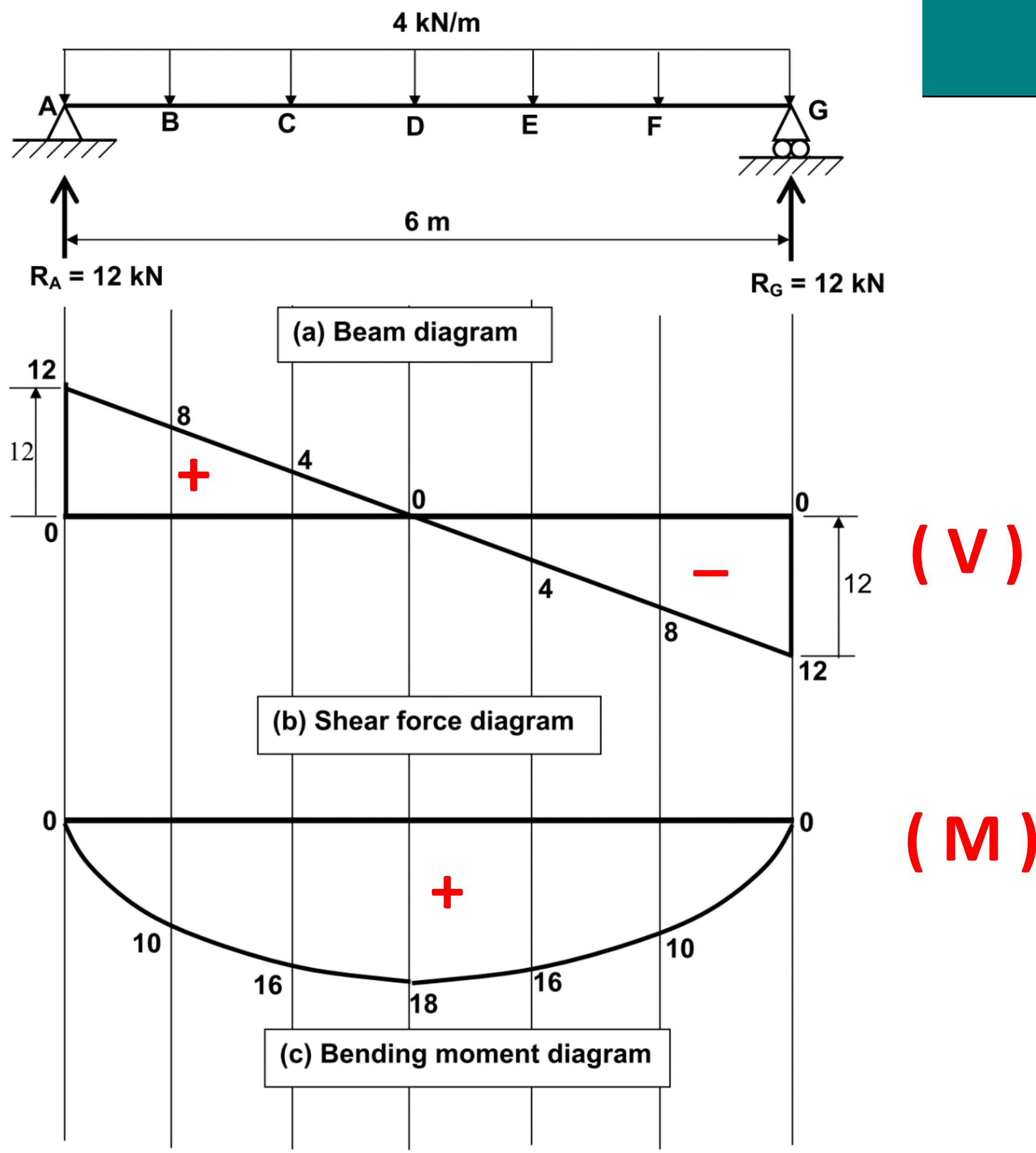
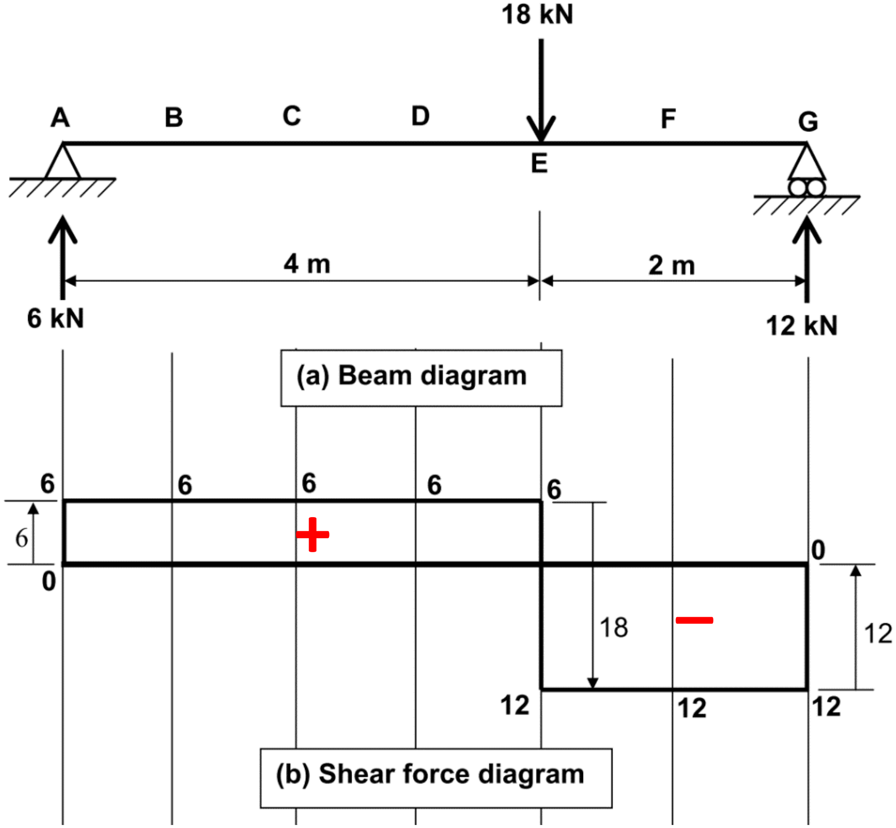



**Fig. 16.10** Example 16.3: Shear force and bending moment diagrams: uniformly distributed load example.

## Shear Force Diagram of the point loaded beam - Follow the arrows

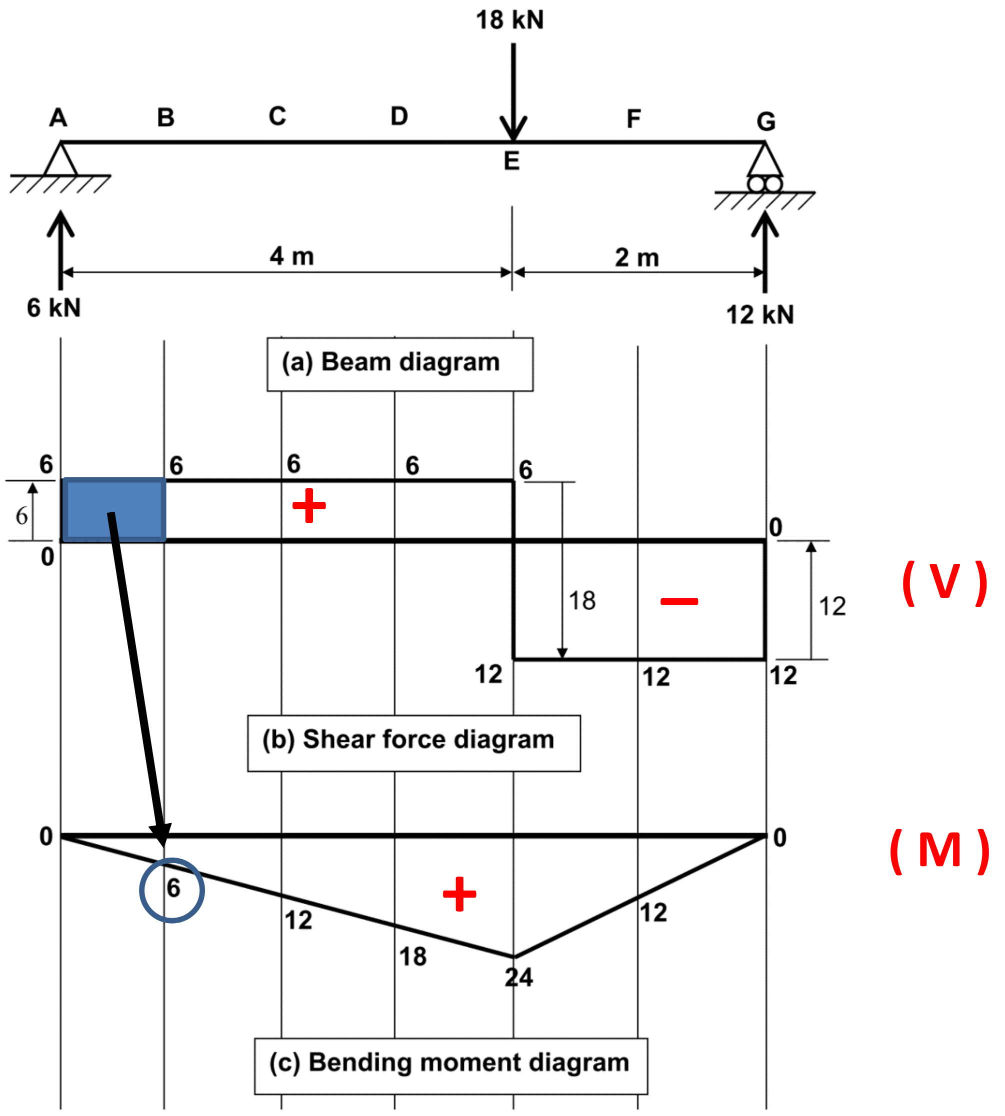
All we've done is follow the arrows. To summarise: if a force goes upwards (for example, the 6 kN reaction at A), then the shear force diagram goes up by that amount. On the other hand, if a force goes downwards (for example, the 18 kN force at E), then the shear force diagram jumps downwards at that point, again by the same amount.



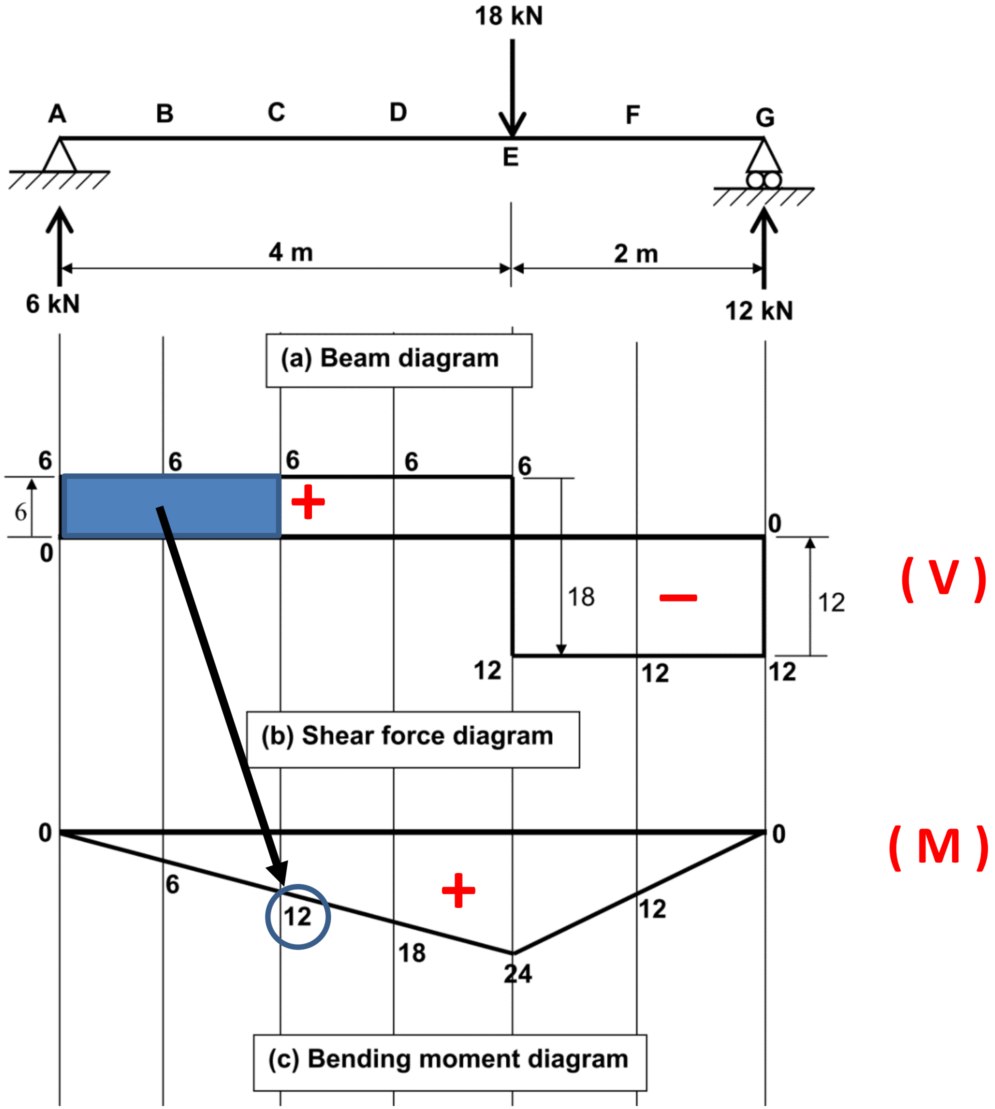

Note that:
Between point forces shear force diagram shows constant variation

**( V )**

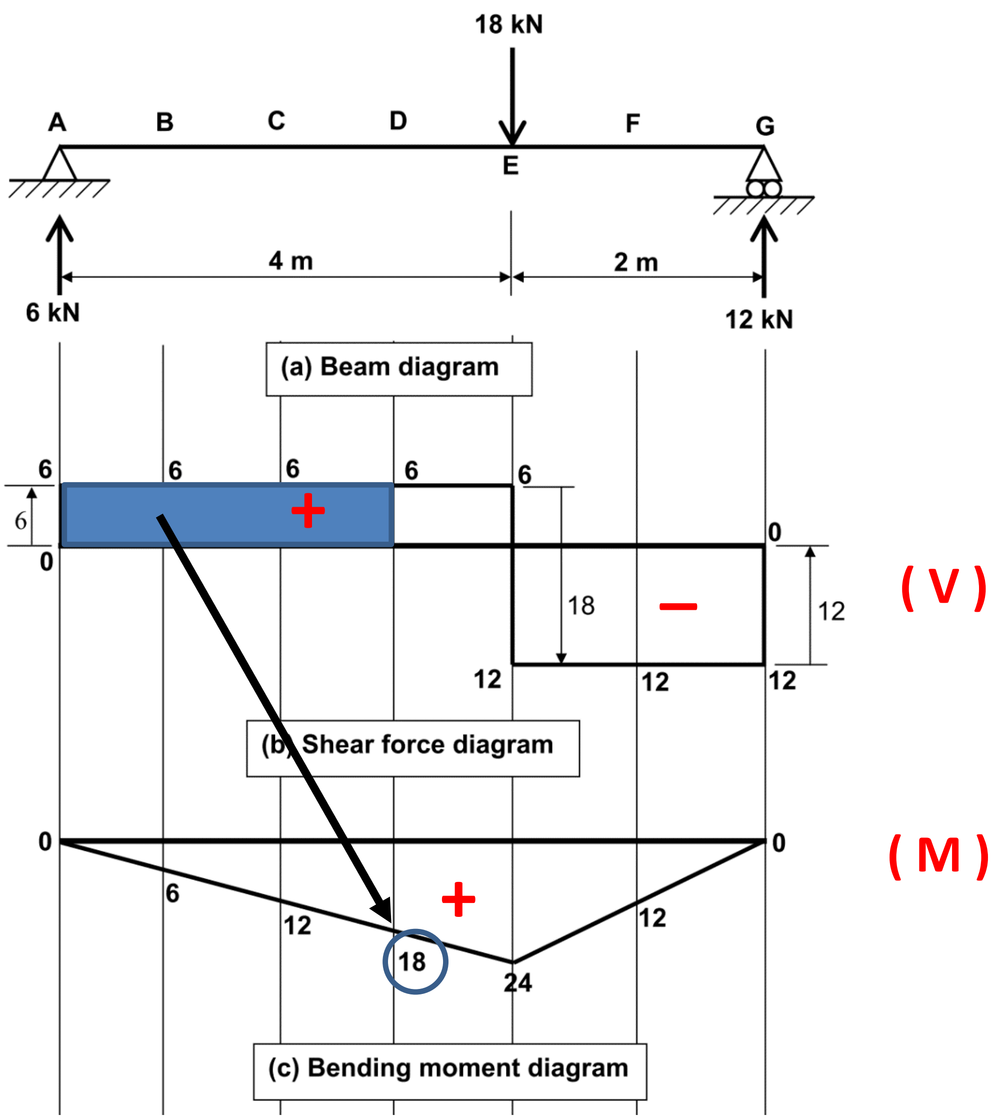
## Bending Moment Diagram – Area under shear force diagram



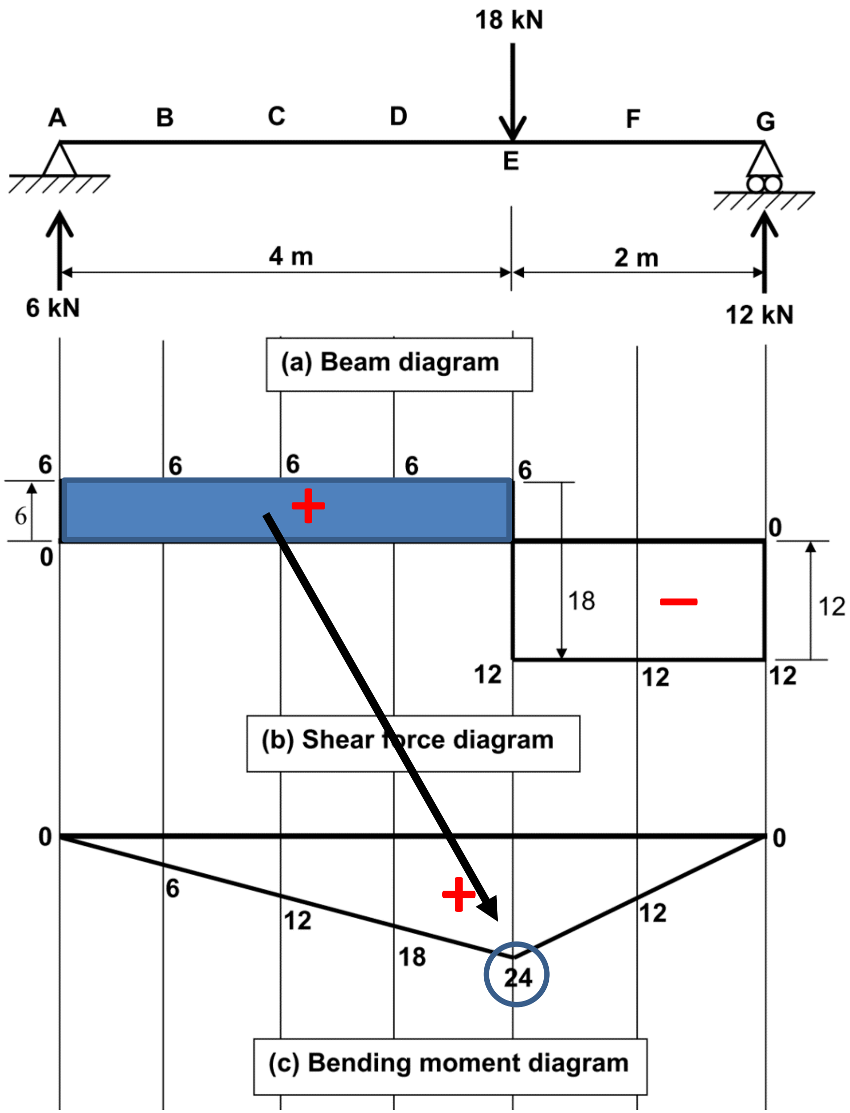
# Bending Moment Diagram – Area under shear force diagram



## Bending Moment Diagram – Area under shear force diagram



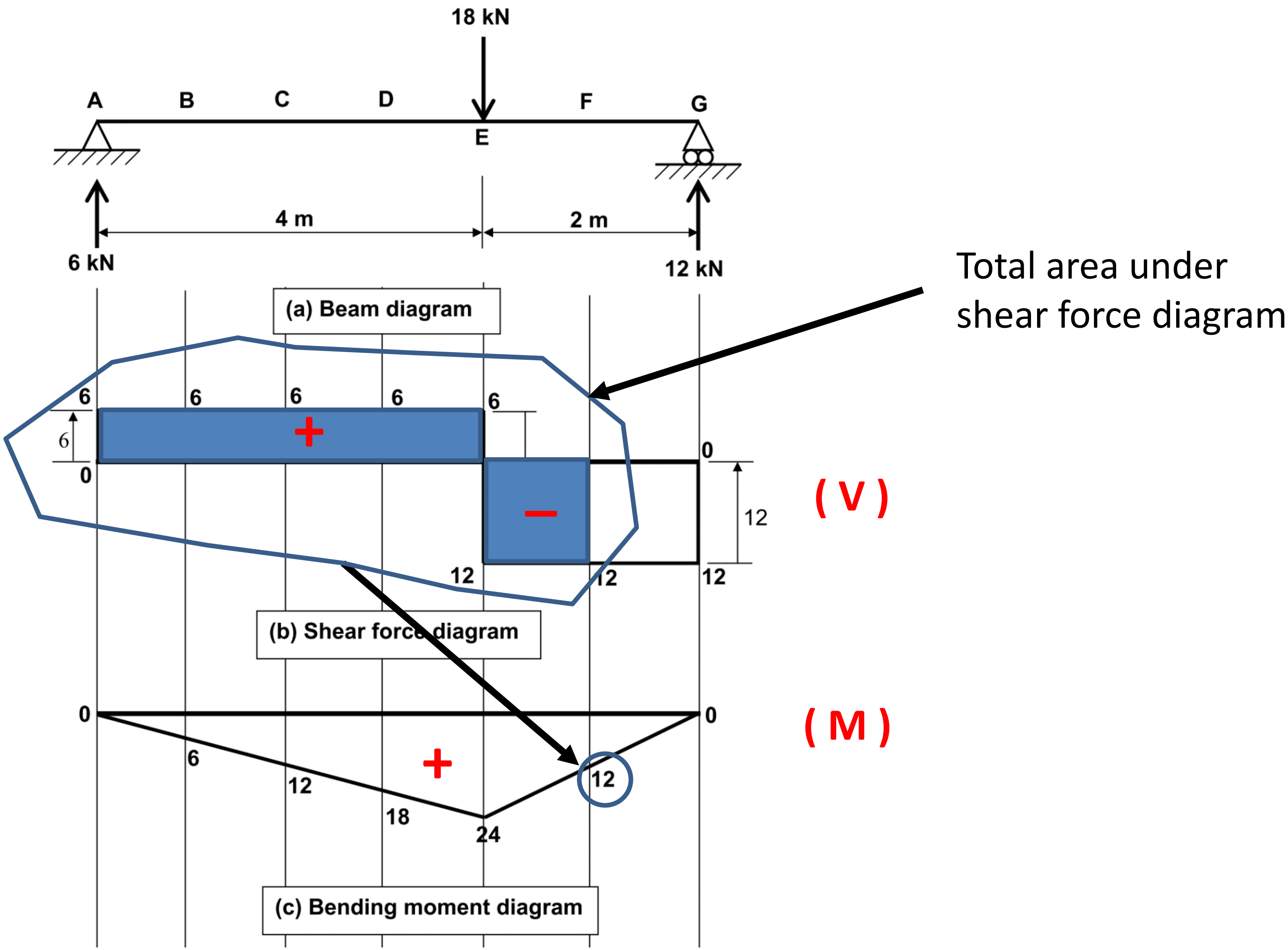
# Bending Moment Diagram – Area under shear force diagram



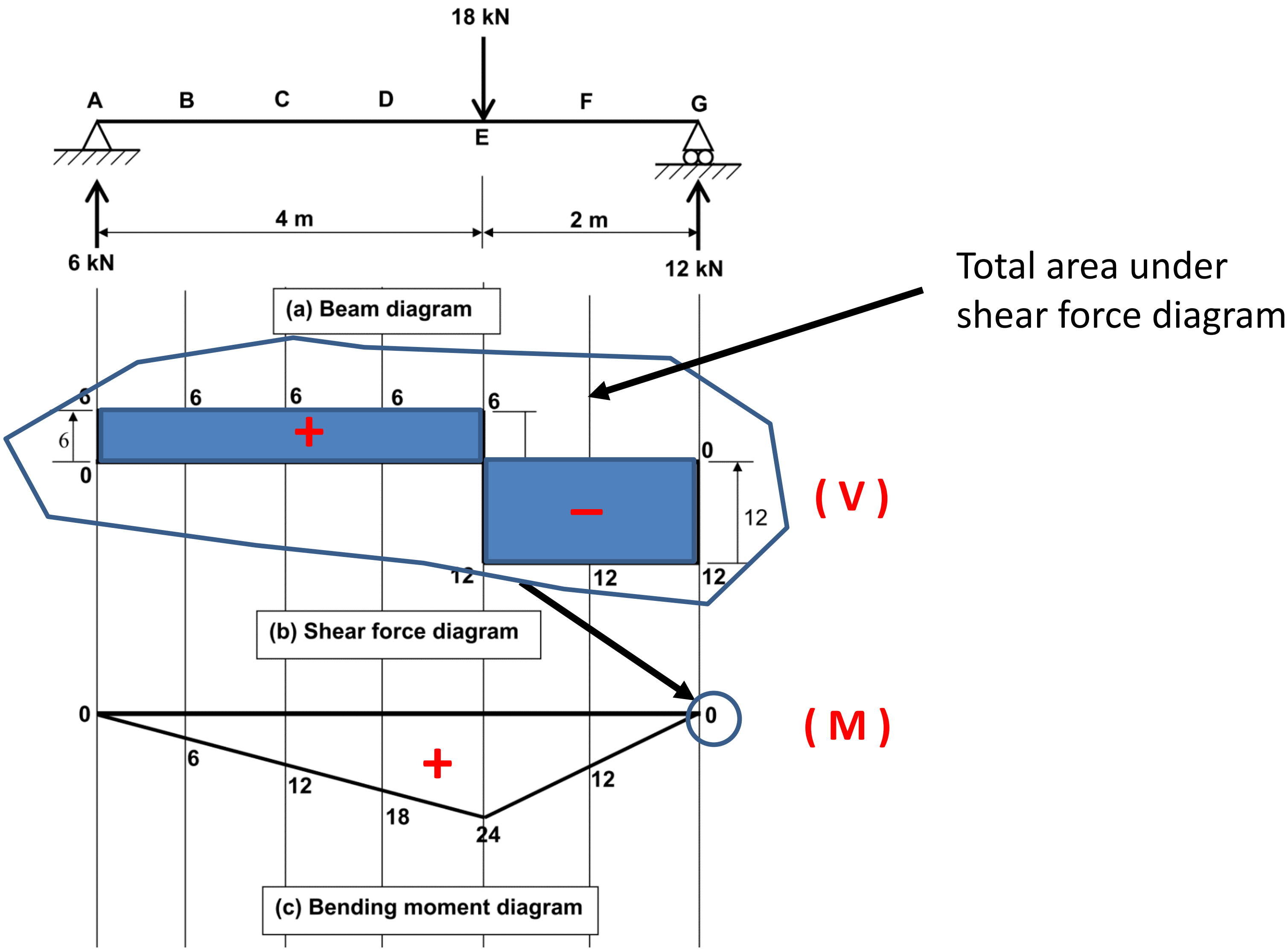

( V )

( M )

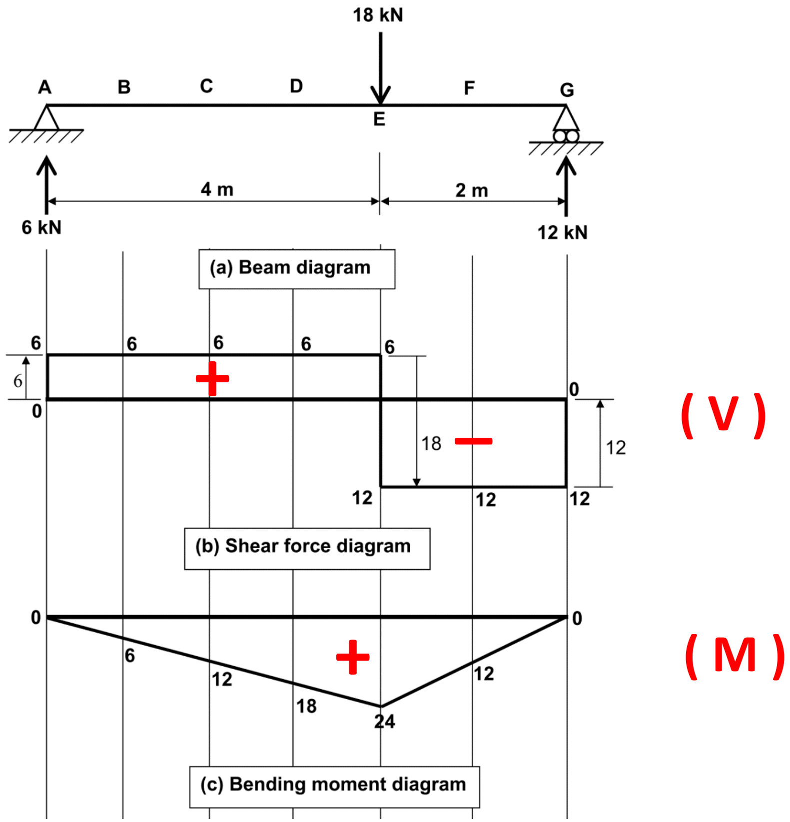
# Bending Moment Diagram – Area under shear force diagram



# Bending Moment Diagram – Area under shear force diagram



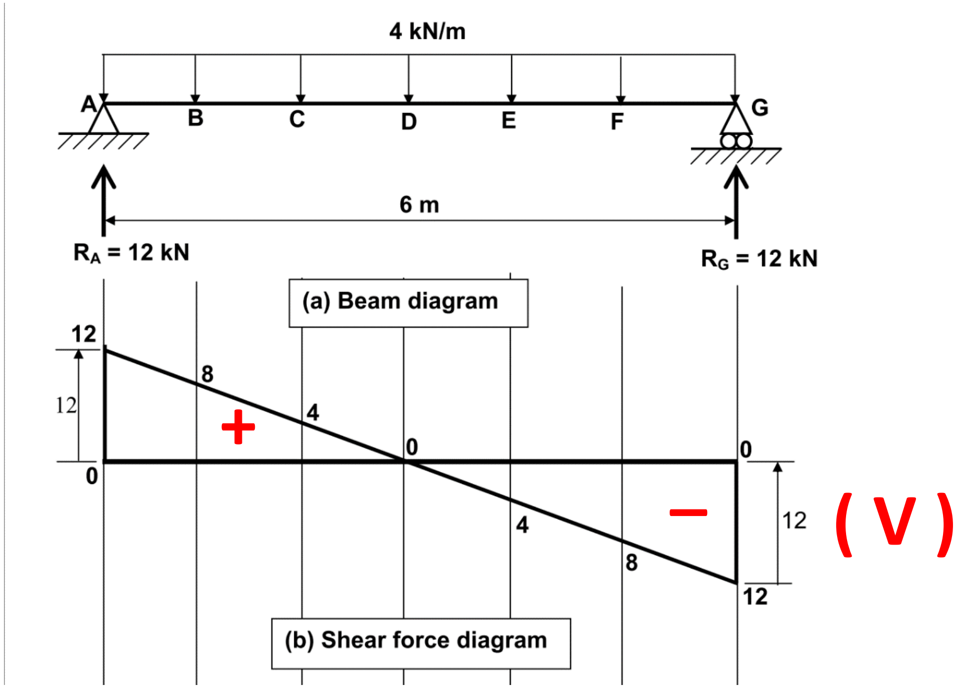
## Shear Force and Bending Moment Diagrams

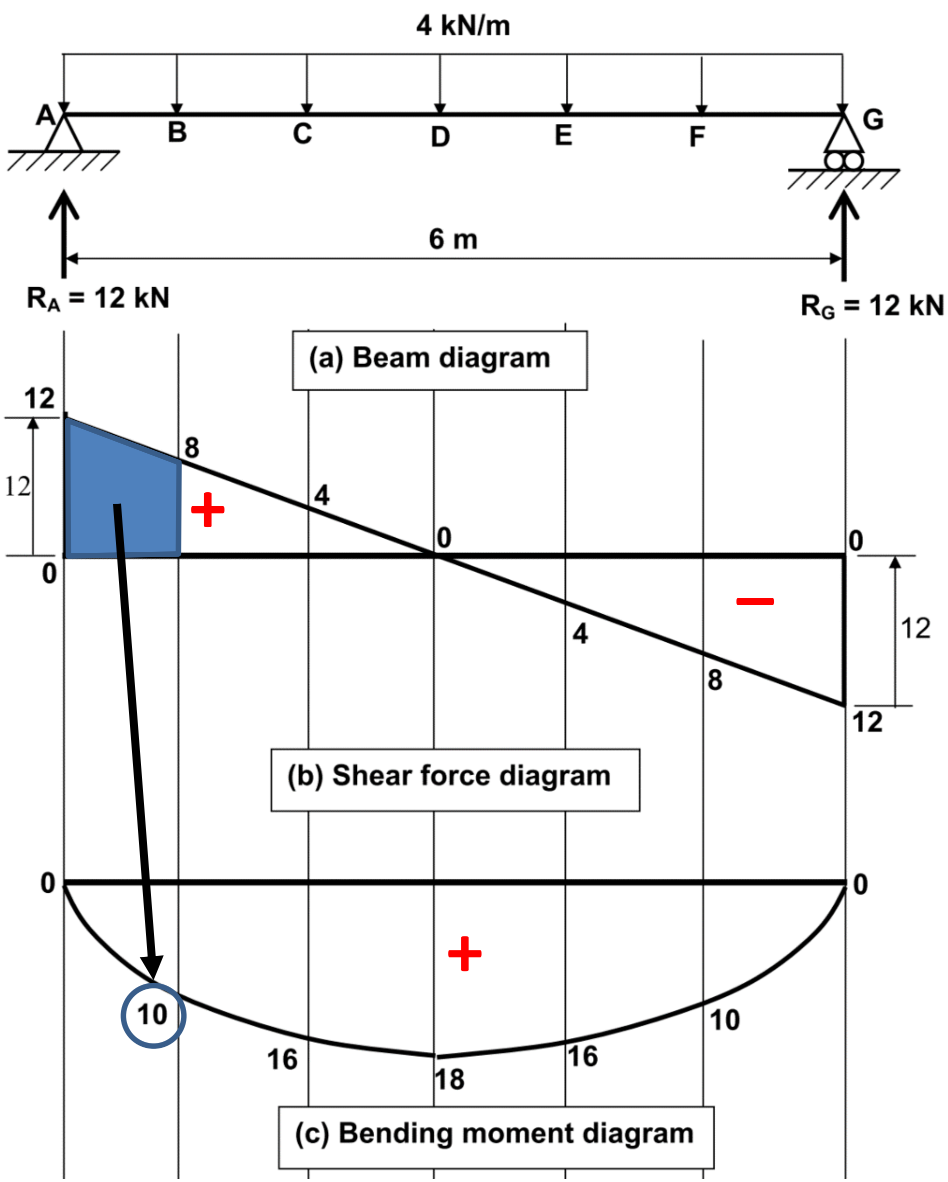



- The shear force diagram is a series of 'steps'; in other words, it contains horizontal and vertical straight lines only.
- The bending moment diagram comprises sloping straight lines.

## Shear Force Diagram of the uniform distributed loaded beam

For this load condition shear forces are decreasing linearly between point forces. To begin The diagram follow the arrow at point A, 12kN than calculate the shear force variation due To distributed load. shear forces decreasing linearly (4kN/m x 6m=24kN) from +12kN at point A to (+12-24 = -12kN) -12kN at point G. At the right support close the Shear force diagram with the force RG=12kN.

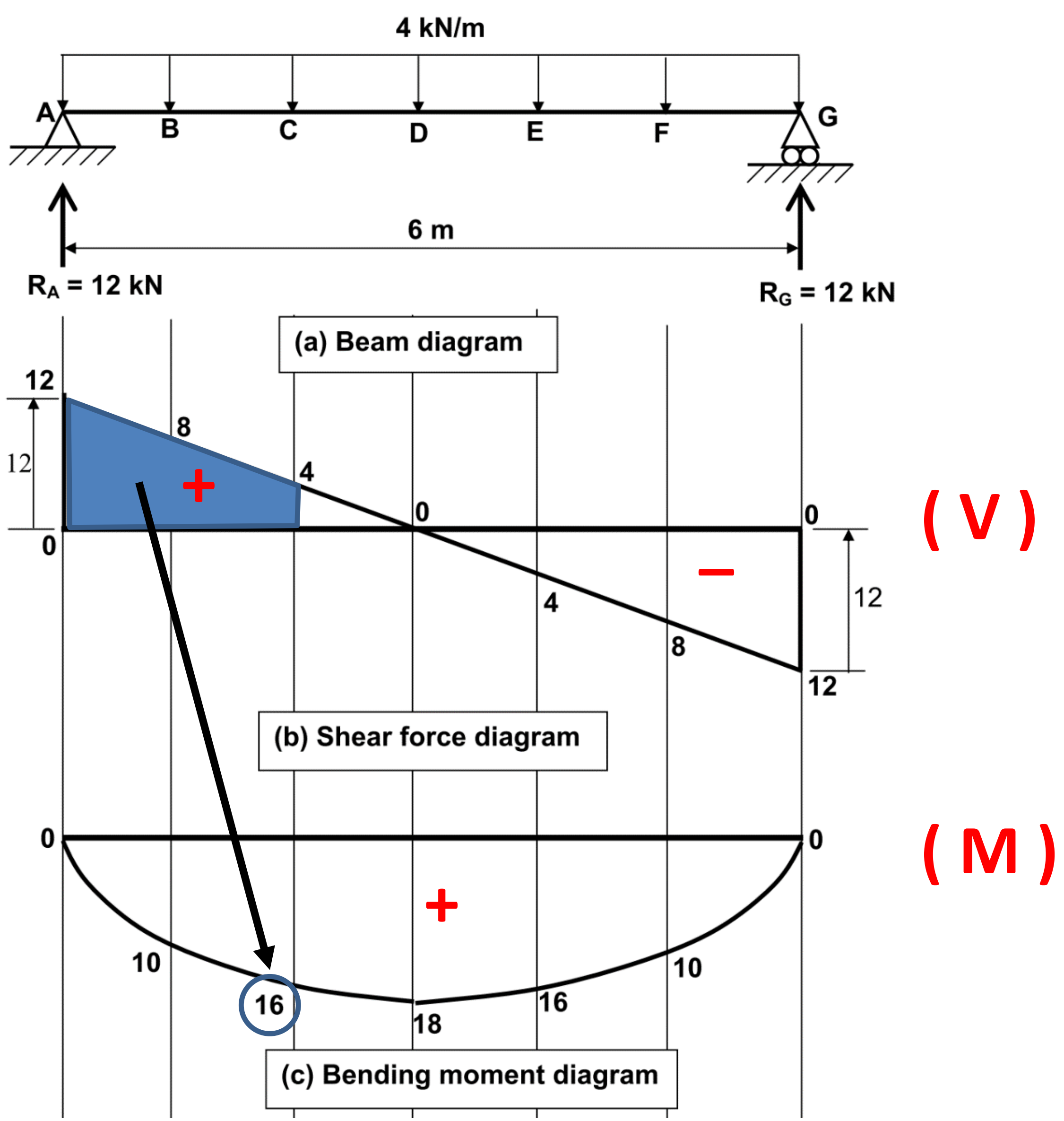




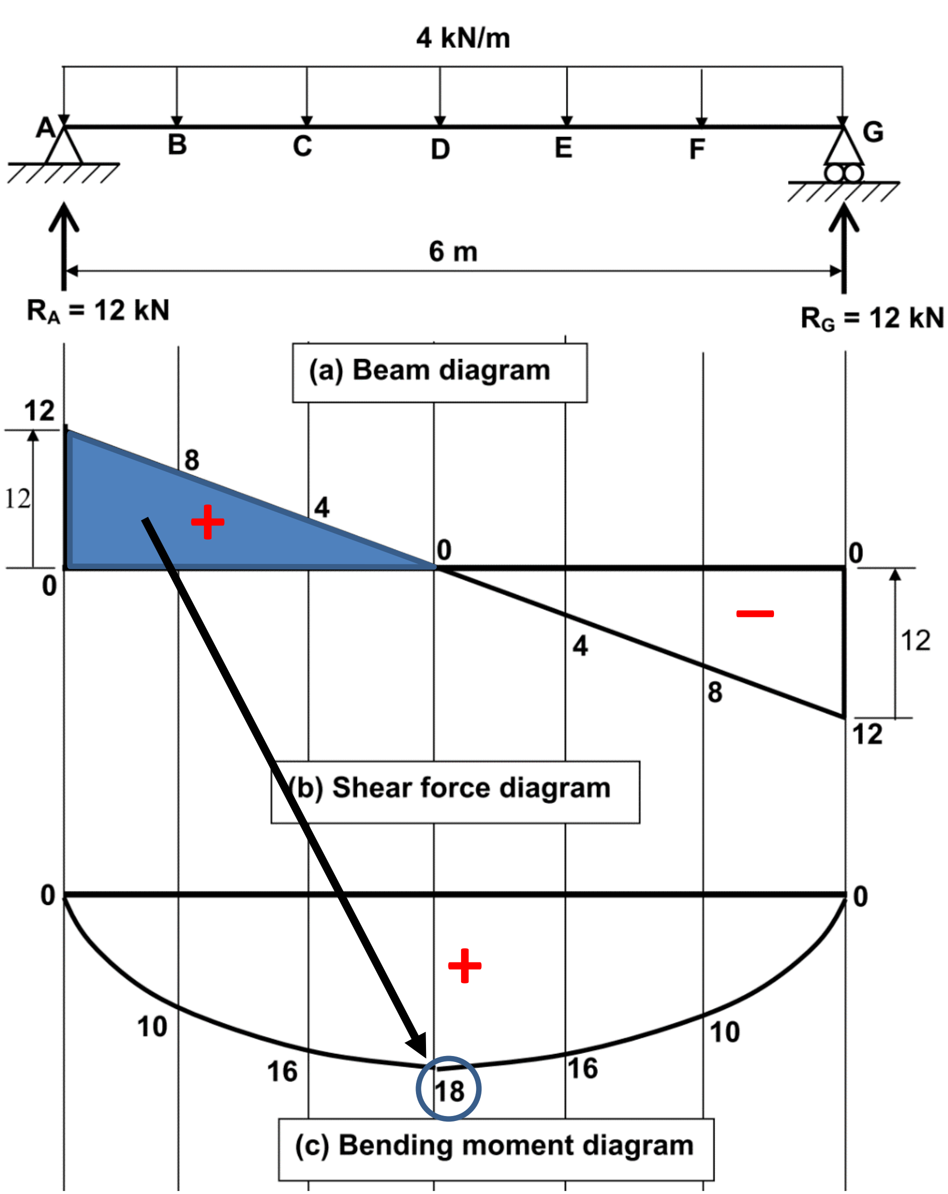

(V)

(M)

$M_B$=(12+8)x0.5x1
=10kNm



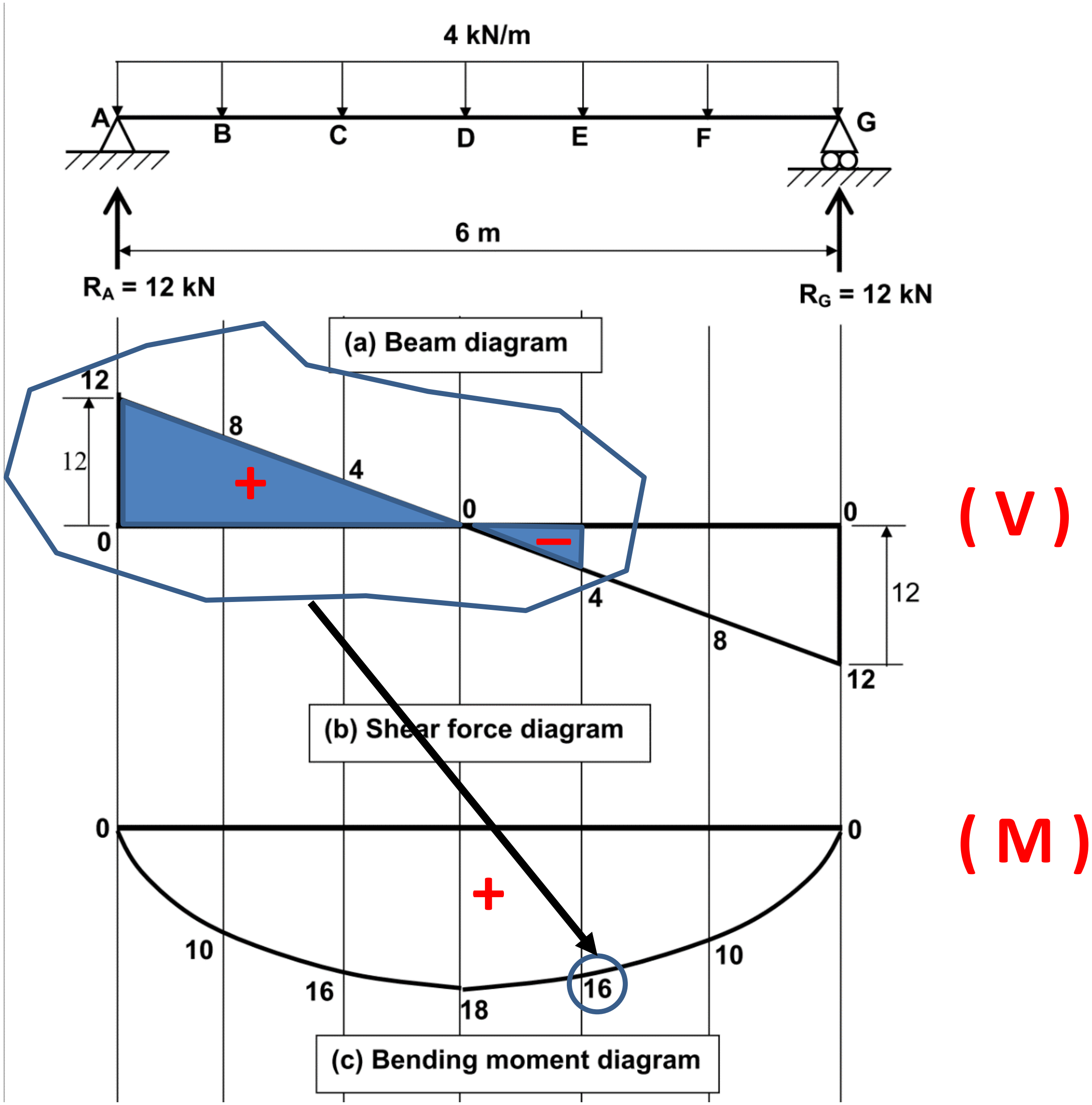

$$M_C = 10 + (8+4) \times 0.5 \times 1$$
$$= 16\text{kNm}$$



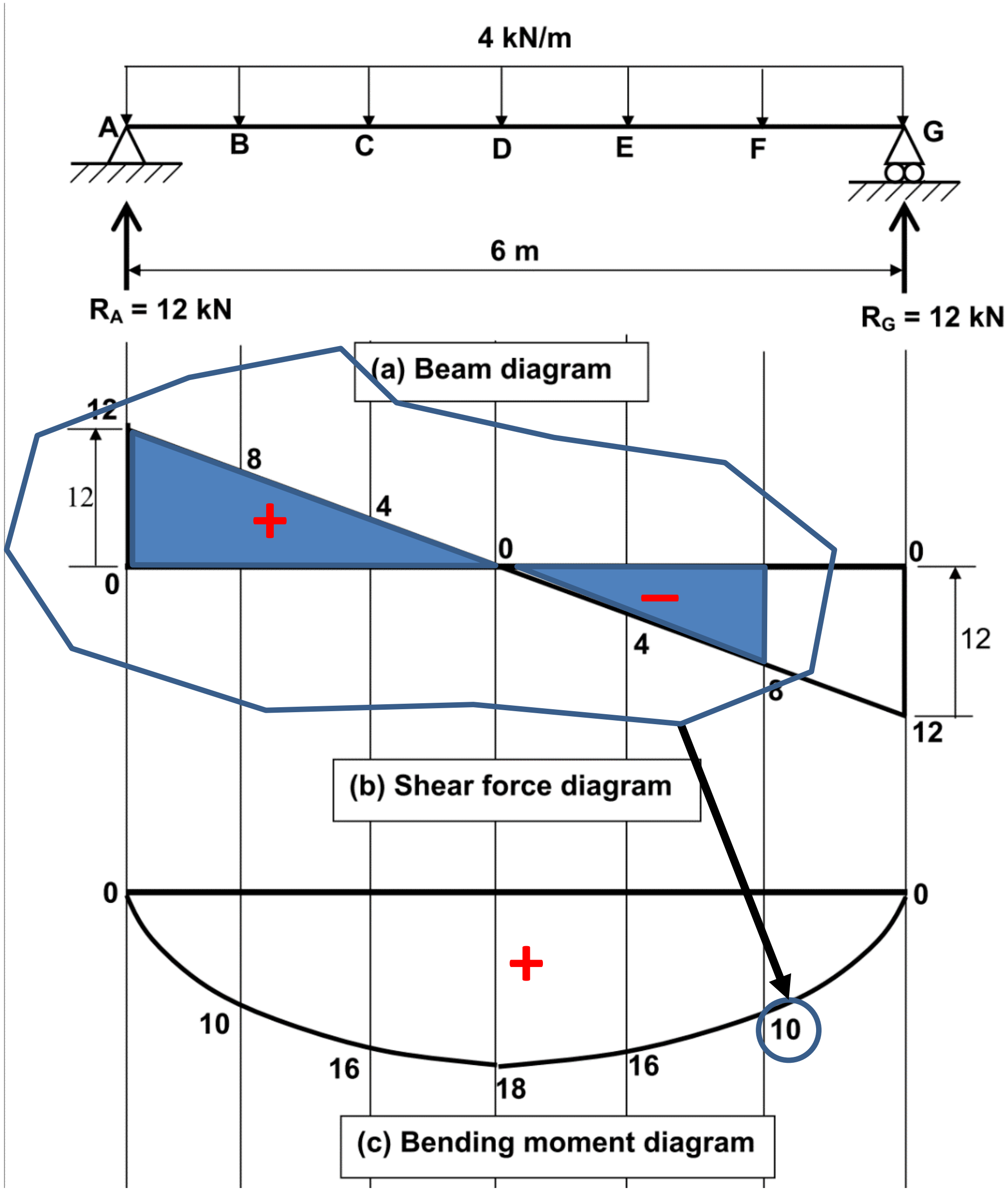

( V )

( M )

$M_D = 16 + 4 \times 1 \times 0.5$
$= 18 kNm$



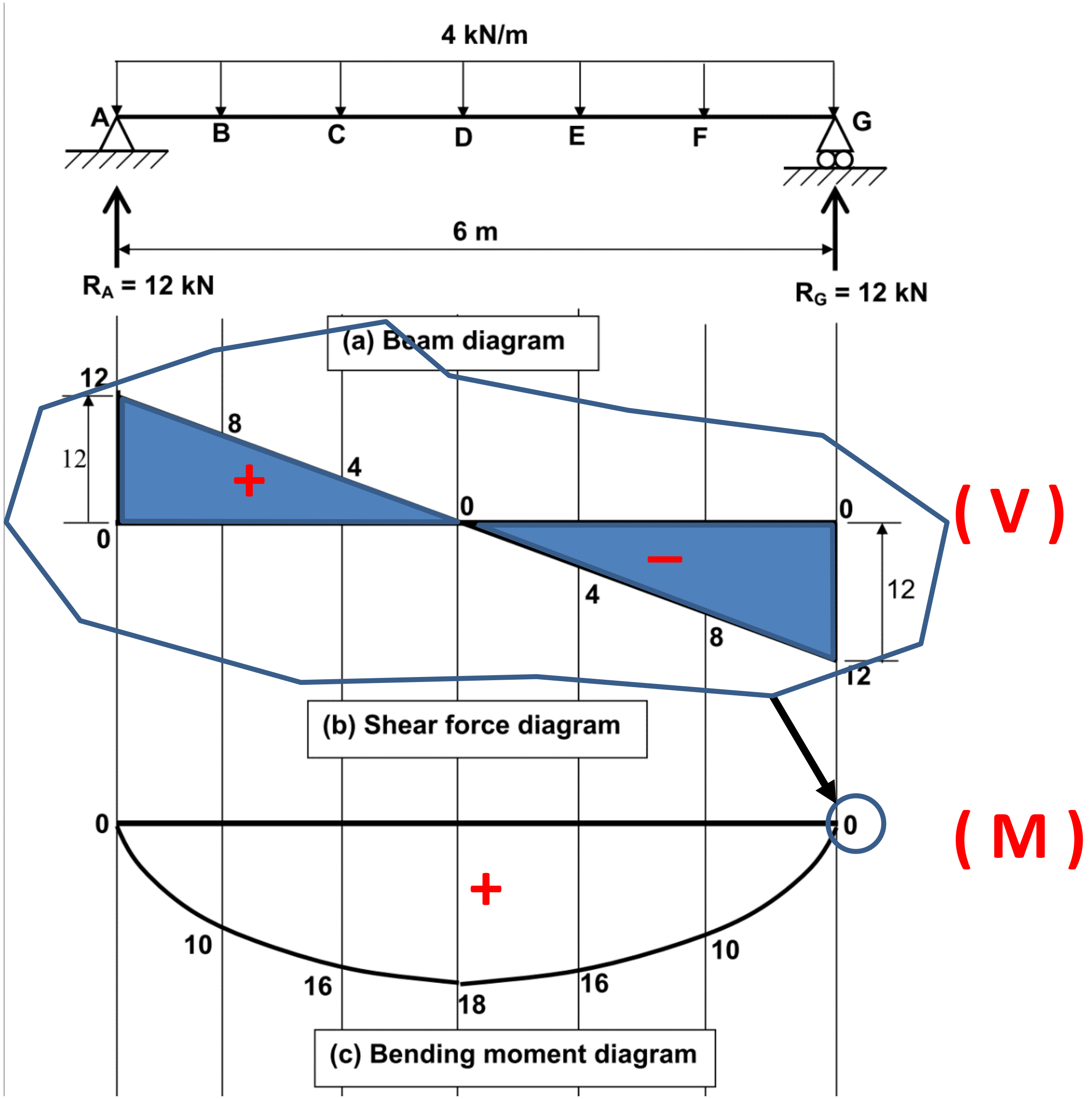

$M_D$=18-4x1x0.5
=16kNm




**( V )**

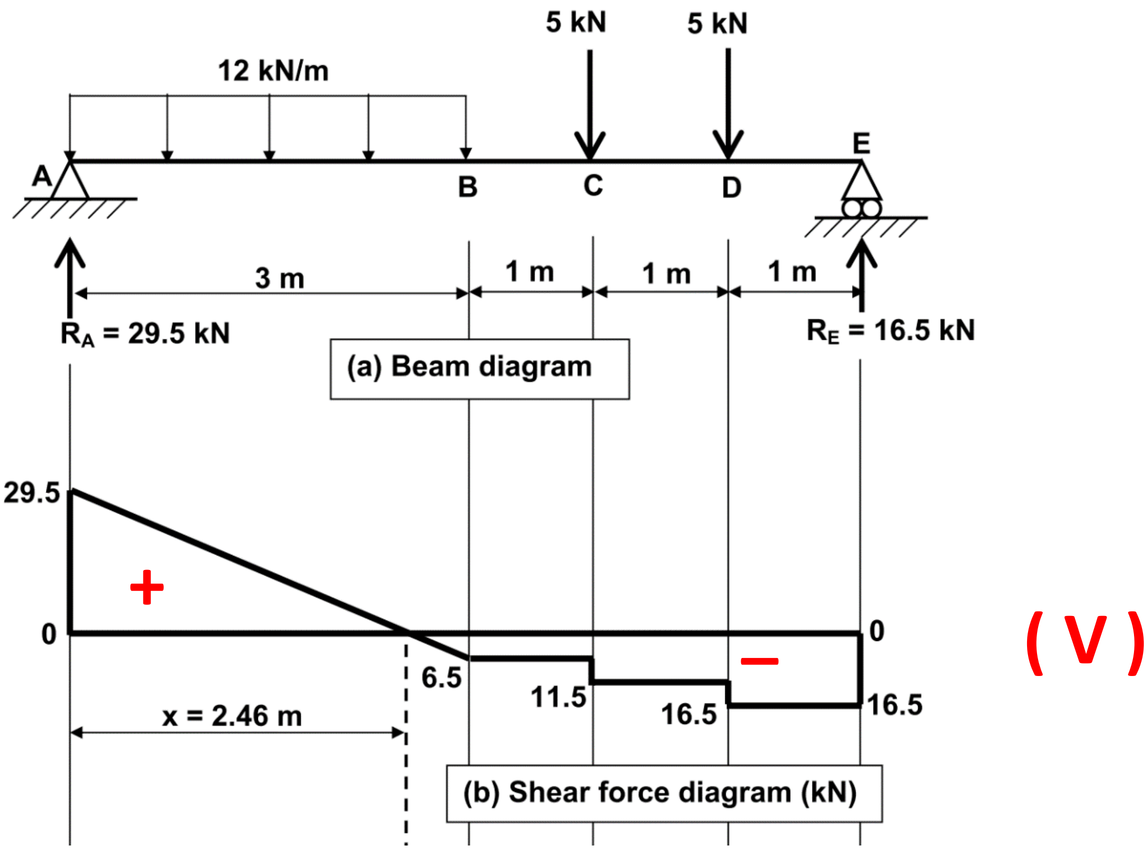
**( M )**

$M_E = 16 - (4+8) \times 1 \times 0.5$
$= 10 \text{kNm}$




$M_E = 10 - (12+8) \times 1 \times 0.5$
$= 0$

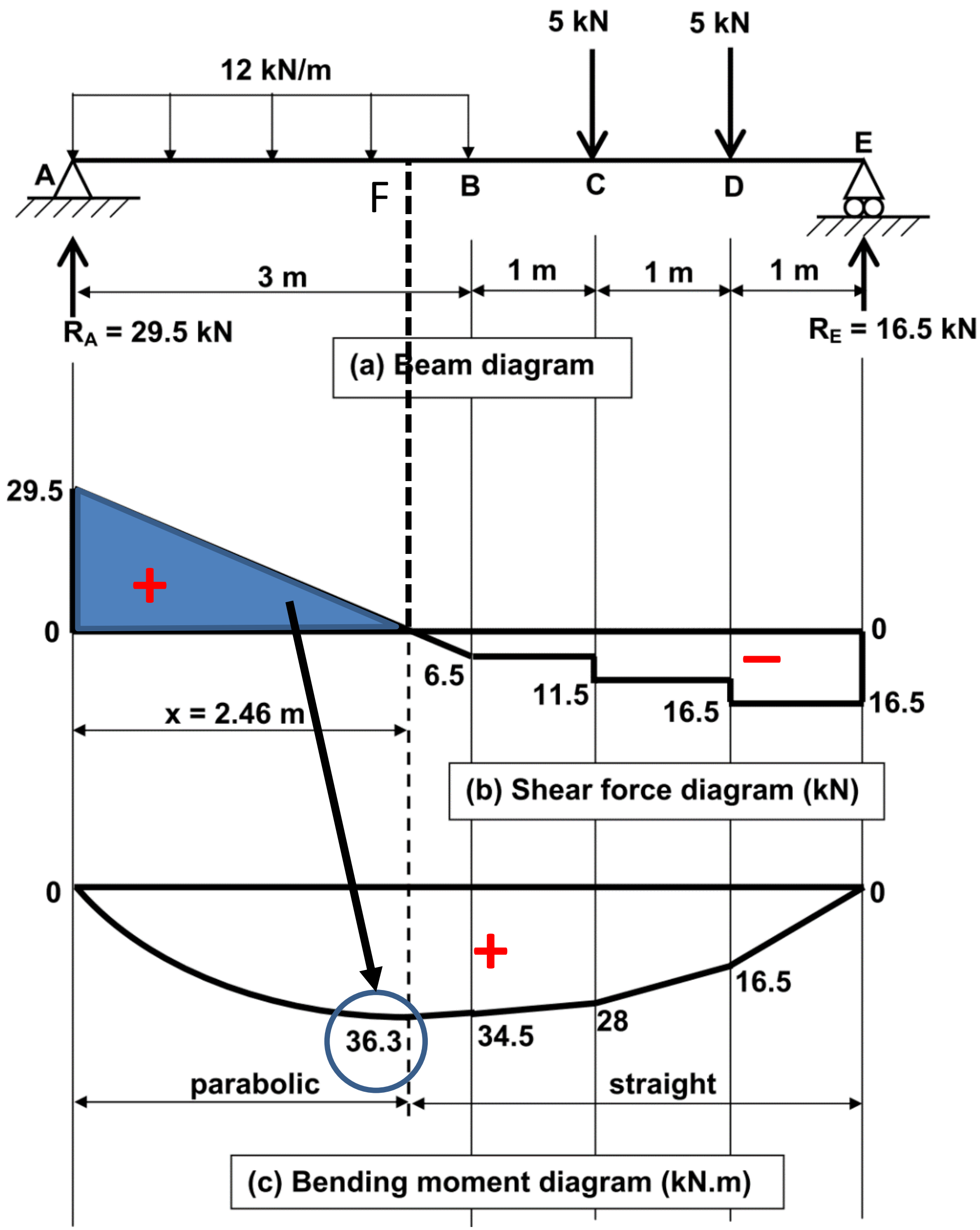
EXAMPLE:




$V_B$=29.5-12x3=-6.5kN

$V_C$=-6.5-5=-11.5kN

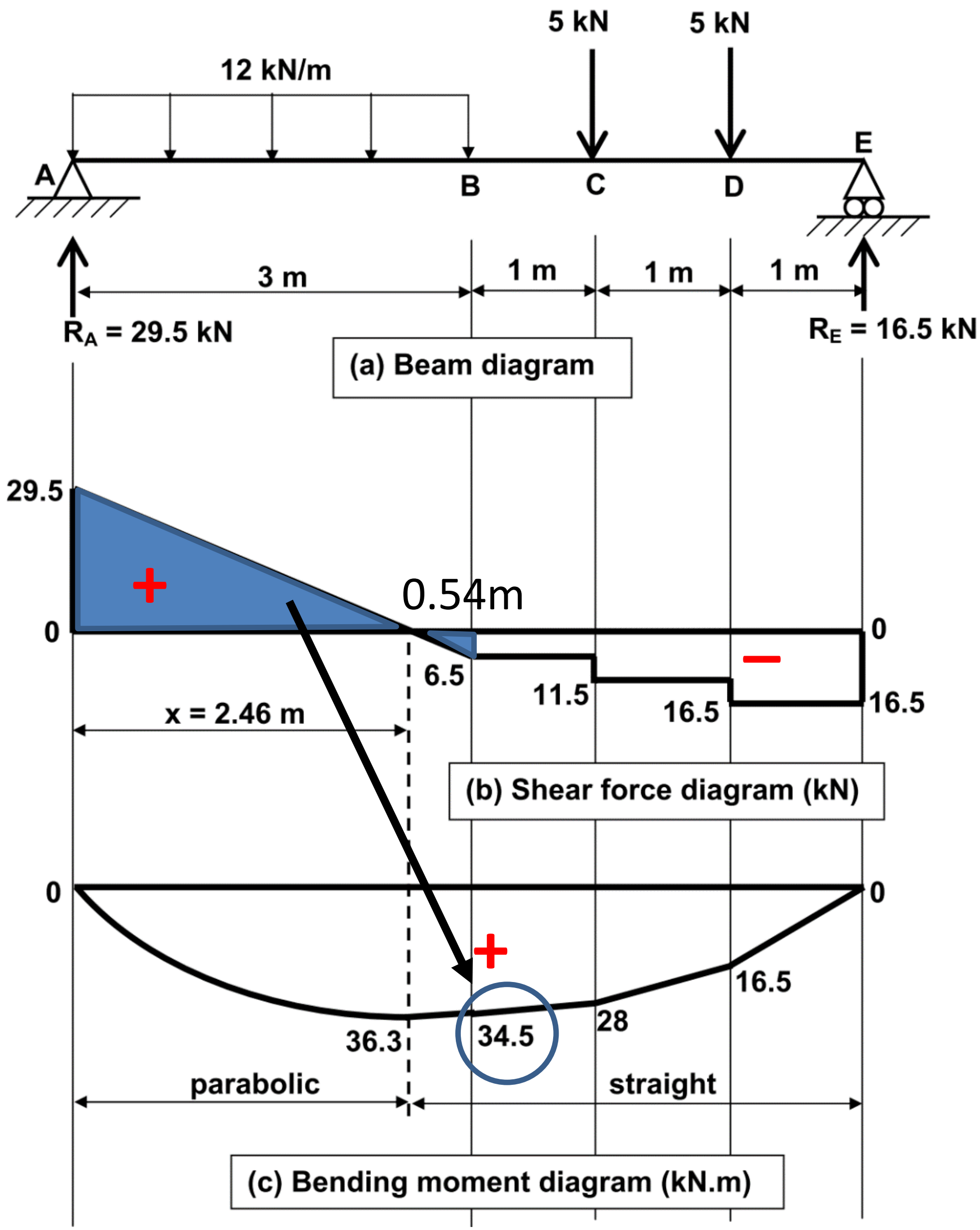
$V_D$=-11.5-5=-16.5kN




$M_F = 29.5 \times 2.46 \times 0.5$
$= 36.3 \text{kNm}$

**( V )**

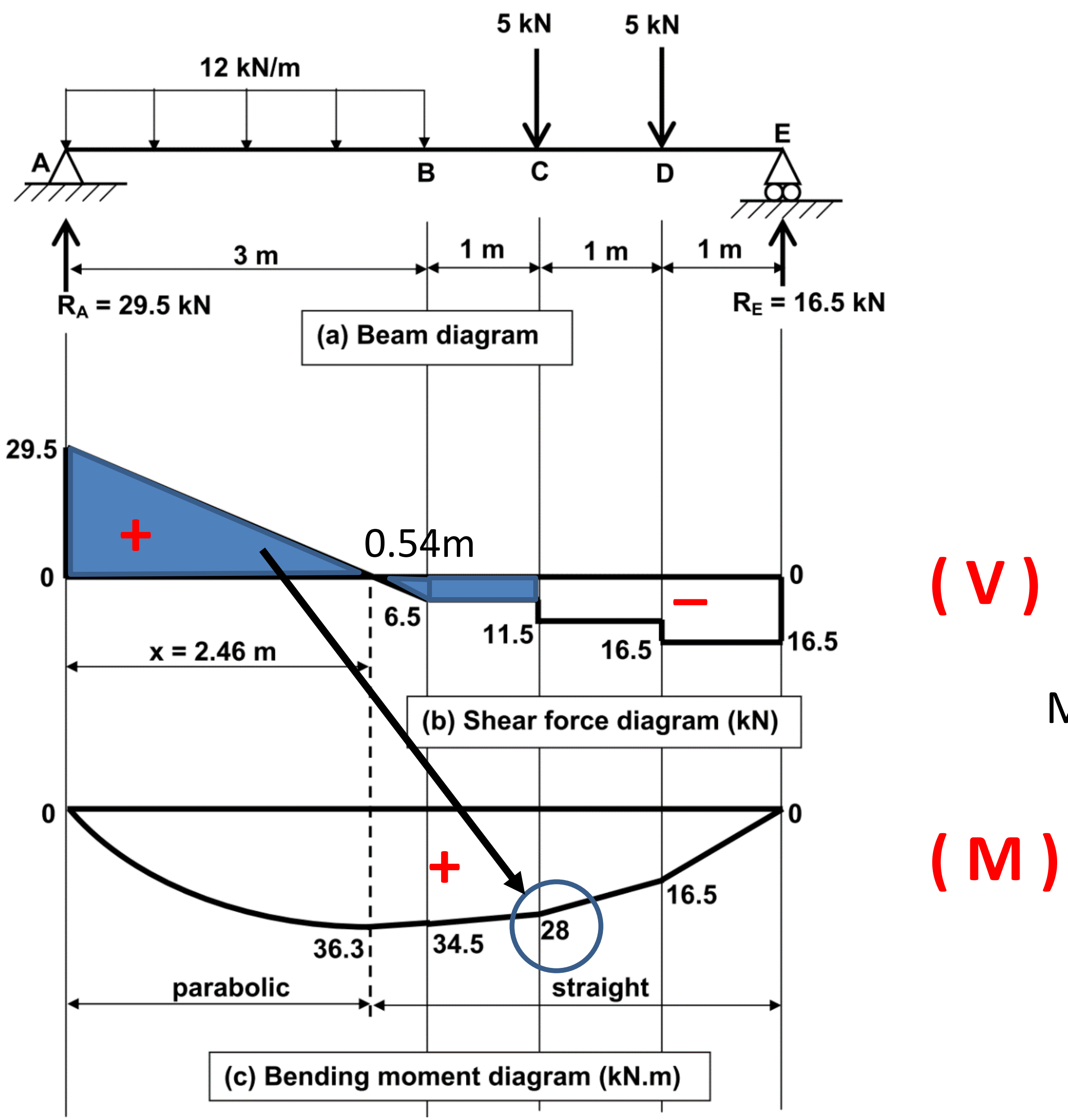
**( M )**




**( V )**

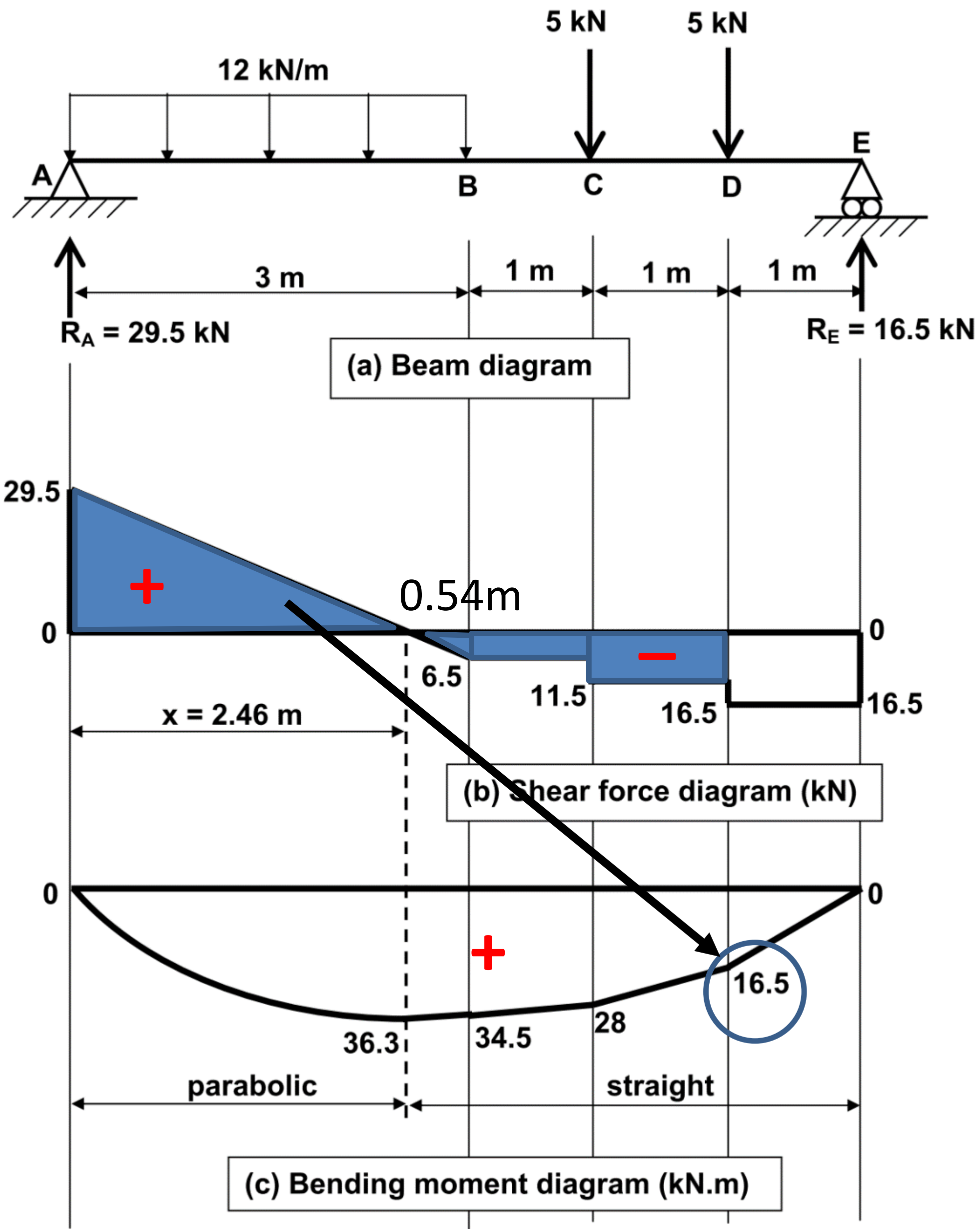
$M_B = 36.3 - 6.5 \times 0.54 \times 0.5$
$= 34.5 kNm$

**( M )**




$M_B = 34.5 - 6.5 \times 1$

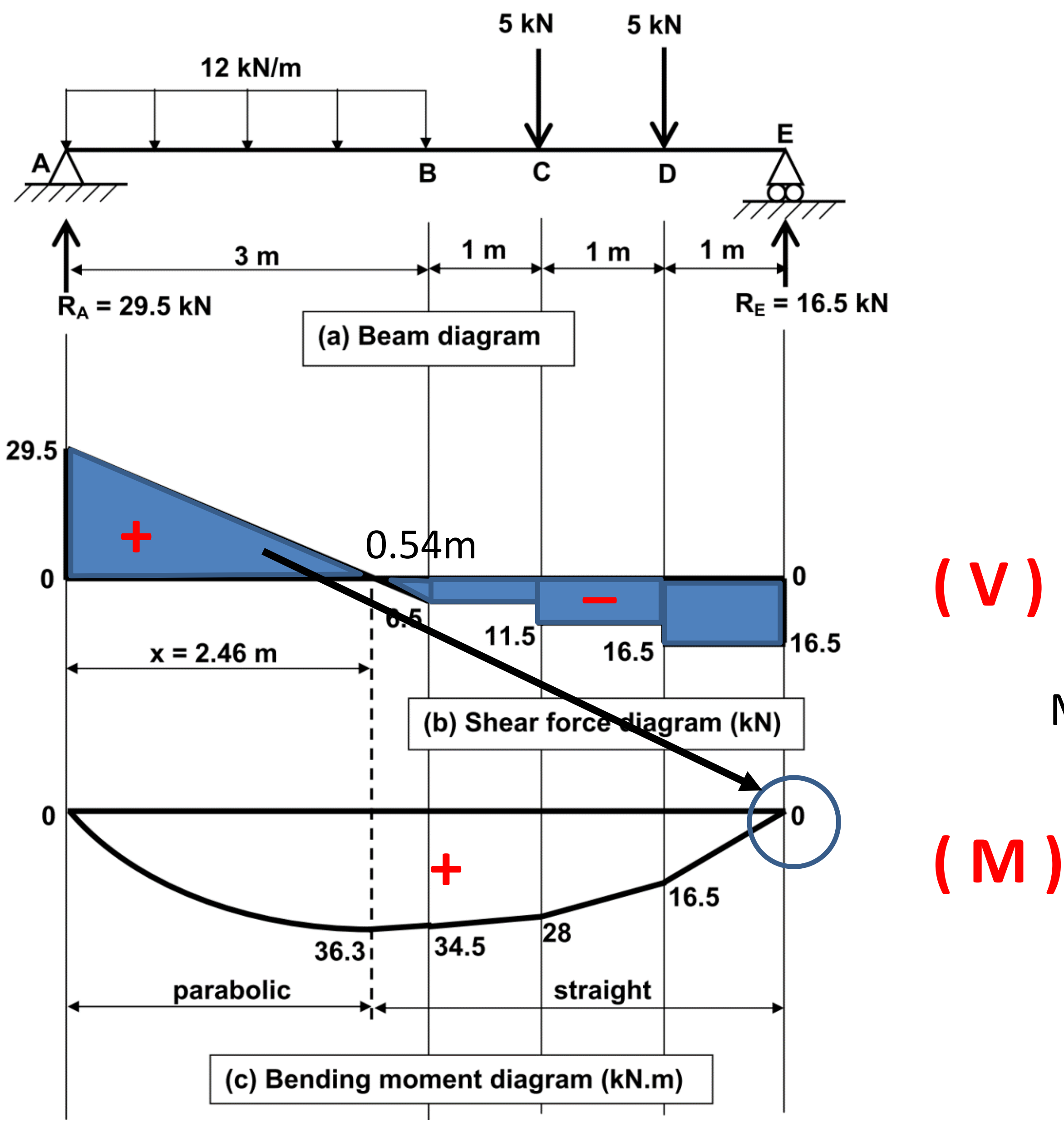
$= 28 kNm$




**( V )**

$M_B = 28 - 11.5 \times 1$
$= 16.5 kNm$

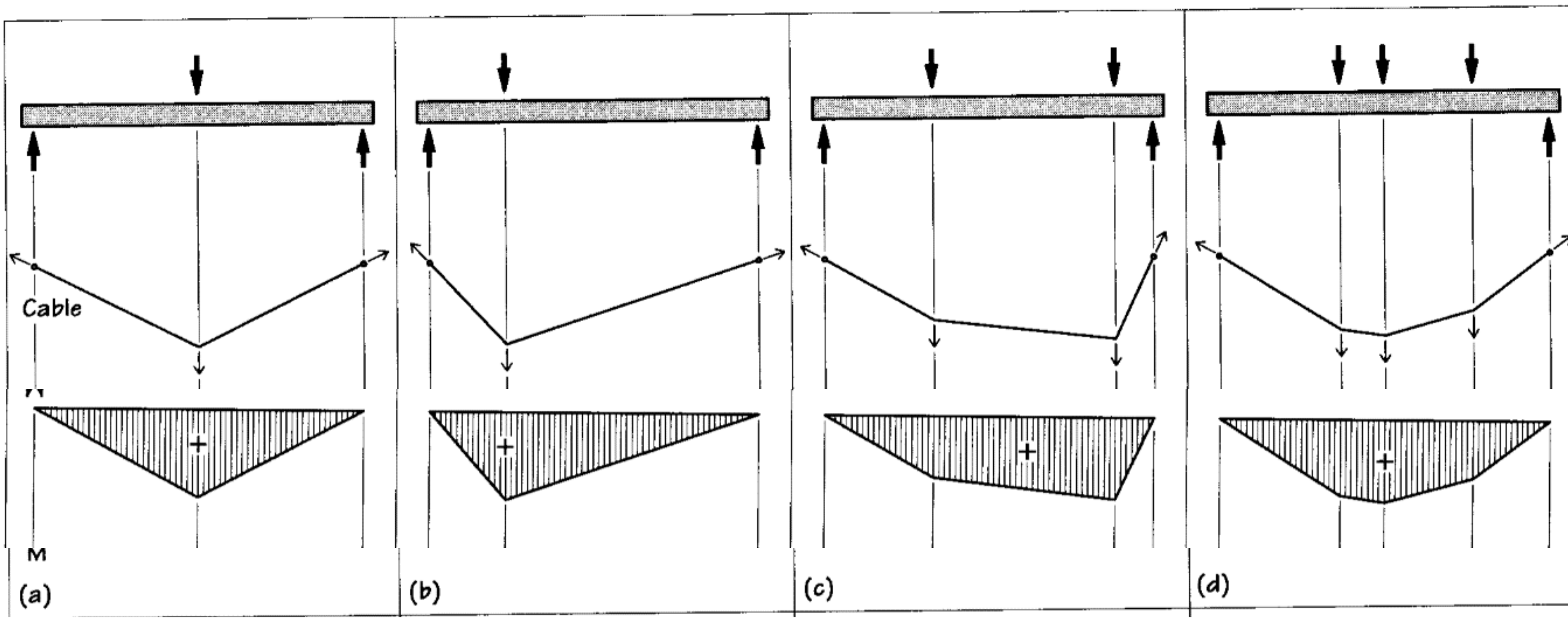
**( M )**




$M_B$=16.5-16.5x1
=0

## LETS REMEMBER

- The value of shear force at any point in a beam = the algebraic sum of all upward and downward forces to the left of the point.
- A beam will fail in either bending or shear. Which occurs first can only be determined by calculation.
- The bending moment is the magnitude of the bending effect at any point in a beam. The value of bending moment at any point on a beam = the sum of all bending moments to the left of the point.
- Shear force and bending moment diagrams are graphical representations of shear force and bending moment and their variation along a beam.
- The bending moment diagram is drawn either above or below the zero line, dependent on whether the beam experiences tension in the top or bottom at the point concerned (top: above the line, bottom: below the line).
- Where the shear force is zero, the bending moment is either a local maximum, a local minimum or zero. It follows from this that the position of maximum bending moment can be determined from drawing the shear force diagram first.


Cable
+
+
+
+
M
(a)
(b)
(c)
(d)

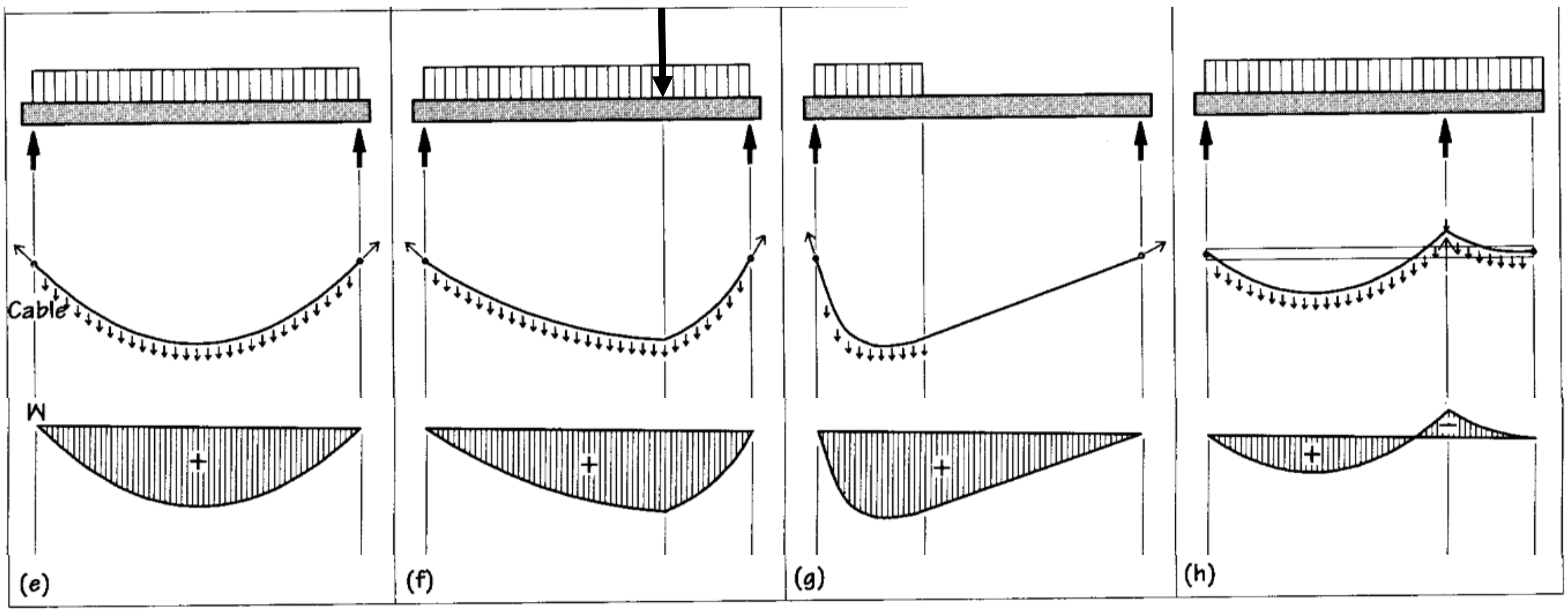



**Figure 16.18** The shape of a moment diagram is always the inversion of the shape taken by a hanging cable with the same support and loading conditions.