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INTEGRATION OF BAROMETRIC AND GPS ALTIMETERS VIA ADAPTIVE DATA FUSION ALGORITHM

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***Abstract* -** An adaptive integrated navigation system, consisting of barometric altimeter (Baro) and GPS, is presented. The integration is achieved by using an adaptive data fusion algorithm with the filter gain correction. Proposed adaptive data fusion algorithm utilizes the measurement noise scale factor in order to reduce the effects of the incorrect barometric altimeter measurements on the estimation procedure. By using the algorithm, incorrect baro measurements are corrected by the system without any impact on the good estimation behavior. Performance of the proposed Baro/GPS integrated system is examined by simulations for altitude estimation of an aircraft.

Key words: Integrated Navigation Systems; Baro altimeter; GPS; Adaptive data fusion; Robust filtration, Integrated altimeter.

1. Introduction

In air-vehicles flying at low altitudes, it is very important to provide an accurate altitude information continuously. For this purpose, barometric altimeters (Baro) are commonly used nowadays. They can measure the atmospheric pressure value according to the relationship between the height and the air data such as pressure and temperature, and can calculate the height indirectly. As known [1], the barometric altimeter is an excellent sensor for vertical height measurement during straight-and-level flight segments and provides short-term accurate vertical altitude information. Although, the barometric altitude information is very accurate in short term (once it is calibrated), it is inaccurate in long term (because of weather changes).

The altitude accuracy at the new location depends on both time and weather. Barometric altimeters are affected by weather since the barometric pressure will change either due to the changes in elevation or the changes in weather. In general, barometers are initialized using the height, temperature, and pressure at the starting point of the vehicle where the barometer is installed. However, as time elapses and as the air-vehicle moves to another place, the characteristics of the atmosphere around the vehicle are changed to produce large barometer errors [2]. To surmount this problem, baro-altimeter must be aided with other altitude sensors such as GPS, radio altimeter, etc.

As is known [3], GPS cannot totally meet the need of safety-of-life applications in civilian aviation especially the vertical accuracy and integrity. It is widely accepted that integrating GPS with barometric altimeter can improve geometry and consequently provide performance enhancement especially in the vertical direction. A data fusion algorithm of Baro and GPS based on delta-altitude is presented in [3]. The simulation results show that that algorithm can substantially improve navigation accuracy performance of the Baro/GPS integrated system, especially in the vertical direction.

The performed investigations in [4, 5, 6] show that the barometric altimeter demonstrated significantly higher levels of noise in the long term than GPS, and the accuracy of the barometric altimeters is significantly poor than the accuracy results presented in [1]. As it is known [6], the barometric altimeter error sources are complicated and various. Till now there is not a widely accepted accurate barometric altimeter error model that reflects these errors [6].

As pointed out above, barometric altitude information is very accurate in short term but inaccurate in long term. On the other hand, GPS altitude information is pretty inaccurate in the short term, but if average of the altitude is taken over a long time, a pretty accurate number can be obtained. So there are two sources of altitude data: short term and long term accurate. It is required to combine these two sources to produce more accurate elevation data than either one alone.

An effective height sensing system combining GPS and barometric altimeter is proposed in [2]. In the system, the barometer errors are identified and compensated to steadily supply the accurate baro-altitude. For the identification, barometer error models are investigated and based on the fact that the barometer errors mainly consist of bias and scale factor errors, a filtering method based on the zero-scan-back multiple hypothesis filtering concept is derived. For obtaining the computationally effective filter structure, some simplifications using the sigma point hypotheses are introduced. In [5], a filter to estimate the bias and scale factor using the GPS altitude is derived by means of multiple hypothesis Gaussian approximation filter technique. The estimated bias and scale factor is used to compensate the baro-altitude. The proposed filters in [2] and [5] can successfully estimate and compensate the baro-errors, but the filters presented in these works increase the required computation burden for baro-error identification procedures significantly.

[8] presents the results of INS/GPS navigation system augmentation with a barometric altimeter for an autonomous Unmanned Aerial Vehicle (UAV). The INS/GPS system was designed using a loosely coupled integration architecture. The GPS typically shows a poor accuracy in the vertical channel due to the deficient vertical dilution of precision and atmospheric effects. In this study, the vertical channel was stabilized by an augmentation of the baro-altimeter.

[9] presents a multi-sensor INS/GPS/Baro-altimeter integrated navigation system. In the INS, the altitude error diverges exponentially, especially when using low-cost sensors. To suppress the aforementioned divergence of the altitude error, a GPS receiver and a baro-altimeter are utilized. The augmented system's vertical error is principally absorbed using the 3rd order vertical channel damping loop and the application of the adaptive error damp coefficients. This integration is done with an extended Kalman filter (EKF).

In [10], three different integrated altimeters are studied: baro-inertial, inertial-GPS and baro-inertial-GPS. All of them are designed based on using an optimal Kalman filter. As stated in the study, barometrical measurements are more accurate than those ones tracked by GPS. However, the fused system gives better measurements than both sources separately. This is the most relevant and essential outcome of this research.

An integrated navigation system consisting of inertial and radar altimeters is investigated in [11]. The error model of the INS's vertical channel is used for integrating the altimeters. And, the parameters of the error model are estimated as states. The integration is realized using a complementary Kalman filter by adding two blocks. First is the block of comparison of the real and theoretical accuracy of the innovation sequence, and the second is the filter gain correction

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block. By this way, an adaptation is provided for the filter based on the change in the measurement system operation conditions.

In the conference paper [12], barometric altimeter and GPS are integrated by using a data fusion algorithm. However, the data fusion algorithm is not adaptive in that work and can be used only in nominal operations (in the absence of measurement faults) of baro-altimeter. Estimation of the altitude in the presence of incorrect barometric altimeter measurements due to a change in the baro measurement model parameters is not taken into consideration in that paper.

This study, introduces a novel adaptive data fusion algorithm for barometric altimeter and GPS vertical position integration that increases the altitude estimation accuracy significantly in the presence of incorrect barometric altimeter measurements. In the proposed integration structure, a correction for measurement noise covariance of the barometric altimeter depending on the change in the baro measurement model parameter is performed. The integration is achieved by using an adaptive data fusion algorithm with the filter gain correction. The Baro/GPS integrated system based on the developed adaptive filtration algorithm is proposed.

The rest sections of the paper proceeds as follows. In Section 2 the barometric altitude measurement equation is given. The maximum likelihood interpretation of the data fusion of the barometric and GPS altimeters and some properties of the proposed data fusion algorithm are presented in Section 3. Data fusion algorithm based Baro/GPS integrated system is developed in this Section. In Section 4, adaptive altitude estimator with the filter gain correction is developed. In Section 5 performance of the proposed Baro/GPS integrated system based on the adaptive data fusion algorithm is tested through the simulations. Section 6 gives a brief summary of the obtained results and the conclusion.

2. Barometric Altitude Measurement Equation

Assuming that the air is an ideal gas and that the gradient of temperature as a function of altitude (i.e., the atmospheric lapse rate) is known, then altitude can be computed via the following expression [6]:

$$h^B = \frac{T_0}{T_{grad}}\left[1-\left(\frac{p}{p_0}\right)^{\frac{T_{grad}R}{g}}\right] \tag{1}$$

where h^B is the barometric altitude, T_0 is temperature at sea level, T_{grad} is the lapse rate, p_0 is pressure at sea level, p is the pressure measured at the altitude of h^B, R is the universal gas constant, g is the local gravity.

The conversion of measured air pressure to altitude given in (1) is based on a theoretical standard atmosphere and the assumption that air is an ideal gas. More precisely, the standard atmosphere is defined as follows [13]:

- air is an ideal gas with gas constant $R = 287 J/(Kg\cdot K)$
- the pressure at sea level is $p_0 = 29.92 \ in.-Hg = 101.325 kPa$
- the air temperature at sea level is $T_0 = 15^\circ C = 288{,}15 K$
- the temperature gradient (lapse rate) is $T_{grad} = 0.0065^\circ C/m$.

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The pressure at sea level p_0 is not constant but varies from day to day and location to location. In practice, therefore, a value of p_0 which makes the altitude estimated using (1) correct is input into the altimeter. This value of p_0 is known as the altimeter setting.

3. Development of the Data Fusion Algorithm for Baro/GPS Integration

We call $h^B(k)$ reading from the baro altimeter and $h^{GPS}(k)$ reading from the GPS vertical position indicator. It is known that the baro altimeter has an error modeled by a Gaussian with standard deviation σ_B. The error of the GPS vertical channel is also normally distributed around zero with standard deviation σ_{GPS}. We would like to combine both measurements into a single estimation and to form a weighted average of both measurements to generate an estimate of altitude $h(k)$ which we call $\hat{h}(k)$.

The measurement equations are,

$$h^B(k) = h(k) + \xi_B(k) \tag{2}$$

$$h^{GPS}(k) = h(k) + \xi_{GPS}(k) \tag{3}$$

where $h^B(k)$ and $h^{GPS}(k)$ are the baro and GPS vertical position measurements, $h(k)$ is the actual value of altitude, $\xi_B(k)$ and $\xi_{GPS}(k)$ are the baro and GPS vertical position measurements noises respectively.

The random noises $\xi_B(k)$ and $\xi_{GPS}(k)$ are normally distributed with the following statistical characteristics:

$$E[\xi_B(k)]=E[\xi_{GPS}(k)]=0, \quad D[\xi_B(k)]= R_B(k) = constant, \;\; D[\xi_{GPS}(k)]= R_{GPS}(k) = constant$$

where E is the mathematical expectation, D is the variance.

Then, the appropriate probability density functions of system and measurement noises can be written as [12]:

$$p\left[\xi_B(k)\right] = \frac{1}{\sqrt{2\pi R_B}} \exp\left\{-\frac{\xi_B^2(k)}{2R_B}\right\} \tag{4}$$

$$p\left[\xi_{GPS}(k)\right] = \frac{1}{\sqrt{2\pi R_{GPS}}} \exp\left\{-\frac{\xi_{GPS}^2(k)}{2R_{GPS}}\right\} \tag{5}$$

3.1. Maximum Likelihood Interpretation of the Data Fusion

By substituting $\xi_B(k) = h^B(k) - h(k)$ and $\xi_{GPS}(k) = h^{GPS}(k) - h(k)$ in the expressions (4) and (5) respectively, the likelihood functions $p\left[h^B(k)/h(k)\right]$ and $p\left[h^{GPS}(k)/h(k)\right]$ can be determined as the functions of the estimated altitude as [12]

$$p\left[h^B(k)/h(k)\right] = \frac{1}{\sqrt{2\pi R_B}} \exp\left\{-\frac{\left[h^B(k) - h(k)\right]^2}{2R_B}\right\} \tag{6}$$

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$$p\left[h^{GPS}(k)/h(k)\right]=\frac{1}{\sqrt{2\pi R_{GPS}}}\exp\left\{-\frac{\left[h^{GPS}(k)-h(k)\right]^2}{2R_{GPS}}\right\} \tag{7}$$

Assume that the measurements $h^B(k)$ and $h^{GPS}(k)$ are independent parameters, then the likelihood function $p\left[h^B(k),h^{GPS}(k)/h(k)\right]$ can be expressed in the following form:

$$\begin{aligned} p\left[h^B(k),h^{GPS}(k)/h(k)\right] &= p\left[h^B(k)/h(k)\right]\times p\left[h^{GPS}(k)/h(k)\right] \\ &= \frac{1}{\sqrt{(2\pi)^2 R_B R_{GPS}}}\exp\left\{-\frac{\left[h^B(k)-h(k)\right]^2}{2R_B}-\frac{\left[h^{GPS}(k)-h(k)\right]^2}{2R_{GPS}}\right\} \end{aligned} \tag{8}$$

It is required to find the optimum value for the altitude $h(k)$ from condition of maximum of the likelihood function.

As the maximum likelihood value, the $\hat{h}(k)$ values that make the likelihood function $p\left[h^B(k),h^{GPS}(k)/h(k)\right]$ reach the maximum value are considered.

The condition of extremum for the likelihood function can be written as:

$$\frac{\partial p\left[h^B(k),h^{GPS}(k)/h(k)\right]}{\partial h(k)}=0 \tag{9}$$

For the purpose of decreasing computation burden when the maximum likelihood values are sought, the extremum value for the logarithm of distribution of $p\left[h^B(k),h^{GPS}(k)/h(k)\right]$ is looked for instead of the equation (9). As a logarithm is the monotone function, then the distributions $p\left[h^B(k),h^{GPS}(k)/h(k)\right]$ and $ln\left\{p\left[h^B(k),h^{GPS}(k)/h(k)\right]\right\}$ attain to the extremums at the same values of argument $h(k)$. After differentiating the maximum likelihood equation, the following expression will be obtained,

$$\frac{\left[h^B(k)-h(k)\right]}{R_B}+\frac{\left[h^{GPS}(k)-h(k)\right]}{R_{GPS}}=0 \tag{10}$$

As a result of the required mathematical transformations, the expression for the sought estimation value is obtained:

$$\hat{h}(k)=\frac{R_{GPS}h^B(k)+R_B h^{GPS}(k)}{R_B+R_{GPS}} \tag{11}$$

Let us check that the point $\hat{h}(k)$ either corresponds to maximum or not. For this purpose the second derivative $\partial^2 ln\left\{p\left[h^B(k),h^{GPS}(k)/h(k)\right]\right\}/\partial h^2(k)$ can be used:

$$\frac{\partial^2 ln\left\{p\left[h^B(k),h^{GPS}(k)/h(k)\right]\right\}}{\partial h^2(k)}=-\frac{1}{R_B}-\frac{1}{R_{GPS}} \tag{12}$$

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As the second derivative is negative, then the value $\hat{h}(k)$ will be the maximum point.

The variance of the obtained estimation altitude (11) is equal to the following expression [12]:

$$D=\left\{-E\left[\frac{\partial^2 ln\left\{p\left[h^B(k),h^{GPS}(k)/h(k)\right]\right\}}{\partial h^2(k)}\right]\right\}^{-1}=\frac{R_B R_{GPS}}{R_B+R_{GPS}} \tag{13}$$

It can be easily shown that the estimates found via the expression (11) are not biased

$$E\left[\hat{h}(k)\right]=\frac{E\left[R_{GPS}h^B(k)+R_B h^{GPS}(k)\right]}{E\left[R_B+R_{GPS}\right]}=h(k) \tag{14}$$

Rewriting the equation for the state estimation (11) and performing necessary mathematical transformations, the following expressions can be obtained:

$$\hat{h}(k)=h^B(k)+K(k)\left[h^{GPS}(k)-h^B(k)\right] \tag{15}$$

where

$$K(k)=\frac{R_B}{R_B+R_{GPS}} \tag{16}$$

is the gain coefficient of the filter. Taking into consideration (16), the variance of the estimation error (13) can be rewritten in the following form

$$P(k)=R_{GPS}K(k) \tag{17}$$

The residual of the filter can be expressed as

$$\Delta(k)=h^{GPS}(k)-h^B(k) \tag{18}$$

The residual variance is

$$P_\Delta(k)=R_B(k)+R_{GPS}(k) \tag{19}$$

The expressions (15) - (19) can be presented as the altitude estimation algorithm based on the baro-altimeter and GPS vertical position measurements.

3.2. Properties of the Data Fusion Algorithm

The result obtained from (16) is:

$$0<K(k)<1, \quad R_B\neq 0, R_{GPS}\neq 0. \tag{20}$$

The result from (17) and (20) is:

$$0<P(k)<R_{GPS} \tag{21}$$

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It is seen from (17) and (20) that the Baro/GPS altitude error is always smaller than the GPS error. Therefore, the accuracy obtained from Kalman filter is greater than GPS measuring accuracy.

Let us describe (17) in the following form

$$P(k)=\frac{R_B R_{GPS}}{R_B+R_{GPS}} \tag{22}$$

It is apparent from (22) that

$$R_B-P(k)=\frac{P(k)R_B}{R_{GPS}}>0. \tag{23}$$

Taking into account that $P(k)>0, R_B>0, R_{GPS}>0$ from (23) we obtain

$$R_B>P(k). \tag{24}$$

Therefore, the accuracy obtained from the data fusion filter is also greater than Baro measuring accuracy.

If the condition $R_B >> R_{GPS}$ is satisfied, the following results can be obtained:

$$K\approx 1; \quad P(k)\approx R_{GPS} \tag{25}$$

On the other hand, if the condition $R_B << R_{GPS}$ is satisfied then $K\approx 0$ and $P(k)\to 0$.

4. Adaptive Altitude Estimator with the Filter Gain Correction

The barometric altimeter and GPS altitude data fusion algorithm above directly introduce altitude information into integrated system. This algorithm requires an accurate error model of the barometric altimeter. However, the barometric altimeter error sources are complicated and various. There is not a widely accepted accurate barometric altimeter error model to reflect these errors [9]. The important errors of the barometric altimeter are system and pressure errors. The system error covers the pressure sensor error, the altitude display error, and the barometric setting error. The barometric altimeter system error is very small when compared to other error sources. This system error will either be provided by the manufacturers or can be calibrated ahead of time [9].

The barometric altimeter pressure error includes the pressure variation error and the pressure difference error. The pressure variation error is the difference between the actual pressure obtained from the altitude relationship and the theoretical standard atmosphere assumed in (1). The pressure difference error is the difference between the pressures at the level of references at different locations. The pressure error is the major error source in barometric altimeters. A more detailed treatment of barometric altimeter errors can be found in [9].

The pressure at sea level p_0 is not constant but varies from day to day and location to location. In practice, therefore, a value of p_0 which makes the altitude estimated using (1) correct is input into the altimeter. This value of p_0 is known as the altimeter setting. The correct value of p_0 will change as an aircraft travels from its departure point towards its destination. Thus, for airplanes flying at an altitude less than 18,000 ft above sea level, the normal procedure is to adjust the barometric altimeter to the local barometric pressure provided by air traffic

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control. For airplanes flying at an altitude greater than 18,000 ft above sea level, the normal procedure is to adjust the p_0 to a standard pressure of 29.92 in of mercury [9].

The Altitude Estimator designed in this study is adaptive in the case where the correct values of T_0 and p_0 will change as an aircraft travels from its departure point towards its destination. The following approach for the solution of the altitude estimation problem is proposed for this case.

In the case when the parameters T_0 and p_0 do not change, the filter works according to the conventional algorithms (15)-(19). But if the values of parameters T_0 and p_0 do not correspond to the model (1) used in the synthesis of filter, then based on the proposed adaptive procedure in [11], the gain of filter automatically changes due to a change in the variance of the residual (19) according to the following rule

$$P_\Delta(k) = S(k)R_B(k) + R_{GPS}(k) \tag{26}$$

In which adaptive factor (weight coefficient) $S(k)$ is calculated from the residual (18) analysis. The filter gain in this case can be written in the following form,

$$K(k) = \frac{S(k)R_B(k)}{S(k)R_B(k) + R_{GPS}(k)} \tag{27}$$

According to the proposed approach the gain is changed when the following condition is valid

$$\Delta^2(k) \ge E\left[\Delta^2(k)\right] = E\left\{\left[h^{GPS}(k) - h^B(k)\right]^2\right\} = R_{GPS}(k) + R_B(k) \tag{28}$$

When a significant change is observed in the model (1), the baro altimeter measurements $h^B(k)$ will considerably differ from the observed GPS vertical position results $h^{GPS}(k)$. Consequently, the discrepancy squares on the left side of (28) will characterize the real filtration error, while the right side determines the theoretical accuracy of the residual sequence, obtained on the basis of a priori information. If condition (28) is met, then the real filtration error exceeds the theoretical error. Therefore, it is necessary to correct the filter gain beginning from this moment. Regarding the expression (26), the adaptive factor $S(k)$ can be obtained from the equality

$$\Delta^2(k) = S(k)R_B(k) + R_{GPS}(k) \tag{29}$$

Hence the following formula for the adaptive factor $S(k)$ is obtained:

$$S(k) = \frac{\Delta^2(k) - R_{GPS}(k)}{R_B(k)} \tag{30}$$

Using (26), (27) and (30) in the optimal estimation algorithm (15) - (19) gives the possibility to accomplish an adaptation of filter to the change of the parameters of model (1). If the left side of the expression (28) is greater than the right side, the adaptive factor value $S(k)$ will increase. This corresponds to the beginning of adaptation of filter. Consequently according to the expressions (26) and (27) respectively the variance of the residual $P_\Delta(k)$ increases, and the filter gain $K(k)$ decreases, which will cause strengthening of the corrective influence of the residual (18) in the estimation algorithm and approach the estimation value $\hat{h}(k/k)$ to the actual value $h(k)$. This will lead to the decrease of residual $\Delta(k)$ and adaptive factor $S(k)$, weakening of the corrective influence of residual, etc.

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The final expressions of the proposed adaptive filtration algorithm with the filter gain correction can be written in the following form:

$$\hat{h}(k/k) = \hat{h}(k/k-1) + K(k)\Delta(k)$$

$$\hat{h}(k/k-1) = \hat{h}^B(k) = \frac{T_0(k-1)}{T_{grad}}\left[1 - \left(\frac{p(k)}{p_0(k-1)}\right)^{\frac{T_{grad}R}{g}}\right]$$

$$\Delta(k) = h^{GPS}(k) - h^B(k) \tag{31}$$

$$K(k) = \frac{S(k)R_B(k)}{S(k)R_B(k) + R_{GPS}(k)}$$

$$P(k) = \sigma_{GPS}^2 K(k)$$

$$S(k) = \frac{\Delta^2(k) - R_{GPS}(k)}{R_B(k)}$$

In contrast to the optimal filtration algorithm (15)-(19) in which the filter gain $K(k)$ is constant, in the proposed algorithm current measurements have larger weight, since the gain coefficient of filter $K(k)$ is corrected by the results of each observation. This algorithm is adapted to the model (1) parameters operating conditions by the approximation of theoretical variance $P_\Delta(k)$ to the real variance of the residual (18), according to changing adaptive factor $S(k)$. The mentioned change is accomplished based on the discrepancy squares $\Delta^2(k)$, which characterizes the real filtration error. Proposed adaptive data fusion algorithm will ensure the adaptation of the filter to the change of the model (1) parameters, in particularly, to the change of the correct values of T_0 and p_0 as the aircraft travels from its departure point towards its destination.

5. Simulations

In this study, Baro/GPS integrated navigation system for altitude determination of an aircraft is simulated. The following are the necessary data and the initial conditions for the integrated system [14]:

$p_0 = 101.325kPa$; $T_0 = 15^\circ C$; $T_{grad} = 0.0065^\circ C/m$; $g = 9.81m/s^2$; $R = 287J/(Kg \cdot K)$

$R_{GPS} = 169m^2$; $R_B = 81m^2$

In simulations the altitude of the aircraft is changed from 10894 meters to the sea level approximately. To do this, the pressure p measured at the current altitude is increased from $23kPa$ (corresponds to the altitude 10894m) to $99.85kPa$ (corresponds to the altitude 123.47m) with the step $0.05kPa$ during 1560 iterations. The calculated altitudes using formula (1) are taken as the actual altitudes of the aircraft. The graph of the actual altitude h is shown in Fig.1.

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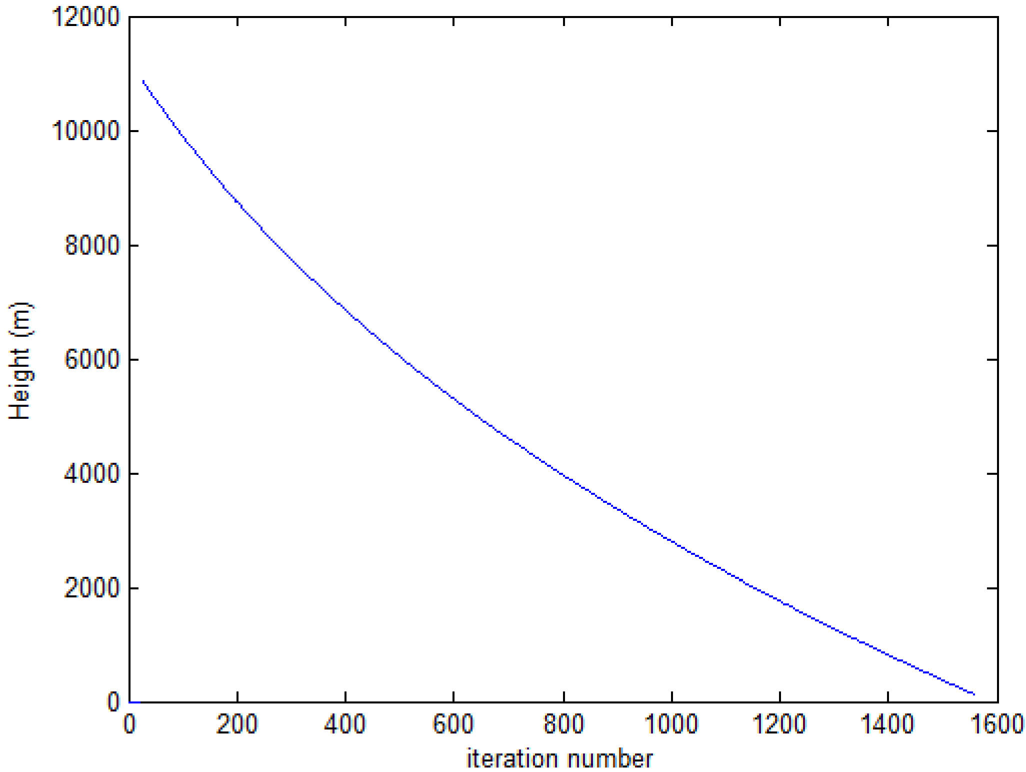

Fig.1. The simulated actual altitude of aircraft.

The simulation of the Baro and GPS altitude measurements are performed according to the following expressions:

$$h^{B}(k) = h(k) + \sigma_{B} randn \tag{32}$$

$$h^{GPS}(k) = h(k) + \sigma_{GPS} randn \tag{33}$$

where σ_B and σ_{GPS} are the standard deviations of the measurement noises of the baro altimeter and GPS vertical position respectively, *randn* is the normalized Gaussian random number with zero mean and variance 1.

The graphs of the absolute errors of the baro-altimeter and GPS vertical position are given in Figs. 2 and 3 respectively.

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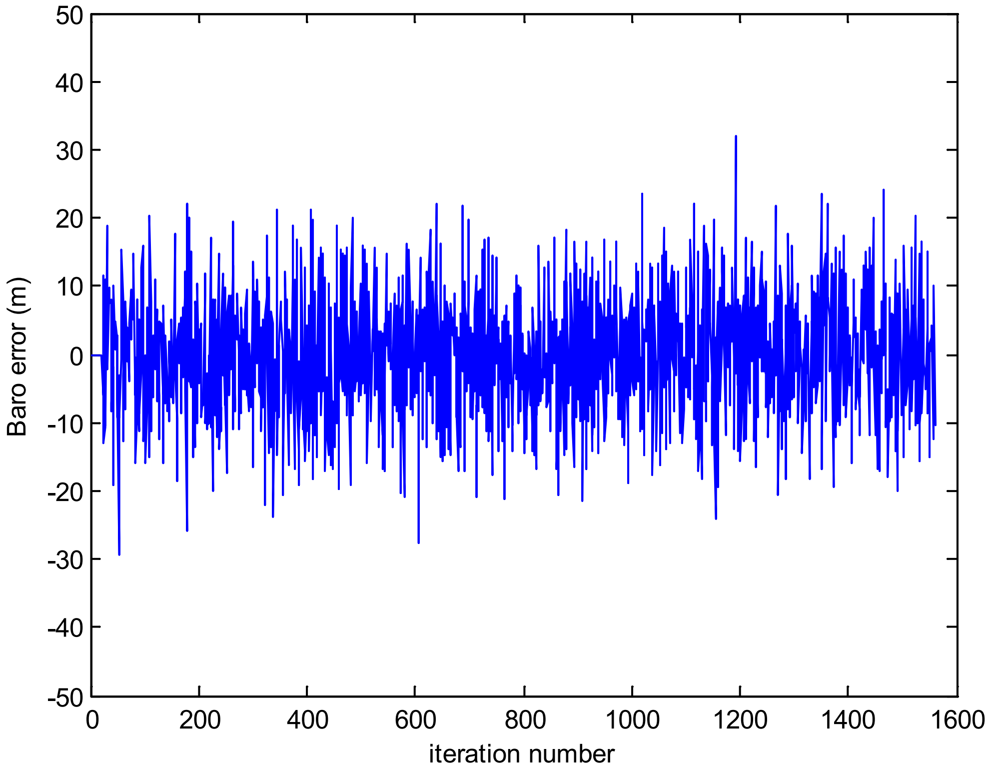

Fig.2. Behavior of the absolute errors of the baro altimeter.

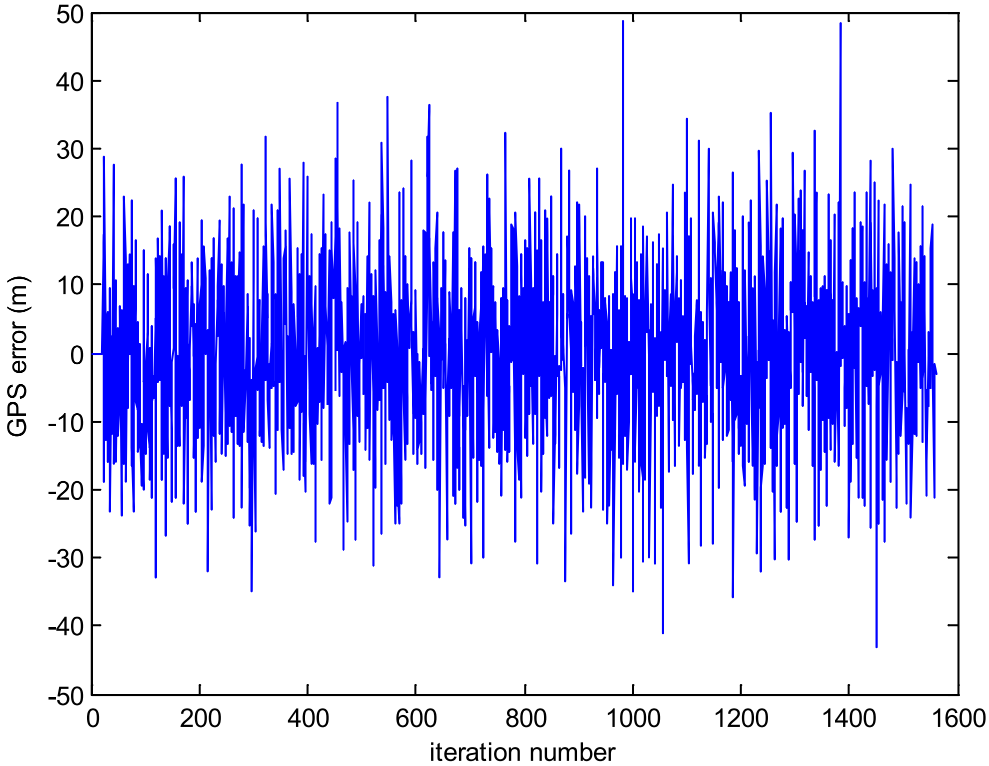

Fig.3. Behavior of the absolute errors of the GPS vertical position.

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The data fusion simulation results are shown in Figs. 4 and 5. In Fig. 4 the absolute errors of the Baro/GPS integrated altimeter are presented. As seen from Fig. 4 the integrated altimeter provides better altitude accuracy than both the baro altimeter and GPS vertical position channel.

The error variance of the Baro/GPS integrated altimeter is obtained as $54.756m^2$, which is significantly lesser than the error variances of both the baro altimeter and GPS vertical position. The graph of the normalized residual

$$\tilde{\Delta}(k)=\frac{h^{GPS}(k)-h^{B}(k)}{\sqrt{R_B+R_{GPS}}} \tag{34}$$

of the proposed filter is given in Fig. 5.

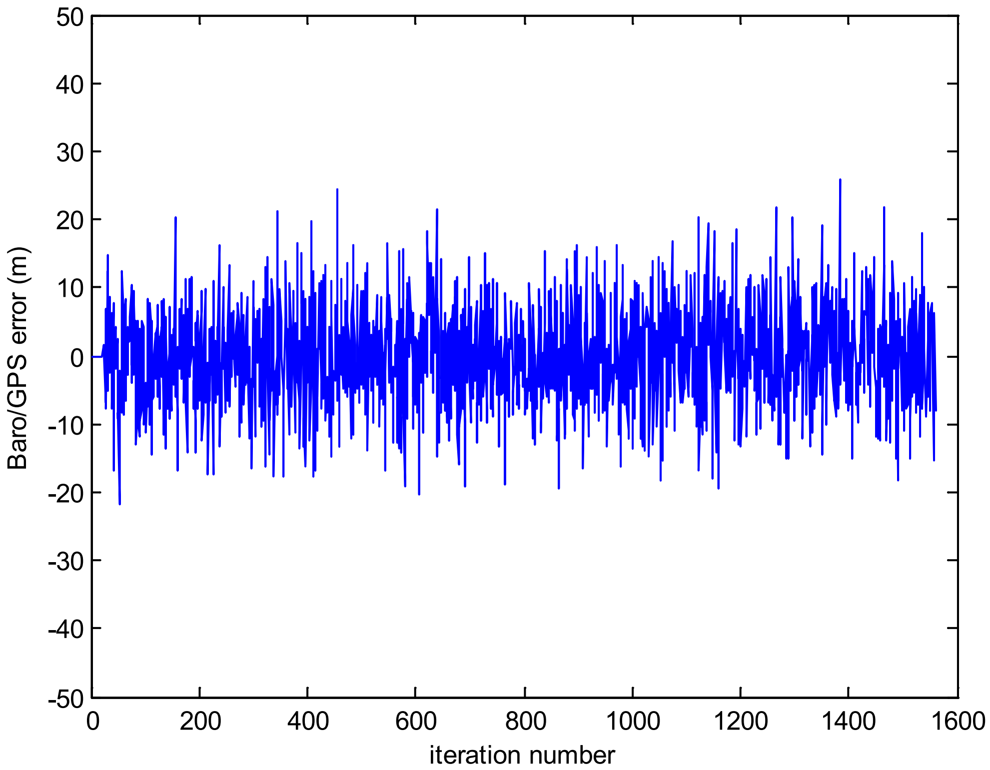

Fig.4. Behavior of the absolute errors of the Baro/GPS integrated altimeter.

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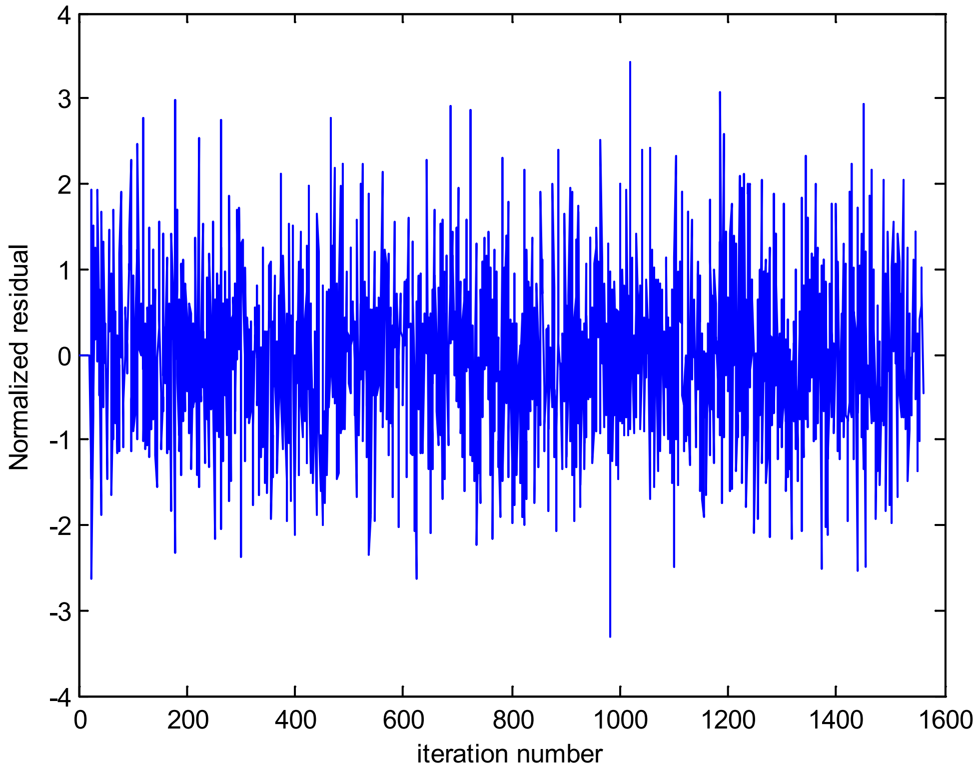

Fig.5. Normalized residual of the data fusion filter.

The data fusion algorithm based Baro/GPS integrated altimeter gives good altitude estimation results if the coefficients p_0 and T_0 are constant. But as well known, the pressure p_0 and temperature T_0 at sea level are not constant and varies from day to day and location to location. In this case the accuracy of the Baro/GPS altimeter becomes bad. The simulation results, when the coefficients p_0 and T_0 differentiated from iteration k=23 to k=1600 as

$$
\begin{aligned}
p_0(k) &= 101{,}325 kPa - k0{,}01 kPa \\
T_0(k) &= 288{,}15 K - k0{,}01 K, \qquad k \geq 23
\end{aligned}
$$

are presented in Figs.6 and 7. As seen from the presented results in Fig. 6, the accuracy of the Baro and Baro/GPS altimeters in this case is considerably worse than the GPS vertical position accuracy. The corresponding results for normalized residual of the data fusion filter in this case are shown in Fig. 7.

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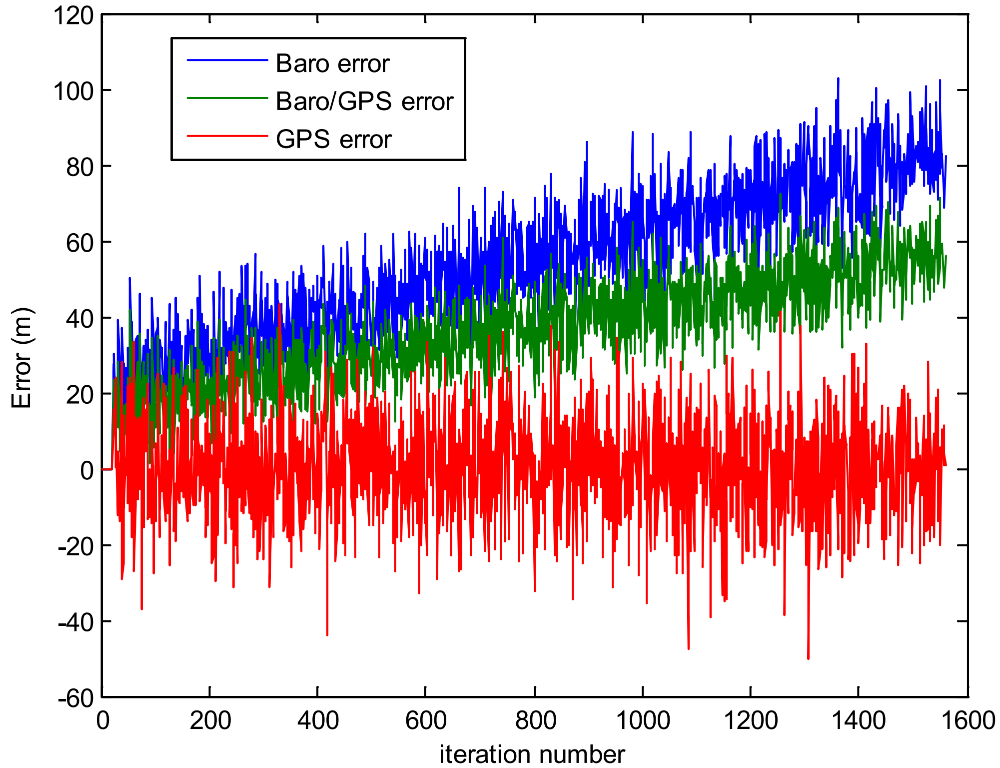

Fig.6. Absolute errors of the Baro, GPS and Baro/GPS altimeters in the case of change in coefficients p_0 and T_0 (non-adaptive filter is used)

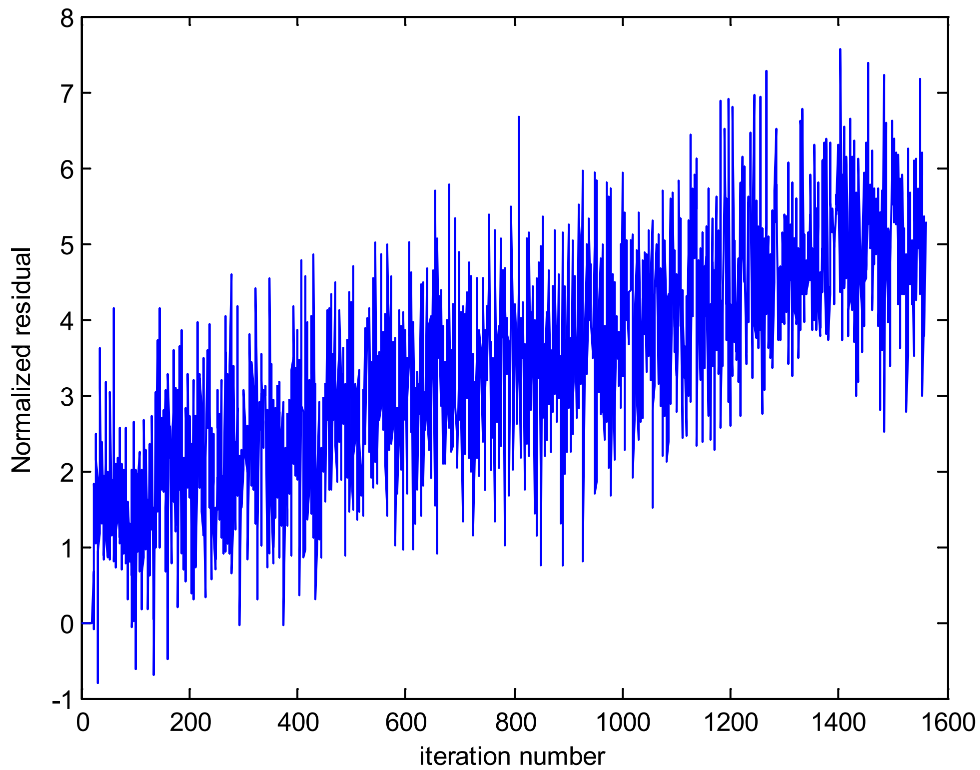

Fig.7. Normalized residual of the data fusion filter in the case when the coefficients p_0 and T_0 change

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The adaptive data fusion filter based Baro/GPS integration simulation results when the coefficients p_0 and T_0 are changed from iteration k=23 to k=1600 are presented in Figs. 8 - 10.

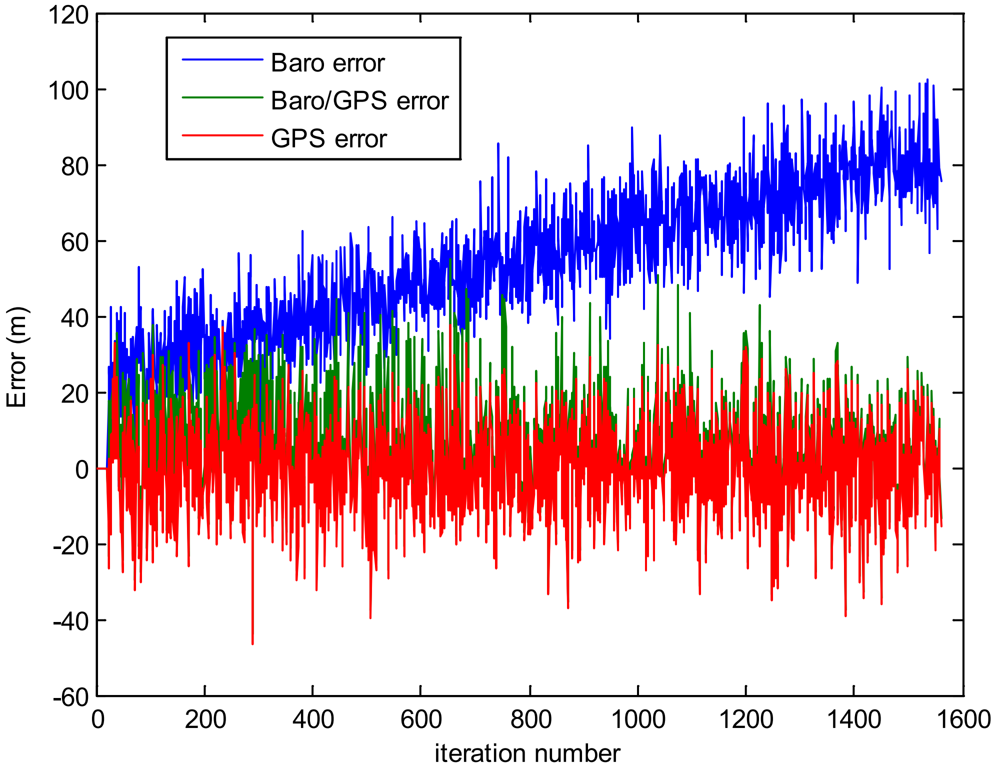

Fig.8. Absolute errors of the Baro, GPS and Baro/GPS altimeters in the case of coefficients p_0 and T_0 change (adaptive filter is used)

As seen from presented graphs in Fig. 8 and Fig.9 the adaptive data fusion filter based Baro/GPS integrated altimeter gives sufficiently good altitude estimation results when the coefficients p_0 and T_0 change. Unlike the non-adaptive Baro/GPS altimeter, the proposed adaptive Baro/GPS integrated altimeter remains robust to sea level pressure and temperature changes, and the adaptiveness property of the altimeter is secured (see Fig. 9). The non-adaptive Baro/GPS altimeter errors in this case increase and go to larger values. We can state that the adaptive Baro/GPS integrated altimeter gives sufficiently good altitude estimation results than the non-adaptive altimeter when the coefficients p_0 and T_0 change.

The variances of the altitude errors in this case are given in Table 1.

Table 1. Error variances of the adaptive data fusion based Baro/GPS altimeter

Iteration	400	600	800	1000	1300	1500
P, m^2	160.693	159.417	163.447	159.054	164.408	164.772

As seen from the results presented in Table 1, the error variance of the Baro/GPS altimeter is lesser than the error variances of the GPS vertical position. The behavior of the adaptive factor $S(k)$ is shown in Fig. 10.

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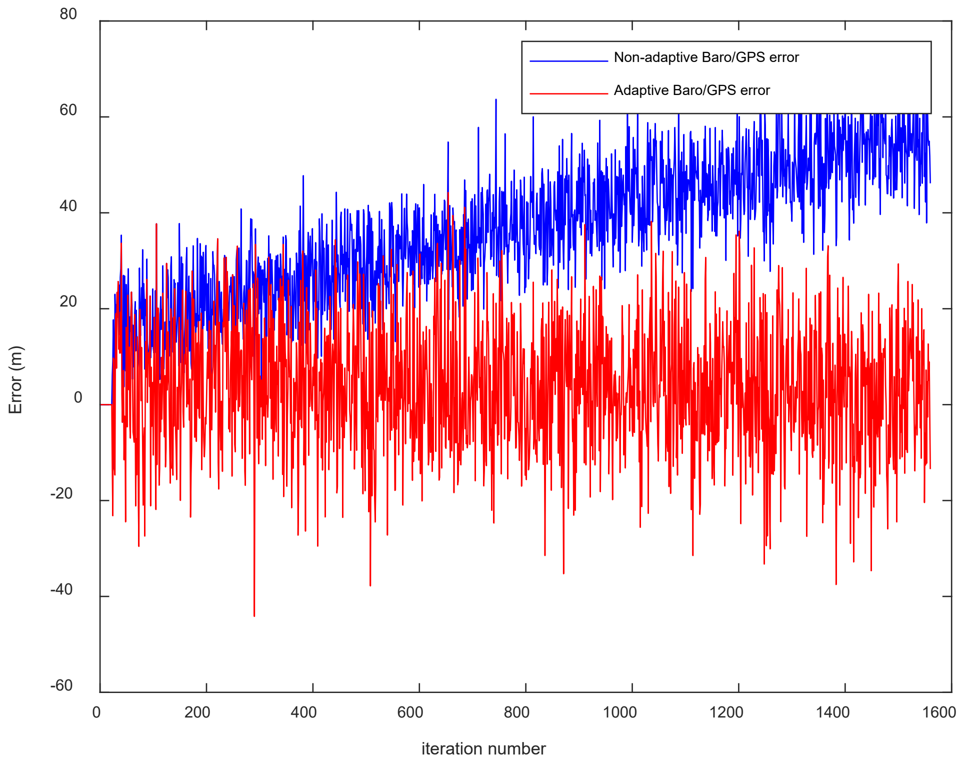

Fig.9. Absolute errors of the adaptive data fusion based Baro/GPS altimeter versus non-adaptive Baro/GPS altimeter in the case when the coefficients p_0 and T_0 change

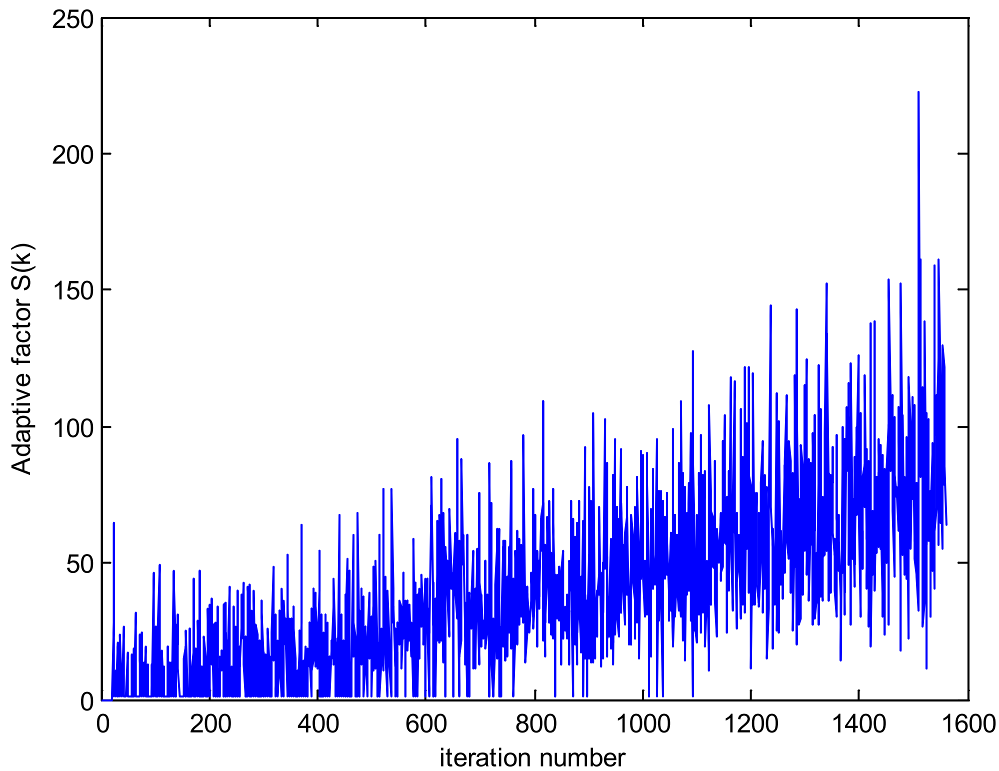

Fig.10. Adaptive factor $S(k)$

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The computational burden of the proposed algorithm and the conventional algorithm is examined in a computer having "Intel (R) Core (TM) 2 Quad CPU Q 9400" by determining the required computing time to perform the algorithms. The elapsed times are:

- 0.021502 s for the conventional non-adaptive algorithm.
- 0.024544 s for the proposed adaptive algorithm.

As seen, the proposed adaptive algorithm does not require an excessive amount of computing time but it takes slightly more time than the conventional algorithm.

6. Conclusions

In this paper Baro/GPS integrated altimeter, robust to the change in the baro altitude model parameters, is developed. The integration is achieved by using an adaptive data fusion filter with the filter gain correction. In contrast to the non-adaptive filtration algorithm in which the filter gain is constant, in the proposed adaptive algorithm, the current measurements have larger weights, since the gain coefficient of filter is corrected by the results of each observation. By the use of defined variable, named as measurement noise scale factor, incorrect baro altimeter measurements are taken into consideration with small weight and the altitude estimations are corrected without affecting the characteristics of the accurate ones. In case of a change in the sea level pressure and the temperature, the proposed adaptive Baro/GPS integrated altimeter remains robust to these changes, unlike the non-adaptive Baro/GPS altimeter, and the adaptiveness property of the altimeter is secured. The simulation results show that the proposed adaptive data fusion algorithm for barometric altimeter and GPS vertical position integration makes the altitude estimation accuracy better significantly in the precence of incorrect barometric altimeter measurements.

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