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Satellite Attitude Estimation Using SVD-Aided EKF with Simultaneous Process and Measurement Covariance Adaptation

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Abstract: The attitude estimation of a spacecraft in low Earth orbit is considered with the design of two different adaptation rules in the extended Kalman filter (EKF) algorithm. The adaptations are designed for compensating both the measurement faults and external disturbances by updating the noise covariances of the Kalman filter. First, the measurement noise covariance (R) adaptation is introduced by using the Singular Value Decomposition (SVD) as a preprocessing step in EKF design. Second, the process noise covariance (Q) adaptation rule is incorporated into the previous filter design. The rules are set simultaneously so that the filter has the capability to be robust against initialization errors, satellite dynamics modeling errors, and measurement malfunctions. Numerical simulations based on several scenarios are employed to investigate the robustness of the filter.

Keywords: Singular value decomposition, extended Kalman filter, attitude estimation, adaptive filtering, rate gyro, small satellite.

1. Introduction

Attitude and angular rate of a satellite can be estimated using a conventional Kalman filter based on nonlinear measurements of reference directions (e.g. Earth magnetic field direction, Sun direction, etc.) (Belokonov et al., 2020; Hajiyev and Cilden-Guler, 2017; Ivanov et al., 2018; Markley and Crassidis, 2014; Mashtakov et al., 2020; Sekhavat et al., 2007; Halil Ersin Soken and Hajiyev, 2014; Springmann and Cutler, 2014; Vinther et al., 2011). The measurement models in this filter are generated based on the nonlinear models of the reference directions. So, the measurements and the states are related through nonlinear equation systems. The initial values for the Kalman filter can be obtained using a single-frame method when the traditional attitude filtering approach is used (Lee et al., 2018).

In the nontraditional approach based on the linear measurements, the attitude angles are first found by using the vector measurements and applying a suitable single-frame attitude determination method (Cilden et al., 2017; Hajiyev et al., 2016; Hajiyev and Bahar, 2003; Hajiyev and Cilden-Guler, 2017; Mimasu and Van der Ha, 2009) at each recursive step. Then, these attitude angles are directly used as measurement inputs for an attitude filter such as Extended Kalman filter (EKF) or Unscented Kalman filter (UKF). The measurement model is linear in this case, since the states are directly measured. As an example, in (Hajiyev and Bahar, 2003), authors present an integrated satellite attitude determination system in which the single-

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frame estimator and EKF algorithms are combined to estimate the attitude angles and angular velocities, respectively. The single-frame attitude estimator uses the TRIAD (Three-Axis Attitude Determination) method. This method is based on computing any two analytical vectors in the reference frame and measuring the same vectors in the body coordinate system. The magnetometers, sun sensors, and horizon scanners/sensors are used as measurement devices, and three different TRIAD algorithms based on the Earth's magnetic field, sun vector, and nadir vector are proposed. In order to obtain the satellite's angular motion parameters with a desired accuracy, an EKF, whose measurement inputs are the estimates of the TRIAD algorithm, is designed.

As the measurement model is linear in the nontraditional attitude filter, the overall computational load for the attitude estimation algorithm can be decreased at a great extent. In addition, if the filter is correctly tuned the impact of failures may be reduced compared to the traditional filter. Because of these benefits the nontraditional approach for attitude filtering is a promising method for attitude estimation of small satellites including the nanosatellites (Cilden et al., 2017, 2015; Hajiyev et al., 2016; Hajiyev and Cilden-Guler, 2017; Mimasu and Van der Ha, 2009; Soken et al., 2015).

The nontraditional approach has the ability to be robust against measurement faults and has a better accuracy performance compared to a filter that adapts the measurement noise covariance matrix (R matrix) based on the innovation sequence (Cilden et al., 2017; Hajiyev and Cilden-Guler, 2017). In (Cilden et al., 2017), a nontraditional filter that integrates Singular Value Decomposition (SVD) method with the UKF, and its R-adaptive version are compared for different fault scenarios. The SVD-aided UKF is used without any further modification for sensor faults. It is concluded that the nontraditional approach is inherently robust against the sensor faults as the vector measurements are first pre-processed in the SVD before running the UKF and the R matrix for the UKF is autonomously tuned depending on the covariance for the SVD estimates.

One of the difficulties for the non-traditional attitude filters that were met in previous investigations is the tuning of the filter in terms of the process noise covariance. Variations in the disturbance torques such as the residual magnetic torque, modeling errors in the satellite's dynamics and any fault in the actuators, they all cause changes in the process noise covariance of the filter. Despite being robust to any kind of variation in the measurements due to its nature, the nontraditional attitude filter is not capable of tuning itself against this kind of process noise covariance variations. The traditional Kalman filtering algorithms including the EKF, also have the same drawback when using any of them for attitude estimation: they are not robust against the environment changes. In case of such changes, the estimation accuracy of the filter deteriorates and if the faults are long lasting, the filter may diverge. For any satellite, harsh operation conditions and anomalies in the geomagnetic field and sun are very likely in space environment. This situation leads us to a search for an Adaptive EKF (AEKF) algorithm that can be implemented in the nontraditional attitude filter.

Various algorithms are proposed for adapting the Kalman filter. One of these ideas is to scale the covariance of the filter. The parameters that are used for scaling can be calculated using different means (Hajiyev, 2007; Hajiyev and Soken, 2012; Kang et al., 2014).

In (Scardua and da Cruz, 2017), the state estimation performance of the UKF has been improved by appropriately tuning both the unscented transform parameters and the process and measurement noise covariance matrices. The tuning problem is solved with a stochastic search algorithm and a standard model-based optimizer. Optimization is performed offline by allowing the UKF to retain low original computational complexity. Therefore, the numerical results indicate how important to tune the UKF parameters properly for the UKF state estimation performance.

In (Sun et al., 2018), a variational Bayesian-based Adaptive EKF algorithm is proposed for cooperative navigation of master-slave autonomous underwater vehicles (AUVs). The predicted error covariance and measurement noise covariance matrix modeled as Inverse Wishart priors are calculated using variational Bayesian approximation along with the system states. The utilization of the predicted error covariance matrix, instead of process noise parameters, enables the estimation of the state to reflect the noise change as well as the variation of the predicted covariance. Thus, the proposed filter improves the robustness of unknown or time-varying noises accordingly.

An AEKF algorithm is proposed by (Huang et al., 2018) for solving the problem of unknown noise covariance matrices in the cooperative localization of AUVs. The predicted error covariance matrix and measurement noise covariance matrix are adaptively estimated based on an online expectation-maximization approach.

The paper (Zheng et al., 2018) proposes a robust adaptive UKF (RAUKF) to improve the accuracy and robustness of state estimation with having uncertain noise covariance. An online fault-detection mechanism is adopted to decide whether there is a need to update the current noise covariance or not. If necessary, innovation-based method and residual-based method are used to calculate the estimations of current noise covariance of process and measurement, respectively. The filter combines the last noise covariance matrices with the estimations for the new noise covariance matrices by using a weighting factor.

Like space anomalies the marine environment is also very complex; therefore, it is difficult to accurately obtain the system noise. The acoustic observations usually have gross errors, which affects the positioning accuracy of classical UKF. An RAUKF algorithm is proposed in (Wang et al., 2019) based on the Sage-Husa filter. The RAUKF can adaptively adjust the system noise to solve the divergence problem, and control the effects of gross measurement errors on dynamic state estimates.

In (Soken and Hajiyev, 2010), the authors propose a RUKF algorithm for pico-satellite attitude estimation. The algorithm is based on a covariance matching method with a set of scale factors calculated to tune the measurement noise covariance matrix of the UKF. First, the fault in the measurements is detected by means of a defined statistical function and then the filter is made robust using the calculated scale factors. In (H. E. Soken and Hajiyev, 2014), this algorithm is also tested for an EKF and the results are compared with those of the UKF. Lastly in (Hajiyev and Soken, 2014), the authors proposed an integrated algorithm which adapts both the measurement and process noise covariance matrices of the UKF for satellite attitude estimation. In all these studies (Hajiyev and Soken, 2014; H. E. Soken and Hajiyev, 2014; Soken and Hajiyev, 2010), the traditional approach for attitude estimation is used.

In this study, we extend and add on to our previous research about non-traditional filtering by proposing an adaptation method for tuning the Q matrix of the filter. The SVD method runs using the magnetometer and Sun sensor measurements as the first stage of the algorithm, and

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estimates the attitude of the satellite, giving one estimate at a single-frame. Then, these estimated attitude terms are used as inputs to the Q-adaptive EKF (AEKF). It is first shown that the SVD aided EKF approach is robust against noise increment type of fault on the measurements. Moreover, since the measurement model becomes linear when processing the measurements using the SVD, a straightforward covariance matching based adaptation can be applied on the Q matrix. As the nontraditional attitude filtering algorithm is inherently adaptive against especially noise-increment type of measurement faults, adapting the Q matrix also provides an algorithm capable of adapting the process and measurement noise covariance matrices simultaneously.

In different from the previous works of authors (Hajiyev et al., 2019, 2017), where SVD-Aided UKF adaptation is performed using single scale factors, in this study adapting the attitude filter is fulfilled using multiple scale factors. Considering that the fault may be only in one of the sensors and not all of them at the same time, a more reasonable way of adapting the attitude filter can be using multiple scale factors, instead of a single one. The multiple scale factors-based adaptation scheme has advantages because in this case the filters are modified such that the unnecessary information loss is prevented by disregarding only the data of the faulty sensor.

The rest of the paper proceeds as follows. In Section 2, the mathematical models for the nanosatellite are presented. The SVD-aided EKF algorithm is introduced in Section 3 in detail for each sub-algorithm. The proposed filter adaptation method is also presented in this section. In Section 4, the simulation results of the proposed SVD-aided AEKF algorithm for a hypothetical nanosatellite are given. In Section 5, a brief summary and conclusions are given.

2. Attitude Determination based on Vector Observations

2.1. Singular Value Decomposition Method

Singular value decomposition (SVD) method is one of the single-frame methods (SFMs) used for minimization of Wahba's loss function (Wahba, 1965) in determining the attitude of a spacecraft. At least two vectors need to be measured in order to determine the attitude using the optimal attitude matrix. The loss function is defined as (Markley and Crassidis, 2014),

$$L(\mathbf{A}) = \frac{1}{2}\sum_i a_i \,|\mathbf{b}_i - \mathbf{A}\mathbf{r}_i|^2 \tag{1}$$

$$\mathbf{B}_{\text{svd}} = \sum_i a_i \mathbf{b}_i \mathbf{r}_i^T \tag{2}$$

$$L(\mathbf{A}) = \lambda_0 - \text{tr}\left(\mathbf{A}\mathbf{B}_{\text{svd}}^T\right) \tag{3}$$

where $\mathbf{b}_i$ is the measurement vector in body coordinate system and $\mathbf{r}_i$ is the vector in reference coordinate system, $\mathbf{A}$ is the attitude transformation matrix from orbit to body coordinates, a_i is the non-negative weight, and λ_0 is the sum of non-negative weights. $\mathbf{B}_{\text{svd}}$ is a matrix which is composed of both model and measurement vectors with the weights. The matrix $\mathbf{B}_{\text{svd}}$ defined in Equation (2) has singular value decomposition:

$$\mathbf{B}_{\text{svd}} = \mathbf{U}\mathbf{S}\mathbf{V}^T = \mathbf{U}\text{diag}\left|S_{11}\; S_{22}\; S_{33}\right|\mathbf{V}^T \tag{4}$$

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$$\mathbf{A}_{opt} = \mathbf{U}\mathrm{diag}[1 \quad 1 \quad \det(\mathbf{U})\det(\mathbf{V})]\mathbf{V}^T \tag{5}$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal left and right matrices, $\{S_{11}, S_{22}, S_{33}\}$ are the primary singular values and $\mathbf{A}_{opt}$ is the optimal attitude transformation matrix determined by SVD. To examine the rotation angle error, covariance matrix can be defined as,

$$\mathbf{P}_{\mathrm{svd}} = \mathbf{U}\mathrm{diag}[(\mathrm{s}_2+\mathrm{s}_3)^{-1} \quad (\mathrm{s}_3+\mathrm{s}_1)^{-1} \quad (\mathrm{s}_1+\mathrm{s}_2)^{-1}]\mathbf{U}^T \tag{6}$$

where $s_1 = S_{11}$, $s_2 = S_{22}$, $s_3 = \det(\mathbf{U})\det(\mathbf{V})\ S_{33}$. In applications where only two vectors are available and one of them has faulty measurements, the SVD method cannot determine the attitude as at least two vector observations are required for the method to work properly.

2.2. Modeling of Earth's Magnetic Field Vector

In this paper, one vector used in the SFM in order to determine the attitude is the magnetic field vector. International Geomagnetic Reference Field (IGRF) is used as the Earth's magnetic field model in this study (Thébault et al., 2015). Model needs two inputs, position of the near-Earth spacecraft and the time, for calculating the geomagnetic field vector with inputs of r_{sc} the spacecraft's distance from Earth's center in km, $a = 6371.2$ km (magnetic reference spherical radius), θ_{sc} colatitude in deg, ϕ_{sc} longitude in deg of the spacecraft at specified time, t. From the model, the geomagnetic field vector can be obtained in orbit coordinate system as $\mathbf{B}_o$.

The magnetometer measurements can be defined using the transformed magnetic field model using the attitude transformation matrix $\mathbf{A}$ as,

$$\mathbf{B}_m(k) = \mathbf{A}(k)\mathbf{B}_o(k) + \mathbf{v}_B(k) \tag{7}$$

where $\mathbf{B}_m$ is the magnetometer measurement vector and $\boldsymbol{\upsilon}_B$ is the zero-mean Gaussian noise of the magnetometer measurements.

2.3. Modeling of Sun Direction Vector

Another vector used in this study for the attitude determination based on SFM is the sun direction vector. The reference sun direction can be calculated in the inertial frame at a specified time (Vallado, 2007). The sun sensor measurements can be defined using the transformed sun direction model from orbit to body coordinates using the attitude transformation matrix $\mathbf{A}$ as,

$$\mathbf{S}_m(k) = \mathbf{A}(k)\mathbf{S}_o(k) + \boldsymbol{\upsilon}_S(k) \tag{8}$$

where $\mathbf{S}_m$ is the sun sensor measurement vector, $\mathbf{S}_o$ is the reference sun direction vector in orbit coordinates and $\boldsymbol{\upsilon}_S$ is the zero-mean Gaussian noise of the sun sensor measurements.

2.4. Coordinate Systems

There are several coordinate systems used in this study. When utilizing the geomagnetic field and sun direction vectors, it is necessary to work with different coordinate systems at the same time. The attitude sensors have the measurements in the body coordinate system while the models are defined in the orbit system. They are both centered at the spacecraft's mass center.

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In orbit coordinates, the x-axis is pointing the displacement direction, z-axis is pointing towards the center of the Earth, and the y completes the right hand rule. Body frame deviates from the orbit frame as much as the attitude angles of the spacecraft. The spacecraft's position can be obtained in ECEF (Earth Centered, Earth Fixed). In the ECEF system, the z-axis is along with the spin axis of the Earth and pointing to the north pole. The x-axis points towards the intersection of the 0º latitude and 0º longitude and y-axis completes the right-hand rule. The ECI (Earth Centered Inertial) frame is centered at the Earth's center and z-axis pointing towards the North Pole. The x-axis points toward the vernal equinox, and y-axis completes the right hand cartesian coordinate system. Additionally, for the geomagnetic field models, MAG (Geomagnetic) coordinates are also used but not seen in the final equations. In the MAG system, the z-axis aligns with the dipole axis and the y-axis is perpendicular to the plane containing the dipole axis and the rotation axis of the Earth. The x-axis completes the right-handed system (Russel, 1971). More details about coordinate systems and transformations can be found in (Hapgood, 1992; Russel, 1971; Wertz, 2002).

3. Mathematical Models

3.1. Satellite's Rotational Motion

The kinematics equation of motion of the satellite using Euler angle representation is given as (Wertz, 2002),

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{9}$$

where ϕ is the roll angle about x axis, θ is the pitch angle about y axis, ψ is the yaw angle about z axis, p, q, r are the components of $\boldsymbol{\omega}_{BR}$ vector in body frame with respect to the reference frame. The body angular rate with respect to the inertial frame is expressed as,

$$\boldsymbol{\omega}_{BI} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T \tag{10}$$

with the relationship

$$\boldsymbol{\omega}_{BR} = \boldsymbol{\omega}_{BI} - \mathbf{A}\begin{bmatrix} 0 & -\omega_o & 0 \end{bmatrix}^T \tag{11}$$

where ω_o defines the orbital angular velocity with respect to the inertial frame, $\mathbf{A}$ is the attitude transformation matrix composed of Euler angles as,

$$\mathbf{A} = \begin{bmatrix} \cos(\theta)\cos(\psi) & \cos(\theta)\sin(\psi) & -\sin(\theta) \\ -\cos(\phi)\sin(\psi)+\sin(\phi)\sin(\theta)\cos(\psi) & \cos(\phi)\cos(\psi)+\sin(\phi)\sin(\theta)\sin(\psi) & \sin(\phi)\cos(\theta) \\ \sin(\phi)\sin(\psi)+\cos(\phi)\sin(\theta)\cos(\psi) & -\sin(\phi)\cos(\psi)+\cos(\phi)\sin(\theta)\sin(\psi) & \cos(\phi)\cos(\theta) \end{bmatrix} \tag{12}$$

For estimating the attitude and attitude rates together, rate gyro driven kinematics or the kinematic and dynamic equations of the satellite can be used. We use dynamic equations in

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this study as well. The dynamic equations of the satellite are derived based on the Euler's equations as,

$$J_x \frac{d\omega_x}{dt} = N_x + \left(J_y - J_z\right)\omega_y \omega_z , \tag{13a}$$

$$J_y \frac{d\omega_y}{dt} = N_y + \left(J_z - J_x\right)\omega_z \omega_x , \tag{13b}$$

$$J_z \frac{d\omega_z}{dt} = N_z + \left(J_x - J_y\right)\omega_x \omega_y , \tag{13c}$$

where J_x, J_y and J_z are the principal moments of inertia and N_x, N_y and N_z are the terms of the external disturbance torque acting on the satellite.

3.2. Measurement Models

Single-frame method aided filters use the SFM estimates as the observation, and the noise covariance calculations as the measurement noise covariance matrix. Since the SFM determines the coarse attitude angles, the three elements of the measurements for the filter are directly composed of these angles as,

$$\begin{aligned} z_\phi(k) &= \phi_{\text{svd}}(k) + \upsilon_\phi(k) \\ z_\theta(k) &= \theta_{\text{svd}}(k) + \upsilon_\theta(k) \\ z_\psi(k) &= \psi_{\text{svd}}(k) + \upsilon_\psi(k) \end{aligned} \tag{14}$$

where $\upsilon_{(\cdot)}(k)$ is the measurement noise of the attitude angles and $\mathbf{z}_\Phi = \left[z_\phi \quad z_\theta \quad z_\psi \right]^T$. The rate gyros are used as the measurement input to the filter as well,

$$\mathbf{z}_\omega(k) = \boldsymbol{\omega}_{BI}(k) + \boldsymbol{\upsilon}_\omega(k) \tag{15}$$

where $\boldsymbol{\omega}_{BI}$ is the body angular rate with respect to the inertial frame defined in (11), $\boldsymbol{\upsilon}_\omega$ is the zero-mean Gaussian noise of the rate gyro measurements and $\mathbf{z}_\omega = \left[z_{\omega_x} \quad z_{\omega_y} \quad z_{\omega_z} \right]^T$. The mathematical expectations and variances of the measurement noises are,

$$E\left[\upsilon_{(\cdot)}(k)\right] = 0,\ E\left[\upsilon_\phi^2(k)\right] = \text{Var}\left(\upsilon_\phi(k)\right),\ E\left[\upsilon_\theta^2(k)\right] = \text{Var}\left(\upsilon_\theta(k)\right),\ E\left[\upsilon_\psi^2(k)\right] = \text{Var}\left(\upsilon_\psi(k)\right),$$

$$E\left[\upsilon_{\omega_x}^2(k)\right] = \text{Var}\left(\upsilon_{\omega_x}(k)\right),\ E\left[\upsilon_{\omega_y}^2(k)\right] = \text{Var}\left(\upsilon_{\omega_y}(k)\right),\ E\left[\upsilon_{\omega_z}^2(k)\right] = \text{Var}\left(\upsilon_{\omega_z}(k)\right). \tag{16}$$

4. SVD-Aided EKF for Attitude and Rate Estimation

4.1. Statement of the Filtering Problem

For the full-state estimation of the spacecraft, the state vector $(\mathbf{x})$ is composed of attitude angles and angular rates as,

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$$\mathbf{x} = \begin{bmatrix} \phi & \theta & \psi & \omega_x & \omega_y & \omega_z \end{bmatrix}^T \tag{17}$$

The process and measurement models can be expressed in the discrete-time as,

$$\hat{\mathbf{x}}(k+1) = \mathbf{f}\left[\hat{\mathbf{x}}(k), k\right] + \mathbf{w}(k) \tag{18}$$

$$\mathbf{z}(k) = \mathbf{H}\ \hat{\mathbf{x}}(k) + \boldsymbol{\upsilon}(k) \tag{19}$$

where $\mathbf{f}[\cdot]$ is the nonlinear state transition function mapping the previous state to the current state, which is presented in (9) and (13) for each state, $\mathbf{w}$ is the zero-mean Gaussian noise of the process, $\mathbf{z}$ is the 6x1 measurement vector that is composed of (14) and (15), $\mathbf{H}$ is 6x6 measurement matrix, and $\boldsymbol{\upsilon}$ is the zero-mean Gaussian noise of the measurements.

It is assumed that both measurement and process noise vectors $\boldsymbol{\upsilon}$(k)=$\begin{bmatrix} \upsilon_\phi(k) & \upsilon_\theta(k) & \upsilon_\psi(k) & \upsilon_{\omega_x}(k) & \upsilon_{\omega_y}(k) & \upsilon_{\omega_z}(k) \end{bmatrix}^T$ and $\mathbf{w}(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean and the corresponding covariances are,

$$E\left[\mathbf{w}(i)\mathbf{w}^T(j)\right] = \mathbf{Q}(i)\delta(ij),\ \ E\left[\boldsymbol{\upsilon}(i)\boldsymbol{\upsilon}^T(j)\right] = \mathbf{R}(i)\delta(ij) \tag{20}$$

where $\delta(ij)$ is the Kronecker symbol. It is assumed that the process and measurement noises are uncorrelated as,

$$E\left[\mathbf{w}(i)\boldsymbol{\upsilon}^T(j)\right] = 0,\ \forall i, j \tag{21}$$

4.2. SVD-Aided EKF Algorithm

The state vector given in (17) can be estimated based on the given data in the statement of the filtering problem by the filtering technique given in this subsection.

The state vector estimate is,

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{x}}(k+1/k) + \mathbf{K}(k+1) \times \left\{\mathbf{z}(k) - \mathbf{H}\ \hat{\mathbf{x}}(k+1/k)\right\} \tag{22}$$

where $\mathbf{z}(k) = \begin{bmatrix} \mathbf{z}_\Phi(k) & \mathbf{z}_\omega(k) \end{bmatrix}^T$ is the measurement vector, $\mathbf{H}$ is the measurement matrix. For our case, the measurement matrix is a 6x6 unit matrix. The extrapolation value is,

$$\hat{\mathbf{x}}(k+1/k) = \mathbf{f}\left[\hat{\mathbf{x}}(k), k\right] \tag{23}$$

The gain of the filter is,

$$\mathbf{K}(k+1) = \mathbf{P}(k+1/k)\mathbf{H}^T \times \left[\mathbf{H}\ \mathbf{P}(k+1/k)\mathbf{H}^T + \mathbf{R}(k)\right]^{-1} \tag{24}$$

The covariance matrix of the extrapolation error is,

$$\mathbf{P}(k+1/k) = \frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} \mathbf{P}(k/k) \frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k), k]}{\partial \hat{\mathbf{x}}(k)} + \mathbf{Q}(k) \tag{25}$$

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where $\frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial \hat{\mathbf{x}}(k)} = \left(\frac{\partial f_i[\hat{\mathbf{x}}(k),k]}{\partial \hat{x}_j(k)} : i = \{1,...,6\}; j = \{1,...,6\} \right)$ with i representing the state transition function number and j representing the state number with the sequence given in (17).

The covariance matrix of the filtering error is,

$$\mathbf{P}(k+1/k+1) = \left[\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}\right]\mathbf{P}(k+1/k) \tag{26}$$

The filter is represented by the Equations (22) – (26). Here, the key parameters for the adaptations in the filter are $\mathbf{R}$, which is the measurement noise covariance matrix, and $\mathbf{Q}$, which is the process noise covariance matrix. SVD method is used in the first stage to determine the attitude measurements contained in $\mathbf{z}$ vector, and updating the measurement noise covariance matrix, $\mathbf{R}$. The determined attitude measurements can also be used as the initial states in the filtering stage. By this way, there is no need to use an arbitrarily chosen initial attitude.

The difference between the actual system and the predicted system based on the process model represents the process noise, whereas, the additive term in the sensor measurements on top of the reference model in the same coordinate system represents the measurement noise (Venkatasubramanian et al., 2003; Villez et al., 2011). As known (Höfling and Pfeufer, 1994), two main groups, additive and multiplicative faults are distinguished. The faults adding to the signal, are generally called additive or bias type faults. The additive fault leads to a change in the expected value of the signal. The faults multiplying with the signal are called multiplicative or noise increment type faults. These faults lead to changes in the covariance of the signal.

One of the difficulties for the nontraditional attitude filters is the tuning of the filter's process noise covariance matrix, $\mathbf{Q}$ (Hajiyev et al., 2017). Especially, the process noise covariance needs to be tuned when the environment changes (Hajiyev et al., 2019). Here, any condition change is referred to as a change that may affect the process model of the filter. These might include uncertainties in the inertia parameters, disturbance torques (as it may happen when the satellite goes in/out of the eclipse) as well as the changes in the controller parameters (such as a malfunction) although a controller is not considered in this paper.

5. SVD-Aided Q-Adaptive EKF

The SVD method runs as the first stage of the algorithm and estimates the attitude of the satellite giving one estimate at a single-frame. Then, these estimated attitude terms are given as inputs to the EKF. This forms the SVD-Aided EKF scheme. An adaptation using process noise covariance matrix is also added to this filter called SVD-Aided AEKF. The SVD-Aided AEKF algorithm scheme is summarized in the block diagram in Fig. 1.

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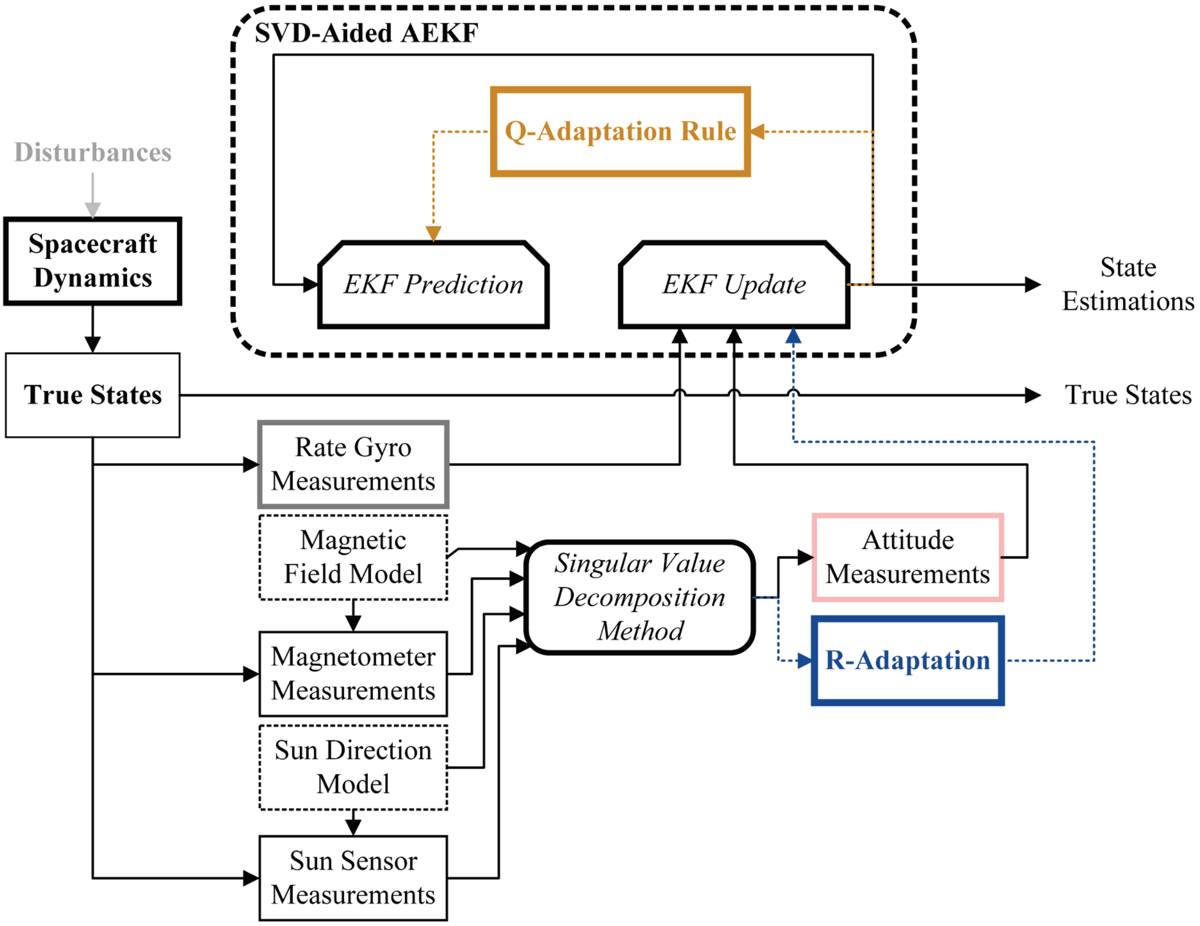

Fig. 1. Attitude and angular velocity estimation using SVD-Aided AEKF algorithm.

Here, there are two measurements used in SVD for attitude determination: magnetic field and sun direction. The determined attitude and corresponding covariance ($\mathbf{R}$) are the filter's inputs from the SVD for the inherently R-adaptive filtering capabilities. In addition to SVD's observations, rate gyro measurements are used as well in the full-state estimation process. Also, one more adaptation rule (Q-adaptation) is defined for making the filter adaptive to the environmental changes. This way, the filter is expected to be robust against process noises and measurement faults at the same time. It should be noted that $\mathbf{Q}$ matrix is updated within the filter not in the SVD method. Only the measurement noise covariance ($\mathbf{R}$) adaptation is computed in SVD.

In case of process noise covariance change, the real error of the innovation covariance will exceed the theoretical one. Thus, the basic of the Q-adaptation is to obtain an appropriate multiplier matrix for the $\mathbf{Q}$ matrix such that the real and theoretical values of the innovation covariance could match.

The innovation of the EKF in Equations (22) - (26) is,

$$\tilde{\mathbf{e}}(k) = \mathbf{z}(k) - \mathbf{H}\ \hat{\mathbf{x}}(k+1/k) = \mathbf{z}(k) - \mathbf{H}\ \mathbf{f}\left[\hat{\mathbf{x}}(k), k\right] \tag{27}$$

In order to adapt the covariance, an adaptive scaling factor is put into the procedure. Hence, if a scaling matrix ($\mathbf{\Lambda}$) built of multiple scaling factors, is added in to the algorithm as,

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$$\mathbf{P}(k+1/k)=\frac{\partial\mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\mathbf{P}(k/k)\frac{\partial\mathbf{f}^T[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}+\mathbf{\Lambda}(k)\mathbf{Q}(k) \tag{28}$$

The scaling matrix $\mathbf{\Lambda}$ can be determined from equality (Hajiyev and Soken, 2012),

$$\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}\tilde{\mathbf{e}}(j)\tilde{\mathbf{e}}^T(j)\approx\mathbf{H}\;\mathbf{P}(k+1/k)\mathbf{H}^T+\mathbf{R}(k) \tag{29}$$

The left side of equality (29) represents the real, and the right side represents the theoretical value of the covariance of the innovation sequence.

The innovation covariance can be expressed in the form below by substituting (28) into (29),

$$\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}\tilde{\mathbf{e}}(j)\tilde{\mathbf{e}}^T(j)=\mathbf{H}\left[\frac{\partial\mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\mathbf{P}(k/k)\frac{\partial\mathbf{f}^T[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}+\mathbf{\Lambda}(k)\mathbf{Q}(k)\right]\mathbf{H}^T+\mathbf{R}(k) \tag{30}$$

or

$$\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}\tilde{\mathbf{e}}(j)\tilde{\mathbf{e}}^T(j)=\mathbf{H}\left[\frac{\partial\mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\mathbf{P}(k/k)\frac{\partial\mathbf{f}^T[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\right]\mathbf{H}^T+\mathbf{H}\;\mathbf{\Lambda}(k)\;\mathbf{Q}(k)\;\mathbf{H}^T+\mathbf{R}(k) \tag{31}$$

Here, μ is the width of the moving window.

From (31), it can be written,

$$\mathbf{H}\Lambda(k)\mathbf{Q}(k)\mathbf{H}^T=\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}\tilde{e}(j)\tilde{e}^T(j)-\mathbf{H}\left[\frac{\partial\mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\mathbf{P}(k/k)\frac{\partial\mathbf{f}^T[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\right]\mathbf{H}^T-\mathbf{R}(k) \tag{32}$$

Multiplying by $\mathbf{H}^T$ on the left and by $\mathbf{H}$ on the right (32), then

$$\begin{gathered}\mathbf{H}^T\mathbf{H}\Lambda(k)\mathbf{Q}(k)\mathbf{H}^T\mathbf{H}=\mathbf{H}^T\times\\ \left\{\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}\tilde{\mathbf{e}}(j)\tilde{\mathbf{e}}^T(j)-\mathbf{H}\left[\frac{\partial\mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\mathbf{P}(k/k)\frac{\partial\mathbf{f}^T[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\right]\mathbf{H}^T-\mathbf{R}(k)\right\}\mathbf{H}\end{gathered} \tag{33}$$

Then, the scale matrix $\mathbf{\Lambda}$ can be determined from (33) as,

$$\begin{gathered}\mathbf{\Lambda}(k)=\left[\mathbf{H}^T\mathbf{H}\right]^{-1}\mathbf{H}^T\times\\ \left\{\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}\tilde{\mathbf{e}}(j)\tilde{\mathbf{e}}^T(j)-\mathbf{H}\left[\frac{\partial\mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\mathbf{P}(k/k)\frac{\partial\mathbf{f}^T[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\right]\mathbf{H}^T-\mathbf{R}(k)\right\}\\ \times\mathbf{H}\left[\mathbf{Q}(k)\;\mathbf{H}^T\;\mathbf{H}\right]^{-1}\end{gathered} \tag{34}$$

For a specific case where the all states are measured ($\mathbf{H}=\mathbf{I}$) as in case, (34) reduces to

$$\mathbf{\Lambda}(k)=\left\{\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}\tilde{\mathbf{e}}(j)\tilde{\mathbf{e}}^{T}(j)-\left[\frac{\partial\mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\mathbf{P}(k/k)\frac{\partial\mathbf{f}^{T}[\hat{\mathbf{x}}(k),k]}{\partial\hat{\mathbf{x}}(k)}\right]-\mathbf{R}(k)\right\}\mathbf{Q}^{-1}(k) \tag{35}$$

If the process noise does not change, the scale matrix will be a unit matrix as $\mathbf{\Lambda}=\mathbf{I}$, since the innovation values already match. Here, $\mathbf{I}$ represents the unit matrix.

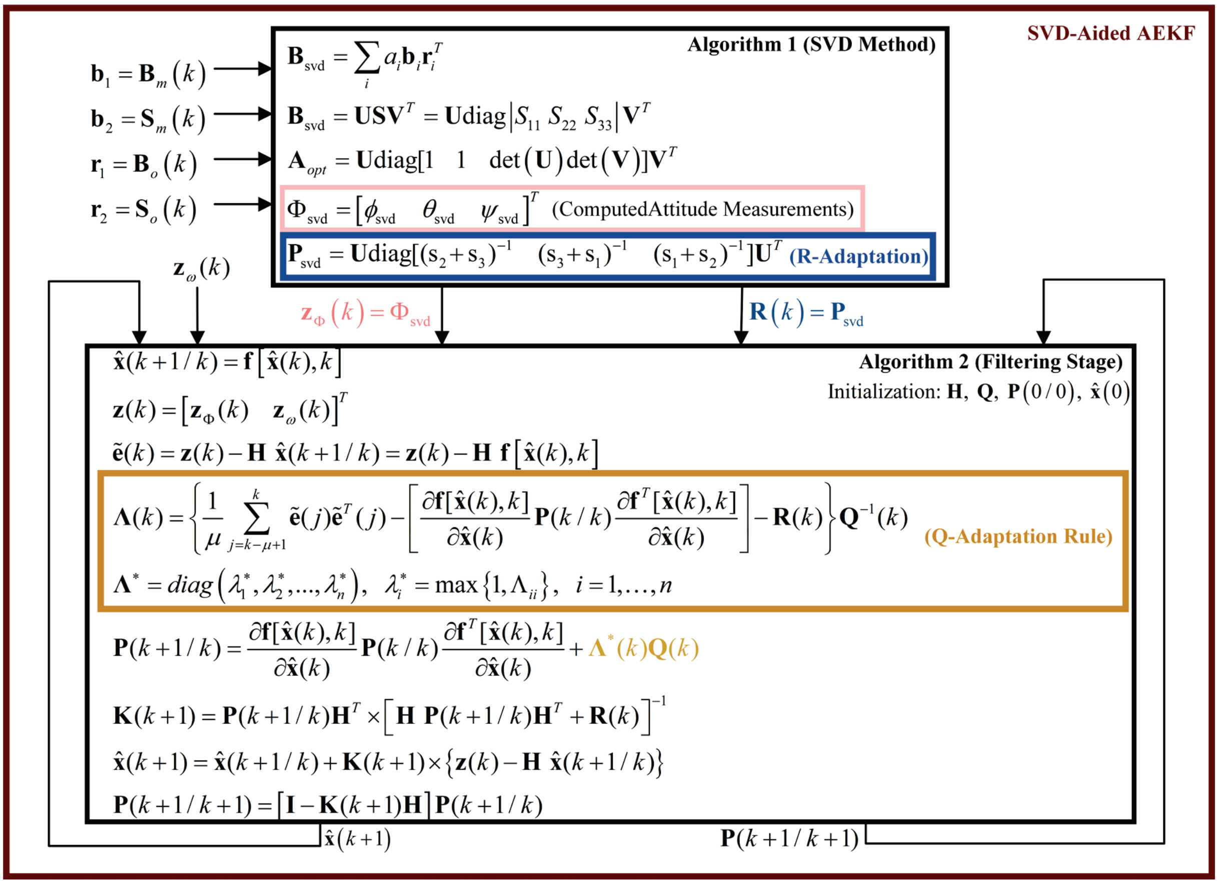

Fig. 2. Detailed SVD-Aided AEKF algorithm flowchart.

The obtained scaling matrix should be diagonalized since the $\mathbf{Q}$ matrix must be a diagonal, positive definite matrix. Remark that, as μ is a limited number because of the number of the measurements and the computations performed with the computer implies errors such as the approximation errors and the round off errors; $\mathbf{\Lambda}$ matrix, found by using Equations (34) or (35), may not be diagonal and may have diagonal elements which are "negative" or less than "one" (although the computer simulations may bring out such results that is physically impossible in reality). Therefore, in order to avoid such situation, composing the multiple measurement noise scale factor (MMNSF) by the following rule is suggested by (Hajiyev and Soken, 2012) as,

$$\mathbf{\Lambda}^*=diag\left(\lambda_1^*,\lambda_2^*,...,\lambda_n^*\right), \tag{36}$$

where,

$$\lambda_i^*=\max\{1,\Lambda_{ii}\}\qquad i=1,\ldots,n. \tag{37}$$

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Here, Λ_{ii} represents the i^{th} diagonal element of the matrix $\mathbf{\Lambda}$. The $\mathbf{Q}$ matrix multiplied with a scaling matrix is always diagonal in this way. Apart from that point, if there is an actuator fault in the system, Λ_k^* must be included in the estimation process as,

$$\mathbf{P}(k+1/k)=\frac{\partial \mathbf{f}[\hat{\mathbf{x}}(k),k]}{\partial \hat{\mathbf{x}}(k)}\mathbf{P}(k/k)\frac{\partial \mathbf{f}^T[\hat{\mathbf{x}}(k),k]}{\partial \hat{\mathbf{x}}(k)}+\mathbf{\Lambda}^*(k)\mathbf{Q}(k) \tag{38}$$

Therefore, when process noise changes, the related element of the scale matrix, which corresponds to the changed component of the state vector, increases and that brings out a smaller Kalman gain, which reduces the effect of the innovation on the state update process. As a result, the robustness of the filter against the process noise change is ensured and deterioration of the estimation procedure caused by the fault is prevented. A detailed flow chart of the algorithms used in the SVD-aided AEKF is shown in Fig. 2 with related formulas and calculations.

6. Simulation Results

A low-Earth orbiting spacecraft with the principal moment of inertia $\mathbf{J}=\mathrm{diag}\left(2.1\times10^{-3}\quad 2.0\times10^{-3}\quad 1.9\times10^{-3}\right)\ \mathrm{kg\cdot m^2}$ is considered in the simulations. The spacecraft is tumbling during the simulations with an initial state of $\mathbf{x}_0=\left[0.03\text{ rad}\quad 0.02\text{ rad}\quad 0.01\text{ rad}\quad 0.001\text{ rad/s}\quad 0.0015\text{ rad/s}\quad 0.002\text{ rad/s}\right]$. The sun sensors are processed at 1 Hz and corrupted by Gaussian zero-mean noise with a normalized standard deviation of 2% whereas the magnetometers are corrupted by 8%. For the first section, the constant process noise covariance and for the second section the initial matrix is selected as $\mathbf{Q}=10^{-4}\mathbf{I}_{6x6}$. There is an eclipse period along the orbit between 2000 and 3000 seconds, which causes the sun sensors to give zero-output as there is no sunlight during this period of time.

The first part of this section is devoted to analyzing the inherently R-adaptive filter (SVD-Aided EKF). The second part analyzes the application of the process noise covariance adaptation on the filter. The simultaneously R- and Q-adaptive filter is called SVD-Aided AEKF in this study.

6.1. SVD-Aided EKF

In this section, the adaptation against measurement faults is investigated by applying two types of fault on the magnetometers as bias and measurement noise increment between 4500 and 5500 seconds.

Measurement Bias: A constant vector $\left[1000\quad 2000\quad 3000\right]^T$ nT is applied on the magnetometer measurements as the bias type of fault. The measurements during the faulty period are simulated by replacing Equation (8) with,

$$\mathbf{B}_m(k)=\mathbf{A}(k)\mathbf{B}_o(k)+\boldsymbol{\upsilon}_B(k)+\left[1000\quad 2000\quad 3000\right]^T,\quad \left(4500\le k\le 5500\right) \tag{39}$$

The diagonal elements of the measurement noise covariance matrix provided by the SVD preprocessing step are presented in Fig. 3. The variance values have a difference between the nominal case and the faulty case which is expected to improve the attitude estimations during the faulty period. As expected, the attitude estimates are improved compared to SVD but the

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biases are not totally compensated (Fig. 4). This means that the filter's adaption for bias type of fault is limited and its ability to improve the estimations is directly related to SVD's measurements and variances. Besides, the filter continues to estimate the attitude during eclipse even using the erroneous SVD measurements. At the beginning, the filter converges to steady state quickly as the initialization of the attitude angles is provided by SVD. And after the eclipse period, sun sensor measurements become available again and the filter converges back to the desired attitude values as well. In Fig. 4, SVD errors around 1000 s increase because most possibly, the magnetic field and sun direction vectors are getting closer to each other. However, the filter copes with the error increase well.

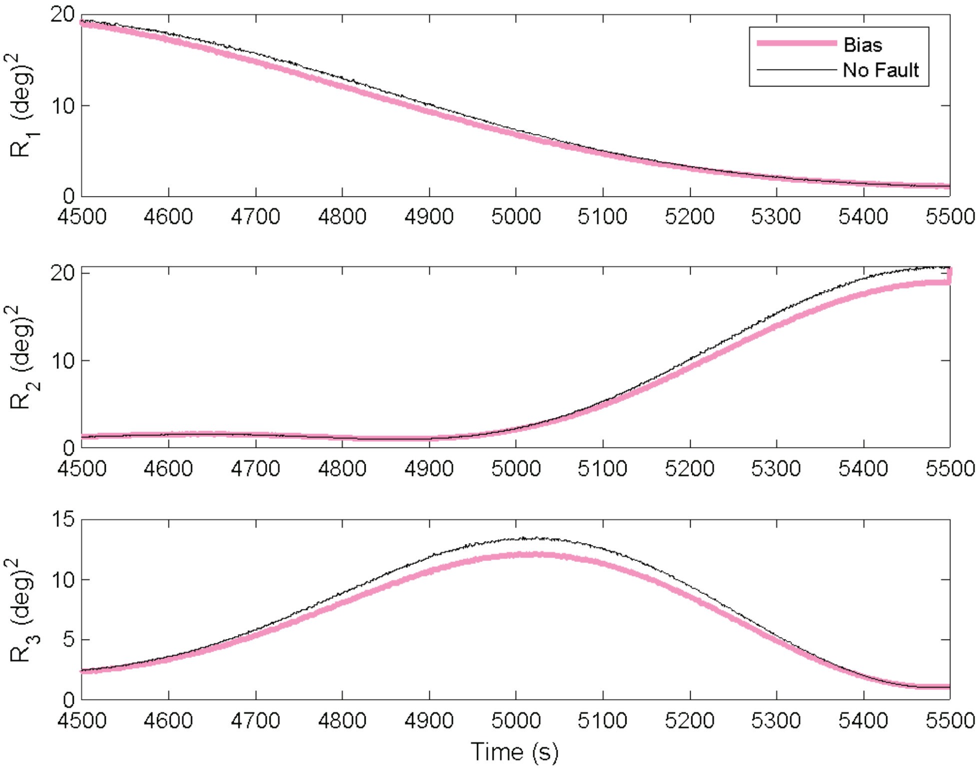

Fig. 3. Diagonal elements of R for the nominal and faulty (bias) cases.

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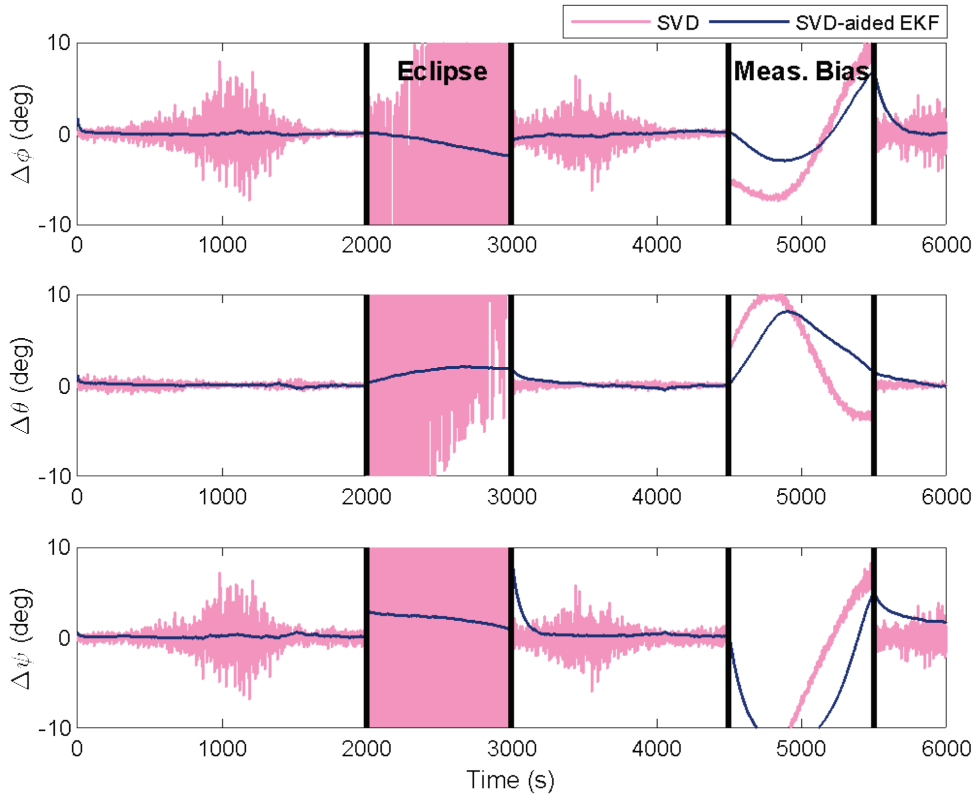

Fig. 4. Attitude estimation errors for faulty (bias) case.

Measurement Noise Increment: An increment is applied on the magnetometer measurements for the second faulty scenario. The measurements during the faulty period are simulated by replacing Equation (8) with,

$$\mathbf{B}_m(k) = \mathbf{A}(k)\mathbf{B}_o(k) + 10 \times \boldsymbol{\upsilon}_B(k), \quad (4500 \le k \le 5500) \tag{40}$$

The diagonal elements of R matrix are presented for nominal and measurement noise increment cases in Fig. 5. As it was mentioned throughout the text, the filter is inherently adaptive against the measurement faults. Here, R matrix adaptation rule could also have been defined. However, simultaneous R- and Q- adaptation would have not been as straightforward as described in this paper. The filter can cope with the noise increment on the measurements better using the R matrix in Fig. 5 and estimates the attitude well as seen in Fig. 6. The filter compensates the fault this time, even though the error levels of the SVD estimations is grossly high.

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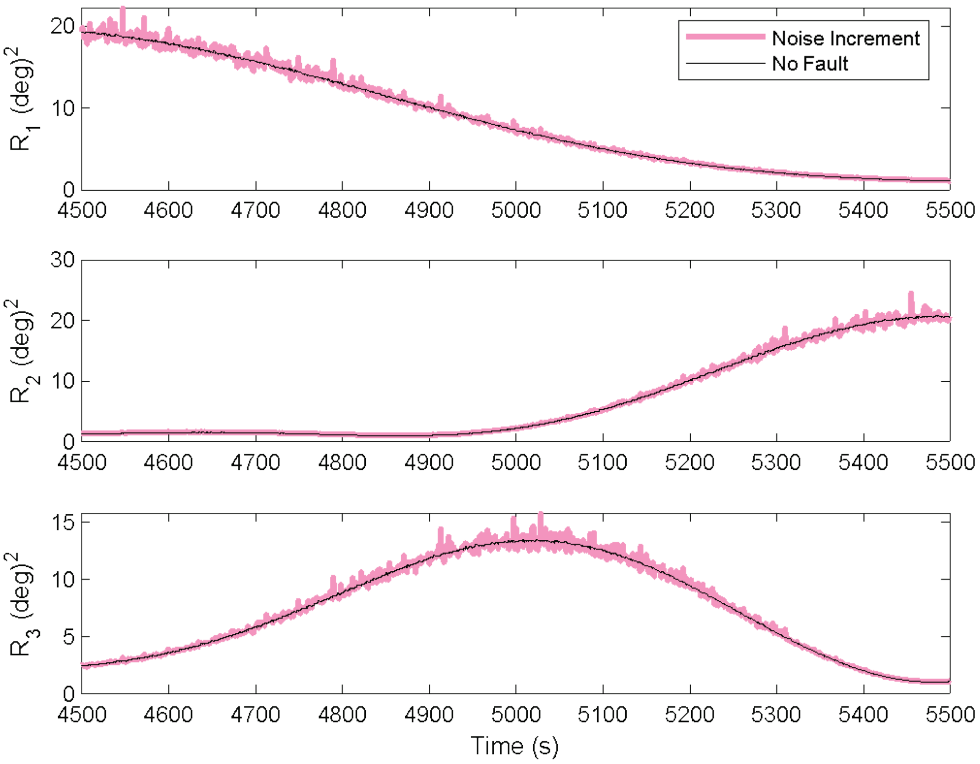

Fig. 5. Diagonal elements of R for the nominal and faulty (measurement noise increment) cases.

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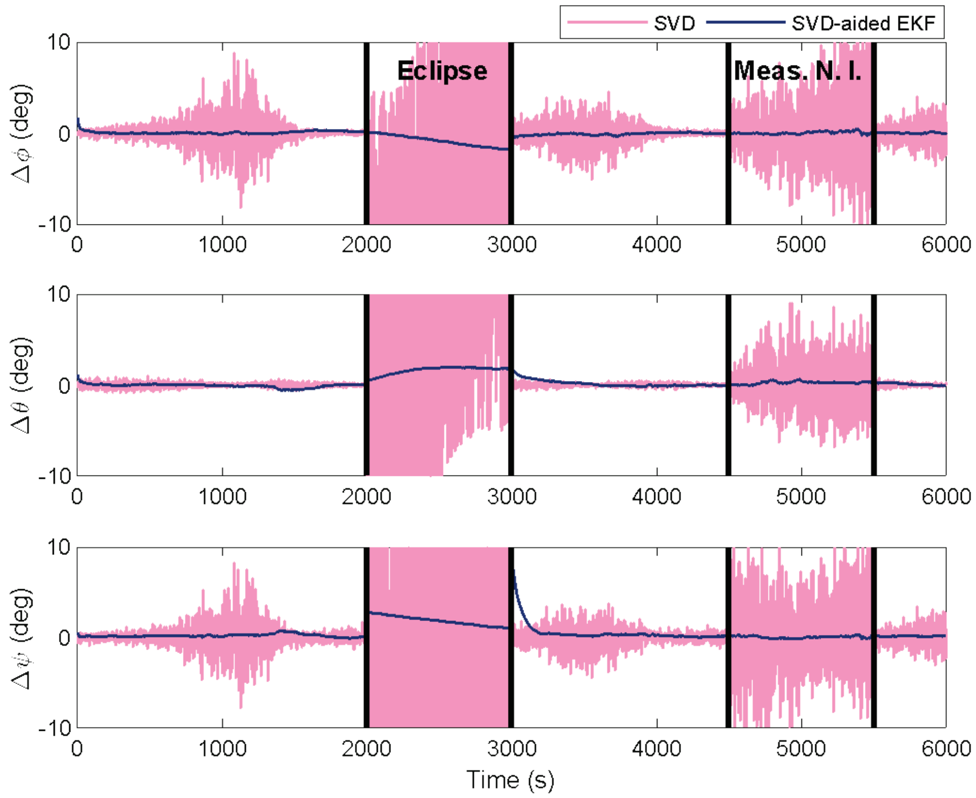

Fig. 6. Attitude estimation errors for measurement noise increment (Meas. N. I.) case.

6.2. SVD-Aided AEKF

In this section, the adaptation against process noise is investigated by applying noise increment on the system with a standard deviation of $\sigma_{1,2,3}^{\text{process}} = 0.01$ rad and $\sigma_{4,5,6}^{\text{process}} = 0.01$ rad/s between 4500 and 5500 seconds. As a complementary section, the eclipse also exists between 2000 and 3000 seconds. The inherently R-adaptive filter called SVD-Aided EKF does not cope well with the process noise increment and deteriorates as seen from Fig. 7 whereas simultaneous R- and Q-adaptive filter called SVD-Aided AEKF estimates more accurate attitude angles. On the other hand, angular velocity estimations deteriorate for both filters as seen in Fig. 8 and are not improved as much as the attitude angles in Q-adaptation.

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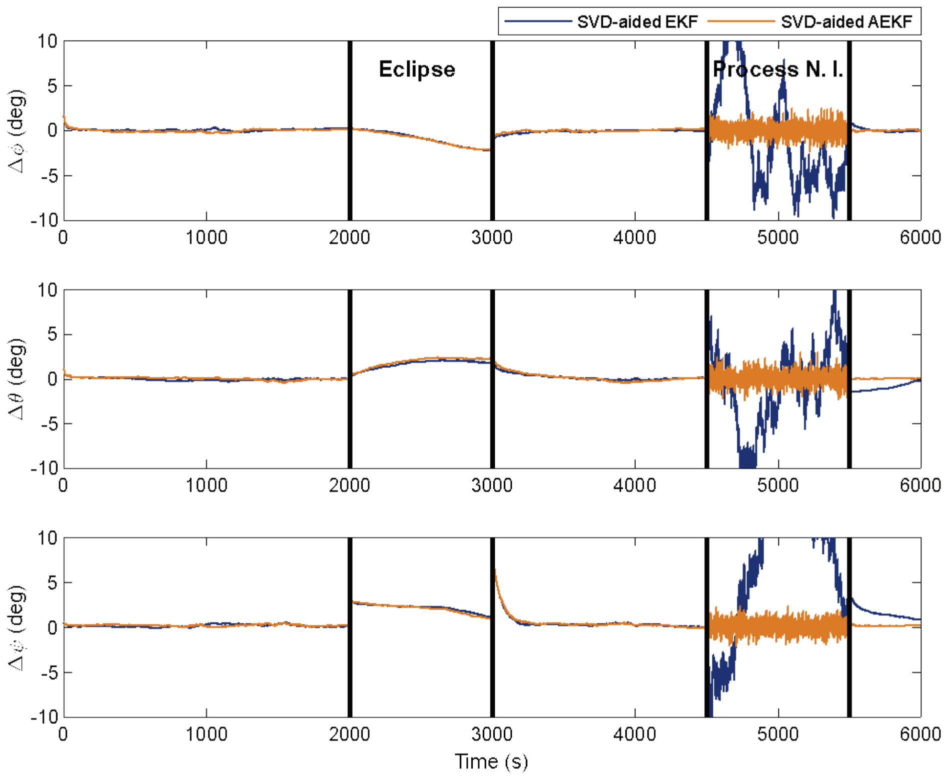

Fig. 7. Attitude estimation errors for process noise increment (N.I.) case.

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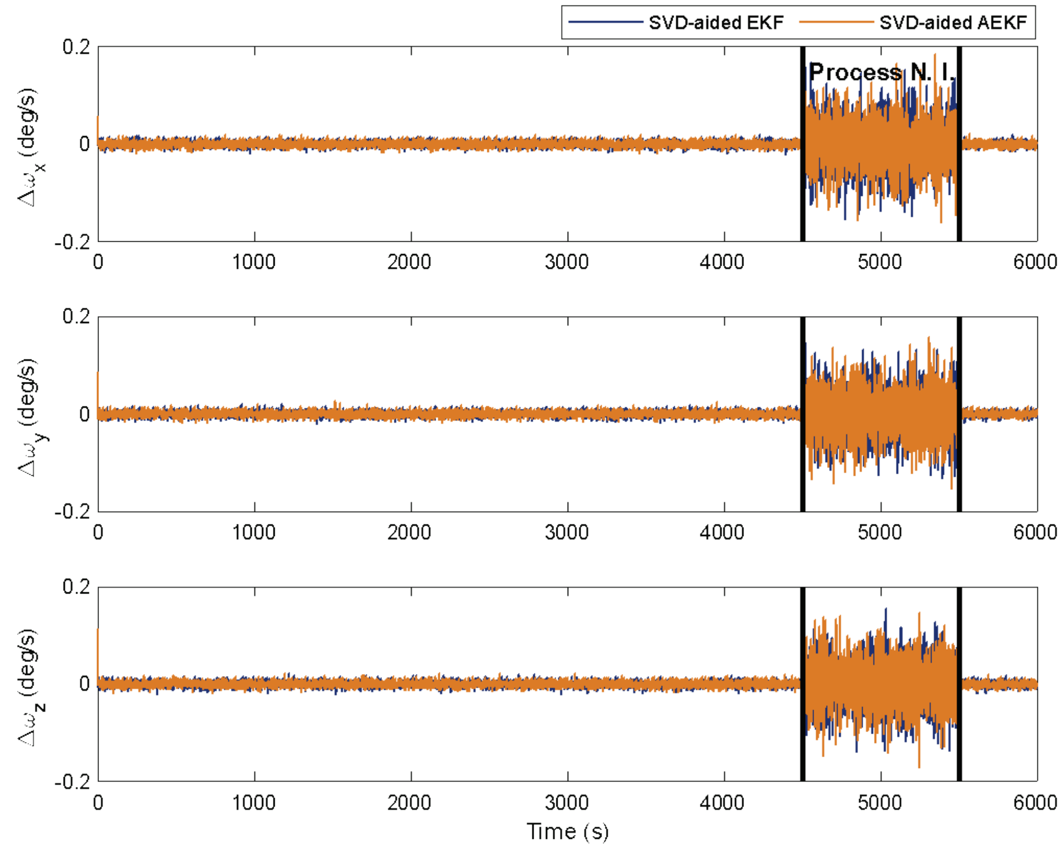

Fig. 8. Angular velocity estimation errors for process noise increment (N.I.) case.

Table 1. RMS errors of SVD-Aided EKF and AEKF.

RMS Error	SVD-aided EKF		SVD-aided AEKF	
	Nominal	Process N.I.	Nominal	Process N.I.
ϕ (deg)	0.1259	6.9793	0.1192	0.7933
θ (deg)	0.2036	5.0703	0.1520	0.7830
ψ (deg)	0.3445	3.4047	0.2191	0.8176
ω_x (deg/s)	0.1453	0.1772	0.1452	0.1750
ω_y (deg/s)	0.1139	0.1493	0.1141	0.1491
ω_z (deg/s)	0.0056	0.0480	0.0056	0.0479

Table 1 is composed of root mean square (RMS) error values of the estimations of the filters considered in this study. N.I. is abbreviated from Noise Increment. The column 'Nominal' takes the period outside of the eclipse and outside of the process noise increment into account. The results do not differentiate much between method-to-method as there is no fault considered in

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this period. On the other hand, SVD-Aided AEKF improves the attitude results with respect to SVD-Aided EKF around 4 to 8 times for the process noise increment interval. Fig. 9 demonstrates the adaptive factor used in the Q-adaptation process, which seems to be successful in compensating the noise on the process for estimating the attitude angles.

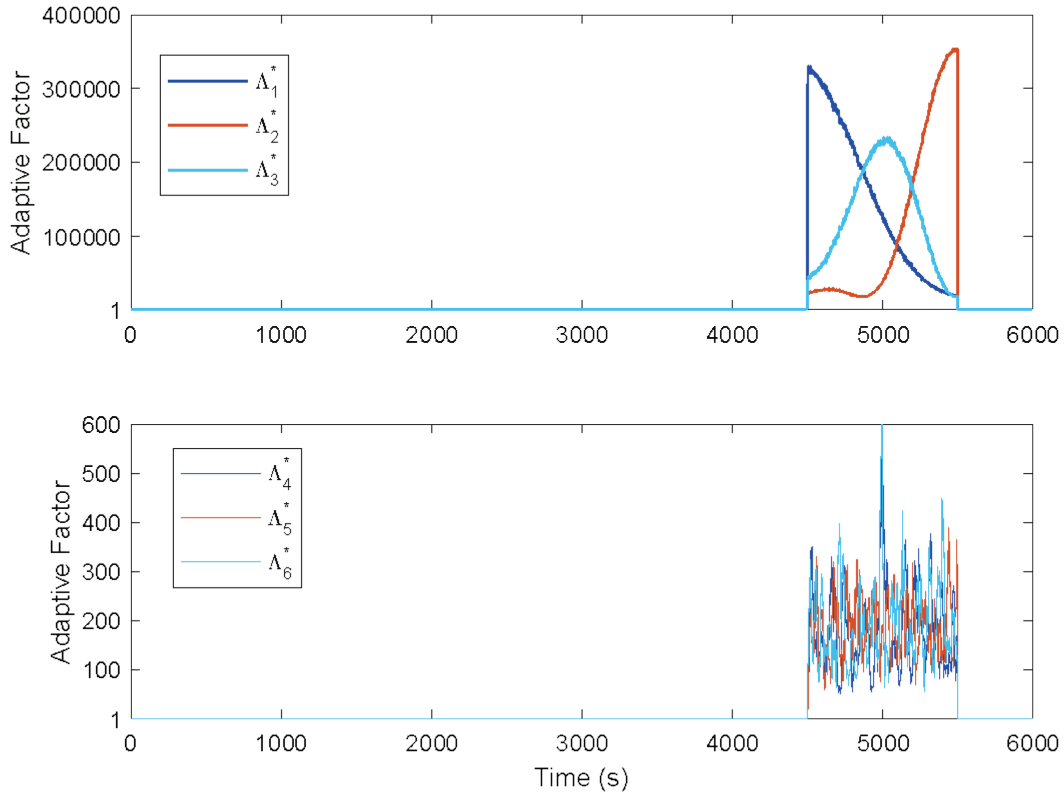

Fig. 9. Adaptive factor of the SVD-aided AEKF.

The adaptive algorithm performs the correction only when the real values of the process noise covariance does not match with the model used in the synthesis of the filter. Otherwise, the filter works in a regular manner.

7. Conclusions

This paper presents the design and numerical analyses of R- and Q- adaptive extended Kalman filtering algorithm. An integrated attitude estimation algorithm is presented first in which the SVD and EKF algorithms are combined to estimate the attitude angles and angular velocities. Processing of the sun sensor and magnetometer measurements within the SVD algorithm computes the Euler angle measurements and their variances, and provides them to the filter as inputs. Here, the filter adapts its measurement noise covariance at each step and becomes an inherently R-adaptive filter. This filter also eliminates the possibility of poorly-chosen initial attitude as it initializes the attitude using SVD method.

Specifically, the Q adaptation method is proposed. In case of process noise increment, which may be caused by the changes in the environment or satellite dynamics, performance of the Q-adaptive EKF is investigated. To adapt the EKF to the changing conditions its Q matrix is tuned

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by an adaptive method. Since the measurement model of the SVD-aided EKF algorithm is linear, the adaptation rule is similar to linear KF and easy to apply.
The performance of the filter is analyzed through two types of measurement fault applications in addition to eclipse period: bias and noise increment. The filter improves the state estimates especially in the noise increment scenario. For investigating the system modelling errors, one more scenario is implemented by applying a noise increment on the system. Here, the Q-adaptation rule is also introduced within the R-adaptive filter. Thus, the attitude and attitude rates of the satellite can be estimated accurately in spite of any change in the process and measurement noise covariance. This approach does not require to purposely select one of the adaptation rules depending on the scenario but provides two different adaptations at the same time.

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References

Belokonov, I. V., Kramlikh, A. V., Melnik, M.E., 2020. Application of artificial intelligence technology in the nanosatellite attitude determination problem, in: IOP Conference Series: Materials Science and Engineering. IOP Publishing Ltd, p. 012036. https://doi.org/10.1088/1757-899X/984/1/012036

Cilden, D., Hajiyev, C., Soken, H.E., 2015. Attitude and attitude rate estimation for a nanosatellite using SVD and UKF, in: RAST 2015 - Proceedings of 7th International Conference on Recent Advances in Space Technologies. https://doi.org/10.1109/RAST.2015.7208431

Cilden, D., Soken, H.E., Hajiyev, C., 2017. Nanosatellite attitude estimation from vector measurements using SVD-AIDED UKF algorithm. Metrol. Meas. Syst. 24, 113–125. https://doi.org/10.1515/mms-2017-0011

Hajiyev, C., 2007. Adaptive Filtration Algorithm with the filter gain correction applied to integrated INS/Radar altimeter. Part G J. Aerosp. Eng. 221, 847–885. https://doi.org/10.1243/09544100JAERO173

Hajiyev, C., Bahar, M., 2003. Attitude Determination and Control System Design of the ITU-UUBF LEO1 Satellite. Acta Astronaut. 52, 493–499. https://doi.org/10.1016/S0094-5765(02)00192-3

Hajiyev, C., Cilden-Guler, D., 2017. Review on Gyroless Attitude Determination Methods for Small Satellites. Prog. Aerosp. Sci. 90, 54–66. https://doi.org/10.1016/j.paerosci.2017.03.003

Hajiyev, C., Cilden, D., Somov, Y., 2016. Gyro-free attitude and rate estimation for a small satellite using SVD and EKF. Aerosp. Sci. Technol. 55. https://doi.org/10.1016/j.ast.2016.06.004

Hajiyev, C., Soken, H.E., 2014. Robust Adaptive Unscented Kalman Filter for Attitude Estimation of Pico Satellites. Int. J. Adapt. Control Signal Process. 28, 107–120. https://doi.org/10.1002/acs.2393

Hajiyev, C., Soken, H.E., 2012. Robust Estimation of UAV Dynamics in the Presence of

Measurement Faults. J. Aerosp. Eng. 25, 80–89. https://doi.org/10.1061/(ASCE)AS.1943-5525.0000095

Hajiyev, C., Soken, H.E., Cilden-Guler, D., 2019. Nontraditional Attitude Filtering with Simultaneous Process and Measurement Covariance Adaptation. J. Aerosp. Eng. 32, 04019054. https://doi.org/10.1061/(ASCE)AS.1943-5525.0001038

Hajiyev, C., Soken, H.E., Cilden-Guler, D., 2017. Q-Adaptation of SVD-aided UKF Algorithm for Nanosatellite Attitude Estimation. IFAC-PapersOnLine 50, 8273–8278. https://doi.org/10.1016/j.ifacol.2017.08.1399

Hapgood, M.A., 1992. SPACE PHYSICS COORDINATE TRANSFORMATIONS : A USER GUIDE. Plan. Sp. Sci 40, 71–717.

Höfling, T., Pfeufer, T., 1994. Detection of Additive and Multiplicative Faults - Parity Space vs. Parameter Estimation. IFAC Proc. Vol. 27, 515–520. https://doi.org/10.1016/s1474-6670(17)48078-5

Huang, Y., Zhang, Y., Xu, B., Wu, Z., Chambers, J.A., 2018. A New Adaptive Extended Kalman Filter for Cooperative Localization. IEEE Trans. Aerosp. Electron. Syst. 54, 353–368. https://doi.org/10.1109/TAES.2017.2756763

Ivanov, D., Ovchinnikov, M., Roldugin, D., 2018. Three-Axis Attitude Determination Using Magnetorquers. J. Guid. Control. Dyn. 41, 2455–2462. https://doi.org/10.2514/1.G003698

Kang, C.H., Kim, S.Y., Park, C.G., 2014. A GNSS Interference Identification and Tracking based on Adaptive Fading Kalman Filter. IFAC Proc. Vol. 47, 3250–3255. https://doi.org/10.3182/20140824-6-ZA-1003.01374

Lee, D.Y., Par, H., Romano, M., Cutler, J., 2018. Development and Experimental Validation of a Multi-Algorithmic Hybrid Attitude Determination and Control System for a Small Satellite. Aerosp. Sci. Technol. 78, 494–509. https://doi.org/10.1016/J.AST.2018.04.040

Markley, F.L., Crassidis, J.L., 2014. Fundamentals of Spacecraft Attitude Determination and Control. Springer, New York. https://doi.org/10.1007/978-1-4939-0802-8

Mashtakov, Y., Ovchinnikov, M., Wöske, F., Rievers, B., List, M., 2020. Attitude determination & control system design for gravity recovery missions like GRACE. Acta Astronaut. 173, 172–182. https://doi.org/10.1016/j.actaastro.2020.04.019

Mimasu, B.Y., Van der Ha, J.C., 2009. Attitude Determination Concept for QSAT. Trans. Japan Soc. Aeronaut. Sp. Sci. Aerosp. Technol. Japan 7, 63–68. https://doi.org/10.2322/tstj.7.Pd_63

Russel, C.T., 1971. Geophysical Coordinate Transformations. Cosm. Electrodyn. 2, 184–196.

Scardua, L.A., da Cruz, J.J., 2017. Complete Offline Tuning of the Unscented Kalman Filter. Automatica 80, 54–61. https://doi.org/10.1016/J.AUTOMATICA.2017.01.008

Sekhavat, P., Gong, Q., Ross, I.M., 2007. NPSAT 1 Parameter Estimation using Unscented Kalman Filter, in: Proceedings of the 2007 American Control Conference. New York, USA, pp. 4445–4451. https://doi.org/10.1109/ACC.2007.4283031

Soken, H.E., Cilden, D., Hajiyev, C., 2015. Attitude Estimation for Nanosatellites Using Singular Value Decomposition and Unscented Kalman Filter, in: International Symposium

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on Space Technology and Science (ISTS). Kobe, Hyogo, Japan.

Soken, Halil Ersin, Hajiyev, C., 2014. Estimation of Pico Satellite Attitude Dynamics and External Torques via Unscented Kalman Filter. J. Aerosp. Technol. Manag. 6, 149–157. https://doi.org/10.5028/jatm.v6i2.352

Soken, H. E., Hajiyev, C., 2014. REKF and RUKF for Pico Satellite Attitude Estimation in the Presence of Measurement Faults. J. Syst. Eng. Electron. 25, 288–297. https://doi.org/10.1109/JSEE.2014.00033

Soken, H.E., Hajiyev, C., 2010. Pico Satellite Attitude Estimation via Robust Unscented Kalman Filter in the Presence of Measurement Faults. ISA Trans. J. Autom. Meas. 49, 249–256.

Springmann, J.C., Cutler, J.W., 2014. Flight Results of a Low-cost Attitude Determination Systems. Acta Astronaut. 99, 201–214. https://doi.org/10.1016/j.actaastro.2014.02.026

Sun, C., Zhang, Y., Wang, G., Gao, W., 2018. A New Variational Bayesian Adaptive Extended Kalman Filter for Cooperative Navigation. Sensors 18, 2538. https://doi.org/10.3390/s18082538

Thébault, E., Finlay, C.C., Beggan, C.D., Alken, P., Al., E., 2015. International Geomagnetic Reference Field: the 12th generation. Earth, Planets Sp. 67:69. https://doi.org/10.1186/s40623-015-0228-9

Vallado, D.A., 2007. Fundamentals of Astrodynamics and Applications (3rd Ed.), Space Technology Library. Microcosm Press/Springer, USA.

Venkatasubramanian, V., Rengaswamy, R., Yin, K., Kavuri, S.N., 2003. A review of process fault detection and diagnosis part I: Quantitative model-based methods. Comput. Chem. Eng. 27, 293–311. https://doi.org/10.1016/S0098-1354(02)00160-6

Villez, K., Srinivasan, B., Rengaswamy, R., Narasimhan, S., Venkatasubramanian, V., 2011. Kalman-based strategies for Fault Detection and Identification (FDI): Extensions and critical evaluation for a buffer tank system. Comput. Chem. Eng. 35, 806–816. https://doi.org/10.1016/j.compchemeng.2011.01.045

Vinther, K., Jensen, K.F., Larsen, J.A., Wisniewski, R., 2011. Inexpensive Cubesat Attitude Estimation Using Quaternions And Unscented Kalman Filtering. Autom. Control Aerosp. 4.

Wahba, G., 1965. Problem 65-1: A Least Squares Estimate of Satellite Attitude. Soc. Ind. Appl. Math. Rev. 7, 409.

Wang, J., Xu, T., Wang, Z., 2019. Adaptive Robust Unscented Kalman Filter for AUV Acoustic Navigation. Sensors 20, 60. https://doi.org/10.3390/s20010060

Wertz, J.R., 2002. Spacecraft Attitude Determination and Control, Astrophysics and Space Science Library. D.Reidel Publishing Company, Dordrecht, Holland.

Zheng, B., Fu, P., Li, B., Yuan, X., 2018. A Robust Adaptive Unscented Kalman Filter for Nonlinear Estimation with Uncertain Noise Covariance. Sensors 18, 808. https://doi.org/10.3390/s18030808