Analyzing Different Neuron Models with XPPAUT

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Abstract- Even though Hodgkin-Huxley model is the best known neuron model, there are numerous neuron models which are simpler. Simplicity of models is desired since behavior of a group of neurons is getting more important especially for computational neuroscience and analyzing large number of neurons is more plausible with simpler neuron models. In this work, three such simple neuron models are considered and their analyze are done using XPPAUT.

Index Terms- neuron models, phase portrait, XPPAUT, Hodgkin-Huxley, Izhikevich, FitzHugh-Nagumo models

I. INTRODUCTION

Modelling systems and networks taking part in cognitive processes is a major interest of neuroscience. Neuroscience has undergone significant development, especially in recent years [1. It is an interdisciplinary area which is related to different areas like biology, computer science, statistics, physic, and mathematics. Mathematical models of neuron and their dynamical behaviors became a hot topic as systems and networks composed of neurons begin to drive more and more attention [2]. Simple neuron models is especially important in modeling systems and networks, thus the subject of this study is to introduce a well-known tool used in dynamical system analysis, XPPAUT, in investigating the behavior of simple neuron models. The first and best known mathematical model of a neuron is the famous Hodgkin-Huxley neuron model. In 1963, Nobel Prize in Physiology/ Medicine is given to A. L. Hodgkin and A. F. Huxley due to this neuron model, which is biophysically accurate and computationally tractable. Though, Hodgkin-Huxley (HH) model is biologically accurate as it is 4^{th} order differential system, it is not much suitable for modeling high order systems and networks [5]. Thus models that are simpler but still capable of creating important dynamical behavior of neurons as bursting, spiking is needed. Two of such models are FitzHugh-Nagumo and Izhikevich model will be investigated in this work and their dynamical behavior will be exploited using XPPAUT [6]. In order to compare the behavior of these neuron models, HH model will also be investigated.

II. MATHEMATICAL NEURON MODELS

In mathematical neuron models, the dynamical behaviors as bursting and spiking can be observed on some circumstances when the systems are not hyperbolic and parameters fulfill some conditions which can be investigated with bifurcation diagrams. Spiking is defined a single response of extended exciting, also bursting is defined a periodic response which has definite frequency [1]. In the following three different neuron models will be given and to give an idea about bifurcation due to exciting current phase portraits with different currents will be shown.

A. Hodgkin-Huxley model of spiking neurons

Hodgkin and Huxley experimented on the giant squid axon which is a large part of nerve tissue suitable for experimentation given the technology of the time. Based on their experiments, they constructed a model and gave a mathematical explanation for the neuron dynamical behavior [4]. Their hypothesis is based on the sodium and potassium channel system. The equations are:

$$v' = (I - g_{na}h(v - v_{na})m^3 - g_k(v - v_{vk})n^4 - g_l(v - v_1))/c \quad (1)$$

$$m' = (a_m(v))(1 - v) - (b_m(v))m \quad (2)$$

$$h' = (a_h(v))(1 - v) - (b_h(v))h \quad (3)$$

$$n' = (a_n(v))(1 - v) - (b_n(v))n \quad (4)$$

Where, v is membrane potential of the neuron, m, h *and* n are membrane gating variables, I is changeable electrical current, c is membrane capacity (1μF/cm^2) , g_{na} ,g_k ,g_l are transmit currents with a maximum conductance, v_{na} is parameter of shifted Nerst equilibrium (-12 mV), v_{vk} is parameter of shifted Nerst equilibrium (120 mV), v_l is parameter of shifted Nerst equilibrium (10.6 mV), $a_m(v)$, $b_m(v)$, $a_h(v)$, $b_h(v)$, $a_n(v)$, $b_n(v)$ are parameters of membrane potential shifted by approximately 65 mV.

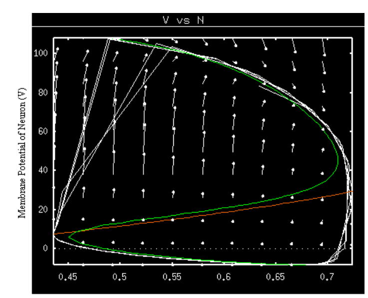

Fig 1. The Phase Portraits of Hodgkin-Huxley neuron model in XPPAUT.

The membrane potential of the neuron, gives us spiking mechanism as in Fig 2.

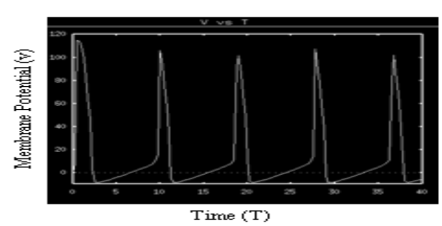

Fig 2. The solution of Hudgkin-Huxley neuron model obtained with XPPAUT.

B. Izhikevich simple model of spiking neurons

Eugene M. Izhikevich reduced the Hodgkin-Huxley neuron models to a two dimensional system of ordinary differential equations. The equations are:

$$v' = 0.04v^2 + 5v + 140 - u + I \tag{5}$$

$$u' = a(bv - u) \tag{6}$$

$$\begin{cases} v = c \\ u = u + d \end{cases} \text{ if } v \geq +30mV \tag{7}$$

Where v is membrane potential of the neuron, u is membrane recovery variable, I is changeable electrical current, a, b, c and d are dimensionless parameters.

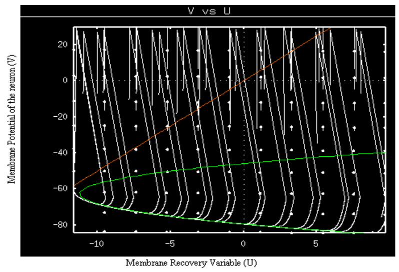

Fig 3. The Phase Portraits of Izhikevich Neuron model in XPPAUT. The solution solved in obtained with I=5 mA.

Izhikevich neuron model gives the known behavior of neuron dynamics, where different spiking and bursting signals can be observed [3]. The Izhikevich's neuron model has some opportunities, it is not only able to produce very rich neuron behavior but also can be easily used in investigating large group of neurons as much as 1000 with excitatory and inhibitory connection [3]. Another advantage is the model has real time properties.

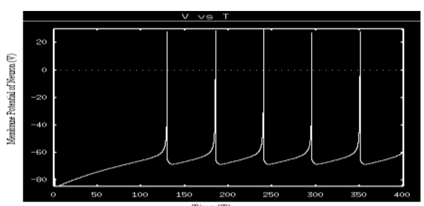

Fig 4. The solutions of Izhikevich neuron model depends on time in XPPAUT.

C. FitzHugh-Nagumo neuron model

The FitzHugh-Nagumo model is a general model which is simplified of Hodgkin-Huxley equations. The model based on two basic approaches, one of them is isolating the conceptually mathematical properties of excitation and the other one is also isolating the propagation from the electrochemical properties of sodium and potassium ion flow [7]. The model is able to give some opportunities. Such as electrical impulses, which are along nerve and cardiac fibers, are observed with this model. The Fitzhugh-Nagumo model is also versatile in explaining the spiking mechanism. The equations are:

$$v' = v(1-v)(v-a) - w + I \tag{8}$$

$$w' = a(v - bw) \tag{9}$$

Where v is fast variable of membrane potential, w is slow variable of membrane potential, I is changeable electrical current, a and b are dimensionless parameters. When we examine the equations in the XPPAUT, the model gives us various solutions which also depend on the parameter values. Especially, the electrical current value affects the solution as shown in the Fig 5.

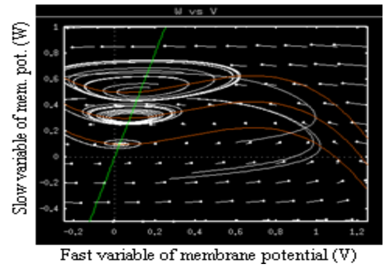

Fig 5. The solutions of FitzHugh-Nagumo neuron model depends on different electrical currents values in XPPAUT

The FitzHugh-Nagumo model has some advantages and disadvantages. The model is simple rather than HH model so this situation caused that the model is used widely. The model explains the excitation phenomenon, dynamical mechanism of spiking and the phenomenon of post-inhibitory spikes as well as HH model. There are also some disadvantages such as; The FitzHugh-Nagumo cannot explain the firing threshold exactly [7]. In Fig 3, the spiking type of solution obtained with FitzHugh-Nagumo model is given.

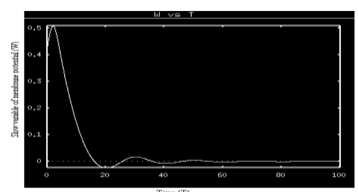

Fig 6. The solutions of FitzHugh-Nagumo neuron model depends on time in XPPAUT.

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