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SVD-Aided EKF Attitude Estimation with UD Factorized Measurement Noise Covariance

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Abstract: This paper describes singular value decomposition (SVD) aided extended Kalman filter (EKF) for nanosatellite's attitude estimation. The development of the filter kinematic/dynamic model, and the measurement models of the sun sensors, and the magnetometers which are used to generate vector measurements is presented. Vector measurements are used in SVD for satellite attitude determination purpose. In the proposed method, EKF inputs are coming from SVD as the linear measurements of attitude angles and their error covariance. In this step, UD is factorizing the attitude angles error covariance with forming the measurements in order to obtain the appropriate inputs for the filtering stage. Results are presented and analyzed in addition that the necessity of the sub-step which is the UD factorization on the measurement covariance is discussed. The accuracy of the estimation results of the SVD-aided EKF with and without UD factorization is compared for the estimation performance.

Keywords: UD factorization; nanosatellite; SVD; EKF; attitude estimation; rate gyro.

I. Introduction

Satellites need to be oriented in space and depending on the mission requirements they can have specific restrictions for the attitude accuracy. Especially the nanosatellites are required to keep the attitude as desired with micro sized attitude sensors because they are cheap, simple, light etc. as needed. Mostly, the nanosatellites are having the magnetometers and sun sensors onboard which are very common with limited achievable attitude accuracy due to the unavailability of the data. For attitude determination, reference directions should be described and those vectors as sensor output and the models related to them can be placed in a single-frame algorithm to process the results in order to improve under the extended Kalman filter (EKF) after the coarse attitude determination phase. Single-frame methods' aim is to minimize the Wahba's loss function defined in 1965 [1], [2]. By minimizing the loss between

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the measurements and the models for the sun and magnetic field directions, attitude and their covariance can be determined. In this paper, singular value decomposition (SVD) is used as the single-frame method in the first step because of its robustness performance. In [3], [4], studies make a critical comparison of those estimators in the sense of robustness in which the SVD and the q method are found to be the most robust in principle.

Basically, attitude estimation of the satellite can be performed using the EKF with assigning initial values, integrating the measurements under the propagation model of the satellite dynamics and estimate the attitude of the satellite. The traditional approaches to the design of Kalman filter for satellite attitude and rate estimation use the nonlinear measurements of reference directions [5]–[8]. However, in this paper non-traditional Kalman estimation technique has been executed. In [9], a review is carried out on gyroless attitude determination methods for small satellites. In that paper, both traditional and non-traditional Kalman filters and their comparisons are presented. The non-traditional approach based on the linear measurements is used in several studies [10]–[12]. In this case, determined coarse attitude angles, and covariance from one of the single-frame methods is used directly in the filter which shapes the second part of the non-traditional algorithm.

At Kyushu university, weighted least square method have been integrated to linearized Kalman filter for developed microsatellite having the same attitude sensors and it is stated that the estimator decreases the random noise effect and determine the attitude in accordance with the mission requirements in [13].

An extended Kalman filter is proposed by [14] for real-time estimation of solid body orientation in 2001 using developed MARG (Magnetic, Angular Velocity and Gravity) sensors which include three-axis magnetometer, three-axis angular velocity sensor, and three-axis accelerometer. The modelled system in the paper, converts angular velocities to quaternion rates and obtains quaternion ratios and integrates them to obtain quaternions. The Gauss-Newton iteration algorithm is used to find the optimal quaternion. The quaternion is used as part of the measurements of the Kalman filter which is a non-traditional form of the Kalman filter. The authors tested the proposed algorithm for different cases include high noise levels as well as major initial faults and resulted that the filter achieves very good estimations.

In the research presented by [15], the problem of attitude estimation is considered for unmanned aerial vehicles (UAV) using inertial measurement unit (IMU) in Kalman filter. The kinematic model of aircraft behavior is not very linear; therefore, a version of the Kalman filter is proposed, which can handle nonlinearities. A common solution to satellite attitude estimation as TRIAD algorithm, which is an observation model in the filter. Using the TRIAD algorithm, it is easy to select the most reliable sensors at different stages of a flight.

Another non-traditional approach is presented by [16] which integrates the q-method with an EKF to generate qEKF filter. In the filter, the attitude vector measurements are first processed using the q method, which is a single-frame method solves the

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Wahba's problem directly, without nonlinear assumptions. The remaining measures are processed for updating obscure situations using the conventional EKF algorithm. The authors are stated to confirm the validity of the proposed approach by numerical simulations and the comparison to the conventional EKF.

In [10], authors use a gyro-free attitude estimation system having magnetometer and sun sensor in the non-traditional Kalman filter and show the superiority of the non-traditional approach to the traditional ones for the attitude accuracy of the satellite. The authors considered the non-diagonal elements of the covariance matrix of the SVD to be small compared to the diagonal elements. Therefore, the non-diagonal elements are neglected, and the error covariance matrix considered diagonally [10], [17]. In this study, an SVD-aided EKF method using UD factorization is considered as an extended version of [18]. As the simulations are performed for nanosatellites, EKF is selected because they are light and easy to implement compared to UKF [19], [20]. In case of using UD in the algorithm, error covariance matrix is factorized without using the assumption of neglecting the non-diagonal elements and the new measurement vector with the uncorrelated error components are redefined. The algorithm is shaped in two phases as the first step, SVD and second step, EKF forms the nanosatellite's non-traditional attitude estimation algorithm. For non-traditional approach of the Kalman filter, inputs from SVD are the attitude angles as the linear measurements and their error covariance values in a matrix form computed every step. However, in order to achieve the required inputs for EKF algorithm, decomposition of the attitude angles error covariance matrix from SVD into diagonalized covariance matrix should be performed. In order to achieve the diagonalization, UD decomposition method [21] is presented and applied on to the first step before the filtering stage. Moreover, the measurement vector introducing the attitude angles which will be having uncorrelated error components is redefined for the filter input.

The structure of this paper is as follows. Section 2 gives mathematical models of the satellite's rotational motion and sensor measurements. The details of the magnetic field direction vector, sun direction vector measurement model are presented in this section. In Section 3, the Wahba's optimization problem and its solution by SVD method are given. In Section 4, SVD aided EKF for satellite attitude estimation based on linear measurements is presented with the introduction of UD decomposition of the measurement noise covariance matrix and details of EKF design. The simulation results of the non-traditional approach including the UD factorized measurements are presented in Section 5. The last section gives a brief summary of the obtained results and conclusions.

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II. Mathematical Models

A. Satellite's Rotational Motion

If the kinematics of the small satellite is derived in the base of Euler angles, then the mathematical model can be expressed with a 6 dimensional system vector which is made of attitude Euler angles (φ is the roll angle about x axis; θ is the pitch angle about y axis; ψ is the yaw angle about z axis) vector and the body angular rate vector with respect to the inertial axis frame,

$$x = \begin{bmatrix} \varphi & \theta & \psi & \omega_x & \omega_y & \omega_z \end{bmatrix}^T . \tag{1}$$

Also for consistency with the further explanations, the body angular rate vector with respect to the inertial axis frame should be stated separately as;

$$\omega_{BI} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T , \tag{2}$$

where ω_{BI} is the angular velocity vector of the body frame with respect to the inertial frame. Besides, dynamic equations of the satellite can be derived by the use of the angular momentum conservation law;

$$J_x \frac{d\omega_x}{dt} = N_x + \left(J_y - J_z\right)\omega_y \omega_z , \tag{3.a}$$

$$J_y \frac{d\omega_y}{dt} = N_y + \left(J_z - J_x\right)\omega_z \omega_x , \tag{3.b}$$

$$J_z \frac{d\omega_z}{dt} = N_z + \left(J_x - J_y\right)\omega_x \omega_y , \tag{3.c}$$

where J_x, J_y and J_z are the principal moments of inertia and N_x, N_y and N_z are the terms of the external moment affecting the satellite.

Kinematic equations of motion of the nanosatellite with the Euler angles can be given as,

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s(\varphi)t(\theta) & c(\varphi)t(\theta) \\ 0 & c(\varphi) & -s(\varphi) \\ 0 & s(\varphi)/c(\theta) & c(\varphi)/c(\theta) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} . \tag{4}$$

Here $t(\cdot)$ stands for tangent function and p, q and r are the components of ω_{BR} the vector which indicates the angular velocity of the body frame with respect to the reference frame. ω_{BI} and ω_{BR} can be related via,

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$$\omega_{BR} = \omega_{BI} - A\begin{bmatrix} 0 \\ -\omega_o \\ 0 \end{bmatrix} \tag{5}$$

where A is the transformation matrix, ω_o denotes the angular velocity of the orbit with respect to the inertial frame, found as $\omega_o = \left(\mu / r_0^3\right)^{1/2}$.

B. *Measurement Models*

Two attitude sensors are considered in this study to estimate the attitude using SVD-aided EKF. Magnetometer and sun sensor measurements should be modelled in order to obtain the first-step-measurements by SVD.

IGRF model defines the magnetic field direction vector [22] in orbit frame. Here, the magnetometer measurements are needed to be modelled too. Three onboard magnetometers of the satellite measure the components of the magnetic field vector in the body frame. Therefore, for the measurement model, which characterises the measurements in the body frame, gained magnetic field terms must be transformed by the use of direction cosine matrix, A . Overall measurement model may be given as;

$$B_m(k) = A(k)\,\mathrm{B}_o(k) + v_B(k)\,, \tag{6}$$

where $B_m\left(k\right)$ is the measured Earth magnetic field vector as the direction cosines in the body frame, $v_B(k)$ is the magnetometer measurement noise. The Sun direction vector measurements can be expressed in the following form:

$$S_m(k) = A(k)S_o(k) + v_S(k)\,, \tag{7}$$

where $S_m\left(k\right)$ is the measured Sun direction vector as the direction cosines in the body frame, $S_0\left(k\right)$ represent the Sun direction vector in the orbit frame as a function of time and orbit parameters, $v_S(k)$ is the sun sensor measurement noise.

III. Singular Value Decomposition (SVD)

In this section, the SVD method, the prefix of the non-traditional approach, is briefly described. After defining Wahba's optimisation problem, two or more vectors can be used in statistical methods to reduce losses the most [1]. In Equation (8), the loss can be seen as the difference between the measurements and the models found in the unit vectors.

$$L(A) = \frac{1}{2}\sum_i a_i \,|\, \mathrm{b}_i - \mathrm{Ar}_i \,|^2 \tag{8}$$

$$B = \sum a_i b_i r_i^T \tag{9}$$

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$$L(A) = \lambda_0 - tr\left(AB^T\right) \tag{10}$$

where b_i (set of unit vectors in body frame) and r_i (set of unit vectors in reference frame) with their a_i (non-negative weight) are the loss function variables obtained for instant time intervals and λ_0 is the sum of non-negative weights. Further, the matrix '*B*' is defined to reduce the loss function into equation (10). Here, maximizing the trace $\left(tr(\mathrm{AB}^T)\right)$ means minimizing the loss function (L). In this study, the Singular Value Decomposition (SVD) method was chosen to reduce the loss function to the minimum [4], [23].

$$B = USV^T = Udiag\left|S_{11}\ S_{22}\ S_{33}\right|V^T \tag{11}$$

$$A_{opt} = Udiag[1 \quad 1 \quad \det(U)\det(V)]V^T \tag{12}$$

The U, V matrices are orthogonal left and right matrices respectively and the primary singular values $\left(S_{11},\ S_{22},\ S_{33}\right)$ can be calculated in the algorithm. To find the angle of rotation of the satellite, the transformation matrix must first be found in equation (12) with an equation. The "*diag*" operator returns a square diagonal matrix with the elements of the vector in the main diagonal. Rotation angle error covariance matrix (P_{SVD})

$$P_{SVD} = Udiag[(\mathrm{s}_2+\mathrm{s}_3)^{-1} \quad (\mathrm{s}_3+\mathrm{s}_1)^{-1} \quad (\mathrm{s}_1+\mathrm{s}_2)^{-1}]U^T \tag{13}$$

is required to determine the instantaneous times that give higher error results than desired. Here $s_1 = S_{11}$, $s_2 = S_{22}$, $s_3 = det\left(U\right)det\left(V\right)\ S_{33}$.

IV. SVD-Aided Extended Kalman Filter

For non-traditional approach of the Kalman filter, inputs from the single-frame methods are the attitude angles as the linear measurements

$$\begin{aligned} z_\varphi(k) &= \varphi(k) + \mathrm{v}_\varphi(k), \\ z_\theta(k) &= \theta(k) + \mathrm{v}_\theta(k), \\ z_\psi(k) &= \psi(k) + \mathrm{v}_\psi(k) \end{aligned} \tag{14}$$

and their error covariance values in a matrix form computed every step. In Eq. (14), $\varphi(k)$, $\theta(k)$ and $\psi(k)$ are the attitude angles determined by SVD method, $\mathrm{v}_{(\cdot)}(k)$ is the measurement noise of the attitude angles.

In order to achieve the required inputs for EKF algorithm, decomposition of the P_{SVD} into diagonalized covariance matrix should be performed. For this purpose, UD decomposition method is presented and applied in the first step before the filtering stage. After UD factorization of the P_{SVD} matrix, the diagonal measurement noise covariance matrix $\tilde{R}$ is obtained and used in the filter as the input from the SVD. Furthermore, the measurement vector with the uncorrelated error components should be redefined. Consequently, in this study SVD gives the uncorrelated observation inputs to the EKF framework.

A. UD Decomposition of the Measurement Noise Covariance Matrix

With the decomposition of the attitude angles measurement error covariance matrix P_{SVD}, measurement vector which will be having uncorrelated error components should be redefined. Also, because of the new uncorrelated measurement vector definition measurement error covariance will be updated as diagonal matrix format [21], [24].

If the measurement vector z coming from the firstly introduced single-frame method is defined as,

$$z = Hx + v \tag{15}$$

with measurement matrix H and measurement noise v.

Corresponding covariance matrix which is not diagonal matrix can be represented as,

$$E\left[vv^T\right] = P_{SVD} = R \tag{16}$$

UD can also be called modified Cholesky decomposition seen in elemental matrix form as,

$$\begin{array}{ccccccc} \mathrm{P}_{SVD} & = & U & \times & D & \times & U^T \\ \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} & = & \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} & & \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} & & \begin{bmatrix} 1 & 0 & 0 \\ u_{21} & 1 & 0 \\ u_{31} & u_{32} & 1 \end{bmatrix} \end{array} \tag{17}$$

The loop can be composed using i and j variables with the row/column size of n for R matrix.

INPUT AND INITIAL VALUES	FIRST ASSIGNMENT	LOOP	$\begin{pmatrix} R = UDU^T \\ D = diag\left(\bar{D}\right) \end{pmatrix}$
$R_{n\times n}$ $U_{n\times n} = zeros\left(n\right)$ $\bar{D}_n = zeros\left(1,n\right)$	$\bar{D}\left(n\right) = R\left(n,n\right)$ $U\left(:,n\right) = R\left(:,n\right)./\,\bar{D}\left(n\right)$	$FOR\ j = n\text{-}1:\text{-}1:1$ $\bar{D}\left(j\right) = R\left(j,j\right) - \sum\left(\bar{D}\left(j+1:n\right)\cdot U\left(j,j+1:n\right)^2\right)$ $U\left(j,j\right) = 1$ $FOR\ i = j\text{-}1:\text{-}1:1$ $U\left(i,j\right) = R\left(i,j\right) - \sum\left(\bar{D}\left(j+1:n\right)\cdot U\left(i,j+1:n\right)\cdot U\left(j,j+1:n\right)\right)$ END END	$D = diag\left(\bar{D}\right)$ $\tilde{R} = D$ $\tilde{z} = U^{-1}z$

Fig. 1 UD decomposition algorithm structure.

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$$
\begin{aligned}
&n = length(R) \\
\text{Init:}\quad &U = D = zeros(n) \\
&D_{nn} = R_{nn} \\
&U_{in} = \begin{cases} i & i = n \\ R_{in} / D_{nn} & i = n-1, n-2, \ldots, 1 \end{cases}
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&j \in \{n-1, n-2, \ldots, 1\} \\
&D_{jj} = R_{jj} - \sum_{k=j+1}^{n} D_{kk} U_{jk}^2 \\
&U_{ij} = \begin{cases} 0 & i > j \\ 1 & i = j \\ \left(R_{ij} - \sum_{k=j+1}^{n} D_{kk} U_{ik} U_{jk} \right) & i = j-1, \ldots, 1 \end{cases}
\end{aligned}
\tag{19}
$$

Covariance matrix (R) can be factored in order to find diagonalized and uncorrelated matrix (D) consisting of the pivots in addition to the unit upper triangular part (U) whose diagonal entries are equal to 1 and forms the measurement vector again.

$$
R = UDU^T \tag{20}
$$

$$
\tilde{z} = U^{-1} z = U^{-1} (Hx + v) = \left(U^{-1} H \right) x + \left(U^{-1} v \right) = \tilde{H} x + \tilde{v} \tag{21}
$$

As it is clear from (7), the "new" measurement $\tilde{z}$ has measurement matrix $\tilde{H} = U^{-1} H$ and measurement noise $\tilde{v} = U^{-1} v$. The covariance matrix $\tilde{R}$ of the measurement noise $\tilde{v}$ is determined as [21],

$$
\tilde{R} = E\left[\tilde{v} \tilde{v}^T \right] = D \tag{22}
$$

Newly composed $\tilde{R}$ and $\tilde{z}$ can be used in the Kalman filter in order to be updated automatically by the single-frame output. The algorithm structure for the UD decomposition and the newly composed measurement vector can be seen in Fig. 1.

B. EKF Design for Attitude and Rate Estimation

If the state vector is arranged as (17) and the mathematical model of the LEO satellite's rotational motion about its centre of mass, is linearized using quasilinearization method. We will consider a real-time linear Taylor approximation of the system function at the previous state estimate. The Kalman Filter which is obtained will be called the Extended Kalman Filter (EKF). Filter algorithm, in this case as, is given below:

Innovation sequence and the equation of the estimation value,

$$
\Delta(k+1) = \left\{ \tilde{z}(k+1) - H\hat{x}(k+1/k) \right\} \tag{23a}
$$

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$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1) \times \Delta\left(k+1\right) \tag{23b}$$

Here $\tilde{z}(k+1)$ is the measurement vector, H is the measurement matrix, $\Delta\left(k+1\right)$ is the innovation sequence. In the investigated case the measurement matrix composed of a unit matrix. For the gyroless satellites, $\tilde{z}(k+1) = \left[\tilde{z}_\varphi(k+1) \quad \tilde{z}_\theta(k+1) \quad \tilde{z}_\psi(k+1)\right]$ measurement vector with $H = \text{diag}\left(\left[1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0\right]\right)$ measurement matrix, for the satellites having gyros, $\tilde{z}(k+1) = \left[\tilde{z}_\varphi(k+1) \quad \tilde{z}_\theta(k+1) \quad \tilde{z}_\psi(k+1) \quad \tilde{z}_{\omega_x}(k+1) \quad \tilde{z}_{\omega_y}(k+1) \quad \tilde{z}_{\omega_z}(k+1)\right]$ measurement vector with $H = \text{diag}\left(\left[1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\right]\right) = I_{6\times 6}$ measurement matrix are used. Normalized innovations are obtained as $\left\{\hat{\Delta}(k+1) = \Delta(k+1) \times \left[HP(k+1/k)H^T + \tilde{R}(k)\right]^{-0.5}\right\}$.

Equation of the extrapolation value,

$$\hat{x}(k+1/k) = f\left[\hat{x}(k), k\right] \tag{24}$$

Filter-gain of EKF

$$K(k+1) = P(k+1/k)H^T \times \left[HP(k+1/k)H^T + \tilde{R}(k)\right]^{-1} \tag{25}$$

The covariance matrix of the extrapolation error is,

$$P(k+1/k) = \frac{\partial f[\hat{x}(k), k]}{\partial \hat{x}(k)} P(k/k) \frac{\partial f^T[\hat{x}(k), k]}{\partial \hat{x}(k)} + Q(k) \tag{26}$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1) = \left[I - K(k+1)H\right] P(k+1/k) \tag{27}$$

$\tilde{R}$ is the diagonalized covariance matrix of measurement noise and Q is the covariance matrix of the system noises.

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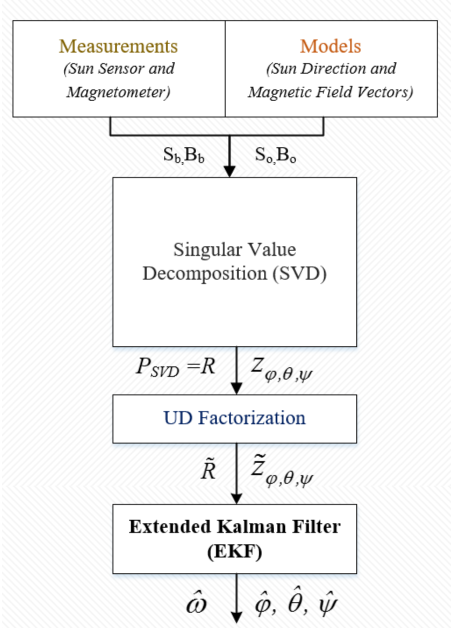

Fig. 2 Integrated SVD/EKF attitude and rate estimation block diagram with UD factorization

Equations given as (23) - (27) represent the Extended Kalman Filter (EKF), which fulfils recursive estimation of the satellite's rotational motion parameters about its mass centre on the linear attitude measurements. The whole algorithm scheme for integrated SVD/EKF attitude and rate estimation can be seen in Fig.2 including the sub-step of UD decomposition.

V. ANALYSIS OF SIMULATION RESULTS

Low Earth orbiting nanosatellite's orbit and structural parameters have been used in order to perform the simulations. The principal moment of inertia, $J = diag\left(2.1\times10^{-3} \quad 2.0\times10^{-3} \quad 1.9\times10^{-3}\right)$ is used which belongs to a nanosatellite. The satellite has attitude sensors as magnetometers and sun sensors. For the magnetometer measurements, the sensor noise is characterized by zero mean Gaussian white noise with a standard deviation of $\sigma_m = 300\ nT$. The standard deviation for the sun sensor noise is taken as $\sigma_s = 0.002$ (for unit vector measurements).

Small satellites especially the nano and pico-satellites have a poor capability on the computations. One of the computationally heavy stages in the algorithms is taking the inverse of the measurement error covariance at each step which also might lead the filter to be unstable at some point. Since the computational sources are not sufficient at small satellites, it is better to use the measurement covariance matrix as diagonal to reduce the computational burden. There are two ways in order to use this matrix diagonal. First is to neglect the non-diagonal terms and the second is to use the UD factorization. For analyzing the simulation results, both methods are considered and compared.

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After the UD factorized $\tilde{R}$ matrix and newly formed measurement vector step, SVD-aided EKF algorithm works recursively. It can be seen in the Fig. 2 that the attitude angles can be estimated by using this sub-step with more realistic way.

A. Simulation Results for the Satellite without Gyros

In Figs. 3-5, actual values and the estimations of the attitude angles by only SVD, integrated SVD&EKF can be seen in the first panels of the figures. The error changes in time also have been plotted as the 2^{nd} panels showing that acceptable attitude estimations can be obtained using the algorithm including the UD decomposed measurement error covariance matrix and newly formed measurement vector updated at each step.

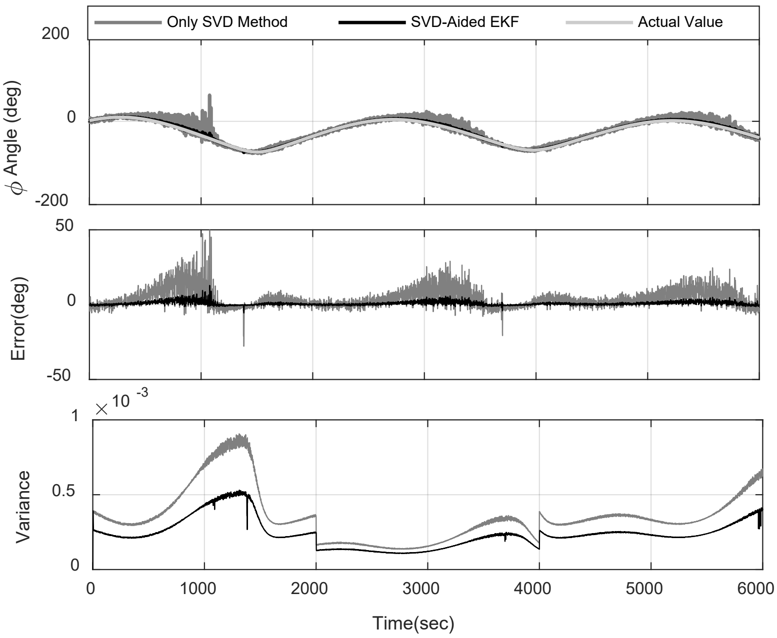

Fig. 3 Roll angle estimations, error and variance of only SVD and SVD-aided EKF using UD factorization (gyro-free).

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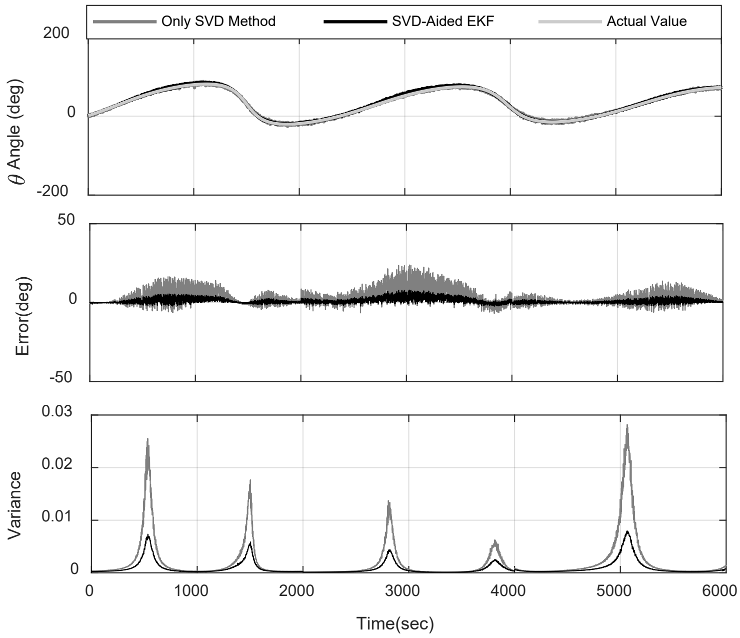

Fig. 4 Pitch angle estimations, error and variance of only SVD and SVD-aided EKF using UD factorization (gyro-free).

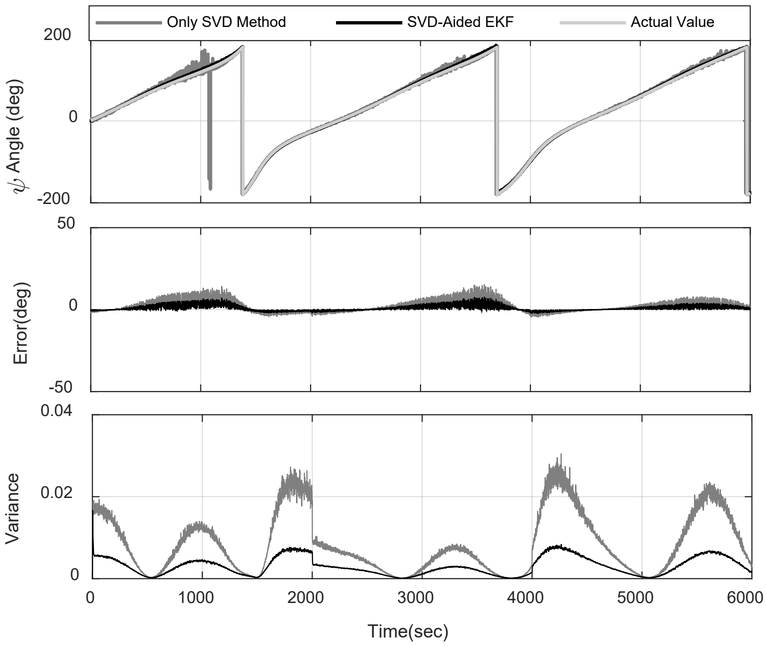

Fig. 5 Yaw angle estimations, error and variance of only SVD and SVD-aided EKF using UD factorization (gyro-free).

The corresponding variance values to roll, pitch and yaw attitude angles are presented in the third panels of Figs 3-5. As it can be seen from the figures, the filter gains an adaptive form by the help of SVD using its variance values and develops it accordingly. Furthermore, in Table I, the RMSE (root mean square error) of attitude angles is presented. The simulation results of the filter that uses UD factorization for the measurement noise covariance diagonalization and measurement update is called “With UD”. On the

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other hand, the simulation results of the filter that assumes the measurements are uncorrelated and eliminate the non-diagonal elements of the measurement noise covariance directly, is called “With Assumption”.

TABLE I. RMSE results of attitude angles (gyro-free).

RMSE (deg)	Roll	Pitch	Yaw
With UD	0.2940	0.0881	0.0635
With Assumption	0.2354	0.0603	0.0426

As seen from Table I, the SVD-aided EKF with assumption gives an unrealistic accuracy of the attitude angles due to neglecting of non-diagonal elements of SVD's angle error covariance matrix in which the measurements are not uncorrelated in reality. In other words, the accuracy is exaggerated or overrated under the assumption of neglecting the non-diagonal elements as it assumes that the errors are not correlated at all but it is not the case in the real applications. Therefore, it is expected to have less accurate results in the simulations for “With UD” but to have more realistic outputs instead.

B. Simulation Results for the Satellite having Gyros

In order to simulate the three-axis rate gyros, the following equation is used,

$$\omega_{BI_m}(k) = \omega_{BI}(k) + v_g(k) \tag{28}$$

For the calibration of the gyroscopes, the biases on gyros’ each axis can be estimated in the filter first as it is done in the studies of [25], [26]. In this study, the gyros are assumed to be calibrated in-orbit before the simulations. The estimations of the roll, pitch and yaw angles can be seen in Figs. 6-8 in the first panels, errors in the second and the related variance in the third. As it can be seen from the figures that the SVD-aided EKF improve the attitude estimations as expected.

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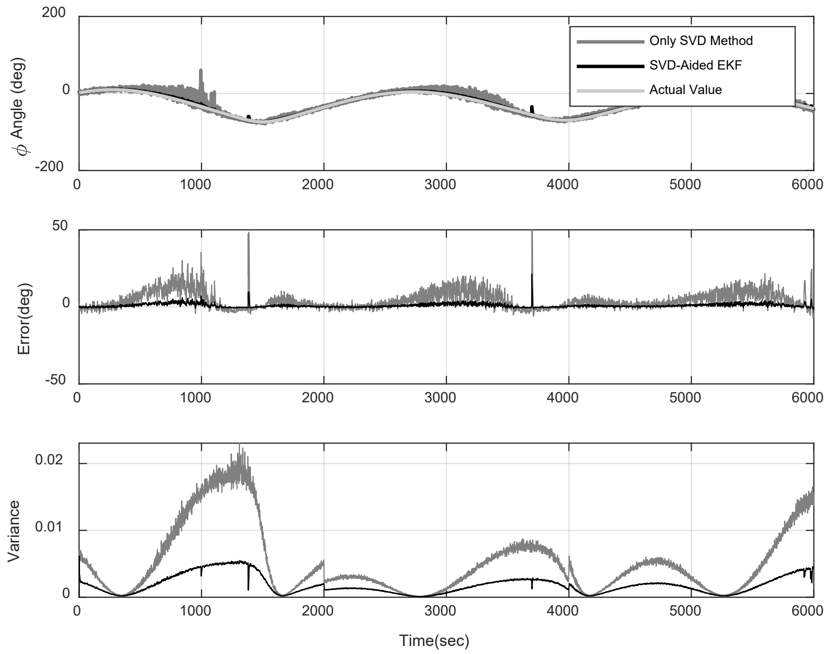

Fig. 6 Roll angle estimations, error and variance of only SVD and SVD-aided EKF using UD factorization (with gyro).

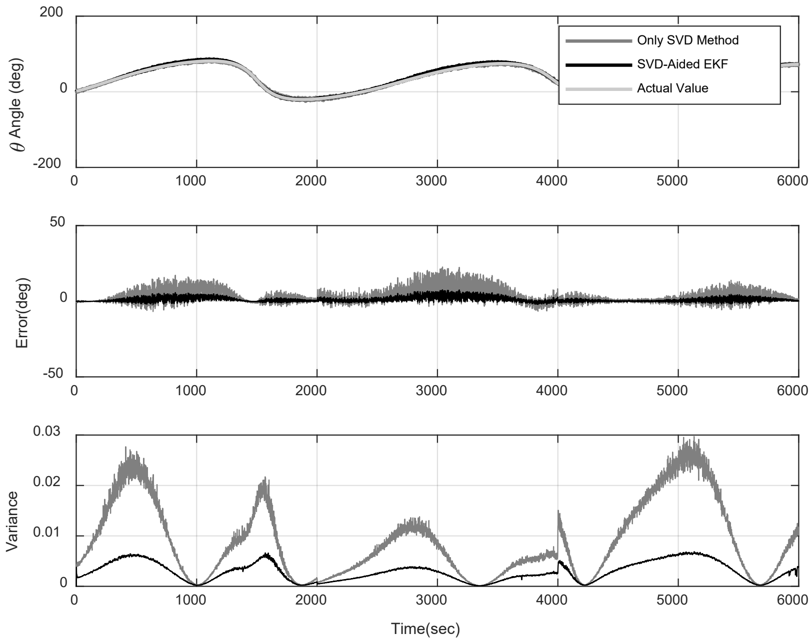

Fig. 7 Pitch angle estimations, error and variance of only SVD and SVD-aided EKF using UD factorization (with gyro).

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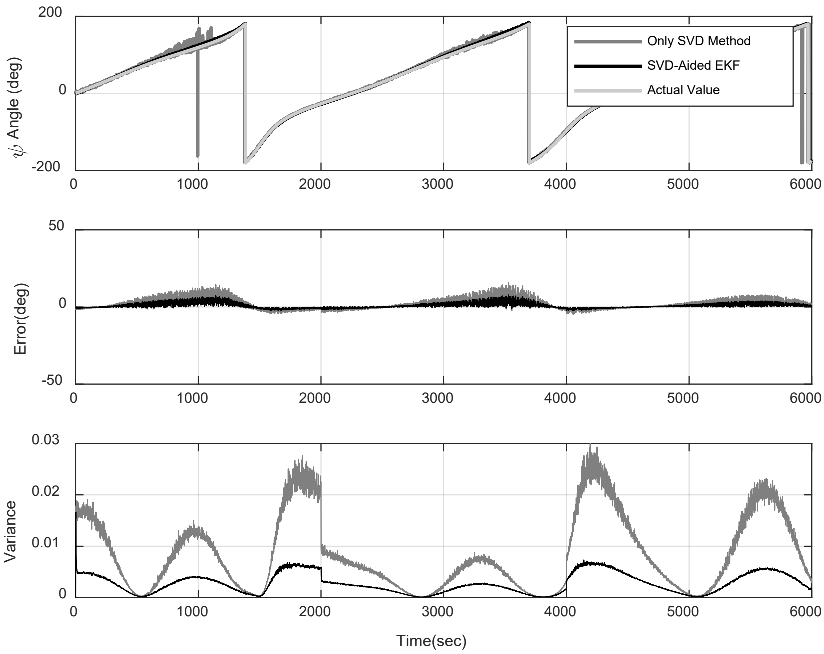

Fig. 8 Yaw angle estimations, error and variance of only SVD and SVD-aided EKF using UD factorization (with gyro).

TABLE II. RMSE results of attitude angles (with gyro).

RMSE (deg)	Roll	Pitch	Yaw
With UD	0.0495	0.0782	0.0669
With Assumption	0.0350	0.0538	0.0614

Table II gives the RMSE of each attitude angle estimation with UD and with assumption. The simulations having UD factorization for the measurement noise covariance diagonalization and measurement update is called “With UD” while the simulations assuming the measurements are uncorrelated and eliminating the non-diagonal elements of the measurement noise covariance, is called “With Assumption”. SVD-aided EKF with assumption gives an accuracy of the attitude angles which are unrealistic. It can be stated that the gyroscopes improve the attitude estimations for both cases (with UD and with assumption cases) if it is compared with Table I.

UD decomposition which is more realistic than neglecting the non-diagonal elements is also considered. It should be noted that for satellites require high accuracy performance and error characteristics for their mission objectives, the UD factorization step is a significant stage. However, the difference between the RMSE results of algorithms using and not using the UD step is very small.

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From this, it can be concluded that the non-diagonal elements of the SVD's angle error covariance matrix can be removed and only the diagonal elements can be considered in the input of the EKF for most of the nanosatellite missions.

CONCLUSION

In this study singular value decomposition aided extended Kalman filter for nanosatellite's attitude estimation is presented. The sun sensors and the magnetometers are used as the attitude sensors in SVD. In the proposed method EKF inputs are coming from SVD as the linear measurements of attitude angles and their error covariance. UD is factorizing the attitude angles error covariance with forming the measurements in order to obtain the appropriate inputs for the EKF. Moreover, gyro and gyro-free cases are considered in the demonstrations. For the integrated SVD/EKF using the newly formed measurements and measurement noise covariance with UD factorization, the whole algorithm is run. Simulation results show that the SVD-aided EKF with assumption which acts that the measurements are uncorrelated and removes the non-diagonal elements of the measurement noise covariance gives an overrated accuracy of the attitude angles. However, the proposed attitude estimation method with UD factorization provides the exact value of attitude accuracy.

The demonstrations show that the difference in the estimation results of the SVD-aided EKF with and without UD factorization is small. Therefore, it can be said that the non-diagonal elements of SVD's angle error covariance matrix can be omitted from the input of the EKF if high accuracy of error characteristics are not required and in this case, it is seen that the error caused by this assumption is small at a negligible level. Besides, if the computational load is a problem, then non-diagonal elements of SVD error covariance matrix can be neglected without UD factorization. If high accuracy performance and error characteristics are required for the satellite mission, UD decomposition is recommended to be used which is an important step. The absence of this step for those missions might cause major problems eventually in attitude control of the satellite.

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