Name:
Student ID:
Lecturer:

Grade

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ISTANBUL TECHNICAL UNIVERSITY CIVIL ENGINEERING FACULTY
DYNAMICS DNK201E / 2nd Midterm Exam / Date: 29/12/2016

Question 1:

A **800-mm-long spring** having a stiffness of **k =20 kN/m** is **compressed** and confined by the plate, which can slide freely along the vertical **700-mm-long rods**. The **50-kg block** is falled with a speed of ***v*** when it is **3-m** above the plate as shown in Figure (1).

If the plate moves downwards 480-mm right after the block momentarily **stops** after striking it, what must **the speed of *v*** become? ($g = 9{,}81\ m/s^2$)

Note: Please neglect the mass and height of plate.

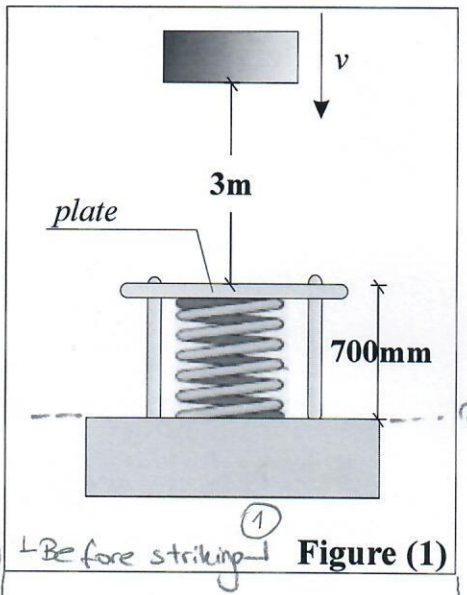


Figure (1)

(25%)

Before striking – (1)

- Potantiel energy of spring: (Spring is compressed 800 - 700 = 100mm = 0,1m initially)

$U_{sp,1} = \frac{1}{2} k x_1^2 = \frac{1}{2} \cdot 20.10^3\ N/m \cdot (0,1)^2 = 100Nm = 100J$

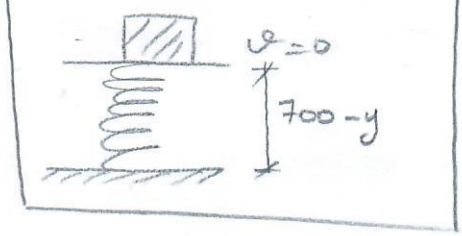
- Potantiel energy of block: ($h_1 = 3 + 0,7 = 3,7m$)

$U_{bl,1} = mgh_1 = 50 \cdot 9,81 \cdot 3,7 = 1814,9J$

- Kinetic energy of block

$V_{bl,1} = \frac{1}{2} m v^2 = 25v^2$

After striking – (2)



After striking – (2)

- Potantiel energy of spring [(0,1+0,48) compressed] $\Rightarrow U_{sp,2}$ $\frac{1}{2} \cdot 20.10^3 \cdot (0,58)^2 = 3364J$
- Potantiel energy of block $\Rightarrow U_{bl,2} = mgh_2 = 50 \cdot 9,81 \cdot (0,7 - 0,48) = 107,91J$
- Kinetic energy of block $\Rightarrow V_{bl,2} = \frac{1}{2} m \cdot 0^2 = 0$ (block momentarily stops.)

Conservation of energy: $U_{sp,1} + U_{bl,1} + V_{bl,1} = U_{sp,2} + U_{bl,2} + V_{bl,2}$

$100 + 1814,9 + 25v^2 = 3364 + 107,91 + 0$

$1914,9 + 25v^2 = 3471,91$

$25v^2 = 1557,01$

$v^2 = 62,28 \rightarrow$ **$v = 7,89\ m/s$**

$v \simeq 8\ m/s$

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Question 2:

A, B and C particles with having masses of **4kg, 6kg** and **10kg**, respectively, are approaching the origin as they slide on a frictionless table. The initial velocities are given as shown in Figure (2).

All particles arrive origin at the same time and they stick together.

a) What must the **x and y components of the initial velocity** v_c be if all three particles are moving at v_{final}=**10 m/s** in the x direction? (20P)

b) What is the **kinetic energy change** of the system? (5P)

(25%)

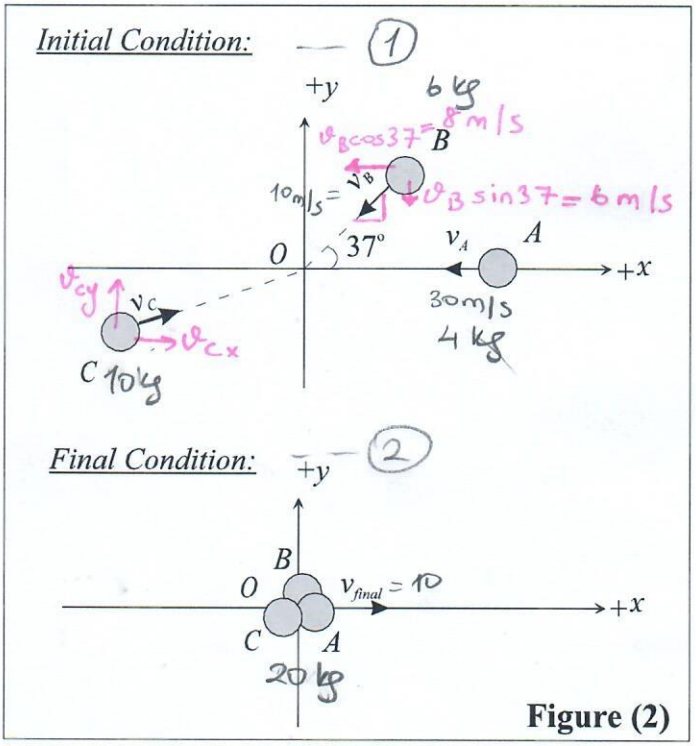

Figure (2)

* Completely inelastic collision

• Kinetic energy decreases, but momentum converges.

a) • $P_1 = P_2$ ⟨ $P_{1x} = P_{2x}$, $P_{1y} = P_{2y}$ • $K_2 < K_1$

$P_{1x} = P_{2x} \longrightarrow m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_t \cdot v_{fx}$

$4(-30) + 6.(-8) + 10(v_{cx}) = 20(+10) \rightarrow v_{cx} = +36{,}8$ m/s

$P_{1y} = P_{2y} \longrightarrow m_A v_{Ay} + m_B v_{By} + m_C v_{cy} = m_t v_{fy}$

($v_{Ay} = 0$, $v_{fy} = 0$)

$6(-6) + 10.v_{cy} = 0 \rightarrow v_{cy} = +3{,}6$ m/s

$v_c = \sqrt{36{,}8^2 + 3{,}6^2} = 36{,}98$ m/s

b) $K_2 = \frac{1}{2} m_t . v_f^2 = \frac{1}{2}.20.10^2 = 1000$ J

$K_1 = K_A + K_B + K_C = \frac{1}{2}.4.30^2 + \frac{1}{2}.6.10^2 + \frac{1}{2}.10.(36{,}98)^2$

$= 1800 + 300 + 6837{,}6 = 8937{,}6$ J

$\Delta K = K_2 - K_1 = 1000 - 8937{,}6 = -7937{,}6$ J (decrease!)

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Question 3:

The uniform slender rod has a **weight of 490.5 N** is shown in Figure (3).

Determine

a) **the horizontal and vertical components of reaction** at the pin O
b) **the angular acceleration** of the rod

just after the cable AB is cut.
($g = 9.81\ m/s^2$)

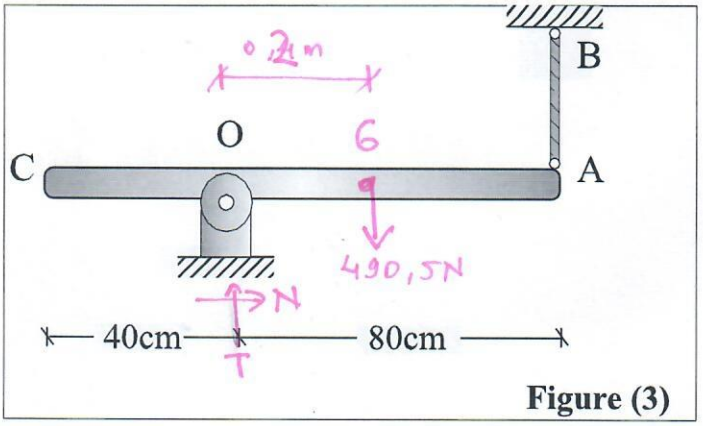

Figure (3)

$\left(I_{G,rod} = \frac{1}{12} m L^2\right)$ **(25%)**

a) $m = 490,5/9,81 = 50\,kg$

$I_O = I_G + md^2 = \frac{1}{12}.50.1,2^2 + 50.(0,2)^2 = 6 + 2 = 8\ kgm^2$

$\circlearrowleft + \Sigma M_O = I_O.\alpha \longrightarrow -490,5.0,2 = 8.\alpha \rightarrow \alpha = -12,26\ rad/s^2$

b) $+\downarrow \Sigma F_t = m(a_G)_t \longrightarrow 490,5 - T = 50.(\alpha.0,2)$

$= m(\alpha r_G)$

$T = 490,5 - 50.(-12,26.0,2)$

$T = 613,1\ N$

$\Sigma F_n = m(a_G)_n \longrightarrow N = 0$

$= m.\underbrace{\omega}_{0}^{2}.r_G$

Adı Soyadı:
Öğrenci No:
Öğretim Üyesi:

Puan ☐

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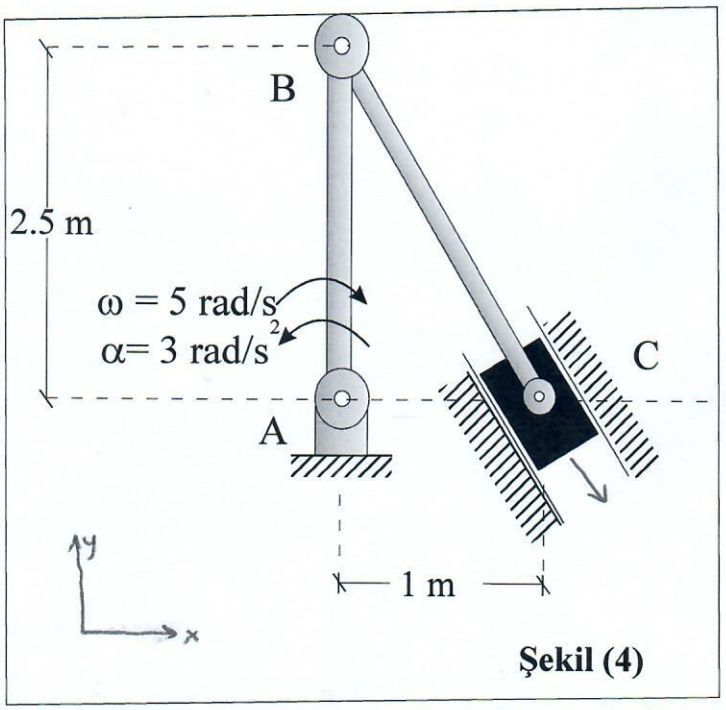
İSTANBUL TEKNİK ÜNİVERSİTESİ İNŞAAT FAKÜLTESİ
DİNAMİK DNK201 Dersi / 2. Ara Sınav / Tarih: 29/12/2016

Soru 4:

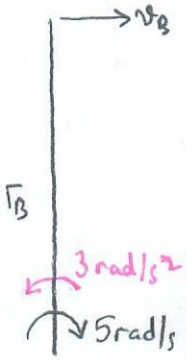
AB elemanı Şekil (4)'te görüldüğü gibi $\omega = 3$ rad/s ve $\alpha = 5$ rad/s^2 ile açısal hareket etmektedir.

Bu anda kayıcı **C bloğunun hızını ve ivmesini** belirleyiniz.



Şekil (4)

1.yol: (Bağıl Hareket) **(25%)**

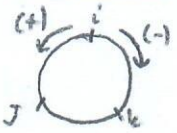


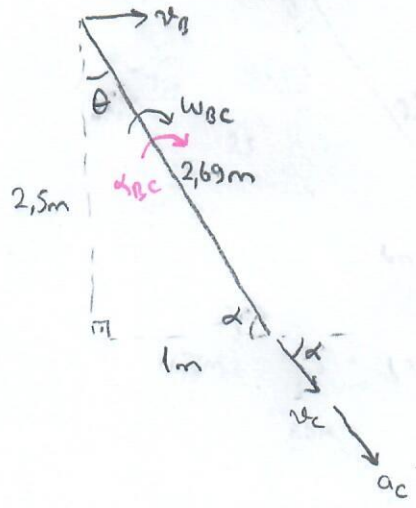
$v_B = \omega_{AB} \cdot r_B$

$= 5 \times 2,5 = \boxed{12,5 \text{ m/s}}$

$\sin\alpha = 0,93$

$\cos\alpha = 0,37$





$v_C = v_B + \omega_{BC} \times r_{C/B}$

$v_C \cdot \cos\alpha\, \vec{i} - v_C \cdot \sin\alpha\, \vec{j} = 12,5\vec{i} + (-\omega_{BC}\cdot\vec{k}) \times (\vec{i} - 2,5\vec{j})$

$0,37 v_C \vec{i} - 0,93 v_C \vec{j} = 12,5\vec{i} - \omega_{BC}\vec{j} - 2,5\,\omega_{BC}\vec{i}$

$0,37 v_C = 12,5 - 2,5\omega_{BC}$

$-0,93 v_C = -\omega_{BC}$

$\boxed{v_C = 1,08\,\omega_{BC}}$

$(0,37) \cdot 1,08\,\omega_{BC} = 12,5 - 2,5\omega_{BC}$

$\boxed{\omega_{BC} = 4,3 \text{ rad/s}}$ $\qquad$ $\boxed{v_C = 4,64 \text{ m/s}}$

$a_B = \alpha_{AB} \times r_{B/A} - \omega^2 \cdot r_{B/A}$

$= 3\vec{k} \times 2,5\vec{j} - (5)^2 \cdot (2,5\vec{j})$

$\boxed{\vec{a}_B = -7,5\vec{i} - 62,5\vec{j}}$

$\vec{a}_C = \vec{a}_B + \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2 \vec{r}_{C/B}$

$0,37 a_C \vec{i} - 0,93 a_C \vec{j} = -7,5\vec{i} - 62,5\vec{j} + (-\alpha_{BC}\vec{k}) \times (\vec{i} - 2,5\vec{j})$

$- (-4,3)^2 \cdot (\vec{i} - 2,5\vec{j})$

$= -7,5\vec{i} - 62,5\vec{j} - \alpha_{BC}\vec{j} - 2,5\,\alpha_{BC}\vec{i} - 18,49\vec{i} + 46,23\vec{j}$

$0,37\,a_c = -7,5 - 2,5\,\alpha_{BC} - 18,49$

$-0,93\,a_c = -62,5 - \alpha_{BC} + 46,23$

$\alpha_{BC} = -16,27 + 0,93\,a_c$

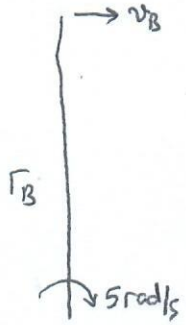
$0,37\,a_c = -7,5 - 2,5(-16,27 + 0,93\,a_c) - 18,49$

$0,37\,a_c = -7,5 + 40,68 - 2,33\,a_c - 18,49$

$2,7\,a_c = 14,69$

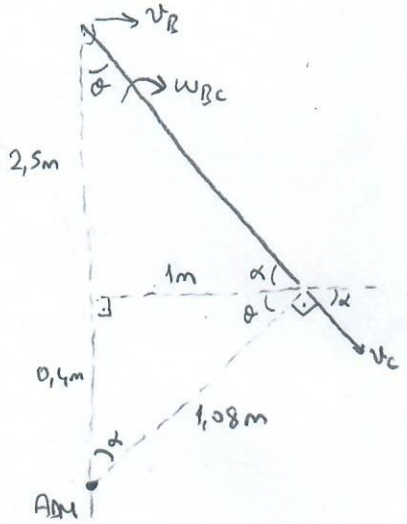
$a_c = 5,44\ m/s^2$

2.yol: (Ani dönme merkezi)



$v_B = \omega_{AB} \cdot r_B$

$= 5 \times 2,5 = \boxed{12,5 \text{ m/s}}$



$\frac{1}{2,5} = \frac{x}{1}$

$\boxed{x = 0,4m}$

$\sin\alpha = 0,93$

$\cos\alpha = 0,37$

$v_B = \omega_{BC} \cdot r_{B/ADM}$

$12,5 = \omega_{BC} \cdot (2,5+0,4)$

$\boxed{\omega_{BC} = 4,3 \text{ rad/s}}$

$v_C = \omega_{BC} \cdot r_{C/ADM}$

$v_C = 4,3 \cdot 1,08$

$\boxed{v_C = 4,64 \text{ m/s}}$

(* ivme aynıdır)