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NANOSATELLITE ATTITUDE ESTIMATION IN SUN AND ECLIPSE PERIODS WITHOUT GYROSCOPES

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Abstract. The paper introduces attitude estimation methods for sun/eclipse periods without gyroscope measurements. Traditional attitude estimation approach with nonlinear measurements and nontraditional approach with linear measurements are investigated relying on sensor measurements from magnetometer, and sun sensor at the first stage. For the nontraditional approach, vectors are placed in Wahba's problem and the loss function is minimized by Singular Value Decomposition (SVD) method. The outputs of the SVD are used in the extended Kalman filter (EKF) in order to estimate the satellite's attitude angles and angular rates. For the second stage in eclipse period, two additional methods are introduced as the prediction method, and traditional EKF based on only magnetometer measurements. For different phases, a switching algorithm is used for better attitude and rate estimation accuracy.

Keywords: eclipse; attitude; estimation; nanosatellite.

AMS Subject Classification: 60G35; 93E10; 93E11.

1. Introduction

Several methods of attitude estimation algorithms for the attitude control system have been developed for small satellites with precision pointing requirements of the mission. Small satellite missions are under budget restriction; therefore, require reliable and inexpensive attitude determination hardware such as coarse sun sensors and magnetometers. Kalman type filters are commonly used in order to estimate the satellite's attitude and angular rates [1–3]. The conventional approach to estimate the rotational motion parameters is to use extended Kalman filter (EKF) based on the nonlinear models of satellite motion and measurements, with Gaussian noises [4]. The flight results of the CubeSat called UVSQ-SAT based on Earth outgoing short- and long- wave radiation measurements are presented using Tri-Axial Attitude Determination (TRIAD) method and one of the Kalman filter extensions during sunlight and eclipse periods [5]. The study in [6] simultaneously estimates the attitude and deformation of a flexible satellite using 2 low-cost attitude sensors: sun sensor and magnetometer. Kalman type filters are employed for the estimations with the eclipse period examination. In [7], various sensor combinations including magnetometer, gyroscope, and sun sensor are investigated with environmental factors on the satellite attitude determination system especially the geospace storms. Different orbits with and without eclipse periods are considered in [8] for attitude estimation of small satellites.

Using at least two vector observations, attitude angles can be determined using deterministic single-frame methods such as q, QUEST, Singular Value Decomposition (SVD) [9, 10]. The vector observations from these methods can be directly used in the Kalman filters with their covariance and they are treated as linear measurements in the filter. This way, the filter gains an adaptive structure because of having variational covariance in time depending on the

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measurements [11–14]. The preprocessing in single-frame methods involves the minimization of Wahba's loss function [15]. In [9], a comparison of minimization methods concludes that the singular value decomposition (SVD) and q methods are the most robust for single-frame attitude estimation methods. The methods based on linear and nonlinear measurements can be called nontraditional and traditional methods respectively. A review of the methods for gyroless attitude determination system of a small satellite is performed in [16].

In [17], a method is proposed based on Gauss-Newton and EKF using the sun and magnetic vector observations in order to provide three-axis attitude estimation in both sun and eclipse periods. The authors concluded that the obtained results show a good performance of the attitude propagation process even in the eclipse phase. However, even the authors use a nontraditional structure for the EKF, the covariance of the measurements are taken as constant; therefore, the proposed filter is not in an adaptive structure. In [18], the prediction method is suggested for the eclipse period besides a nontraditional method in the adaptive scheme for the sun-phase but it is concluded that the accuracy of the results from the algorithm is not sufficient if the eclipse period is long-term.

In [19], an attitude determination concept for QSAT (Kyushu Satellite) micro-satellite was established for the eclipse period by using magnetometer and optionally rate gyros. It is concluded that the gyro rate sensors do not bring many benefits in the sun phase; however, it leads a better accuracy during the eclipse.

In this study, several attitude estimation methods are considered for nanosatellite's rotational motion. In the first stage, in the sun-phase, two filters based on EKF with traditional and nontraditional approaches are taken into account for a nanosatellite having magnetometer and sun sensor. For the second stage, due to the absence of the sun measurements in eclipse, two additional methods are proposed to maintain the attitude and rate estimation during this period. First, prediction method; second, only magnetometer measurements based EKF are considered. After the analysis of the results of the aforementioned methods, the best accurate methods are recommended for different periods of time. In the switching algorithm flow, the eclipse-phase method uses the last estimations of the sun-phase method when it is stopped and works recursively until the end of this period. Then, it switches again to the sun-phase method by using the last information from the eclipse-phase method at this time. By using this structure, it is expected to have a better performance on the attitude and rate estimation accuracy than the conventional methods [20].

The paper structure is organized as follows. Section 2 is for a brief information on the mathematical models for the vector observations. Section 3 and 4 describe the EKF based on nonlinear and linear measurements respectively. Then, the prediction method for satellite's rotational motion is presented in Section 5. The analysis and results are given in Section 6. As the final part of the paper, the concluding remarks are given in Section 7.

2. Mathematical Models For The Vector Observations

Magnetometer and sun sensor measurements are used in this paper as the vector observations. For this purpose, mathematical models of the magnetic field and the sun direction vectors are introduced. For obtaining the magnetic field vector (B_{IGRF}) in the orbital frame in nT, the most updated International Geomagnetic Reference Field (IGRF-12) model is used with the 3 position data and the time variable for every instant time. The IGRF used in the paper is 13th degree and updates its coefficients every 5 years [21]. To be consistent with the sun direction vector, magnetic field vector is normalized; and therefore, unitless.

$$B_o = \frac{B_{IGRF}}{\|B_{IGRF}\|} \tag{1}$$

(1) shows the direction of magnetic field vector at an instant time; therefore, the values of each component are changing between ± 1. In order to simulate the three-axis magnetometer vector observations (B_m) in body coordinates, magnetometer measurement noise (v_B) is assumed to have a zero-mean Gaussian distribution and added on the transformed magnetic field model vector (B_o) using the transformation matrix from orbit to body coordinates (A) and described as,

$$B_m = AB_o + v_B \tag{2}$$

The unit sun direction vector (S_{ECI}) can easily be obtained in Earth Centered Inertial (ECI) frame using only the Julian Day [22]. Then, it can be transformed into the orbital frame (S_o) using the orbital parameters. Similar to the magnetic field observations, three-axis sun direction vector observations (S_m) in body coordinates, sun sensor measurement noise (v_S) is assumed to have a zero-mean Gaussian distribution and added on the transformed sun direction model vector (S_o) using the transformation matrix from orbit to body coordinates (A) and described as,

$$S_m = AS_o + v_S \tag{3}$$

3. Nonlinear Measurements Based EKF

As it is discussed in the introduction part, EKF can be used in two different forms as a linear and nonlinear measurement based. In this section, the traditional EKF, which is based on the nonlinear measurements, is introduced. The noiseless satellite's rotational motion about its mass center is formulated using the discrete-time nonlinear state space model

$$x(k+1) = f[x(k)] \tag{4}$$

$$z(k) = h[x(k)] \tag{5}$$

where $f[\cdot\,]$ and $h[\cdot\,]$ are the nonlinear dynamic and measurement functions respectively, x_k is the n dimensional state vector at time k, $z(k)$ is the d dimensional measurement vector. The system is modeled using zero-mean Gaussian noise $w(k)$ with covariance $Q(k)$, and the measurements using zero-mean Gaussian noise $v(k)$ with covariance $R(k)$. It is assumed that both noise vectors $w(k)$ and $v(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean,

$$E[w(k)] = E[v(k)] = 0, \forall k \tag{6}$$

Filter algorithm based on the described system and measurements in (4) can be given. The approximations in the prediction and update stages of the filter can be found based on the EKF [2]. The estimation value can be found as,

$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1)\left\{z(k+1) - h\left[\hat{x}(k+1/k)\right]\right\} \tag{7}$$

The extrapolation value of the dynamic function can be found as,

$$\hat{x}(k+1/k) = f\left[\hat{x}(k)\right] \tag{8}$$

Filter-gain of the EKF is,

$$K(k+1) = P(k+1/k)H^T(k+1)\big[H(k+1)P(k+1/k)H^T(k+1) + R(k)\big]^{-1} \tag{9}$$

where $H(k+1) = \frac{\partial h[\hat{x}(k+1/k)]}{\partial \hat{x}(k+1/k)}$ is the partial derivatives of the measurement function with respect to the states.

The covariance matrix of the extrapolation error is,

$$P(k+1/k) = \frac{\partial f[\hat{x}(k)]}{\partial \hat{x}(k)} P(k/k) \frac{\partial f^T[\hat{x}(k)]}{\partial \hat{x}(k)} + Q(k) \tag{10}$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1) = [I - K(k+1)H(k+1)]\, P(k+1/k) \tag{11}$$

The filter expressed by the formulas (8)-(11) is called the EKF based on traditional approach. Here, all the sensors are assumed to be calibrated; therefore, there is no bias in the measurements.

The state vector considered in this paper is,

$$x(k) = \begin{bmatrix} \phi(k) & \theta(k) & \psi(k) & \omega_x(k) & \omega_y(k) & \omega_z(k) \end{bmatrix}^T \tag{12}$$

where $\phi(k), \theta(k), \psi(k)$ are the roll, pitch, yaw angles, and $\omega_x(k), \omega_y(k), \omega_z(k)$ are the x, y, z-axes angular velocity components respectively. Here, attitude problem is formulated using Euler angles as it is easy to visualize the three angles of rotation. However, it should be noted that Euler angles might be subject to singularity in some cases. Other attitude representations including quaternions and modified Rodriguez parameters are also used because of the singularity issues.

The dynamic equations of the satellite's rotational motion can be derived by the use of the angular momentum conservation law as,

$$J_x \frac{d\omega_x}{dt} = N_x + (J_y - J_z)\omega_y\omega_z \tag{13}$$

$$J_y \frac{d\omega_y}{dt} = N_y + (J_z - J_x)\omega_z\omega_x \tag{14}$$

$$J_z \frac{d\omega_z}{dt} = N_z + (J_x - J_y)\omega_x\omega_y \tag{15}$$

where J_x , J_y and J_z are the principal moments of inertia and N_x , N_y and N_z are the terms of the external disturbance torque acting on the satellite. If only ideal spherical body gravity of Earth is taken into consideration, the external disturbances are,

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = -3\frac{\mu}{r_0^3} \begin{bmatrix} (J_y - J_z)A_{23}A_{33} \\ (J_z - J_x)A_{13}A_{33} \\ (J_x - J_y)A_{13}A_{23} \end{bmatrix} \tag{16}$$

where μ is the product of the universal gravitational constant and the mass of Earth, and r_0 is the distance between the satellite and Earth's center of mass.

For the magnetometer and sun sensor based EKF, the nonlinear measurement vector $z(k)$ shown in (5) is composed of magnetometer and sun sensor as,

$$z(k) = \begin{bmatrix} B_m & S_m \end{bmatrix}^T \tag{17}$$

For only the magnetometer-based EKF, the nonlinear measurement vector $z(k)$ shown in (5) is composed of only the magnetometer as,

$$z(k) = B_m \tag{18}$$

Related measurement matrix $H(k+1)$ and the measurement error covariance matrix $R(k)$ are formed depending on the specific application.

4. Linear Measurements based SVD/EKF

The nontraditional approach uses a pre-processing step in order to obtain the linear measurements. In this paper, SVD as one of the single-frame methods is used because of its robustness [9]. Once, the SVD processed the measurements, and obtained the coarse attitude angle measurements and related covariance; EKF can use those variables which are linear attitude measurements $z(k)$ and their covariance $R(k)$. The filter is called SVD/EKF which is based on the linear measurements and it is able to estimate the state vector given in (12) as the attitude angles and the angular velocities.

The SVD method composes a matrix $B(k) = \sum_i a_i(k)b_i(k)r_i(k)^T$ where a_i is the non-negative weight, b_i is the observation vectors in body frame, and r_i is the reference vectors in orbit frame for separating it into its singular values by $B(k) = U(k)S(k)V(k)^T =$ $U(k)diag|S_{11}(k), S_{22}(k), S_{33}(k)|V^T$ where U and V are orthogonal left and right matrices, S_{11}, S_{22}, S_{33} are the primary singular values. The optimal transformation matrix can be found by,

$$A_{opt}(k) = U(k)diag[1, 1, det(U(k))det(V(k))]V(k)^T \tag{19}$$

The Euler angle measurements from SVD can be obtained using the optimal transformation matrix as,

$$\begin{bmatrix} \phi_{SVD}(k) \\ \theta_{SVD}(k) \\ \psi_{SVD}(k) \end{bmatrix} = \begin{bmatrix} arctan(-A_{opt}^{21}(k)/A_{opt}^{22}(k) \\ arcsin(A_{opt}^{23}(k) \\ arctan(-A_{opt}^{13}(k)/A_{opt}^{33}(k) \end{bmatrix} \tag{20}$$

The corresponding measurement covariance matrix is calculated by the SVD method as,

$$R(k) = U(k)diag\begin{bmatrix} [S_{22}(k) + det(U(k))det(V(k))S_{33}(k)]^{-1} \\ [det(U(k))det(V(k))S_{33}(k) + S_{11}(k)]^{-1} \\ [S_{11}(k) + S_{22}(k)]^{-1} \end{bmatrix}^T U(k)^T \tag{21}$$

SVD/EKF based on the linear measurements is applied to the same system given in (4) and (5). The new linear (Euler angle) measurements from the SVD by processing the magnetometer and sun sensor measurements can be described as,

$$z(k) = \begin{bmatrix} \phi_{SVD}(k) \\ \theta_{SVD}(k) \\ \psi_{SVD}(k) \end{bmatrix} \tag{22}$$

where $\phi_{SVD}(k), \theta_{SVD}(k), \psi_{SVD}(k)$ are the attitude angles determined by SVD method with $v_{(\cdot)}(k)$ measurement noise of the attitude angles. The measurement error covariance matrix is calculated for every step in the SVD, using this variant $R(k)$ from SVD method makes the filter adaptive. Therefore, there is no need to define a constant matrix or design an adaptive structure for the measurements in SVD/EKF method. As the measurements are linear, the measurement matrix will be a constant matrix defined as,

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

The equation of the estimation value defined in (7) can be replaced for the SVD/EKF as

$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1)\left\{z(k+1) - H\hat{x}(k+1/k)\right\} \tag{24}$$

The extrapolation value of the dynamic function can be obtained using (8). The filter gain of the SVD/EKF is,

$$K(k+1) = P(k+1/k)H^T\left[HP(k+1/k)H^T + R(k)\right]^{-1} \tag{25}$$

where $R(k)$is the covariance matrix of the measurement noise, which has diagonal elements built of the variances of attitude angle measurement noises.

The covariance matrix of the extrapolation error can be found using (9). The covariance matrix of the filtering error is,

$$P(k+1/k+1) = [I - K(k+1)H]\,P(k+1/k) \tag{26}$$

The filter expressed by the above formulas is called the SVD/EKF based on nontraditional approach. Here, all the sensors are also assumed to be calibrated as it was for the traditional EKF method.

5. Prediction of the Satellite's Rotational Motion Parameters

Besides the estimation techniques for the attitude angles and angular velocities of the nanosatellite, prediction method can be used especially in case of any malfunction or absence of the measurements. In this study, nanosatellite has sun and eclipse phases while orbiting around Earth. For these two phases, different methods are evaluated in the sense of their accuracy. Methods used for attitude and rate determination of the nanosatellite in which the phase is appropriate are discussed in the simulation results sections for each phase. The prediction algorithm characterizing the satellite's rotational motion about its center of mass can be derived for the nonlinear and non-stationary system as,

$$x\,(k+1) = f\,[x\,(k)] \tag{27}$$

where $x(k) = \left[\begin{array}{cccccc} \phi(k) & \theta(k) & \psi(k) & \omega_x(k) & \omega_y(k) & \omega_z(k) \end{array}\right]^T$ is the state vector (12). Considering the parameter estimation error small, solution of the (27) can be expressed as

$$x\,(k) \approx \hat{x}\,(k) + \sum_{l=1}^{g} \frac{\partial f}{\partial\,(\Delta x_l\,(k))} \Delta x_l\,(k)\ \ (\mathrm{l} = 1,\ldots,\mathrm{g}) \tag{28}$$

where $\hat{x}\,(k)$ is the solution based on the parameter estimate values of $x\,(k-1)$ vector. $\Delta x_l\,(k)$ is the increment of the state parameters. In the prediction of the parameters of the rotational motion of the satellite, the changes in the external moments that affect the satellite should be taken into account as,

$$x^{pr}\,(k+1) = \hat{f}\,[x^{pr}\,(k)\,, N^{pr}\,(k)] \tag{29}$$

Here, $x^{pr}\,(k)$ is the predicted values of the states,$N^{pr}\,(k) = \left[\begin{array}{ccc} N_x^{pr}\,(k) & N_y^{pr}\,(k) & N_z^{pr}\,(k) \end{array}\right]^T$ is the predicted values of the external moments. The accuracy of the prediction is determined by the covariance matrix elements characterizing the second moment of the distribution of the vector x;

$$E\left[(x - x^{pr})^2\right] = D_x^{pr} = \left(\frac{\partial \hat{f}}{\partial (\Delta x)^T}\right) P_{st} \left(\frac{\partial \hat{f}}{\partial (\Delta x)^T}\right)^T \tag{30}$$

where P_{st} is the covariance matrix of the estimation value errors at the moment that the estimation process of the $x$vector has been stopped. Substituting $\frac{\partial \hat{f}}{\partial (\Delta x)^T} = F_u$ into the expression (30) we obtain the prediction covariance matrix in the following simple form,

$$E\left[(x - x^{pr})^2\right] = D_x^{pr} = F_U P_{st} {F_U}^T \tag{31}$$

6. Analysis and Results

In this section, we give the simulation results obtained. We consider a low Earth orbiting nanosatellite having principal moments of inertia
$J = diag\begin{pmatrix} 2.1 \times 10^{-3} & 2.0 \times 10^{-3} & 1.9 \times 10^{-3} \end{pmatrix} \text{kgm}^2$. The satellite has two attitude sensors as magnetometers and sun sensors. For the magnetometer measurements, the sensor noise is characterized by zero-mean Gaussian white noise with a standard deviation of $300nT$ which is suited to 0.008 for the unit vector measurements. The measurement noise standard deviation for the unit vector is assumed to be constant after the calculations of the worst-case scenario of the direction cosine matrix [23] for less computational burden. By this way, the results are analyzed under a guaranteed error for worst-case for the whole orbit. It should be also noted that the average of the axis values is varying around 3% along the orbit. The standard deviation for the sun sensor noise is taken as 0.002 for the unit vector measurements. The eclipse period is between 3000-5000 sec and analyzed in Section 6.2 The system covariance matrix is taken as $Q = diag\begin{pmatrix} 1 \times 10^{-3} & 1 \times 10^{-3} & 1 \times 10^{-3} & 1 \times 10^{-6} & 1 \times 10^{-6} & 1 \times 10^{-6} \end{pmatrix}$.

6.1. **Simulation Results for the Sun Phase.** The first part of this study is to compare the traditional and nontraditional attitude and rate estimation approaches based on EKF. For this purpose, the nanosatellite, having a magnetometer and sun sensor onboard, has a circular orbit without any eclipse period. The attitude and rate estimations of these filters can be seen in Fig.1 and Fig.2. As it can be seen from the figures, especially the traditional EKF fails in eclipse period. Therefore, the sun phase is investigated in this section. For this purpose, Normalized Root Mean Square (NRMS) percent errors based on the actual values simulated are composed in Table I calculated as $\frac{100}{\bar{x}_{actual}(k)}\sqrt{\frac{\sum_{k=1}^{N}[x_{actual}(k)-x_{estimation}(k)]^2}{N}}$. The table is created based on the average of 10 simulations. The reference values are simulated using kinematics/dynamics equations and can be seen in Fig.1 and Fig.2 as 'Actual'. The jumps seen on the figures are caused from the yaw angle transition from +/-180 degrees, which can be eliminated during the transformations. After averaging the 10 simulation cases and for overall accuracy in the tables, it seems to get better accuracy in SVD/EKF for sun phase.

Table I indicates that both filters perform reasonably well in the sun-phase, with SVD/EKF having the smallest attitude NRMS percent error. Thus, SVD/EKF method is used for the sun phase. However, two additional methods are compared in the next sub-section in order to find efficient estimations for the satellite's attitude and angular rates during the eclipse.

Table 1. RMS errors of the attitude and angular rates for the Sun phase.

NRMS Percent Errors (%)	Traditional Approach (EKF)	Non-Traditional Approach (SVD/EKF)
ϕ	0.069	0.066
θ	0.115	0.070
ψ	0.131	0.124
ω_x	0.022	0.027
ω_y	0.056	0.020
ω_z	0.055	0.016

6.2. **Simulation Results for the Eclipse Phase.** Any satellite experiencing the eclipse during its orbit needs an eclipse operation mode if it has only sun sensors and magnetometers as attitude sensors. In this study, two methods are investigated and compared to the eclipse period. First, the only magnetometer based traditional EKF described in Section 3 is applied right after

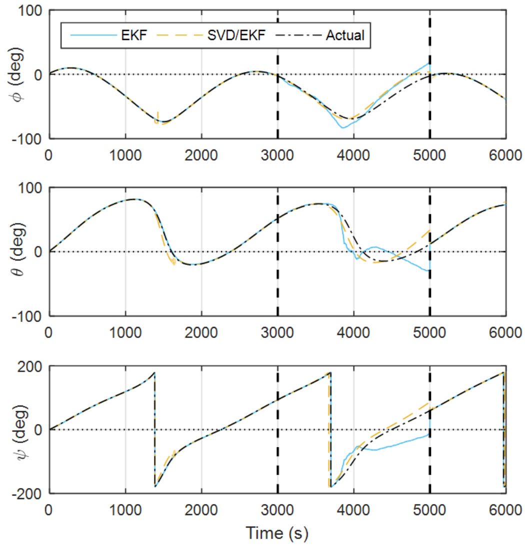

FIGURE 1. Attitude estimations using traditional EKF and nontraditional SVD/EKF wrt actual values of the satellite's attitude angles.

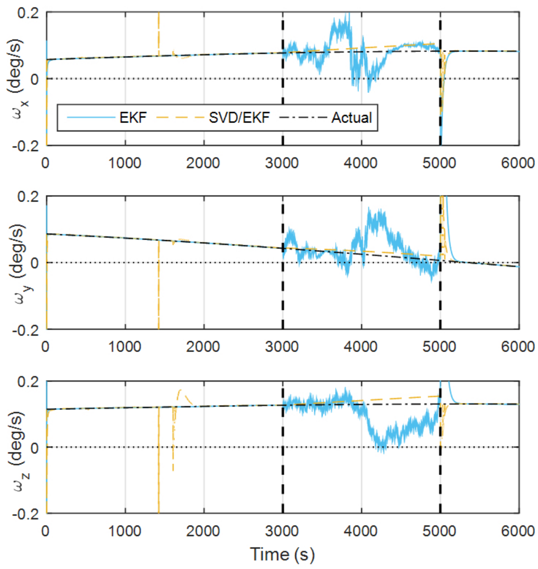

FIGURE 2. Angular rate estimations using traditional EKF and nontraditional SVD/EKF wrt actual values of the satellite's angular velocities.

the SVD/EKF method is stopped for the eclipse period. It uses the initial conditions as the last estimations of the SVD/EKF before the eclipse. For the whole eclipse period, it runs recursively. Then, in the sun-phase, SVD/EKF become active again for being switched from the only magnetometer based traditional EKF. For the second method, a prediction algorithm described in Section 5 is applied. Here, SVD/EKF is also presented in Fig.3-4 as the base model of this study with using both measurements from sun sensor (zero-output) and magnetometer. Prediction method and traditional EKF method use the initial conditions at the beginning of the eclipse from SVD/EKF. The prediction does not use any measurements whereas traditional EKF uses only magnetometers during the presented period of time.

The absolute errors of attitude and rate determination using SVD/EKF, prediction, and EKF based on only magnetometer measurements are seen in Fig.3-4. As it can be seen from the figure, only magnetometer measurements based EKF is not affected from the eclipse since it does not use any information from the Sun. The results of the algorithm running on the satellite for the eclipse period are improved by the prediction method with respect to SVD/EKF; however, if the eclipse period were expanded, the results would be deteriorated in time.

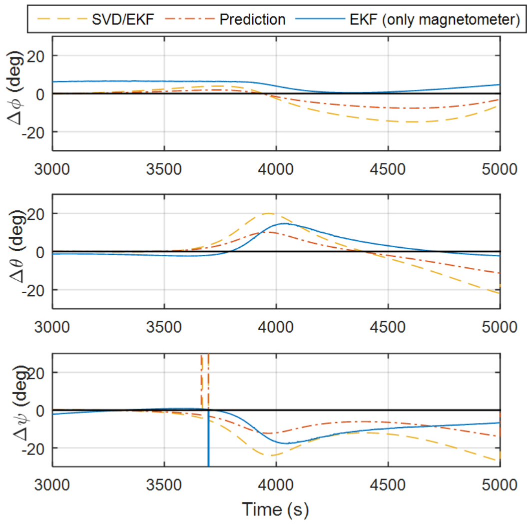

FIGURE 3. Absolute errors of the attitude angles by SVD/EKF, prediction, and only magnetometer measurements based EKF in eclipse period.

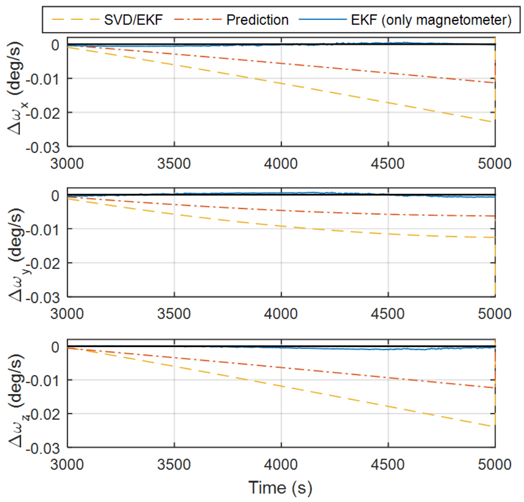

FIGURE 4. Absolute errors of the angular velocities by SVD/EKF, prediction, and only magnetometer measurements based EKF in eclipse period.

A comparison between the methods is performed for short-long term periods of the eclipse as 1000 and 2000 seconds along one orbit. Short term period is between 3000th and 4000th seconds, long-term between 3000th and 5000th seconds. NRMS percent errors for the short-long term eclipse phases can be seen in Table II-III. The values are also averaged over 10 simulations here. The attitude determination is achieved using both of the methods in each period of time considered. However, depending on the orbital parameters of the satellite one of the methods can be selected. As our case has a long-term eclipse period, we suggest using the only magnetometer based traditional EKF in the eclipse phase and SVD/EKF for the sun phase. So, in eclipse, the algorithm is switched into the only magnetometer based traditional EKF method from the base nontraditional attitude estimation approach which is using the covariance knowledge. However, if the eclipse period is not very long, the prediction of attitude parameters based on the mathematical model of the rotational motion of the satellite's center of mass can also be used.

Table 2. NRMS percent errors of the attitude for the eclipse phase.

NRMS Percent Errors (%)	Long-term			Short-term		
	ϕ	θ	ψ	ϕ	θ	ψ
SVD/EKF	0.121	0.133	0.238	0.032	0.104	0.32
Prediction	0.120	0.118	0.215	0.032	0.102	0.31
EKF (mag only)	0.063	0.074	0.079	0.084	0.052	0.09

Table 3. NRMS percent errors of the angular rates for the eclipse phase.

NRMS Percent Errors (%)	Long-term			Short-term		
	ω_x	ω_y	ω_z	ω_x	ω_y	ω_z
SVD/EKF	0.126	0.206	0.089	0.074	0.137	0.048
Prediction	0.124	0.206	0.093	0.071	0.138	0.053
EKF (mag only)	0.004	0.009	0.004	0.005	0.008	0.001

The difference of the switching algorithm using the nontraditional SVD/EKF as a base method can be seen in Fig. 5. Finally, the suggested switching algorithm based on SVD/EKF and EKF based on only magnetometer methods is presented in Fig. 6. The proposed switching algorithm flow can be seen in Fig. 7.

As a summary, both estimation filters based on linear and nonlinear measurements use two vector observations (sun sensor and magnetometer) in sun-phase for fair comparison. However, traditional filtering fails during eclipse which makes the SVD/EKF more attractive. The eclipse period is not widely investigated for the presented filters. Therefore, SVD/EKF is compared with two additional methods for eclipse. First one is the magnetometer based traditional filter and the second one is the prediction method, which does not use any measurements. As the eclipse period depends on the orbital parameters of the satellite, the method to be chosen also depends on this. If the eclipse is short-term for the specific application, prediction or SVD/EKF can be used for the attitude estimation purposes.

The errors in eclipse are accumulating while the eclipse period if sun sensor measurements are used in the filter. Therefore, the EKF based on only magnetometer measurements is suggested for this period. However, the accuracy of this method may be improved if an additional sensor is added such as rate gyros.

7. Conclusion

In this study, gyro-free attitude and angular rate estimation methods are introduced with taking into account of the sun and eclipse phases of a nanosatellite. For the sun-phase of the orbit, traditional EKF and nontraditional SVD/EKF based methods are considered in the first stage. Since they both failed in the eclipse period in which the sun sensors have zero-output

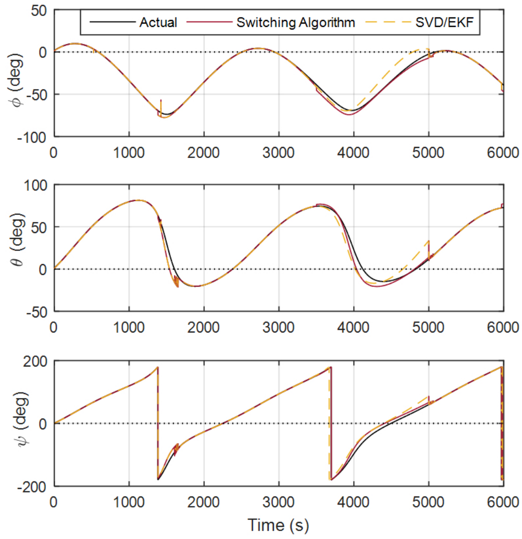

FIGURE 5. Attitude estimations using the switching algorithm, and SVD/EKF for one-orbit.

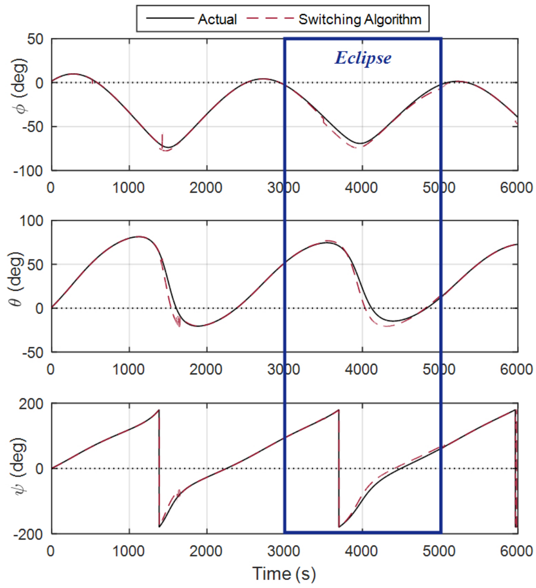

FIGURE 6. Attitude estimations using the switching algorithm.

failure, prediction method and only magnetometer based EKF method are considered in the second stage of the analysis. For this purpose, the SVD/EKF with better accuracy than the traditional EKF is stopped and the other method considered for the eclipse is stepped into until the eclipse phase is over.

The drawback of the prediction in eclipse is that it can be used for only short time intervals. If the errors are accumulated in time while there is no observation from Sun, it is recommended that only magnetometer measurements based EKF can be preferred. By using two methods, a switching algorithm is introduced with the best performance among others.

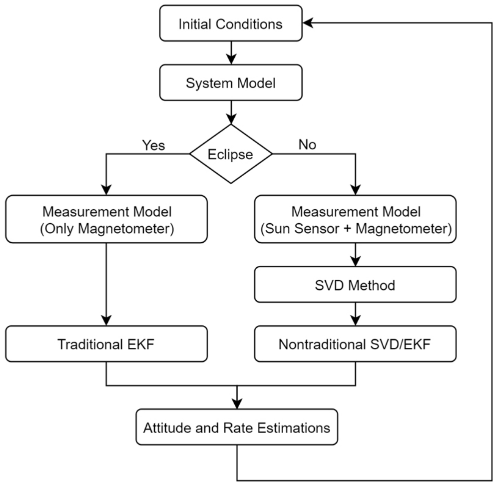

Figure 7. The sketch of the switching algorithm flow.

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