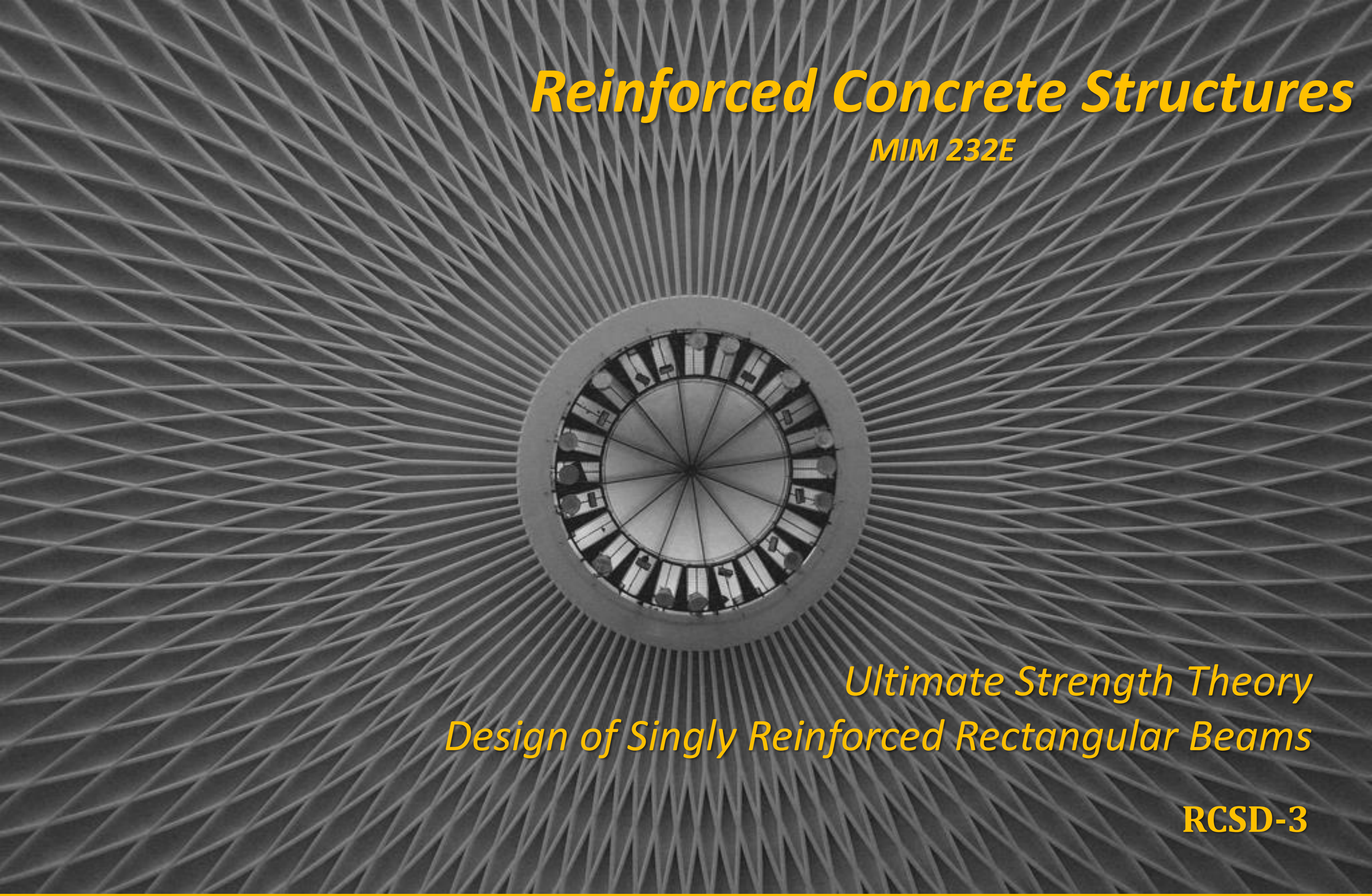


Reinforced Concrete Structures

MIM 232E

Ultimate Strength Theory

Design of Singly Reinforced Rectangular Beams

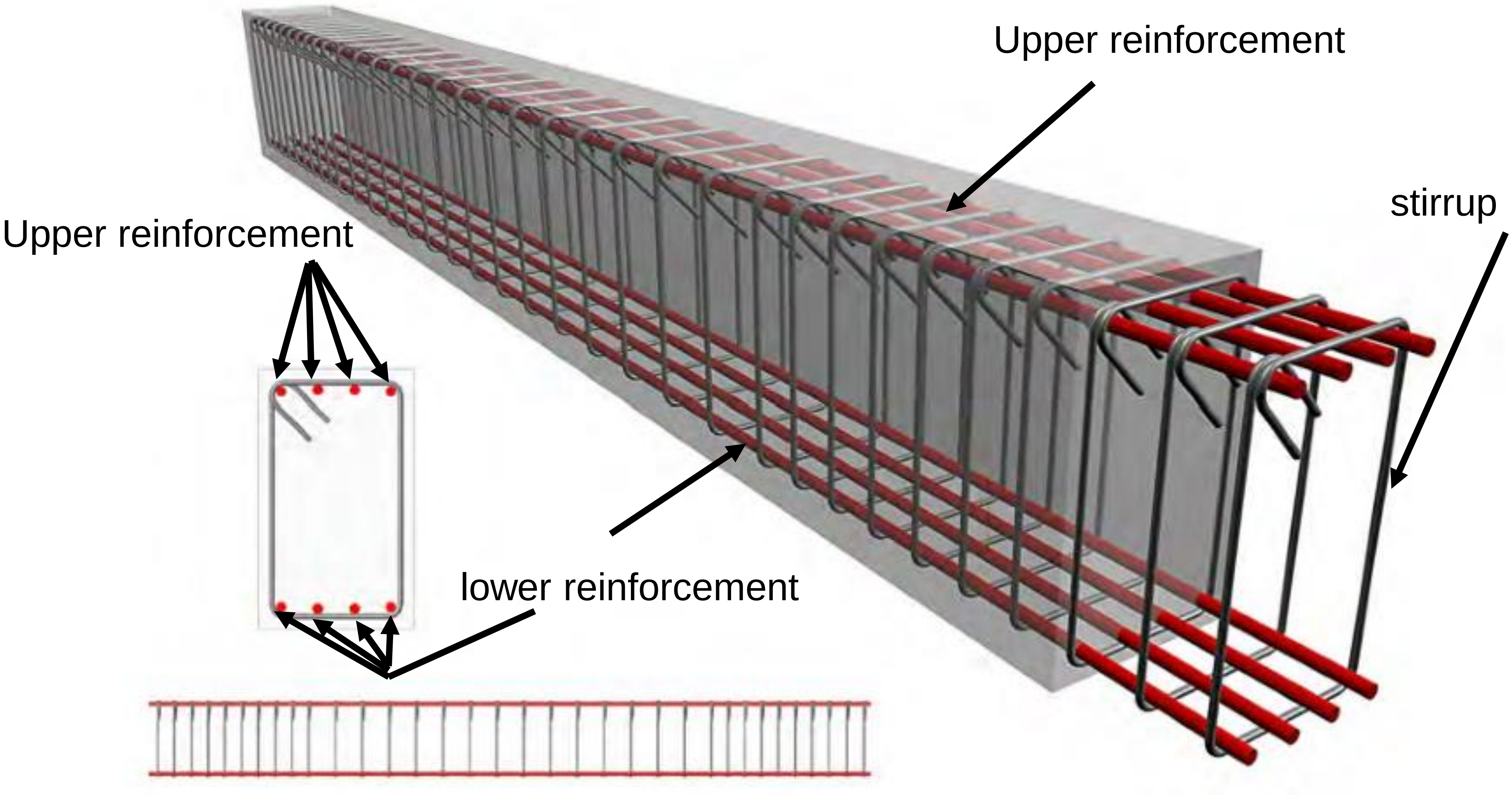
RCSD-3

Dr. Haluk Sesigür

I.T.U. Faculty of Architecture

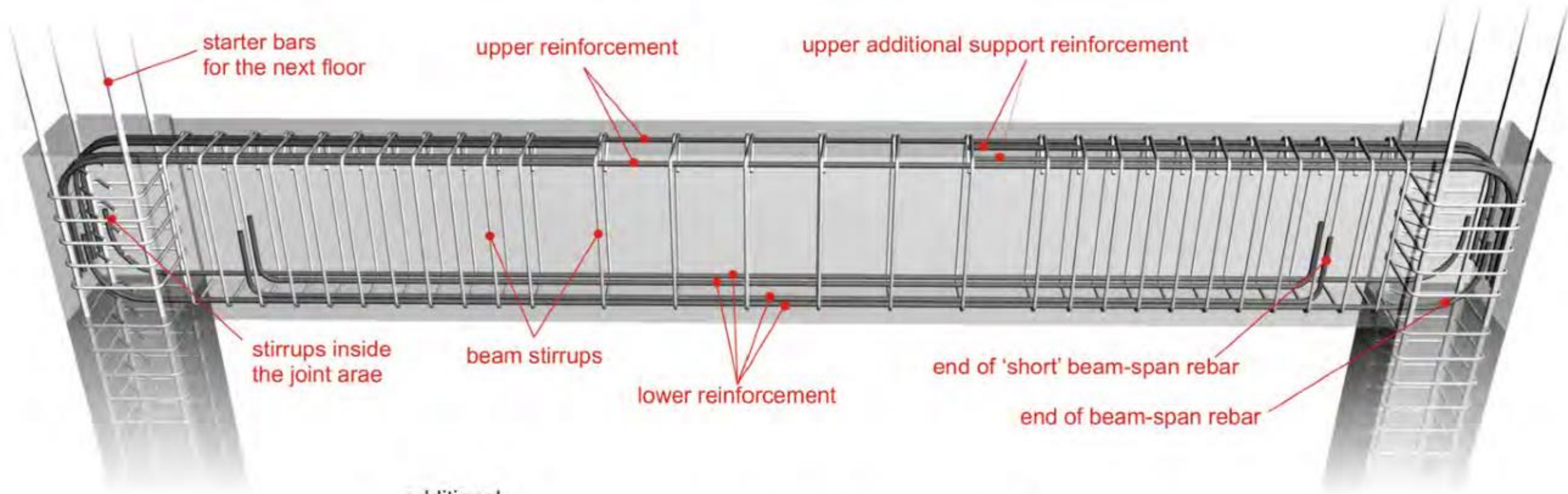
Structural and Earthquake Engineering WG

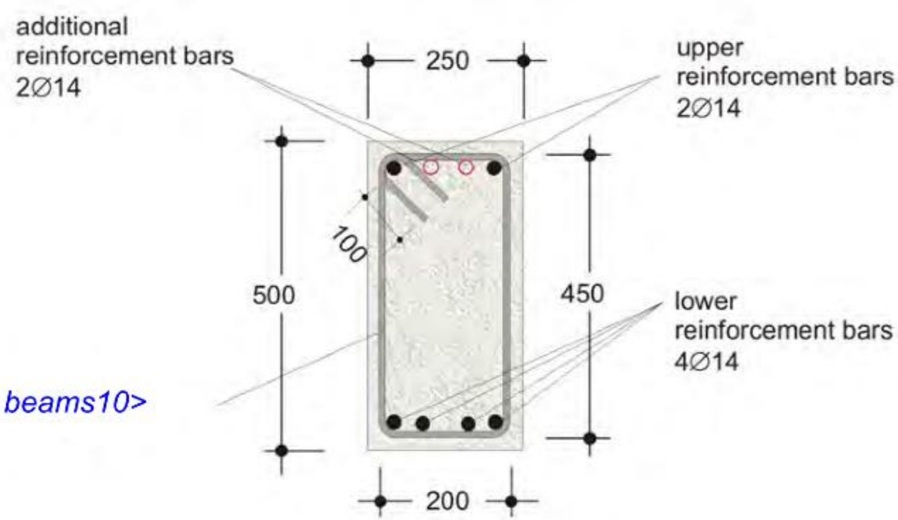
RC Beam Design



Beam with ductility requirements

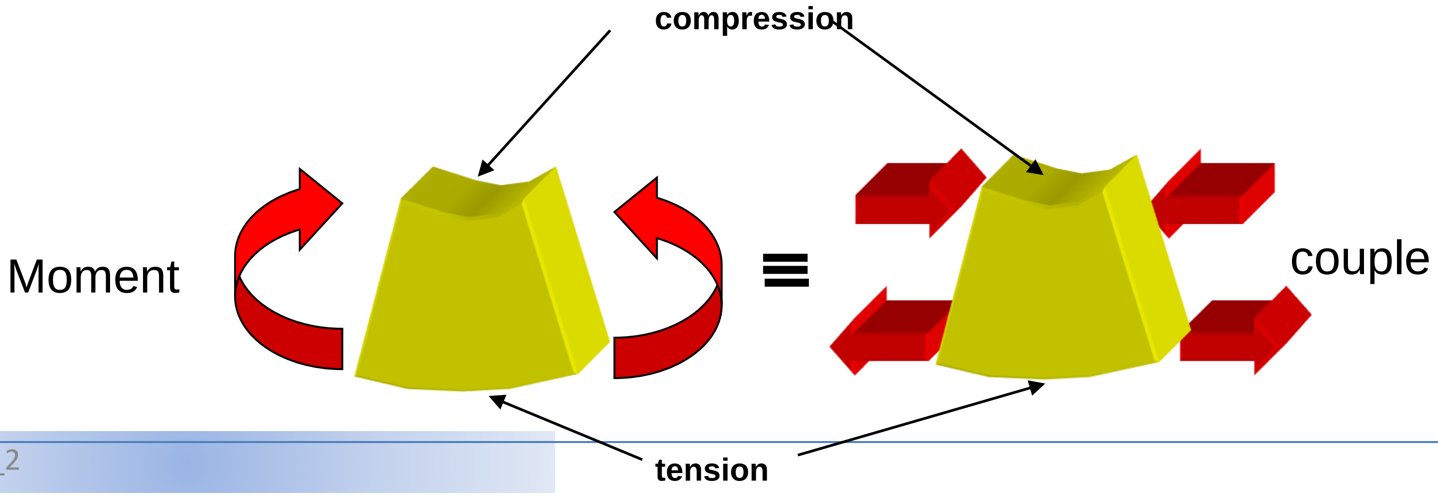
RC Beam Design





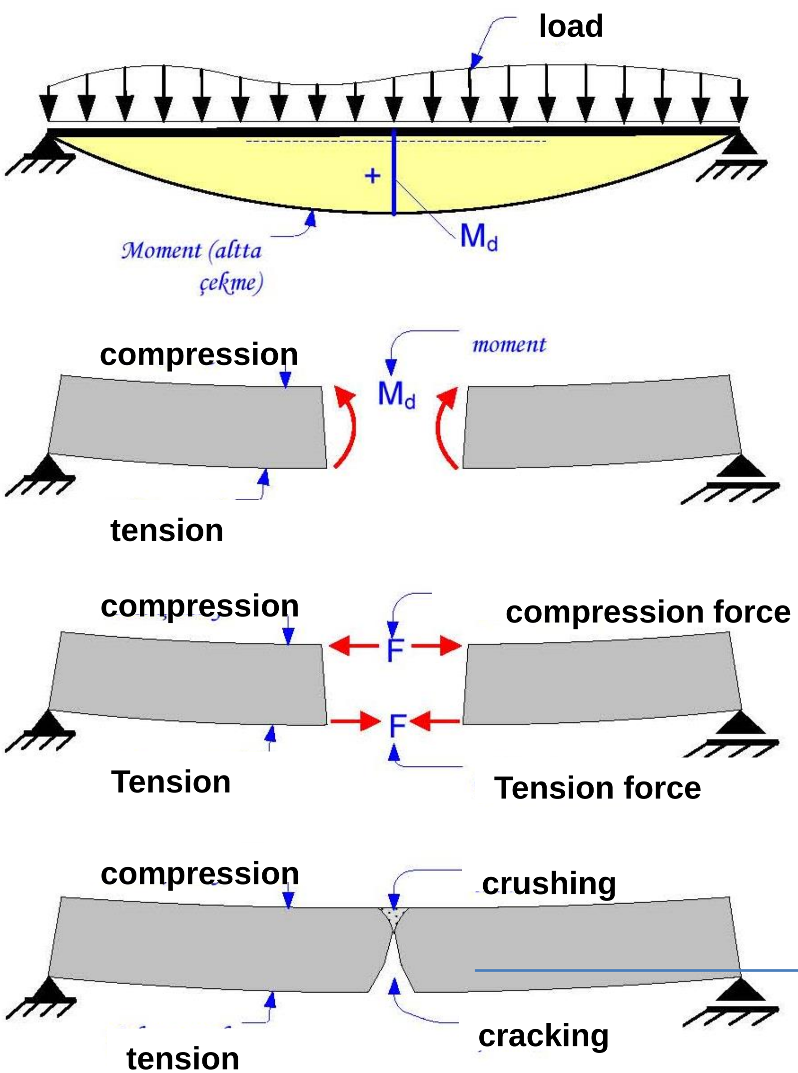
Beam 250 mm x 500 mm *<project: beams10>*

Ultimate Strength Theory



Ultimate Strength Theory

Pure bending



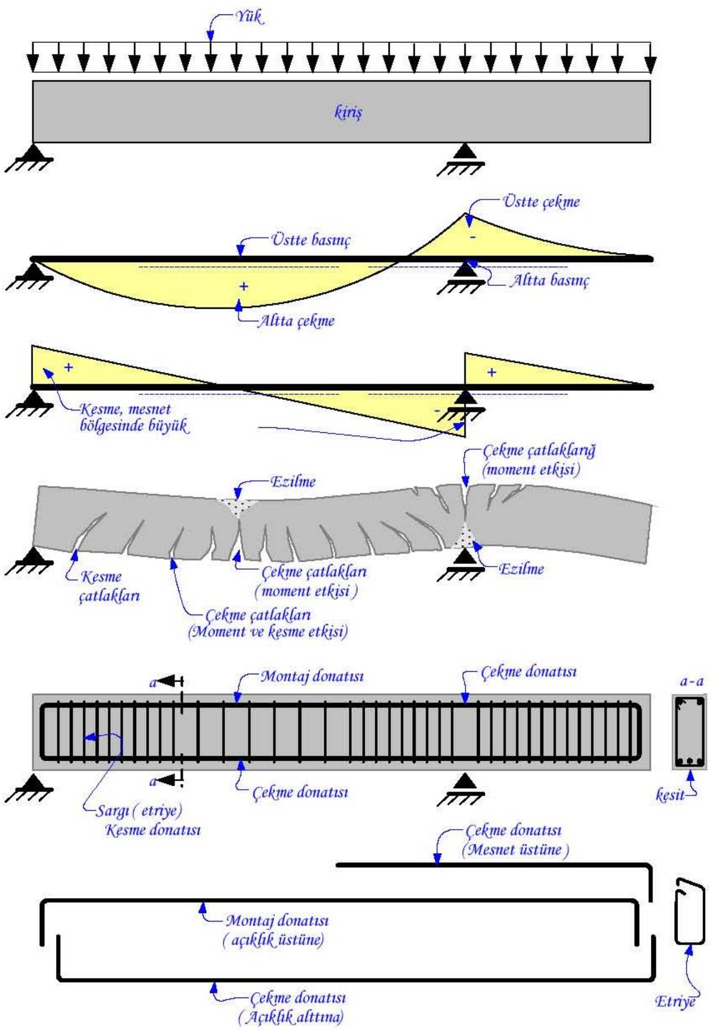
Moment : couple (tension and comp. force)

Concrete is good in compression;

Reinforcment to resist tension at the bottom

Ultimate Strength Theory

Necessity for reinforcement



M

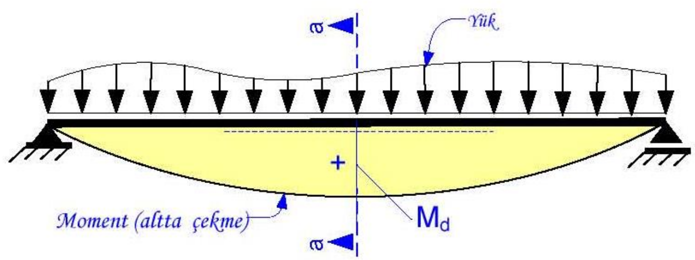
V

Video

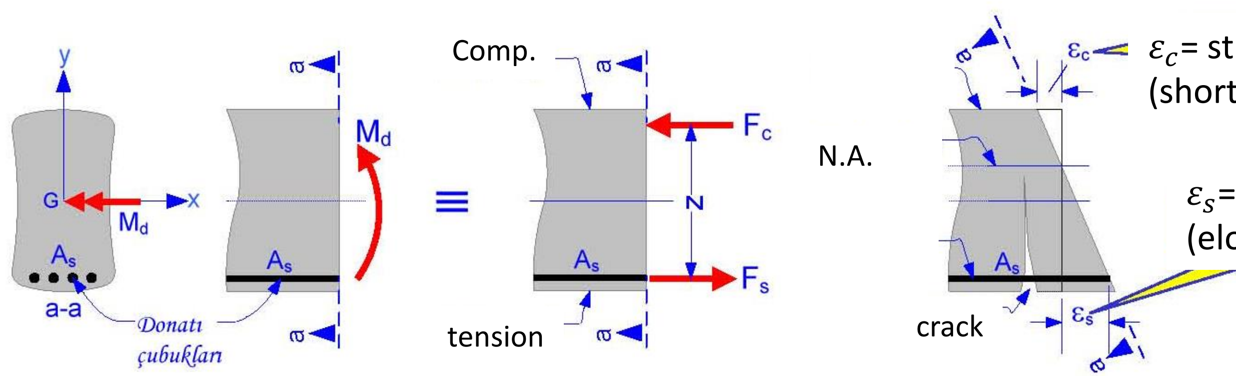
Deformation;
cracking at tension region
Cruching at compression region

Deformation

Moment → *couple*



As: area of tension reinf.
Fs: tension force of steel
Fc: compression force of concrete
z: moment arm
ε_c= strain of concrete (shortening)
ε_s= strain of steel (elongation)
Horizontal equilibrium ; Fc=Fs



Cross-setion **İnternal forces** **deformation**

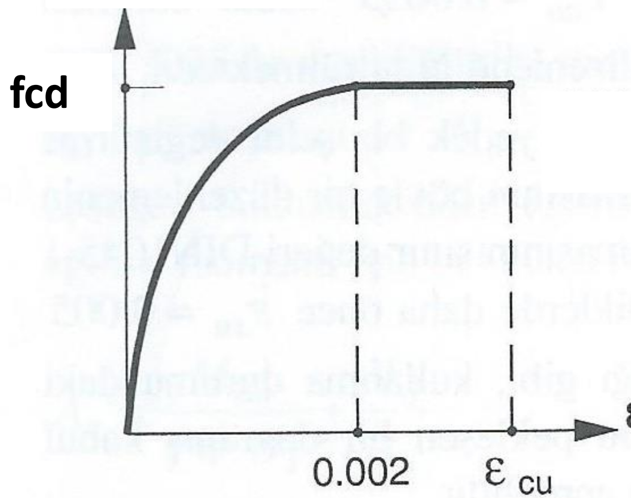
ε_c= strain of concrete (shortening)

ε_s= strain of steel (elongation)

Ultimate Strength Theory

Idealized Stress-Strain (σ-ε) Relationships

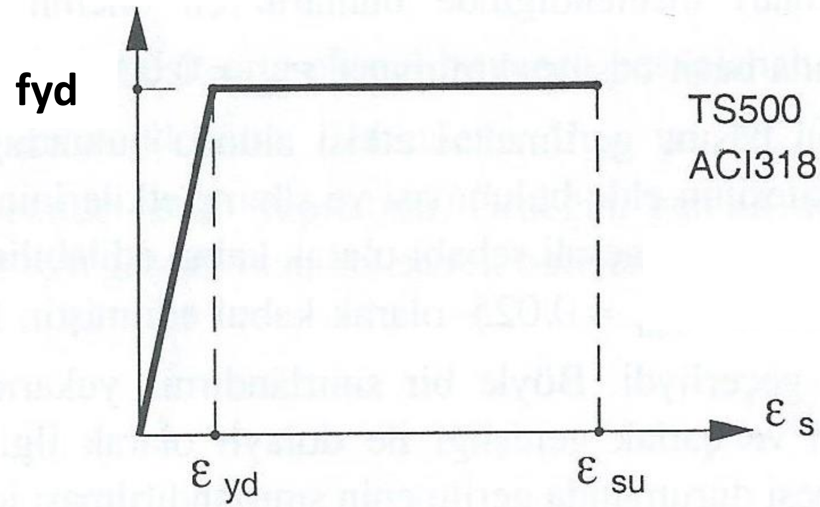
concrete



=0.003

crushing

steel



=0.01

rupture

$$\varepsilon_{yd} = \frac{fyd}{E_s}$$

E_s=210000MPa (modulus of elasticity)

$$fcd = \frac{fck}{1,5}$$

$$fyd = \frac{fyk}{1,15}$$

Ultimate Strength Theory

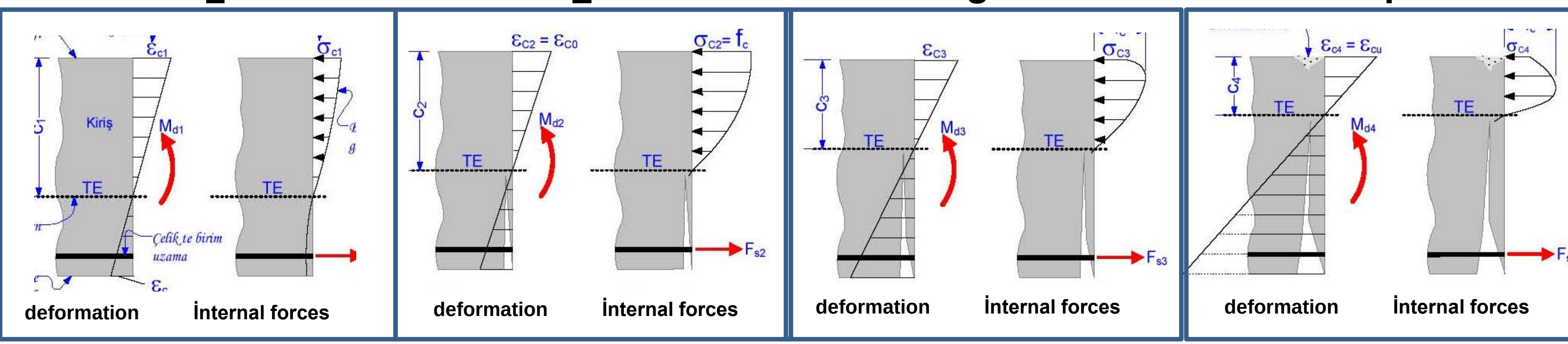
general

- Md moment → copule (Fc;Fs) Fc=Fs
- Compression at top; tension at bottom
- Fc can be carried (f_{cd} is good);
  Fs cannot be carried (f_{ctd} is poor) → cracking occur
- Fs should be complately carried by As (no concrete contribution is considered)
- ε_c shortening at top; ε_s elongation at bottom; by rotation of cross section

Deformation & strain

Moment (Md) increases

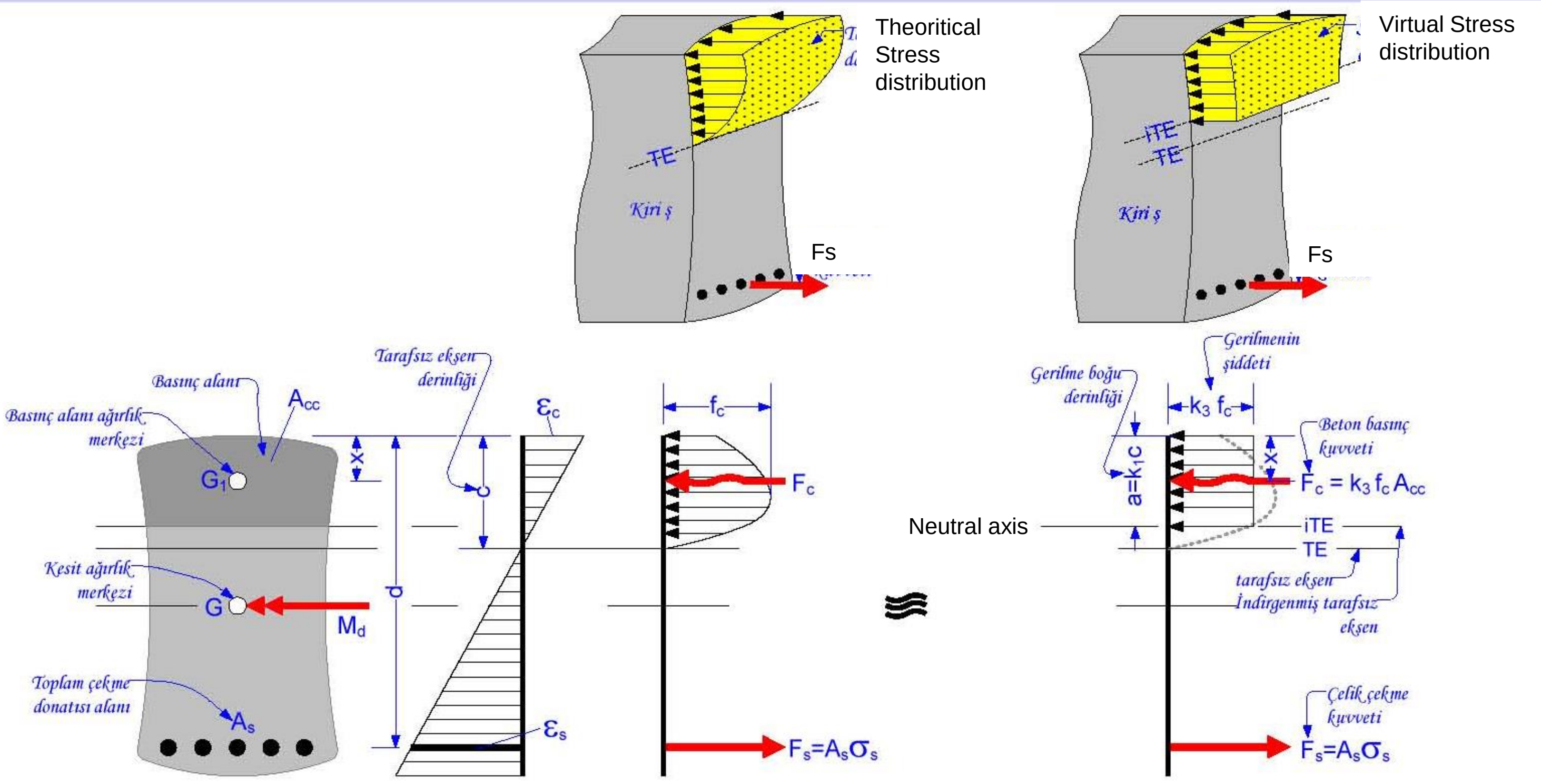
1 2 3 4 *crushing*



cracking *cracking* *cracking*

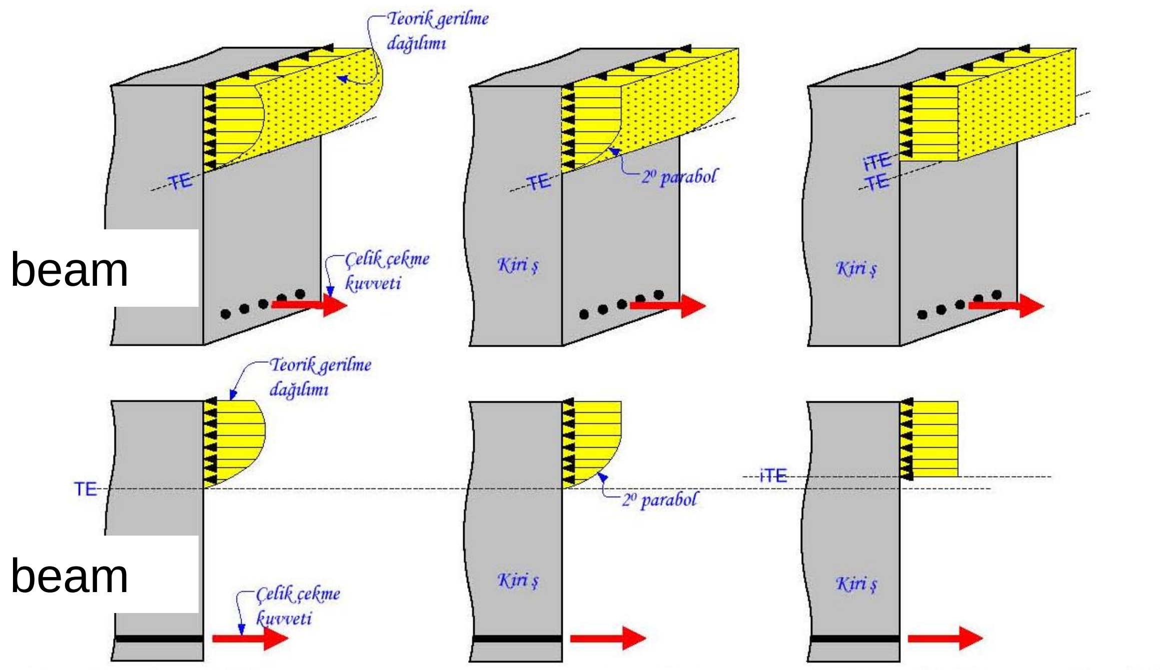
Starin increases

compressive stress block



Theoric distribution **(TS 500)**

compressive stress block



Theoritical CEB model ACI TS5000

Stress distribution at the crushing/failure of concrete

Ultimate Strength Theory

Assumptions (TS500)

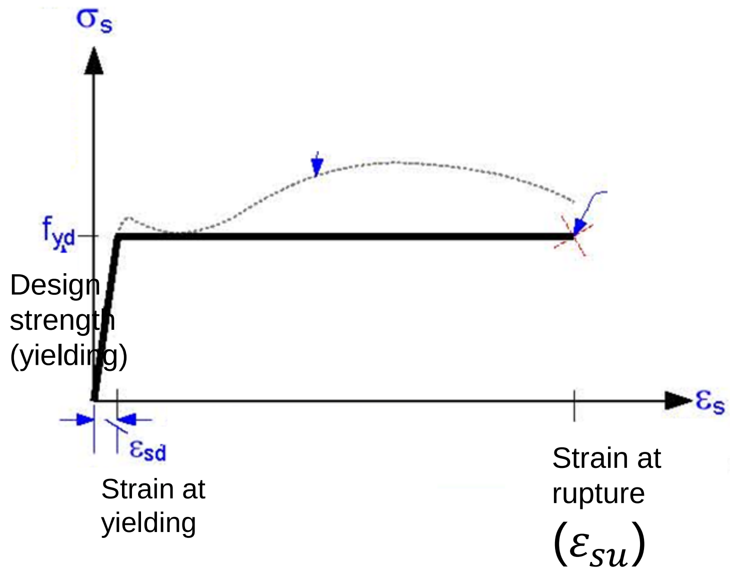
1- Strain distribution is lineer (plane sections remain plane-Bernoulli/Navier)
2- Tensile strength of concrete is neglected (~0)
3- full adherence
4- Elasto-plastic curve

$(\sigma_s - \varepsilon_s)$

$\sigma_s = E_s \varepsilon_s$ $\quad (\varepsilon_s < \varepsilon_{sd}$, before yielding) HOOKE

$\sigma_s = f_{yd}$ $\quad (\varepsilon_s \geq \varepsilon_{sd}$, after yielding)

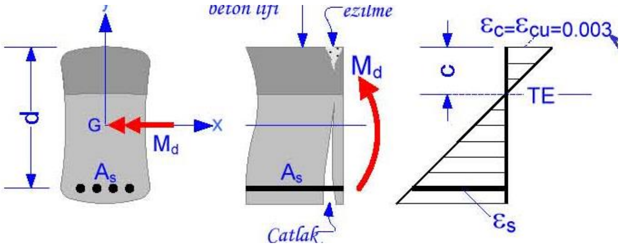
$$\sigma_s = E_s \varepsilon_s \leq f_{yd}$$



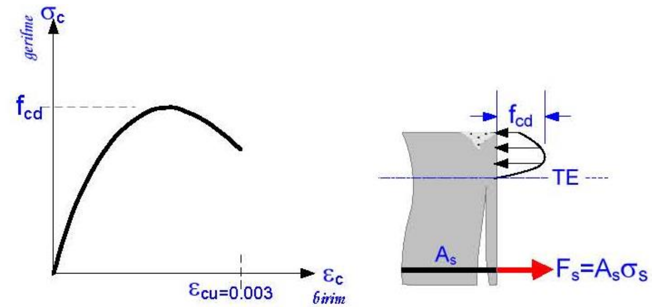
Ultimate Strength Theory

Assumptions (TS500)

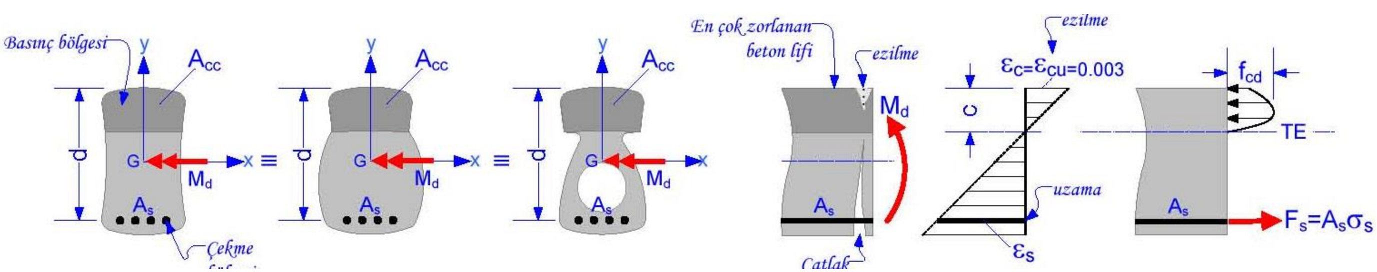
- 5- at ultimate strength;
  $\varepsilon_c = \varepsilon_{cu} = 0.003$



- 6- at ultimate strength;
  theoric σ_c -ε_c



- 7- shape of tensional region is not important



Equivalent sections

Assumptions (TS500)

- Failure/fracture types;

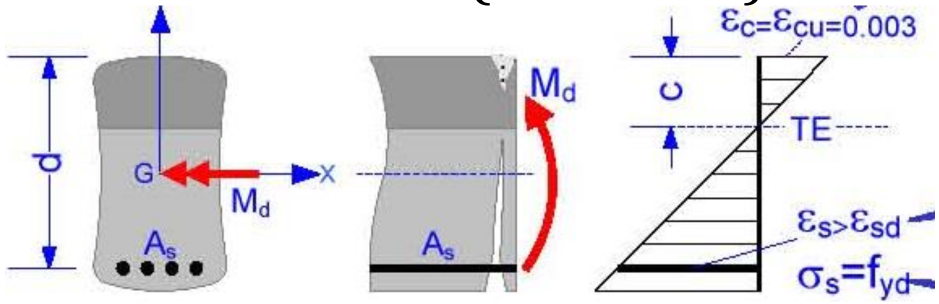
  $\varepsilon_c = \varepsilon_{cu} = 0.003 \Rightarrow$ concrete crushing

  Mr is reached failure condition limit;

At this point; there are 3 failure types due to yielding of reinforcement;

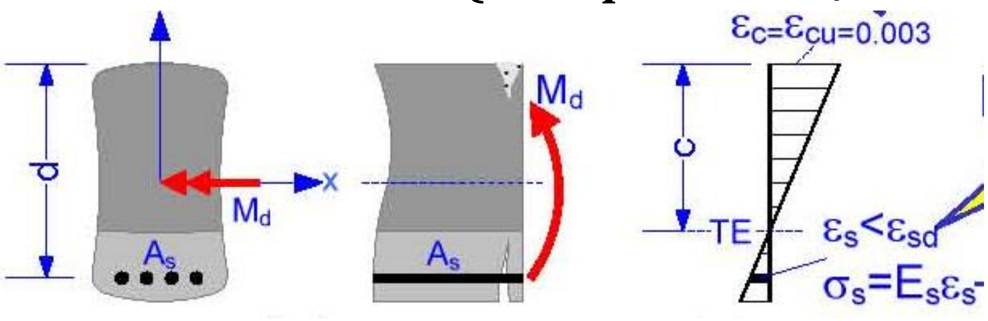
- Ductile failure (tensional)
- Brittle failure (compression)
- Failure at balance (brittle)

1. Ductile failure (tensional)



Section deformation

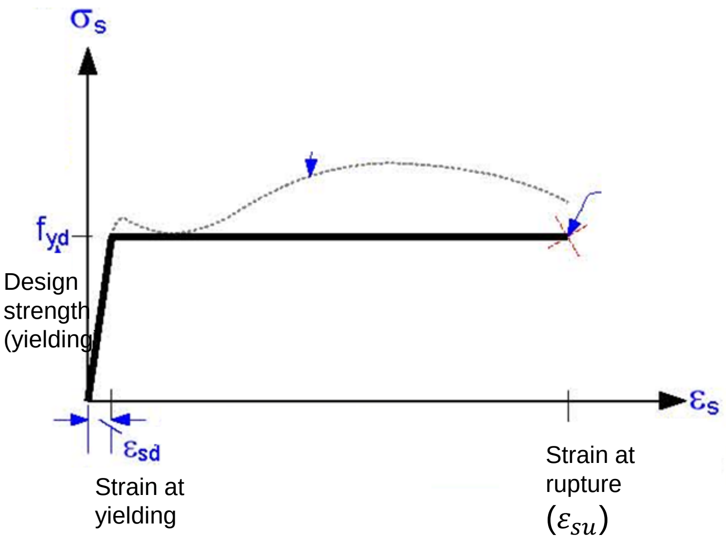
2. Brittle failure (compression)



Section deformation

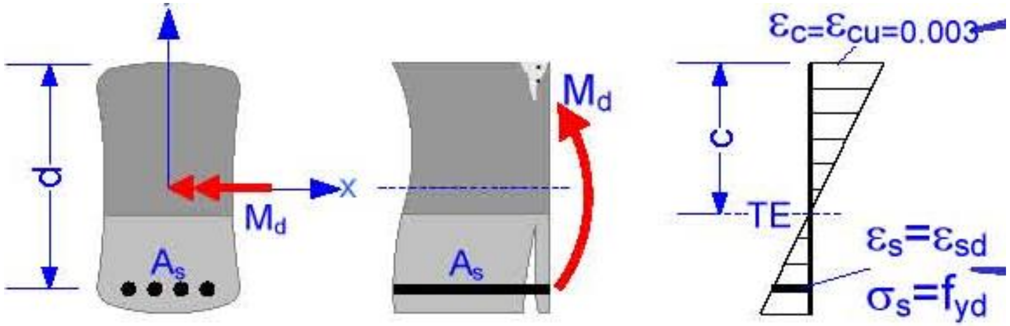
Before $\varepsilon_c = \varepsilon_{cu} = 0.003$; reinforcing bar yields ($\varepsilon_s \geq \varepsilon_{sd}$)

Firstly bar yields;
then $\varepsilon_c = \varepsilon_{cu}$ and concrete crushes.
Ductile Failure



If $\varepsilon_s < \varepsilon_{sd}$ *(before yielding)* ;
ε_c *becomes* $\varepsilon_{cu} = 0.003$; brittle failure occurs.
Firstly concrete cruches, bar does not yield.

3. Failure at balance (brittle)



Section deformation

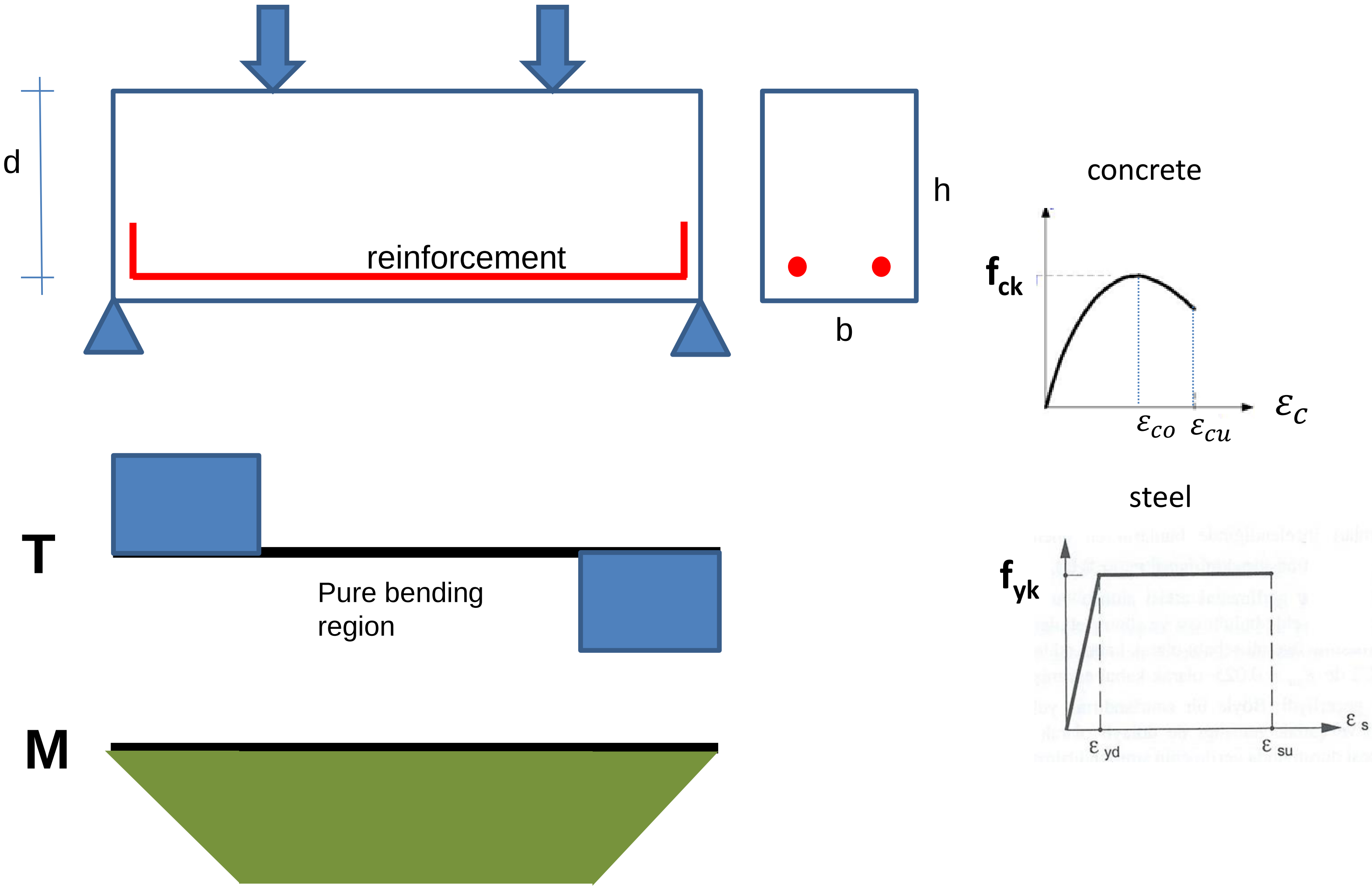
It is a special case.

When $\varepsilon_s = \varepsilon_{sd}$; $\varepsilon_c = \varepsilon_{cu} = 0.003$
Concrete cruching and yielding of bar are in the same time

No more elongation is available in Reinf. Bar.
Brittle failure

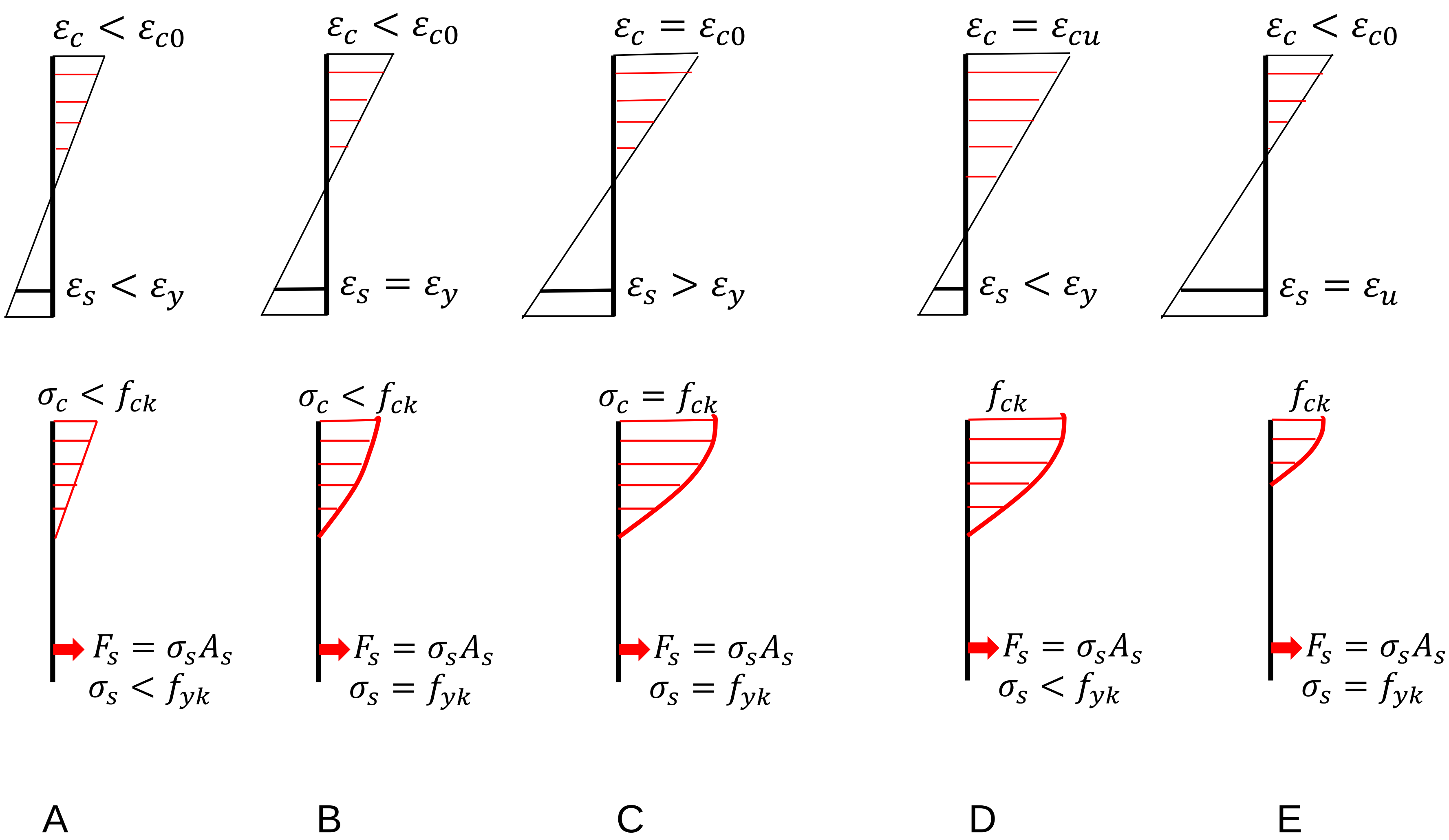
Pure bending

General



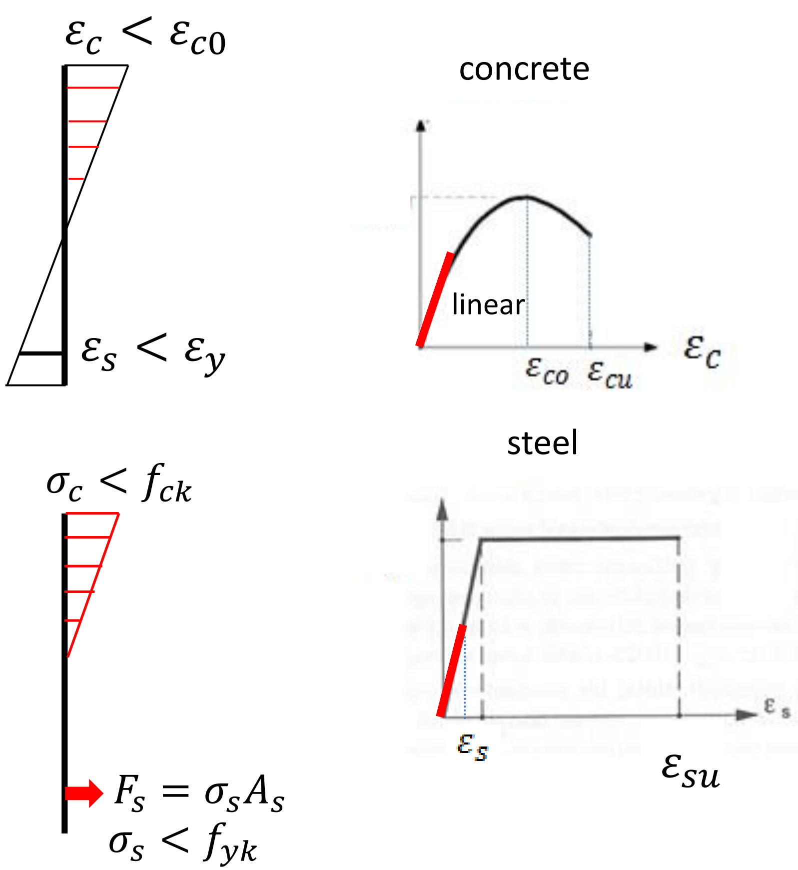
Pure bending

Stress-strain / cases

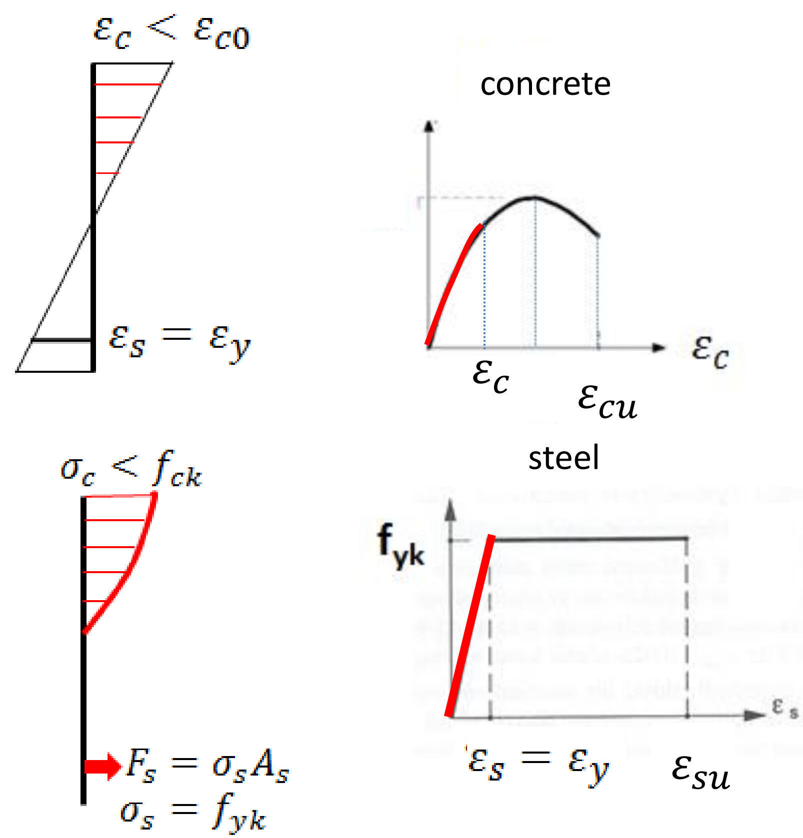


Pure bending

A : *Moment small*
Bernoulli/Novier (elastic behaviour)



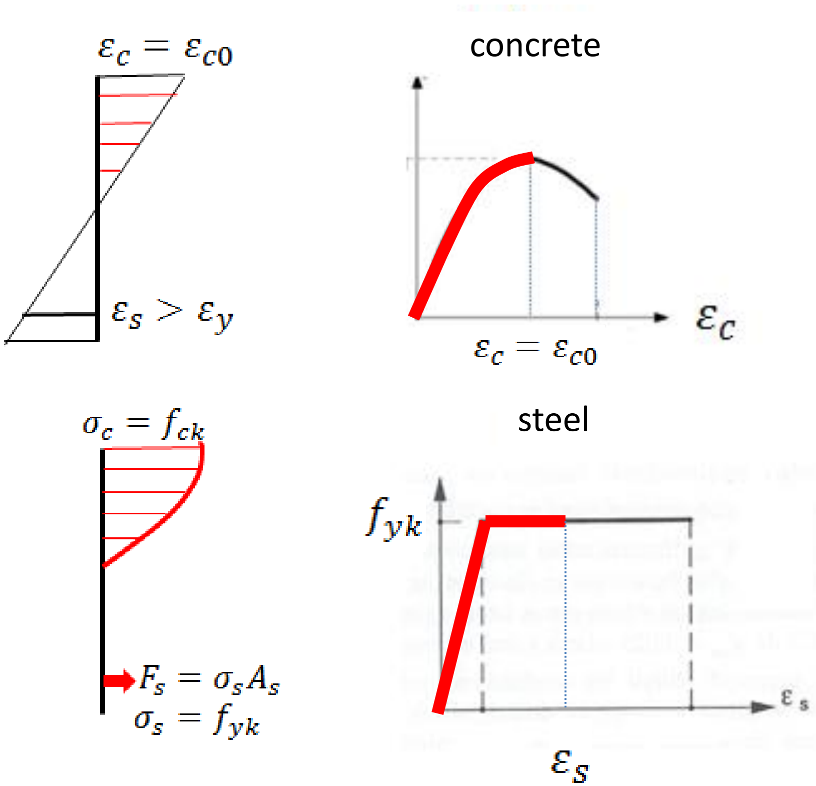
B : *Moment increases*



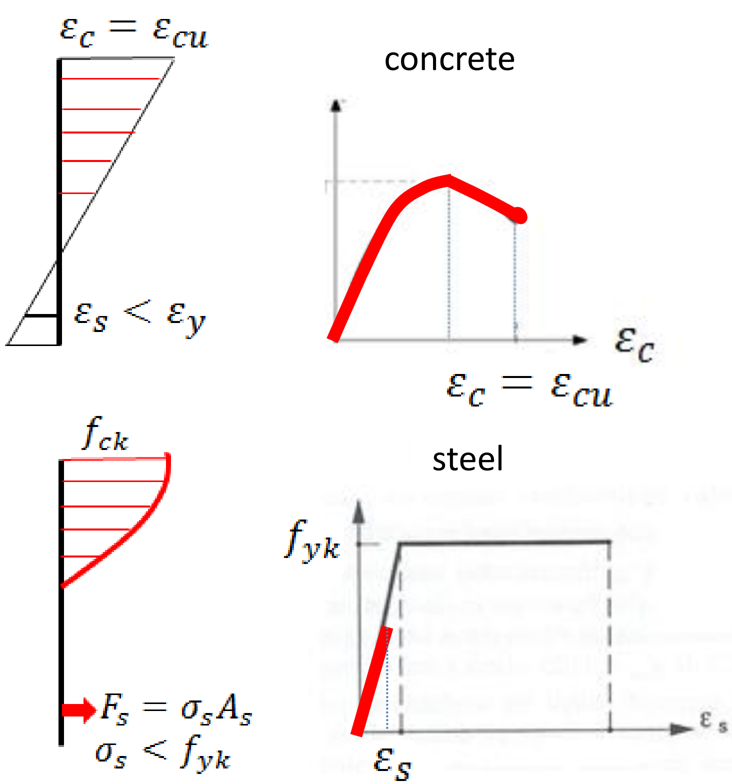
Stress-strain / cases

C : *Moment increases*

$$\varepsilon_c = \varepsilon_{c0}$$

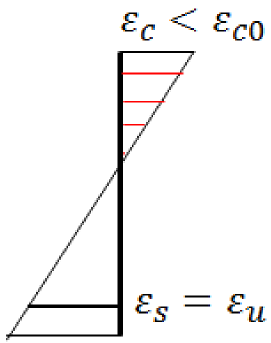


D : beam with heavy reinforcement
Brittle failure

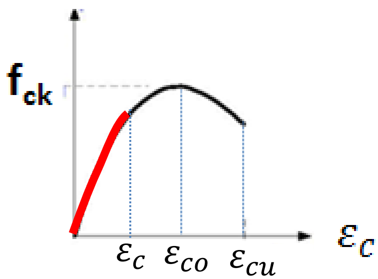


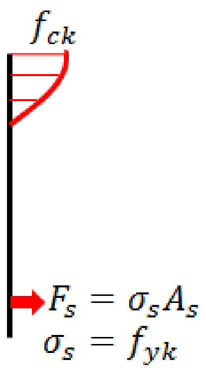
Stress-strain / cases

E: reinforcement ratio is small,
beam with weak reinforcement ratio

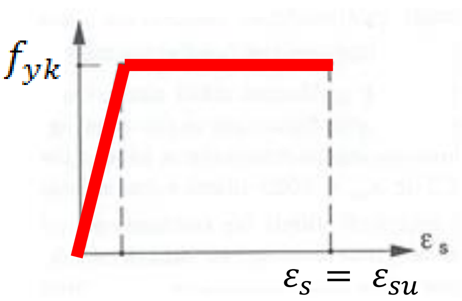


concrete

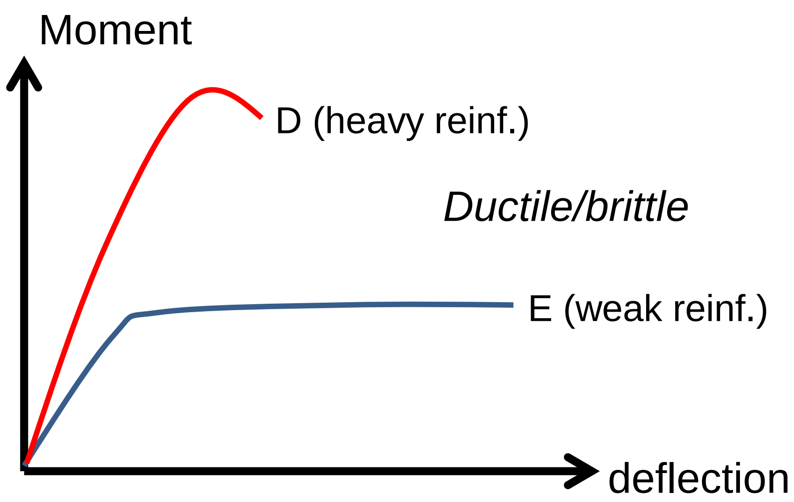


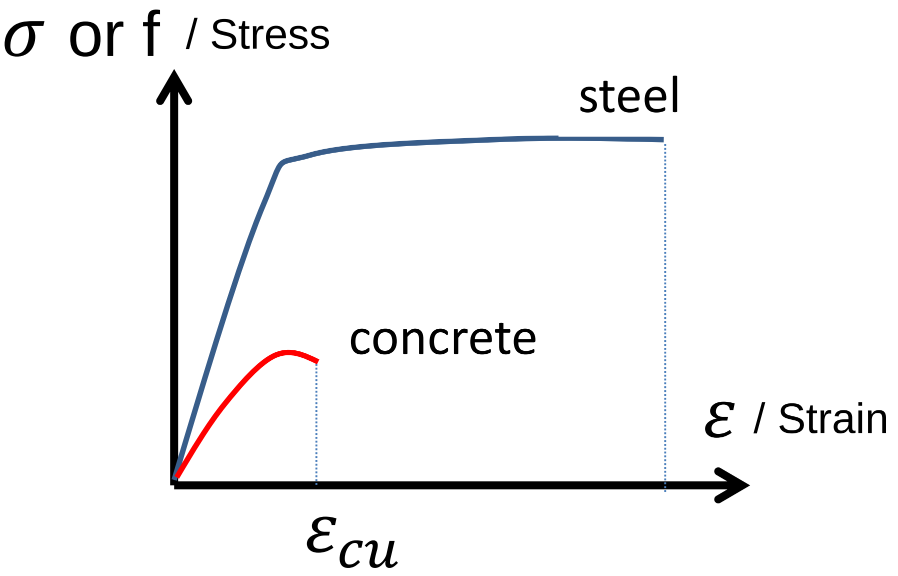


steel



Pure bending

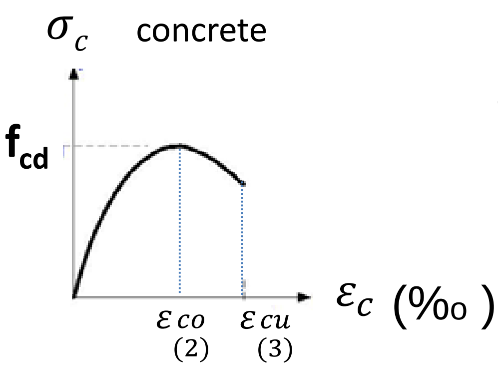


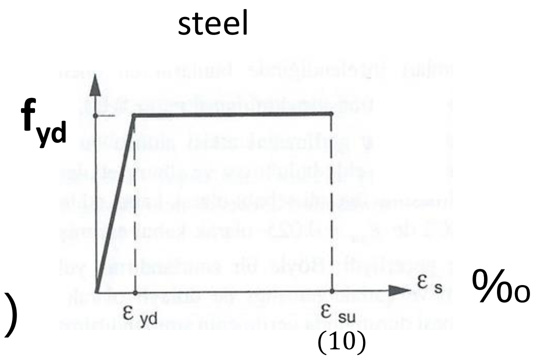


Dimensioning:

TS500; concrete ε_{cu}= %o 3

ε_{su}= %o 10





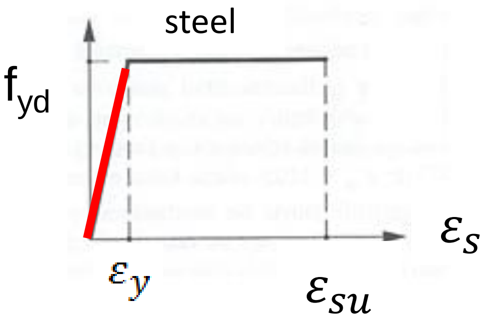
Pure bending

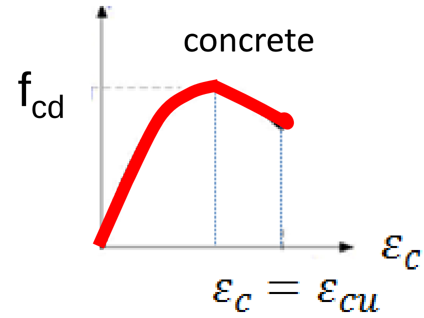
Moment capacity of rectangular section

Brittle Failure:

beam with heavy reinforcement

$\sigma_s < f_{yd}$





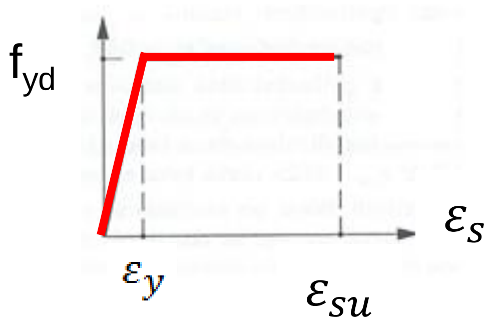
ε_{cu}= ‰ 3

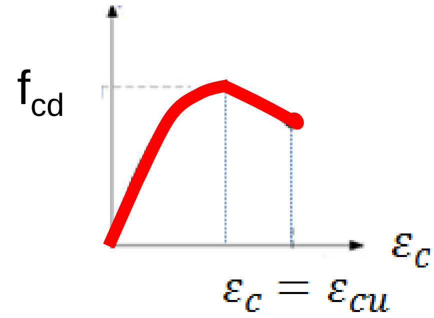
Ductile Failure:

$\sigma_s = f_{yd}$

beam with weak reinforcement

$\varepsilon_{su} > \ \varepsilon_s > \varepsilon_y$



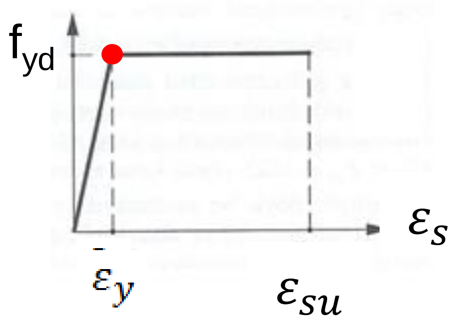


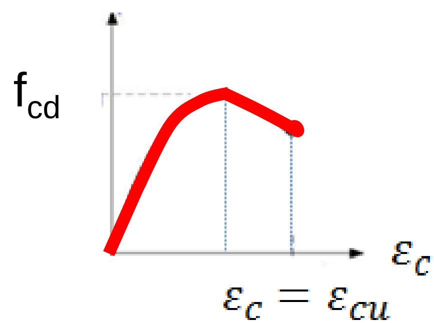
ε_{cu}= ‰ 3

Failure at Balance / Brittle Failure:

$\sigma_s = f_{yd}$

$\varepsilon_s = \ \varepsilon_y$



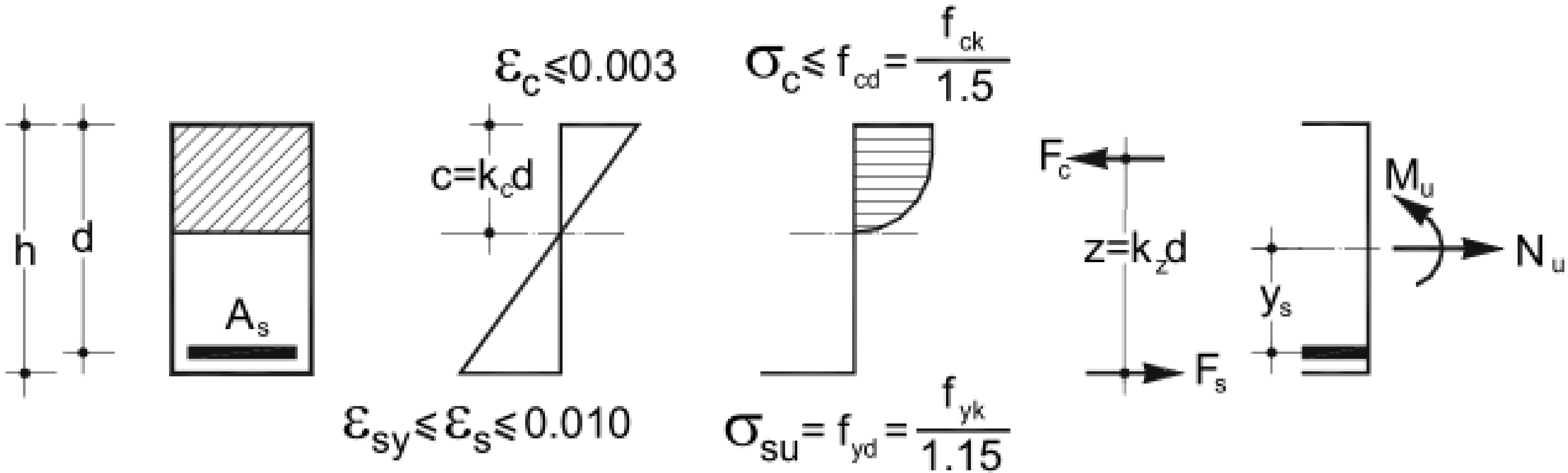


ε_{cu}= ‰ 3

Note that all the failures govern by concrete crashing

Pure bending

Moment capacity of rectangular section



Fc: compression stress resultant in concrete
Fs: tension force in reinforcement

- *0,85fcd ; (specimen size/experimental cond.)*
- *Fc =0,661. b . x . fcd = 0,661. b. d. kx. fcd* *kx=x/d ; Fs = fyd. As*
- *Equilibrium; Outer moment acts on cross-section;* *Fc = Fs*
- $Mr = b.d^2.fcd.kx.(0,661 - 0,268kx)$
- $Mr = \frac{b.d^2}{K}$; $K = \frac{1}{fcd.kx.(0,661-0,268kx)}$ *cm²/t*

Pure bending

Moment capacity of rectangular section

Moment for any point;

$$Mr = Fs.z \qquad z = \frac{Mr}{Fs} \qquad j = \frac{z}{d} \quad (dimensionless)$$

$$j = \frac{Mr}{d.Fs} = \frac{b.d^2.fcd.kx.(0{,}661 - 0{,}268kx)}{d.As.fyd}$$

$$Mr = Fs.z = As.fyd.j.d \qquad As = \frac{Mr}{fyd.j.d}$$

$$As = \frac{Mr}{fyd.j.d} = ks.\frac{Mr}{d}$$ ***ks depends on stress and strain***

$$\rho = \frac{As}{b.d}$$ ***reinforcement ratio*** $$\rho_{\min} = \frac{As}{b.d} = \mathbf{0.8}\frac{f_{ctd}}{f_{yd}}, \rho_{\max} = 0.85\rho_b$$

Ex. C20, S420 fctk=1.6MPa, fyk=420MPa

fctd=fctk/1.5=1.07MPa, fyd=fyk/1.15=365MPa

$\rho_{\min}$ =***0.002***

Pure bending

Tables for solution

- *Parameters **ks, K** are dimensional.*

For $\varepsilon_c , \varepsilon_s$; kx and ks coefficients are calculated and

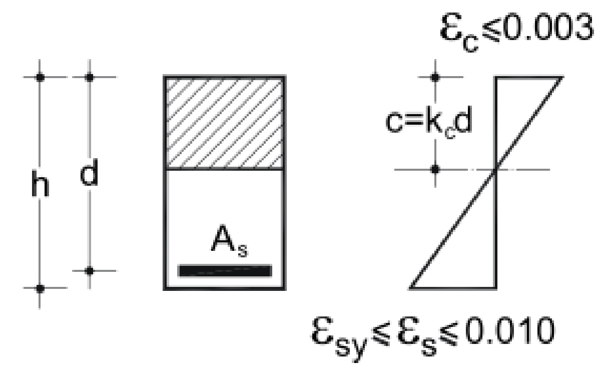
For each (fcd) K or kd; For each steel quality (fyd) ks are calculated and tabulated.

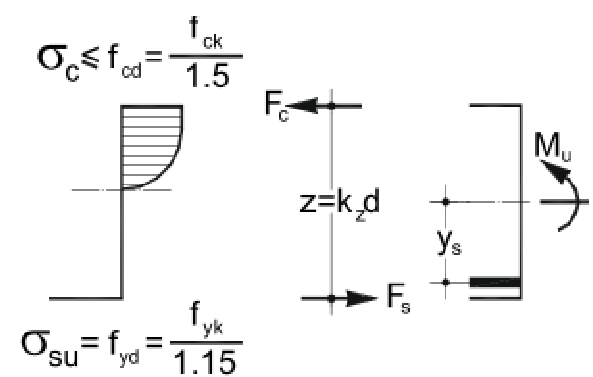
- *In tables;*
  - *the values for section at balance; last 3 row below the line.*
  - *23,24,25. rows are K* values for S500,S420 ve S220 respectively.*
    - *If K<K* shows brittle failure; increase of section or compressive reinforcement is required.*
    -
  - *0,85% of «reinf. at balance» should not be exceeded (underlined ks values)*

Tables for solution

	$K=k_d^2$					k_s			k_c	k_z	ε_c	ε_s
	C14	**C16**	**C18**	**C20**	**C25**	**S220**	**S420**	**S500**			‰	‰
1	6694.4	5857.6	5206.7	4686.1	3748.9	0.526	0.276	0.232	0.020	0.993	0.2	10
2	1778.9	1556.6	1383.6	1245.2	996.2	0.530	0.277	0.233	0.038	0.987	0.4	10
3	841.1	735.9	654.2	788.8	471.0	0.533	0.279	0.235	0.057	0.981	0.6	10
4	503.8	440.8	391.8	352.7	282.1	0.536	0.281	0.236	0.074	0.974	0.8	10
5	343.7	300.7	267.3	240.6	192.5	0.540	0.283	0.238	0.091	0.968	1.0	10
6	254.8	222.9	198.2	178.3	142.7	0.543	0.285	0.239	0.107	0.962	1.2	10
7	200.1	175.1	155.6	140.0	112.0	0.547	0.286	0.241	0.123	0.956	1.4	10
8	164.0	143.5	127.6	114.8	91.8	0.550	0.288	0.242	0.138	0.950	1.6	10
9	139.0	121.6	108.1	97.3	77.8	0.554	0.290	0.244	0.153	0.944	1.8	10
10	121.0	105.9	94.1	84.7	67.8	0.558	0.292	0.245	0.167	0.938	2.0	10
11	107.7	94.2	83.8	75.4	60.3	0.561	0.294	0.247	0.180	0.931	2.2	10
12	97.5	85.3	75.8	68.2	54.6	0.565	0.296	0.249	0.194	0.925	2.4	10
13	89.4	78.2	69.5	62.6	50.1	0.569	0.298	0.250	0.206	0.919	2.6	10
14	82.9	72.5	64.5	58.0	46.4	0.573	0.300	0.252	0.219	0.913	2.8	10
15	77.5	67.8	60.2	54.2	43.4	0.577	0.302	0.254	0.231	0.907	3.0	10
16	72.1	63.1	56.1	50.5	40.4	0.582	0.305	0.256	0.250	0.899	3.0	9
17	66.8	58.4	52.0	46.8	37.4	0.588	0.308	0.259	0.273	0.890	3.0	8
18	61.5	53.8	47.8	43.0	34.4	0.595	0.312	0.262	0.300	0.879	3.0	7
19	56.2	49.2	43.7	39.3	31.5	0.604	0.317	0.266	0.333	0.865	3.0	6
20	51.0	44.6	39.6	35.7	28.5	0.616	0.323	0.271	0.375	0.848	3.0	5
21	45.8	40.0	35.6	32.0	25.6	0.632	0.331	**0.278**	0.429	0.827	3.0	4
22	40.6	35.6	31.6	28.4	22.8	0.655	**0.343**	0.288	0.500	0.798	3.0	3
23	36.5	32.0	28.4	25.6	20.5	0.683	0.358	0.301	0.580	0.765	3.0	**2.174**
24	34.8	30.5	27.1	24.4	19.5	**0.698**	0.366	0.307	0.622	0.748	3.0	**1.826**
25	30.8	27.0	24.0	21.6	17.3	0.754	0.395	0.332	0.758	0.693	3.0	**0.956**

$$Mr = \frac{b.d^2}{K} \qquad As = ks.\frac{Mr}{d}$$



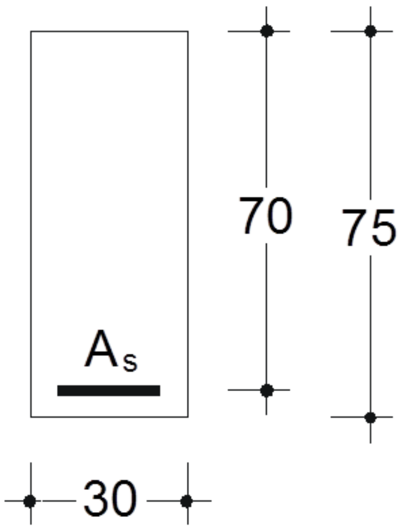


EXPLANATIONS

1. N_u axial force; tension (+), compression (-)
2. f_{yk} characteristic yield strength of reinforcement is 2.2, 4.2 and 5.0 t/cm^2 for steel grades S220, S420, S500, respectively.
3. 23, 24 ve 25. rows correspond to $K^*=k_d^{*2}$ values for balanced cross-sections of steel grades S500, S420 ve S220, respectively.
4. Underlined k_s values indicate the limits for %85 reinforcement of balanced cross-section.
5. Material coefficients are: γ_{mc}=1.50, γ_{ms}=1.15 ; Modulus of elasticity of steel is: E_s=2x10^5 N/mm^2

Examples - 1

Ex. 1:



Md=25tm
As=?
C20/S420
d'=5cm

$$d = h - d'$$

$$= 75 - 5$$

$$= 70 \text{ cm}$$

$$K = \frac{b \cdot d^2}{M_d}$$

$$= \frac{30 \times 70^2}{25 \times 100}$$

$$= 58.80 \quad \text{cm2/t}$$

Pure bending

Examples -1

	$K=k^2_d$					k_s			k_c	k_z	ε_c	ε_s
	C14	C16	C18	C20	C25	S220	S420	S500			‰	‰
1	6694.4	5857.6	5206.7	4686.1	3748.9	0.526	0.276	0.232	0.020	0.993	0.2	10
2	1778.9	1556.6	1383.6	1245.2	996.2	0.530	0.277	0.233	0.038	0.987	0.4	10
3	841.1	735.9	654.2	788.8	471.0	0.533	0.279	0.235	0.057	0.981	0.6	10
4	503.8	440.8	391.8	352.7	282.1	0.536	0.281	0.236	0.074	0.974	0.8	10
5	343.7	300.7	267.3	240.6	192.5	0.540	0.283	0.238	0.091	0.968	1.0	10
6	254.8	222.9	198.2	178.3	142.7	0.543	0.285	0.239	0.107	0.962	1.2	10
7	200.1	175.1	155.6	140.0	112.0	0.547	0.286	0.241	0.123	0.956	1.4	10
8	164.0	143.5	127.6	114.8	91.8	0.550	0.288	0.242	0.138	0.950	1.6	10
9	139.0	121.6	108.1	97.3	77.8	0.554	0.290	0.244	0.153	0.944	1.8	10
10	121.0	105.9	94.1	84.7	67.8	0.558	0.292	0.245	0.167	0.938	2.0	10
11	107.7	94.2	83.8	75.4	60.3	0.561	0.294	0.247	0.180	0.931	2.2	10
12	97.5	85.3	75.8	68.2	54.6	0.565	0.296	0.249	0.194	0.925	2.4	10
13	89.4	78.2	69.5	62.6	50.1	0.569	0.298	0.250	0.206	0.919	2.6	10
14	82.9	72.5	64.5	58.0	46.4	0.573	0.300	0.252	0.219	0.913	2.8	10
15	77.5	67.8	60.2	54.2	43.4	0.577	0.302	0.254	0.231	0.907	3.0	10
16	72.1	63.1	56.1	50.5	40.4	0.582	0.305	0.256	0.250	0.899	3.0	9
17	66.8	58.4	52.0	46.8	37.4	0.588	0.308	0.259	0.273	0.890	3.0	8
18	61.5	53.8	47.8	43.0	34.4	0.595	0.312	0.262	0.300	0.879	3.0	7
19	56.2	49.2	43.7	39.3	31.5	0.604	0.317	0.266	0.333	0.865	3.0	6
20	51.0	44.6	39.6	35.7	28.5	0.616	0.323	0.271	0.375	0.848	3.0	5
21	45.8	40.0	35.6	32.0	25.6	0.632	0.331	**0.278**	0.429	0.827	3.0	4
22	40.6	35.6	31.6	28.4	22.8	0.655	**0.343**	0.288	0.500	0.798	3.0	3
23	36.5	32.0	28.4	25.6	20.5	0.683	0.358	0.301	0.580	0.765	3.0	**2.174**
24	34.8	30.5	27.1	24.4	19.5	**0.698**	0.366	0.307	0.622	0.748	3.0	**1.826**
25	30.8	27.0	24.0	21.6	17.3	0.754	0.395	0.332	0.758	0.693	3.0	**0.956**

Required reinf.

$$A_s = \frac{k_s \cdot M_d \cdot 100}{d}$$

$$= \frac{0.3 \times 25 \times 100}{70}$$

$$= 10.714 \text{ cm}^2$$

Selected reinf.

$6\,\phi\,16$ ($12.1\ \text{cm}^2$)

$\rho_{min} = \mathbf{0.002}$

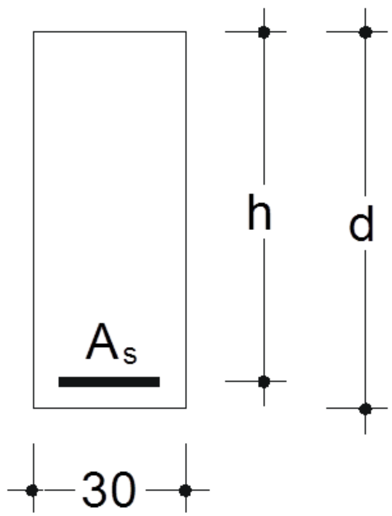
$A_{smin}=0.002.30.70=4.2\text{cm}^2$

$A_{smax}=0.343.2500/70=12.25\text{cm}^2$

Pure bending

Examples -2

Ex. 2:



M_d=25.2tm
$\varepsilon_c/\varepsilon_s$=3/8
d=?
A_s=?
C25/S420

	$K=k^2_d$					k_s			k_c	k_z	ε_c	ε_s
	C14	**C16**	**C18**	**C20**	**C25**	**S220**	**S420**	**S500**			‰	‰
1	6694.4	5857.6	5206.7	4686.1	3748.9	0.526	0.276	0.232	0.020	0.993	0.2	10
2	1778.9	1556.6	1383.6	1245.2	996.2	0.530	0.277	0.233	0.038	0.987	0.4	10
3	841.1	735.9	654.2	788.8	471.0	0.533	0.279	0.235	0.057	0.981	0.6	10
4	503.8	440.8	391.8	352.7	282.1	0.536	0.281	0.236	0.074	0.974	0.8	10
5	343.7	300.7	267.3	240.6	192.5	0.540	0.283	0.238	0.091	0.968	1.0	10
6	254.8	222.9	198.2	178.3	142.7	0.543	0.285	0.239	0.107	0.962	1.2	10
7	200.1	175.1	155.6	140.0	112.0	0.547	0.286	0.241	0.123	0.956	1.4	10
8	164.0	143.5	127.6	114.8	91.8	0.550	0.288	0.242	0.138	0.950	1.6	10
9	139.0	121.6	108.1	97.3	77.8	0.554	0.290	0.244	0.153	0.944	1.8	10
10	121.0	105.9	94.1	84.7	67.8	0.558	0.292	0.245	0.167	0.938	2.0	10
11	107.7	94.2	83.8	75.4	60.3	0.561	0.294	0.247	0.180	0.931	2.2	10
12	97.5	85.3	75.8	68.2	54.6	0.565	0.296	0.249	0.194	0.925	2.4	10
13	89.4	78.2	69.5	62.6	50.1	0.569	0.298	0.250	0.206	0.919	2.6	10
14	82.9	72.5	64.5	58.0	46.4	0.573	0.300	0.252	0.219	0.913	2.8	10
15	77.5	67.8	60.2	54.2	43.4	0.577	0.302	0.254	0.231	0.907	3.0	10
16	72.1	63.1	56.1	50.5	40.4	0.582	0.305	0.256	0.250	0.899	3.0	9
17	66.8	58.4	52.0	46.8	$\boxed{37.4}$	0.588	$\boxed{0.308}$	0.259	0.273	0.890	$\boxed{3.0}$	$\boxed{8}$
18	61.5	53.8	47.8	43.0	34.4	0.595	0.312	0.262	0.300	0.879	3.0	7
19	56.2	49.2	43.7	39.3	31.5	0.604	0.317	0.266	0.333	0.865	3.0	6
20	51.0	44.6	39.6	35.7	28.5	0.616	0.323	0.271	0.375	0.848	3.0	5
21	45.8	40.0	35.6	32.0	25.6	0.632	0.331	**0.278**	0.429	0.827	3.0	4
22	40.6	35.6	31.6	28.4	22.8	0.655	**0.343**	0.288	0.500	0.798	3.0	3
23	36.5	32.0	28.4	25.6	20.5	0.683	0.358	0.301	0.580	0.765	3.0	**2.174**
24	34.8	30.5	27.1	24.4	19.5	**0.698**	0.366	0.307	0.622	0.748	3.0	**1.826**
25	30.8	27.0	24.0	21.6	17.3	0.754	0.395	0.332	0.758	0.693	3.0	**0.956**

Pure bending

Examples -2

$$d = \sqrt{\frac{K \cdot M_d}{b}}$$

$$= \sqrt{\frac{37.4 \times 25.2 \times 100}{30}}$$

$$= 56.050$$

d=56cm

$$A_s = \frac{k_s \cdot M_d}{d}$$

$$= \frac{0.308 \times 25.2 \times 100}{56}$$

$$= 13.860\ \text{cm}^2$$

<u>Selected reinf.</u>

$5\ \phi\ 20$ (15.7 cm^2)

Pure bending

Solution by dimensionless parameters

Cross-sectional areas of reinforcing bars (rebars) [cm²]

ϕ mm	g kg/m	Number of bars 1	2	3	4	5	6	7	8	9	10
8	0.395	0.50	1.01	1.51	2.01	2.51	3.01	3.52	4.02	4.52	5.03
10	0.617	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85
12	0.888	1.13	2.26	3.39	4.52	5.65	6.79	7.92	9.05	10.18	11.31
14	1.21	1.54	3.08	4.62	6.16	7.70	9.24	10.78	12.32	13.85	15.39
16	1.58	2.01	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11
18	2.00	1.54	5.09	7.63	10.18	12.72	15.26	17.81	20.36	22.90	25.45
20	2.47	3.14	6.28	9.42	12.57	$\boxed{15.71}$	18.84	21.99	25.14	28.28	31.42

Width of the beam b_w [cm], minimum stirrup diameter ϕ_t [mm] and rebar area A_s [cm²]

ϕ	ϕ_t	3 bars		4 bars		5 bars		6 bars		7 bars		8 bars	
mm	mm	b_w	A_s	b_w	A_s	b_w	A_s	b_w	A_s	b_w	A_s	b_w	A_s
12	8	11.8	3.4	15.0	4.5	18.2	5.7	21.4	6.8	24.6	7.9	27.8	9.1
14	8	12.4	4.6	15.8	6.2	19.2	7.7	22.6	9.2	26.0	10.8	29.4	12.3
16	8	13.0	6.0	16.6	8.0	20.0	10.0	23.8	12.1	27.4	14.1	31.0	16.1
18	8	13.6	7.6	17.4	10.2	21.2	12.7	25.0	15.3	28.8	17.8	32.6	20.4
20	8	14.6	9.4	18.6	12.6	22.6	15.7	26.6	18.8	30.6	22.0	34.6	25.1
22	8	15.6	11.4	20.0	15.2	24.4	19.0	28.8	22.8	33.2	26.6	37.6	30.4
24	8	16.6	13.6	21.4	18.1	26.2	22.6	31.0	27.1	35.8	31.7	40.6	36.2