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Review on Gyroless Attitude Determination Methods for Small Satellites

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Abstract

This study surveys the developments in the gyroless attitude determination system, especially for small satellites. Two kinds of gyroless satellite attitude determination algorithms were reviewed namely, vector measurements and Kalman filter based methods. Traditional and nontraditional Kalman filters were considered in the Kalman filter based methods including Unscented Kalman Filter (UKF) and Extended Kalman Filter (EKF). Also, robust versions of those Kalman filters, which were incorporated with single, and multiple measurement noise scale factors (SMNSF, MMNSF respectively) are investigated and compared in the presence of measurement faults.

Keywords: gyroless, small satellite, attitude determination, Kalman filter, single-frame method, measurement fault.

Contents

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Nomenclature

Roman Symbols		**Acronyms**	
a_i	Non-negative Weight	GPS	Global Positioning System
b_i	Set of Unit Vectors in the Body Frame	ECI	Earth Centered Inertial
r_i	Set of Unit Vectors in Reference Frame	ESOQ	Estimator Of The Optimal Quaternion
A	Transformation Matrix	EKF	Extended Kalman Filter
L(.), J(.)	Loss Function	LKF	Linearized Kalman Filter
P	Error Covariance Matrix	FOAM	Fast Optimal Attitude Matrix
O_{local}	Observed Stars	IMU	Inertial Measurement Unit
Greek Symbols		LEO	Low Earth Orbit
λ	Wavelength	MEMS	Micro Electrical-Mechanical Systems
ϕ_j	Euler Angles Vector (deg)	MMNSF	Multiple Measurement Noise Scale Factor
φ	Roll Angle (deg)	UKF	Unscented Kalman Filter
θ	Pitch Angle (deg)	QUEST	Quaternion Estimator
ψ	Yaw Angle (deg)	REKF	Robust Extended Kalman Filter
ω_x	Angular Velocity (x direction, deg/s)	RMSE	Root Mean Square Error
ω_y	Angular Velocity (y direction, deg/s)	RUKF	Robust Unscented Kalman Filter
ω_z	Angular Velocity (z direction, deg/s)	SMNSF	Single Measurement Noise Scale Factor
Subscripts		SVD	Singular Value Decomposition
B	Body	TAM	Three-Axis Magnetometer
O	Orbit		
R	Reference		

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1. Introduction

1.1. Why Gyroless Spacecraft?

Attitude determination system is the subsystem of a spacecraft, which is low power consuming, less expensive, and uses non-fragile gyroscopes. Developed micro electrical-mechanical systems (MEMS) are low power consuming, have cheap sensors, but they are inaccurate and have an inadequate resolution for providing the desired performance. In addition, gyroscopes have a tendency to degrade or fail in orbit with time because of their nature.

Three types of rate gyros are used in today's Inertial Measurement Unit (IMU) systems. These are ring laser gyro (RLG), fiber optic gyro (FOG) and MEMS. RLGs may have a "locked-in" condition at very slow rotation rates. FOGs in comparison to RLGs require no mechanical burden for their operation and thus eliminate a troubling noise source. The drawback is that the sensed angular velocity is limited on the phase difference due to the Sagnac effect. Solid-state inertial sensors, such as MEMS devices, have cost, size, and weight advantages. However, their accuracy and resolution are lower than expected to meet most mission requirements which is a great disadvantage [1].

1.2. Gyro Failures

Due to the reasons stated in the previous subsection, gyroless attitude control software is necessary for continuous back up. If a satellite does not have any backup mode for gyro failure, the whole mission may fail. A couple of examples for the gyroscope failures are given as follows. The International Ultraviolet Explorer (IUE) launched in 1978 had six gyroscopes for the designed inertial system. In 1985, one of the gyros failed, and the IUE used only two gyros for the rest of the mission life. To continue operations and achieve all scientific goals of the spacecraft, innovative redesign of specific systems was developed on the ground [2]. The Hubble Space Telescope had six gyros including three redundant gyros. Also, its components have a possibility to be replaced with the new equipment by the astronauts. In 1999, the third gyro of the telescope failed, so the telescope started to use redundant, spare gyroscopes [3]. The gyroscope failures are caused by chemical, mechanical and electrical effects. The satellite operations center of the European Space Agency (ESA) reports that the ESA Remote Sensing Satellites (ERS)-2 required an orbital rescue because of the gyro failure. In 2000, a gyro design for the attitude and orbit control system was improved to minimize the necessary number of gyros to reduce the gyroscope failures [4]. In 2001, after the last gyroscope failed, a method of operating the ERS-2 sensors and actuators in a new way was developed for the gyroless ERS-2.

1.3. Gyro-Free Systems

Because of the failure of the gyros or the necessity of a design at the beginning of a mission, many of the spacecraft are designed gyroless. For both systems, it is needed to have a software that estimates the angular rates. The Solar Heliospheric Observatory (SOHO) lost its control temporarily by the end of its nominal mission in 1998. However, the equipment of SOHO was recovered to extend the mission except for the gyroscopes because of the damage by the extreme thermal stress of the environment. Hence, engineers had to solve the problem without using a gyro. In 1999, the gyro free software was uploaded to the satellite and SOHO became the first ESA 3-axis stabilized gyroless satellite [5]. The Defense Meteorological Satellite Program (DMSP) has a gyroless navigation mode and the satellite uses only Earth and Sun sensor data without gyroscope. The algorithm for yaw error estimates is an innovative software for satisfactory attitude results [6]. This model was successfully operated for 24 hours and thus it can be used for critical time intervals without gyro data.

In reference [7], a magnetometer-only attitude determination algorithm was described. The initial values are obtained from a deterministic method for propagating them in extended Kalman Filter (EKF). With no gyro data, spacecraft states should include not only the attitude but also the rates. To estimate attitude, the time derivative of the magnetic field can be used as the second vector. In that paper, the Solar, Anomalous, and Magnetospheric Particle Explorer (SAMPEX) spacecraft does not have a gyroscope on-board and only relies on a three-axis magnetometer for attitude determination. The described method was tested by using the actual spacecraft data during the eclipse period to simulate possible failure of the digital sun sensor and in the presence of a three axis magnetometer measurement only. The proposed combined algorithm works for 1.5 deg in attitude and 0.01 deg per sec (deg/s) on the angular rates.

Another method is presented in [8] and demonstrated with actual spacecraft data without possessing any attitude rate measurement equipment. Using a gyroless system increases the sensitivity of the estimates on the model uncertainty and measurement noise. As a result, the proposed algorithm in the study uses a Minimum Model Error (MME) approach. The problems resulting from the absence of attitude rate measurements are solved using the MME based approach in the presence of significant model error or noisy data. Spacecraft like SAMPEX which does not have any angular rate devices or spacecraft without any angular rate measurements as a result of any failures in existing gyros are the basis of this paper. Corrected models by using the results of actual flight data from SAMPEX spacecraft indicate that an algorithm related to this problem led to the accurate estimations for either spacecraft's position or attitude rate. In reference [9], the MME approach which is an optimal attitude estimation and smoothing algorithm was developed for spacecraft which do not have a device measuring angular rates as in [8]. Only the attitude sensors such as magnetometers, sun sensors, star sensors, etc. were used for the described model in the paper. The general form of optimal estimation approach using the solution of the nonlinear two-point-boundary-value problem and the linearized solution were considered. The MME based estimation was applied for the spacecraft's attitude.

For a satellite, launch and start of the orbit are two critical time intervals. During these intervals, attitude is estimated by making use of the sensor measurements. In the reference [10], a simple method was suggested for attitude estimation of a low-Earth-orbiting satellite in the sun acquisition mode. Because of the necessity of quick and reliable attitude knowledge for satellite missions, gyroscopes may fail, therefore the satellite may also fail. For reliable and quick results, the paper uses only Sun sensors and magnetometers. Also, the recommended algorithm was compared with the results of Kompsat-I satellite telemetry data for the verification.

Star tracker missions without using gyro rate data are given as an examples in this section. In [11], two different approaches were used for obtaining angular rates. The study considers a case of gyro failure. Attitude and angular rates are estimated by a dynamic model of the spacecraft in the first technique. In the second approach, the spacecraft attitude is independent of the estimation of angular rates. Star camera rates are the basis of the second technique to find spacecraft body angular rates. Thus, the independence of body angular rates makes the second algorithm bias-free from attitude estimates. Reference [12] presents a normal mode attitude control with a complete design of the microsatellite DEMETER. The paper describes three phases. The first step is to recall the characteristics of the satellite, specifications, and constraints of the mission. Then, this is followed by the dual mode control synthesis and designing a conventional filter due to concern for attitude control. A gyroless mission operation mode which is the satellite's normal mode was applied to the algorithm. Also, the attitude of the satellite was determined by using only an autonomous star tracker in the paper.

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2. Gyroless Attitude Determination Methods

Gyroless attitude determination methods can be classified into two different groups. First of these groups is the method, which does not use the kinematics and dynamics of the satellite and are listed as follows:

- Algebraic Method
- SVD Method
- q Method
- QUEST Method
- ESOQ Method
- FOAM Method

Single Frame (Point-by-Point) Methods

- Attitude Determination Based on Star Tracker Measurements
- Attitude Determination Based on GPS Measurements etc.

There are two approaches to the methods which use the kinematics and dynamics of the satellite.

- Traditional Approach to design a Kalman filter for satellite attitude and rate estimation uses the nonlinear measurements
- Non-traditional approach is based on the linear measurements

In the traditional approach, the measurement models in the filter are based on the nonlinear models of the reference directions, and so the measurements and states are related via nonlinear equations. The attitude angles are first found by using the vector measurements and applying a suitable single-frame attitude determination method at each step in the non-traditional approach. Then these attitude angles are directly used as measurement input for the Kalman filter. Hence the measurement model is linear in this case since the states are measured directly. Each of these approaches provides results by using Extended or Unscented Kalman Filters. In addition to the initial attitude angles found by using single-frame methods, attitude error covariance matrix is also the input to the EKF or UKF based on the linear measurements. These approaches include the following algorithms.

Traditional Approach:

- EKF or UKF based on Single Sensor Measurements

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- EKF or UKF based on Double Sensor Measurements
- EKF or UKF based on Multiple Sensor Measurements

Non-traditional Approach:

- Attitude Determination Based on Point-by-Point Methods and EKF
- Attitude Determination Based on Point-by-Point Methods and UKF

The gyroless attitude determination methods can be classified as seen in Table 1.

Table 1. Gyroless attitude determination methods.

<table>
<tr><th colspan="2">With Using
Dynamics and Kinematics of the Satellite</th><th colspan="2">Without Using
Dynamics and Kinematics of the Satellite</th></tr>
<tr><th>Traditional</th><th>Non-Traditional</th><th>Single Frame Methods</th><th>Only One Sensor Based</th></tr>
<tr><td>EKF or UKF based on only Single Sensor Measurements</td><td>Attitude Determination Based on Point-by-Point Methods and EKF</td><td rowspan="3">○ Algebraic Method,
○ SVD Method,
○ q Method,
○ QUEST Method,
○ ESOQ Method,
○ FOAM Method
etc.</td><td rowspan="3">Attitude Determination Based on :
○ Star Tracker,
○ GPS etc.</td></tr>
<tr><td>EKF or UKF based on Double Sensor Measurements</td><td rowspan="2">Attitude Determination Based on Point-by-Point Methods and UKF</td></tr>
<tr><td>EKF or UKF based on Multiple Sensor Measurements</td></tr>
</table>

Paper [13] states that an attitude determination system using star sensor does not require a package of rate gyro based on comparative studies of the gyroless and gyro-aided systems. The accuracy of the gyroless system would meet many satellite mission requirements. Hence, an accurate attitude determination and control system without gyroscopes is developed for environmental satellites in that study. In [14], rate gyro and dynamic gyro performances are compared. The aim of the paper is to investigate the error sources for the dynamic model and estimate the angular rate for multi-body spacecraft. Kalman filter based on the single sensor (star tracker) is used in the study to make the system robust to the error sources. Also, by modeling the disturbance torques, the attitude determination performance is improved.

Magnetometers are common sensors to estimate attitude for small satellites because they are cheap, simple, light and available as commercial-of-the-shelf equipment. Many types of research use magnetometers as their attitude sensors. Magnetic sensors can be used in single sensor based traditional filtering methods. In paper [15], a three-axis magnetometer (TAM) is used to estimate the state of a spacecraft which are three axis attitude and angular rates without any priori information. The authors propose two algorithms based on the deterministic method based on only one sensor and a Kalman Filter without using any gyro on board. The paper presents the test results using in-flight data from SAMPEX (gyroless, sun-pointing) and the Earth Radiation Budget Satellite (ERBS) (gyro-based, Earth-pointing) spacecraft. Also in [16], only geomagnetic field data are required to perform the algorithm which will allow achieving an acceptable attitude and rate error range for low-cost spacecraft or a backup estimator in the case of sensor failures. In the proposed EKF, spacecraft states, therefore angular rates, are estimated (no gyroscope necessary). In [17], magnetometer-based gyroless attitude and angular rates are estimated for the Far Ultraviolet Spectroscopic Explorer (FUSE) spacecraft. The estimation includes two algorithms and their combination. In the first algorithm, the integrated-rate parameters (IRP) approach, which uses the kinematic model for propagation, is used to model the angular acceleration of the spacecraft. The second algorithm is the pseudo-linear extended Kalman filter which uses the dynamic model for rate propagation. These algorithms are combined and called hybrid IRP-Euler algorithms to estimate both attitude and rate of the spacecraft. A predictive filter is designed for the real-time gyroless system of a satellite mission with only attitude sensors (e.g. sun sensor, star tracker) [18]. The study proposes a new algorithm which predicts the required torque modeling error input. The attitude of the SAMPEX spacecraft is estimated by this real-time predictive filter which provides a robust algorithm by using magnetometer measurements only.

Sun sensors are used for different kinds of purposes on a satellite. For attitude determination of a satellite, both coarse and fine sun sensors are available with changing mass and sizes. In several studies, sun sensors and magnetometers are used together as attitude sensors onboard [19-21].

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3. Single Frame Methods

3.1. Two-Vector Algorithm

The attitude of the satellite is found using only two vector observations by creating three orthogonal vectors. These vectors represent both model and sensor. Therefore a transformation attitude matrix is performed. Two vector observations (u and v) can be any vectors which define an orthogonal coordinate system. For example, u and v can be chosen as sun direction and magnetic field vector.

The equations can be defined as follows [22]:

$$\hat{q} = \hat{u} \tag{1}$$

$$\hat{r} = \frac{\hat{u}\, x\, \hat{v}}{|\, \hat{u}\, x\, \hat{v}\, |} \tag{2}$$

$$\hat{s} = \hat{q}\ x\ \hat{r} \tag{3}$$

The reference matrix M_R is obtained using the two reference vectors in orbital coordinates, $\hat{u}_R$ and $\hat{v}_R$. Also, the body matrix M_B is calculated using the two measured vectors in the spacecraft body coordinates, $\hat{u}_B$ and $\hat{v}_B$.

$$M_R = [\hat{q}_R \,\vdots\, \hat{r}_R \,\vdots\, \hat{s}_R] \tag{4}$$

$$M_B = [\hat{q}_B \,\vdots\, \hat{r}_B \,\vdots\, \hat{s}_B] \tag{5}$$

$$AM_R = M_B\,,\quad A = M_B M_R^{-1} \tag{6}$$

From defined reference and body matrices, the attitude matrix (A) is calculated and issued to find the attitude of the satellite. For the error covariance matrix [23] in the Euler form, the Cartesian covariance matrix ($P_{\theta\theta}$) must be known and it is defined as

$$P_{\theta\theta} = \sigma_1^2 I + \frac{1}{|\vec{u}_B x \vec{v}_B|^2}[\left(\sigma_2^2 - \sigma_1^2\right)\vec{u}_B \vec{u}_B{}^T + \sigma_1^2\left(\vec{u}_B . \vec{v}_B\right)\left(\vec{u}_B \vec{v}_B{}^T + \vec{v}_B \vec{u}_B{}^T\right)] \tag{7}$$

where σ_1^2 is the variance of the magnetometer, σ_2^2 is the variance of the sun sensor and I is the unit matrix with the dimension 3x3. The error covariance matrix as a set of Euler angles is given in equation (8) using the matrix H^{-1} as in (9).

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$$P_{\phi\phi} = HP_{\theta\theta}H^T \tag{8}$$

$$H^{-1} = \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial A_k}{\partial \phi_j} x A_k\right)_i \tag{9}$$

Here, ϕ_j are the Euler angles (phi, theta, psi) respectively. The covariance matrix is used as input for filtering algorithms since the matrix for variance changes.

$$W = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}, \quad W = \dot{A}A^{-1} \tag{10}$$

Angular velocity is found from the equation (10) using matrix (A), which is defined as attitude transformation matrix [24].

3.2. Minimization of Wahba's Loss Function

The attitude of a spacecraft is estimated in statistical methods that include unit vectors and found by using the sensor data and existing models such as Earth's magnetic field, Sun direction, nadir direction, and star direction vectors. Grace Wahba (1965) defined a problem to minimize the loss function (L) between sensor and model definitions in order to obtain the transformation matrix between two different coordinate frames [25]. The algorithm of the statistical method is based on the definition to solve the least-squares problem. The recursive algorithm estimates the attitude of the satellite by using statistical methods for model and measurement or using only measurement data. Also, many studies use point methods only for obtaining initial values in the recursive method [26, 27]. Loss function aims to find transformation matrix A:

$$L(A) = \frac{1}{2}\sum_i a_i \left|b_i - Ar_i\right|^2 \tag{11}$$

In the equation (11), b is the set of unit vectors in the body frame (sensor data), r is the set of unit vectors in the reference frame (defined models), and a is the non-negative weight (inverse variances: σ^{-2}) with $\lambda_0 \equiv \sum a_i$.

$$B \equiv \sum a_i b_i r_i^T \tag{12}$$

$$L(A) = \lambda_0 - tr\left(AB^T\right) \tag{13}$$

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The loss function is rewritten in a more convenient way, the equation above (13) with reducing the Wahba's problem into maximizing the trace function (sum of the diagonal elements) of the product of A and transpose of B matrices. There are many methods to solve the problem faster and more robust if they are compared in these aspects. SVD, q, and QUEST methods will be explained because of their advantages in the computational expenses and robust results. Comparison of those methods was performed in several studies [26, 28-31].

3.2.1. Singular Value Decomposition (SVD) Method

The matrix B is expressed in two orthogonal matrices (U, V) by using singular value decomposition [32-34]. Here, S refers to primary singular values defining the secondary ones (s_1, s_2, s_3). B matrix:

$$B = USV^T = Udiag\,|\,S_{11}\;S_{22}\;S_{33}\,|\,V^T \tag{14}$$

where 'diag' indicates a square matrix with zero elements outside the diagonal, U and V are the left and right singular vectors of the B. The matrices U and V are orthogonal, and the primary singular values follow the inequalities $S_{11} \geq S_{22} \geq S_{33} \geq 0$.

From the calculated U and V matrices, the optimal attitude matrix is obtained with maximized trace, and $\det(A) = 1$.

$$A_{opt} = U\;diag\left[1\;1\;\det(U)(detV)\right]V^T \tag{15}$$

From the secondary singular values and left-right singular vectors of the B matrix, the rotation angle error covariance matrix P is found [28].

$$s_1 = S_{11},\; s_2 = S_{22},\; s_3 = det(U)\,det(V)\,S_{33} \tag{16}$$

$$P = Udiag\left[(s_2+s_3)^{-1}(s_3+s_1)^{-1}(s_1+s_2)^{-1}\right]U^T \tag{17}$$

The covariance matrix of the estimation error P has similar trend in-orbit with the absolute error matrix. Recursive methods can minimize these errors arising from unobservable or parallel attitude vectors using the covariance matrix.

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3.2.2. q Method

Quaternions have several advantages in attitude representation. Therefore, the attitude matrix can be parameterized using quaternions [35, 36]. A practical approach with unit quaternion was suggested by Davenport [28, 37, 38].

$$q = \begin{bmatrix} q \\ q_4 \end{bmatrix} \tag{18}$$

$$A(q) = (\mathrm{q}_4{}^2 - |q_v|^2)\mathrm{I} + 2\,\mathrm{q}_v \mathrm{q}_v{}^T - 2q_4[q_v \times] \tag{19}$$

$$tr(\mathrm{AB}^T) = q^T K q \tag{20}$$

The Euler theorem defines a single rotation angle and a fixed rotation axis to perform quaternion parameterization [39]. The quadratic function is given in the above equations that include quaternions. If ‘K’ is defined as a traceless matrix, an equation for ‘A’ and ‘B’ matrix is found. These are defined in Wahba’s Loss Function.

$$K \equiv \begin{bmatrix} S - I\ tr(B) & z \\ z^T & tr(B) \end{bmatrix} \tag{21}$$

$$S \equiv B + B^T \tag{22}$$

$$z \equiv \begin{Bmatrix} B_{23} - B_{32} \\ B_{31} - B_{13} \\ B_{12} - B_{21} \end{Bmatrix} = \sum_i a_i b_i \times r_i \tag{23}$$

$$K q_{opt} \equiv \lambda_{\text{maksimum}} q_{opt} \tag{24}$$

The traceless matrix K includes the 3x3 ‘S’ matrix obtained by the mathematical models and sensor data. The S matrix includes the B matrix and its transpose B^T. The z matrix has three units, and is included in the K matrix like S.

An optimum quaternion is a normalized eigenvector about the maximum eigenvalue. The last equation above (24) represents the solution by using eigenvalues and eigenvectors. There are several software products which have their own packages to obtain eigenvalue and vectors easily. However, there is one problem that if the eigenvectors are equal, then the correct solution cannot be obtained. Markley [28] stated that it is not the issue of the q method but in this case data are not suitable to determine attitude. The results in this case are expected to be similar to the results with Sun Sensor in eclipse going to infinity.

3.2.3. QUaternion ESTimator (QUEST) Method

In the q method, the described characteristic equation is solved with iterative methods such as Newton-Raphson. Besides, some assumptions can be made to obtain the solutions faster. QUEST is one of the methods that uses numerical iterative techniques and uses Gibbs vector as the attitude representation. However, Gibbs, too, has a singularity problem that can be found in detail in [40]. The point here is that the SVD method can minimize the Wahba's loss function with Euler angle representation directly and in addition to quaternions, but q method and QUEST only use quaternions to obtain attitude.

$$\alpha \equiv \lambda_{max}^2 - (trB)^2 + tr(adjS) \tag{25}$$

$$\beta \equiv \lambda_{max} - trB \tag{26}$$

$$\gamma \equiv det\left[\left(\lambda_{max} + trB\right)I - S\right] = \alpha\left(\lambda_{max} + trB\right) - det\,S \tag{27}$$

$$\mathbf{x} \equiv \left(\alpha I + \beta S + S^2\right)\mathbf{z} \tag{28}$$

$$q_{opt} = \frac{1}{\sqrt{\gamma^2 + |\mathbf{x}|^2}}\begin{bmatrix}\mathbf{x}\\ \gamma\end{bmatrix} \tag{29}$$

λ_0 defined in the equation (13) is the initial value to find $\lambda_{\max}$ from the characteristic equation of $det(K - \lambda_{max} I) = 0$ [28]. The variables and notations are the same as in the q method. Moreover, using [41], the covariance matrix was obtained as follows:

$$P = \left[\sum_i a_i \left(I - b_i b_i^T\right)\right]^{-1} \tag{30}$$

As mentioned earlier, filtering approaches like UKF use the covariance matrix as an initial value or variance values for the mission duration. Also, instant results allow to finding the time when the algorithm should be switched to another by using error covariance analysis.

In the case of any parallelism between the vectors, the angles between them are considered and compared if the jumping times of the variance results match each other.

$$\vec{u} \cdot \vec{v} = |u||v| \cos\alpha \tag{31}$$

$$\alpha = \cos^{-1}\left(\frac{u_1 v_1 + u_2 v_2}{\sqrt{u_1 + u_2} \cdot \sqrt{v_1 + v_2}} \right) \tag{32}$$

For the angle of ± 180 degrees between two vectors, algebraic or statistical methods using two vectors do not estimate attitude.

3.2.4. Other Methods

The Fast Optimal Attitude Matrix (FOAM) solves the Wahba's loss function for the optimal attitude matrix directly, and no additional intermediate computations are necessary [28, 42, 43]. Another solution to Wahba's optimization problem is EStimator of the Optimal Quaternion (ESOQ) described in [44] by Mortari. In the paper [45], a new analytical solution is suggested as stable and accurate as QUEST and q method. This analytical method also gives better results than ESOQ method.

3.3. Comparison of Single Frame Methods

Single frame methods are used when at least two vectors are available (only two vectors for the algebraic method). To compare all presented methods, sun sensor and magnetometers have been selected and used in those methods. Their results for different periods of time are listed in Table 2 as the root mean square (RMS) error. 2nd, 5th,8th, 11th, and 14th rows (Error 1) represent the first interval (before the eclipse). Eclipse period is seen as Error 2. The last rows of all methods as Error 3, are the period after the eclipse duration.

As seen from the Table 2, SVD method and q method are the most reliable methods regarding the robustness. If the computational burden is the issue, then the QUEST method or algebraic method is selected as the base method to determine the attitude of the satellite [46].

Table 2. RMS results for attitude angles using different single-frame methods.

RMS Error (deg)		Roll	Pitch	Yaw
Algebraic Method	Error 1	5.53	1.81	5.42
	Error 2	nd	nd	Nd
	Error 3	4.07	1.70	1.14
SVD Method	Error 1	3.38	1.79	0.96
	Error 2	100.09	28.33	200.10
	Error 3	2.69	1.64	1.07
Q Method	Error 1	3.38	1.79	0.97
	Error 2	100.19	30.09	194.19
	Error 3	2.69	1.65	1.07
QUEST Method	Error 1	3.39	1.87	0.98
	Error 2	93.19	30.12	194.21
	Error 3	2.72	1.65	1.11
FOAM Method	Error 1	4.26	2.21	1.00
	Error 2	nd	nd	nd
	Error 3	2.93	0.84	1.98

4. Attitude Determination Based on Only One Sensor

Some sensors have the ability to find all the attitude angles or at least two of them. For this purpose, the problem was reduced into one cost function from the measurements. After obtaining the cost function, attitude is determined by using methods which are also presented in the following sections.

4.1. Star Tracker Measurements

Star trackers have commonly been used as attitude sensors in attitude determination systems of the satellites. In [47], authors survey the algorithms for star identification which are utilized in the star cameras. Selected lost-in-space, recursive, and non-dimensional algorithms were compared in that study. In the paper, it was stated that lost-in-space case from a single image in real-time became possible without a priori information. A real-time attitude estimation algorithm was presented in [48]. In the algorithm presented in

[48] one can accurately estimate the three-axis attitude of a satellite using only star sensor measurements. The requirements of the mission design lead to choosing sufficiently fine Charged Coupled Device (CCD) or Complementary metal–oxide–semiconductor (CMOS) devices. The attitude determination using star trackers as attitude sensors are based on keeping the sensor fixed on the satellite and observing the sky by taking into account the map. The steps for the attitude determination are listed as follows:

- Identification of the observed stars (O_{local}),
- Axis transformation (A) of the observed stars into Earth-Centered Inertial (ECI) frame,
- Comparison of the stars in the catalog (S) and observed stars in ECI frame (O_{ECI}),
- Minimizing the cost function.

This process requires an initial estimation of transformation matrix (A) and correction of the observed stars into the catalogued stars.

$$\begin{aligned} &O_{local_1} \xrightarrow{A} O_{ECI_1} \to S_1 \\ &O_{local_2} \xrightarrow{A} O_{ECI_2} \to S_2 \\ &\quad\vdots \\ &O_{local_n} \xrightarrow{A} O_{ECI_n} \to S_n \end{aligned} \tag{33}$$

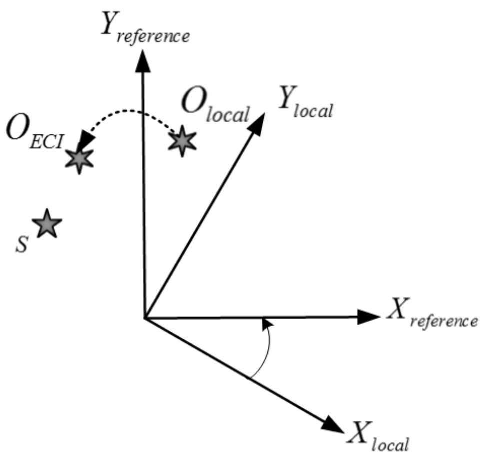

Figure 1. Star observation and catalog view.

After rotating the O_{local} observed stars into O_{ECI} catalogue reference frame (see Fig.1), the attitude matrix A should be updated with the correct one by minimizing the difference between O_{ECI} and S. However, the correction may have some problems caused by the false image of another star, i.e. O_{ECI} can be a different

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star than S. For this purpose, the relative geometry of a set of stars should be taken into consideration in the first step (identification of the observed stars).

$$\begin{aligned} O_{ECI_1} &= AO_{local_1} \\ O_{ECI_2} &= AO_{local_2} \\ &\vdots \\ O_{ECI_n} &= AO_{local_n} \end{aligned} \tag{34}$$

The direction cosine matrix A which characterizes the attitude is determined from the minimization of the cost function.

$$J = \sum_{i=1}^{n} \left(S_i - AO_{local_i} \right) \to \min \tag{35}$$

4.2. GPS Measurements

The Global Positioning System (GPS) is commonly used as satellite navigation system to provide position and time information for Earth orbiting satellites. These sensors are also used to determine the attitude of the satellite [49]. In [50], the attitude of a satellite was determined coarsely by using the Global Positioning System's (GPS) signal-to-noise ratio (SNR) from a single antenna. Authors stated that the performance of the method that they used highly depends on the number of GPS signals available. The method is proposed to be used in case of a backup mode for the satellite attitude determination system. In [51], coarse attitude determination is achieved using a single GPS antenna because attitude sensors of small satellites are limited by their operational skills. The authors evaluated their method using the telemetry data from the Space Flight Laboratory's (SFL) CanX-5 nanosatellite. Here, the eclipse period is also considered with data from the magnetometer, GPS measurements, and no data from the sun sensor. Results showed that the GPS measurements improve the accuracy of the attitude estimate accuracy by two-three times than by the use of magnetometer data alone.

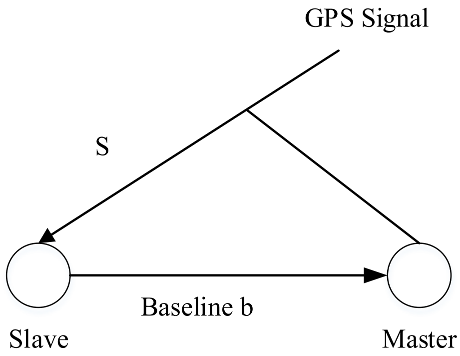

Figure 2. GPS signal and baseline.

Consider a satellite that includes two antennas (Master and Slave) as seen in Fig.2. From the figure, it can be said that the baseline (b) distance is used to detect the orientation of the satellite in space. The process includes first taking the projection of the baseline vector on the direction of incoming GPS signal (S) and then considering the path difference of the signal between two receivers (antennas).

Vector direction $\mathbf{S}$ is obtained in the geocentric reference frame because the position of the GPS satellite and the master antenna are known. Through this, the path difference is calculated in Eq. (36) moreover, transformed into body frame in Eqs. (37)-(38).

$$\Delta r = \mathbf{S}^T b \tag{36}$$

$$\mathbf{S} = AS \tag{37}$$

$$\Delta r = S^T A^T b \tag{38}$$

Most of the time, the phase difference ($\Delta\varphi$) is measured instead of Δr seen in Eq. (39).

$$\Delta\varphi = \Delta r \frac{2\pi}{\lambda} \tag{39}$$

where λ is the wavelength. Rotation around the baseline can be only determined by using at least three independent measurements. This means that two other baselines are necessary for solving the problem. As a result, three antennas on the satellite and two GPS satellites should be available, and their measurements are represented in Table 3.

Table 3. GPS measurements.

	Baseline 1	Baseline 2
GPS Satellite 1	$\Delta r_{11} = S_1^T A^T b_1$	$\Delta r_{12} = S_1^T A^T b_2$
GPS Satellite 2	$\Delta r_{21} = S_2^T A^T b_1$	$\Delta r_{22} = S_2^T A^T b_2$

With the help of two baselines and two GPS satellites, the attitude of an Earth orbiting satellite is determined. This does not mean that having three GPS satellites but only one baseline would allow determining the attitude. After all measurements and calculations, a cost function is defined in Eq. (40).

$$J = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\Delta r_{ij} - S_i^T A^T b_j \right) \tag{40}$$

where m is the number of satellites and n is the number of baselines. From the minimization of the cost function, the direction cosine matrix A is determined.

5. Kalman Filter-Based Methods

Gyroless attitude estimation with magnetometer and sun sensor measurements was addressed in several studies [7-10, 19, 20] and various algorithms that intend improving the estimation accuracy were proposed. A basic solution for the problem is to use a Kalman filtering algorithm for integrating the measurements under the propagation model of the satellite dynamics and estimate the attitude of the satellite possibly along with the sensor biases. The main drawback of the existing algorithms is the degradation in the estimation results when the satellite is in eclipse and sun sensor data were not available.

The aim of these studies is to design an attitude determination system that gives attitude knowledge with the desired accuracy along the whole orbit. To obtain the attitude of the satellite with the desired accuracy, an extended Kalman filter (EKF) and unscented Kalman filter (UKF) for satellite's angular motion parameter estimation were used.

By the use of a Kalman filter algorithm measurement inputs of these sensors are easily integrated to estimate the attitude parameters of the satellite precisely. At this stage, the methods of dynamic filtration (for

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example Kalman filters) may be useful. In general, two types of Kalman filter algorithms will be taken into consideration:

a) Kalman filter based on nonlinear measurements

b) Kalman filter based on linear measurements (nontraditional approach)

For the first case, sun sensor and magnetometer measurements based on traditional approach scheme, which is using nonlinear measurements, are given in Fig.3.

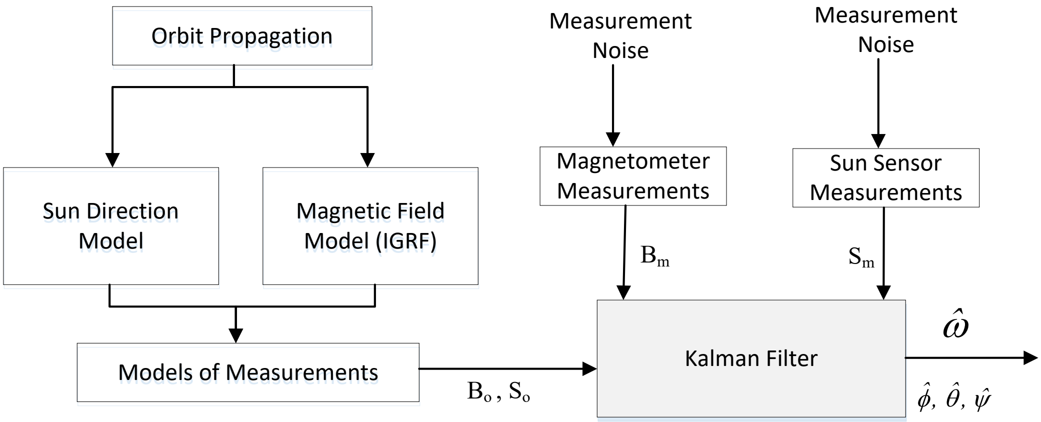

Figure 3. Attitude and rate estimation using traditional approach.

In the first method which is called traditional approach, measurement models are based on nonlinear models of reference directions. Therefore, there is a nonlinear relation between the measurements and the states. Additionally, the nontraditional Kalman filter structure considering the sun sensor and magnetometer measurements is seen in Fig. 4.

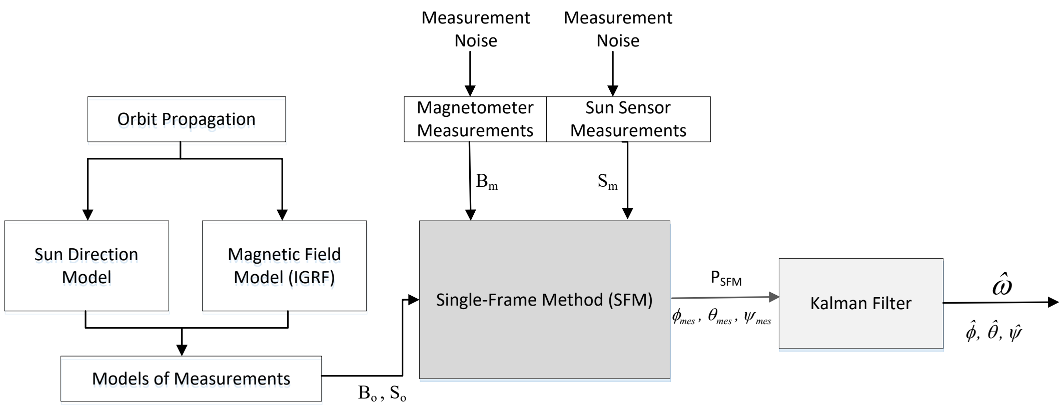

Figure 4. Attitude and rate estimation using the nontraditional approach.

In the second case, seen in Fig.4, based on linear measurements called a nontraditional method, attitude angles are found by attitude determination methods based on vector measurements at each step. Then these are directly used as measurement input in the Kalman filter. Hence the measurement model is linear in this case since the states are measured directly.

5.1. The Traditional Approach

The traditional approaches for the design of a Kalman filter for satellite attitude and rate estimation use the nonlinear vector measurements. Kalman filter plays a major role in the attitude estimation procedure of the spacecraft since it was proposed in [52]. Regarding the obstacles met during the development process of the attitude estimation systems, various types of Kalman filters were developed. One of these difficulties is the inherent nonlinear dynamics and kinematics of the satellites. Extended Kalman Filter (EKF) was proposed so as to overcome this problem and it was used instead of linear Kalman filter for estimating the attitude of the satellite [53].

A three-axis Magnetometer/Kalman Filter based attitude determination system for a spacecraft in low-altitude Earth orbit was developed, analyzed, and simulation tested in [54]. The modified EKF described and analyzed in this paper estimates 3-axis spacecraft attitude, attitude rate, and constant disturbance torques solely from 3-axis magnetometer measurements distributed over one orbit. The filter works for gravity-gradient stabilized spacecraft operating in inclined, low Earth orbits.

The paper in [17] documents the testing and development of magnetometer-based gyroless attitude and rate estimation algorithms for the Far Ultraviolet Spectroscopic Explorer (FUSE). The results of two approaches are presented. The first algorithm which is the integrated rate parameters (IRP) algorithm uses a kinematic approach in modeling the angular acceleration of the spacecraft. The second algorithm, the pseudo-linear extended Kalman filter, relies on the spacecraft dynamics model in the rate propagation. The only attitude sensor available is a magnetometer. Combining the two algorithms into the hybrid IRP-Euler algorithm provides attitude and rate estimates within the accuracy requirements for most maneuver scenarios.

In [55], a computationally efficient, nonlinear estimator, that directly uses vector measurements to estimate both the attitude matrix and the angular velocity, avoiding the need to precompute the temporal derivatives of these noisy measurements was presented. The algorithm was based on the IRP third-order minimal parameterization of the attitude matrix introduced, which is at the heart of its computational efficiency.

Avoiding the use of the uncertain spacecraft dynamic model, the filter uses a polynomial state space model, in which the spacecraft angular acceleration is modeled as an exponential correlation stochastic process, a concept used in tracking theory.

Software considering the gyroless attitude determination algorithms in case of a gyro failure or operational mode itself became a necessity for small, lightweight and inexpensive spacecraft. In particular, the three-axis magnetometers are considered as the primary attitude sensor for most nanosatellites. The paper [56] demonstrated that UKF algorithm could be used for this purpose which is superior to EKF.

In the research of [57], the EKF does not function properly without the angular rates from the gyros, since the filter depends on the gyroscope results to model the satellite dynamics. On the other hand, for the Linearized Kalman Filter (LKF), the rates are not necessarily essential, since the angular rates are modeled by the Euler formulae in the state model. In this paper, there are also two different scenarios depending on the gyroless algorithm as sun sensor only and magnetometer only to analyze the performance of the satellite. Another significant result is achieved in the study that the gyros do not bring many benefits in the sunlight case but give better accuracy of the LKF estimation during the eclipse period.

The performance evaluation of a spacecraft attitude and rate estimation algorithm was presented by [16]. In this work, the EKF used by the algorithm requires only the magnetometer measurements and a reduced state vector. The differential equation satisfied by the state perturbation filter was derived through an explicit approach. A performance assessment was conducted in response to clearly stated different initial conditions of the filter state as well as for un-modelled disturbance torques.

The paper [58] describes an attitude determination system that is based on two vector measurements of non-zero, non-collinear vectors. The algorithm relies on a quaternion formulation of Wahba's problem [25] whereby the error quaternion was taken as the observed state and was placed in the standard linear measurement equation. The proposed attitude determination system was based on two measured quantities which are the Earth's magnetic and gravitational fields. The accelerometers in conjunction with the derivative of GPS velocity provided a measure of the gravitation field vector, and the magnetometers measured the Earth's magnetic field vector. The time-varying Kalman filter implementation of this algorithm was performed on the simulated and real data collected from TRIAD of accelerometers and magnetometers.

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The performance of the Kalman filter algorithm for satellite attitude estimation which uses sun-sensor and magnetometer measurements was investigated in [59]. For magnetometer and sun sensor use there arises a problem when the magnetic field and Sun directions become collinear. In that case, the three-axis attitude is non-observable. Kalman filter accuracy decreases when the angle between two vectors becomes small. In [60], this dependence was obtained analytically and compared with in-flight data experiments from the Chibis-M microsatellite. It should be emphasized that the proposed method was applied only to the quasi-stationary motion when the acting forces and the measurement model are close to being a constant during the time interval between sequential measurements. This approach allows estimating the influence of unaccounted perturbations on attitude determination accuracy. The main advantage of this approach is that it does not require the simulation of Kalman filter. Therefore it requires less computational time. Accuracy dependence on filter parameters and perturbations is derived analytically and is more reliable than the one derived by common investigation approaches. The dependence of the attitude estimation accuracy on the angle between geomagnetic field vector and sun direction was obtained by the method and compared with the actual estimated satellite accuracy.

EKF has some disadvantages, especially for the highly nonlinear systems. This is caused by the mandatory linearization phase of the EKF procedure, and so Jacobians are derived for that purpose. For most of the applications, generation of Jacobians is hard, time-consuming and prone to human errors [61]. Nonetheless, linearization brings about an unstable filter performance when the time step intervals for the update are not sufficiently small and that results in the filter divergence [62]. Per contra, small time step intervals increase the computational load because of the larger number of Jacobian calculations. As a result of these facts, EKF may be efficient only if the system is almost linear on the timescale of update intervals.

A relatively new Kalman filtering technique, which does not have the shortcomings of EKF for nonlinear systems, is Unscented Kalman Filter (UKF). UKF generalizes Kalman filter for both linear and nonlinear systems, and in the case of nonlinear dynamics, UKF may afford considerably more accurate estimation results than the known observer design methodologies such as Extended Kalman Filter. The basic of UKF is the fact that the approximation of a nonlinear distribution is easier than the approximation of a nonlinear function or transformation [63]. UKF introduces sigma points to catch higher order statistics of the system, and by securing higher order information of the system, it satisfies both better estimation accuracy and convergence characteristics [64].

As a spacecraft attitude estimation algorithm, UKF has many implementation examples in the literature. In [65] it is used as a state estimator, while both the states and the parameters of the satellite are estimated by UKF in [66, 67]. Moreover, UKF is used as a part of the attitude control scheme of multibody satellites in [68].

The paper [56] develops an UKF in an attempt to solve the spacecraft attitude estimation and calibration problem based only on magnetometer measurements. Attitude vector was described by the three-component Rodrigues parameters, which avoids the singularity of the covariance matrix when using unit quaternion in attitude determination. To reduce the computational burden of filters, a better-behaved sigma point selection strategy of unscented transformation for UKF, spherical simplex sigma, was investigated. The UKF was tested through numeric simulation of a fully actuated rigid body with the only magnetometer. The results presented in this study clearly demonstrate the UKF is superior to EKF in coping with the nonlinearity of attitude dynamics in the presence of model uncertainties. It was shown that the UKF converges with poor initial estimates of the attitude and calibration, while the EKF was shown to have a greater tendency to diverge. At the same time, under the same condition, the same attitude accuracy was obtained using the different sigma point selection strategy for UKF, symmetric and spherical simplex sigma points, but the computational burden of filters using the latter was reduced a lot.

The study [69] deals with attitude determination, parameter identification and reference sensor calibration simultaneously. A LEO satellite's attitude, inertia tensor and calibration parameters of Three-Axis-Magnetometer were estimated during the mission. For this purpose, kinematic and kinetic state equations of spacecraft motion were augmented for the determination of inertia tensor and TAM calibration parameters including scale factors, misalignments, and biases along the three body axes. As the attitude determination is a nonlinear estimation problem, Unscented Kalman Filter and advanced nonlinear estimation algorithm with good performance were used to estimate the satellite attitude, but its computational cost was considerably larger than the widespread, low accuracy, EKF. Reduced Sigma Points Filters provided good solutions and also decreased the run time of UKF. However, in contrast to the nonlinear problem of attitude determination, parameter identification and sensor calibration have linear dynamics. Therefore, a new Marginal UKF was proposed that combines the utility of Kalman Filter with Modified UKF. Finally, Monte Carlo simulation in this paper demonstrated a good accuracy for concurrent estimation of attitude, inertia tensor and TAM calibration parameters in significantly less time than sole utilization of the UKF.

5.2. Comparison of EKF and UKF Estimation Results in the Presence of Measurement Faults

Based only on magnetometer measurements EKF and UKF estimation results in the presence of measurement faults were compared in [70]. Picosatellite carries three-axis magnetometers onboard as the only measurement device. In that paper, the Euler angles were used as the attitude representation method. To implement Kalman filter algorithms, a measurement model of this sensor should be derived. The instantaneous abnormal measurements type measurement malfunction scenario was taken into consideration. Instantaneous abnormal measurements were simulated by adding a constant term to the magnetic field tensor measurement of one magnetometer at the 300th second.

Table 4 gives the estimation results for optimal EKF and UKF in the case of measurement malfunction. As it is clear from the Table 4, both regular filters fail in case of measurement malfunction. Moreover, as the results show, it takes more time for EKF to regulate the effect of the fault and settle again. Besides as another fact about the results, it may be stated that even in the case of faulty measurements UKF gives more accurate results than EKF.

Table 4. Absolute values of error for EKF and UKF in the case of measurement malfunction [70].

Parameter	Abs. Values of Err. for Regular EKF		Abs. Values of Err. for Regular UKF	
	350^{th} s.	400^{th} s.	350^{th} s.	400^{th} s.
φ(deg)	54,414	26,390	2,2597	1,12762
θ(deg)	183,78	162,82	1,1496	1,41417
ψ(deg)	32,725	6,0517	2,9869	2,39368
ω_x(deg/s)	1,6190	0,3529	0,0037	0,00272
ω_y(deg/s)	0,8424	0,4885	0,0041	0,00157
ω_z(deg/s)	0,0290	0,3323	0,0089	0,00676

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5.3. Nontraditional Approach

In an approach based on the linear measurements, the attitude angles were first found by using the vector measurements and applying a suitable single-frame (point-by-point) attitude determination method [71] at each step. Then these attitude angles are directly used as measurement input for the Kalman filter. Hence the measurement model is linear in this case since the states are measured directly.

An integrated satellite attitude determination system based on the linear measurements was presented by [72-74], in which the algebraic method and the EKF algorithms were combined to estimate the attitude angles and angular velocities respectively. The attitude determination system uses the algebraic method (two-vector algorithm). This method is based on computing any two analytical vectors in the reference frame and measuring the same vectors in the body coordinate system. The magnetometers, sun sensors, and horizon scanners/sensors were used as measurement devices and three different two-vector algorithms based on the Earth's magnetic field, sun vector, and nadir vector were proposed. To obtain the satellite's angular motion parameters with the desired accuracy, an EKF was designed. Here, the measurement inputs for the EKF are the attitude estimates obtained using single-frame methods.

An EKF was proposed for real-time estimation of the orientation of human limb segments by [75]. The filter processes data from small inertial/magnetic sensor modules containing triaxial angular rate sensors, accelerometers, and magnetometers. Quaternion representation was used for representing the rotation in the filter instead of the Euler angles. The QUEST algorithm that solves the attitude based on the acceleration and magnetometer measurements gives the input for the EKF. Thus, the dimension of the state vector reduces and the measurement equations become linear.

The attitude determination concept of mini-satellite QSAT, which is designed at the Space Systems Dynamics Laboratory at Kyushu University, is based on a combination of the Weighted-Least-Square and Linearized-Kalman filter estimation methods. The Weighted-Least-Square method produces the optimal attitude-angle observations at one point in time by using the sun sensor and magnetometer measurements. The recursive Linearized-Kalman filter combines the angular observations with the attitude rate measured by the gyros to produce the optimal attitude solution [76].

In [77] the q-method for quaternion estimation was integrated into an EKF to produce the novel qEKF filter for attitude estimation, which is capable of treating both attitude and non-attitude states without additional numerical iterations. Within the filter, attitude vector measurements were first processed using the q-method, which solves the nonlinear Wahba problem directly without any linearizing assumptions. Remaining measurements were processed to update the non-attitude states using the standard multiplicative EKF algorithm.

A Gauss-Newton and EKF based Attitude Determination System was designed and numerically evaluated in [78] where the obtained results show a good performance of the attitude propagation process during the eclipse phase. The angular rate estimation based on a rough calculation and filter process, as well as the reduction of its bias in sun phases, allowed the propagation of satellite attitude information during the eclipse phase.

A two-phased estimation algorithm was proposed for a small satellite which has magnetometers and sun sensors as the attitude sensors onboard [20]. In the first phase, Wahba's problem, a well-known approach for single frame attitude estimation with vector sun sensor and magnetometer measurements, was solved by the Singular Value Decomposition method and Euler angle-estimations were obtained for the satellite's attitude. Obtained Euler angle-estimations were used as measurement inputs for an EKF, which forms the second phase of the algorithm. The covariance estimation of the SVD was used as the measurement noise covariance matrix of the EKF; this is how the filter was tuned specifically in the eclipse period. The results of the proposed algorithm were compared with a traditional approach using nonlinear measurements.

In [79] an integrated Algebraic method/Extended Kalman filter (EKF) attitude determination system was presented, in which the 2-vector and EKF algorithms were combined to estimate the attitude angles and angular velocities. As a reference direction for the algebraic method, the unit vectors toward the sun and Earth's Magnetic Field were used. The Euler angles produced a 2-vector algorithm, and their error variances were provided as input to the EKF. Then the EKF uses this attitude information as the measurements for providing more accurate attitude estimates even when the satellite is in eclipse. The "attitude angle error covariance matrix" calculated for the estimations of the algebraic method were regarded as the measurement noise covariance for the EKF. The parameters of the satellite's rotational motion (Euler angles and angular velocities) were estimated using EKF. In comparison to more traditional approaches, this preprocessing step significantly reduces the complexity of filter design by allowing the use of linear measurement equations.

The singular value decomposition method and the unscented Kalman filter were integrated to estimate the attitude and attitude rates of a nanosatellite recursively in [80]. First, the SVD method minimizes the Wahba's loss function to find the optimal solution for the attitude by magnetometer and sun sensor vector measurements. Then the UKF uses this attitude information as the measurements for providing more accurate attitude estimates even when the satellite is in eclipse. The rotation angle error covariance matrix calculated for the estimations of the SVD method is regarded as the measurement noise covariance for the UKF. Discussions for the UKF tuning are included specifically for the eclipse period where the SVD method fails, and practically there are no measurements input to the filter.

5.4. Comparison of Traditional and Nontraditional Kalman Filters

As the optimal attitude method, SVD was determined in the light of the results in Table 2, for developing the nontraditional approach SVD is used as the base single frame method. To compare the filter outputs, same conditions, constants, and vectors were selected (sun sensor and magnetometer). One orbital period takes about 6000 seconds. Performances of both algorithms are investigated by dividing the whole orbital period into several time intervals given in Table 5. It can be seen that the integrated SVD/EKF algorithm results are superior in most of the time intervals. Here, the comparison criterion is the Normalized Root Mean Square (NRMS) errors without units. When the SVD has any jumps on the attitude angle determination or covariance (1000-2000 sec), SVD/EKF will be affected, and the affected NRMS error results are seen in the table. Integrated SVD/EKF achieves more accurate attitude results than the traditional approach because of its adaptive way from the covariance values [21].

Table 5. NRMS errors for SVD/EKF and traditional EKF algorithms [21].

Time Interval (sec)	NRMS Errors for Angles					
	SVD/EKF			**Traditional EKF**		
	Phi	Theta	Psi	Phi	Theta	Psi
0-1200	0.0557	0.0044	0.0076	0.1273	0.0269	0.01822
1000-2000	0.0353	0.2143	4.3246	0.0189	0.1326	4.0551
2000-4000	0.0979	0.0414	0.0307	0.0462	0.0287	0.0542
4000-5500	0.0146	0.1604	0.0641	0.0321	0.5350	0.0879

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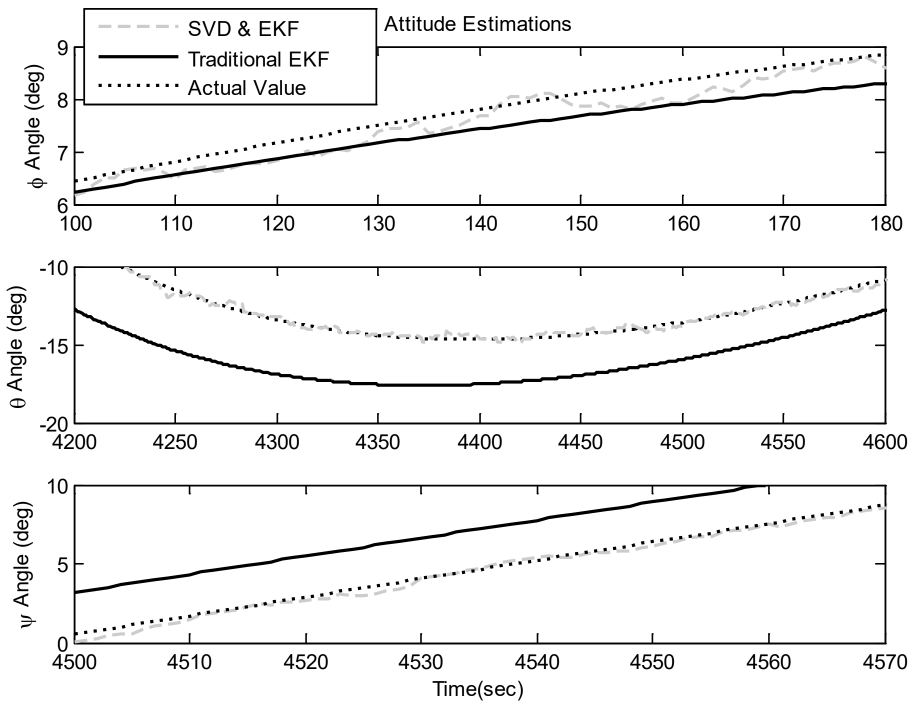

Figure 5. Comparison of the results of the traditional and nontraditional algorithms for specific time intervals [21].

In Fig 5, a zoomed frame of the satellite's attitude dynamics determined by traditional and nontraditional approaches are given. As a result, the figure indicates that the SVD/EKF nontraditional approach can follow the actual attitude angles better than the traditional method.

5.5. Robust and Adaptive Kalman Filters

When a small satellite is under normal operational conditions, whether it is EKF or UKF, a conventional KF gives sufficiently good estimation results. However, if the measurements are not reliable because of any malfunctioning in the estimation system, Kalman filter gives inaccurate results and diverges with time. The regular KF has no capability to adapt itself to the changing conditions of the measurement system. Malfunctions such as abnormal measurements, increase in the background noise, etc. affect instantaneous filter outputs; therefore, the filter may fail. To avoid such conditions, the filter must be operated robustly.

The study [70] compares two different robust Kalman filtering algorithms; Robust Extended Kalman Filter (REKF) and Robust Unscented Kalman Filter (RUKF) to address the measurement malfunctions. In both filters by the use of measurement noise scale factor, the faulty measurements were taken into consideration with a small weight, and the estimations were corrected without affecting the characteristics of the accurate ones. In this case, the filter was adapted by using a Single Measurement Noise Scale Factor (SMNSF) as a corrective term on the filter gain. REKF and RUKF were applied for the attitude estimation process of a pico-satellite, and the results were compared (See Section 5.6).

As discussed in [53], it is possible to adapt the filter by using a single scale factor as a corrective term on the filter gain, but this is not a healthy procedure if the filter performance differs for each state for the complex systems with multivariable [81]. The preferred method is then to use a matrix built of Multiple Measurement Noise Scale Factors (MMNSFs) to fix the relevant terms of the measurement noise–covariance matrix and consequently, the Kalman gain. In [82, 83], a Robust Kalman filtering method based on MMNSF was proposed for the quaternion attitude estimation problem. In these studies, both the EKF and UKF were modified. The new algorithms are robust against measurement faults and called Robust EKF and Robust UKF, respectively. However, the applied adaptation scheme is different from the one given in [70]. Moreover, in the studies [82, 83], the attitude estimation problem was generalized and instead of the Euler angles the quaternions were used as the attitude representation method. Additionally, robust Kalman filters were examined for different measurement system failure cases, and discussions on application were found in [83]. A multiple scale factor based adaptation scheme was applied, so any unnecessary information loss was prevented by disregarding only the data of the faulty sensor. To show the obvious effects of the discarded data from the failed sensor, the authors in [84] performed the adaptation by using both single and multiple scale factors which are two different approaches to the same problem. In the first case, the filter was adapted by using a single scale factor as a corrective term on the filter gain. In the second case, a scale matrix with multiple factors was used to fix the relevant term of the Kalman gain matrix individually. The algorithms proposed in [82-84] were tested for the attitude and attitude rate estimation problem of a small satellite which has only three magnetometers as the attitude reference source. Using only magnetometers is a common preference for the small satellite applications (in particular for the cubic pico-satellites), and limited number of sensors onboard increases the significance of the given robust Kalman filtering methods. A common technique for improving the estimation performance of the Kalman filter and making the filter robust against any faults is to adapt its process and measurement noise covariance matrices. In [85], two practical problems (R-adaptation and Q-adaptation) for a nanosatellite carrying magnetometers onboard were examined, and it was described how to work around this issue using the adaptive Kalman filtering approach. For this, the process noise covariance (Q) and measurement noise covariance (R) adaptation techniques were presented. The Q-adaptation method was used to tune the Q matrix based on the residual series and obtain the optimal Q values. As a result, the attitude estimation and sensor calibration performance of the UKF increased. The innovation based R-scaling method was used to adapt the R matrix and build an UKF that was robust against sensor malfunctions. Then as the next step, an integration scheme for using these two adaptation techniques in a single UKF simultaneously was proposed. The applicability conditions of the new algorithm that is named Robust Adaptive UKF (RAUKF) was demonstrated for attitude estimation of a hypothetical nanosatellite. The simulation results showed that the RAUKF performs well under all conditions including the sensor fault case and gave better estimation results than the regular UKF algorithm. These demonstrations proved that the integration scheme that was proposed for two different adaptation methods works properly.

5.6. Comparison of REKF and RUKF Estimation Results in the Presence of Measurement Faults

5.6.1. REKF and RUKF with Single Measurement Noise Scale Factor

Only magnetometer measurements based REKF and RUKF with SMNSF estimation results in the presence of magnetometer faults were compared in [70]. A picosatellite carries three-axis magnetometers onboard as the only measurement device. To implement Kalman filter algorithms, a measurement model of this sensor should be derived. The instantaneous abnormal measurements type measurement malfunction scenario was taken into consideration. Instantaneous abnormal measurements were simulated by adding a constant term to the magnetic field tensor measurement of one magnetometer at the 300th second.

An important point that stands out when the results were compared with the ones given in Table 6, robust algorithms of both filter compensate the measurement fault and estimate the states in a more accurate way. Nonetheless, a comparison of the robust algorithms reflects that RUKF is better than REKF in point of view of the estimation precision. Consequently, robust Kalman filters are more advantageous for such a pico-satellite in the case of measurement malfunction and RUKF should be preferred rather than REKF because of its better estimation characteristics.

Table 6. Absolute values of error for REKF and RUKF with SMNSF in the case of measurement malfunction [70].

Parameter	**Abs. Values of Err. for REKF**		**Abs. Values of Err. for RUKF**	
	350^{th} s.	400^{th} s.	350^{th} s.	400^{th} s.
φ(deg)	2,0108	3,2613	0,2955	0,20296
θ(deg)	0,4470	2,0493	0,4234	0,39910
ψ(deg)	4,5272	3,748	0,6867	0,67967
ω_x(deg/s)	0,7200	0,3115	0,0006	0,00018
ω_y(deg/s)	0,1368	0,0911	0,0003	0,00048
ω_z(deg/s)	0,1965	0,1871	0,0020	0,00172

5.6.2. REKF and RUKF with Multiple Measurement Noise Scale Factors

In [82-84] REKF and RUKF with MMNSF were tested via simulations for a small satellite model. In addition, the same simulation scenarios were repeated by using the conventional UKF or EKF algorithms, and the results were compared. The simulations were carried out 7000 seconds with a sampling time of $\Delta t = 0.1$sec. This period coincides with approximately one orbit of the satellite. The orbit of the satellite was assumed as circular. Three different scenarios were taken into account for simulating the fault in the measurements; continuous bias, the fault of zero output and measurement noise increment. For each scenario, a series of simulations are run using the REKF, the RUKF and as well using the conventional EKF and UKF algorithms.

To compare the results for two different types of filter, the RUKF and REKF, the root mean square error (RMSE) of the estimations were tabulated (Table 7).

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Table 7. The RMSE for the UKF, EKF, RUKF and REKF in the case of continuous bias fault; the data sampled between the 5000th and 7000th s [83].

	UKF	**EKF**	**RUKF**	**REKF**
$\varphi(^\circ)$	0.3373	0.6866	0.0207	0.3743
$\theta(^\circ)$	0.2868	0.4768	0.0164	0.2634
$\psi(^\circ)$	0.2275	0.3892	0.0218	0.3504
$\omega_x\,(\mathrm{rad/s})$	14.132e-5	24.366e-5	5.3311e-5	23.752e-5
$\omega_y\,(\mathrm{rad/s})$	25.770e-5	32.509e-5	1.5907e-5	7.6111e-5
$\omega_z\,(\mathrm{rad/s})$	23.874e-5	26.903e-5	2.6197e-5	11.745e-5

The table shows that in the case of continuous bias fault in one of the magnetometer measurements, the most efficient filter is the RUKF. As well as the conventional algorithms, RUKF outperforms the REKF. It can be stated that this is an expected result because of the characteristics of the UKF. Since the simulations start with a high initial attitude error, the RUKF achieves better performance than the REKF. This is because it is more robust against such conditions inherently [65]. Nevertheless, supplementary simulations show that when the initial attitude error is small, then the estimation error of the REKF decreases and approaches to the one for the RUKF. The similar results were obtained for the rest of the magnetometer fault scenarios in [82-84]. Main results obtained from the references on Robust and Adaptive Kalman Filters for attitude estimation of small satellites are given below:

- Comparison between the performances of the UKF and EKF (or RUKF vs. REKF) shows that the UKF algorithms outperform the EKF algorithms regarding accuracy and the convergence speed.
- The RUKF outperforms all other filters including the REKF for all simulation cases when the initial attitude error is high.
- Conventional EKF and UKF fail at a given accurate estimation for a longer period than the magnetometer fault itself.
- In the case of magnetometer fault, REKF with MMNSF is superior to REKF with SMNSF regarding the estimation accuracy.

- RUKF with the SMNSF gives satisfactory results for magnetometer fault case in short periods.
- The RUKF with MMNSF is not affected by the magnetometer fault and can perform accurate estimation even when the fault lasts long.

The references [70, 82-84] showed that the robust Kalman filtering is more reasonable in the case of measurement faults. For small satellites the risk of being affected by the external and internal disturbances is high. The interaction between the tightly placed subsystems and the external disturbances such as the ionospheric charges may change the characteristics of the magnetometers, and this is seen in the measurements as additional bias, increase in the noise, etc. For small satellite applications, the magnetometers are usually preferred as the primary sensors since they are light and small, appropriate for the concept of the satellite.

6. Conclusions

Two types of gyroless satellite attitude determination methods were reviewed in this study: single-frame attitude determination methods based on vector measurements and attitude estimation methods based on Kalman filter. Two types of Kalman filter algorithms were taken into consideration as a traditional approach based on nonlinear measurements and nontraditional approach based on linear measurements. Then, those linear measurements were directly used as measurement input for the Kalman filter. However, in the traditional approach, measurement models are based on nonlinear measurements. Therefore, there is a nonlinear relation between the measurements and the states. Also, robust versions of those Kalman filters which are incorporated with single and multiple measurement scale factors (SMNSF, MMNSF respectively) were investigated and compared in the presence of measurement faults.

Comparison studies between single-frame methods showed that SVD method and q method are the most reliable methods in the sense of robustness. However, if the computational burden is the issue, then the QUEST method or algebraic method should be selected for attitude determination of the satellite.

This review arrives at the following conclusions obtained from the Kalman filter based attitude estimation methods.

Comparison between the performances of the UKF and EKF (or RUKF vs. REKF) shows that the UKF algorithms outperform the EKF algorithms regarding accuracy and the convergence speed. Both traditional UKF and EKF fails in case of measurement malfunction. Conventional EKF and UKF fail at a given accuracy for estimations for a longer period than the magnetometer fault itself. Moreover, it takes more time for traditional EKF to regulate the effect of the fault and settle again. It may be stated that even in the case of faulty measurements traditional UKF gives more accurate results than traditional EKF. Robust Kalman filters are more advantageous than the conventional Kalman filters for pico-satellites in the case of measurement malfunction. RUKF should be preferred over REKF in the presence of measurement malfunctions because of its better estimation characteristic. In the case of magnetometer fault, REKF with MMNSF is superior to REKF with SMNSF regarding the estimation accuracy. RUKF with the SMNSF gives satisfactory results for magnetometer fault case in short period. The RUKF with MMNSF is not affected by the magnetometer fault and gives accurate estimation even when the fault lasts long. Hence the single scale factor approach may be useful only for faults which last a short period. On the contrary, the RUKF with the multiple scale factors does not have such limitation and keeps its estimation accuracy without being affected by the fault.

The integrated SVD/EKF which is a nontraditional approach achieves a more accurate attitude estimation than the traditional approach because of its adaptive way for the covariance values. In comparison to more traditional approaches, a preprocessing step in the nontraditional approach reduces the complexity of filter design significantly by allowing the use of linear measurement equations. Integrated SVD/ EKF estimates the attitude in eclipse duration with an increase in measurement covariance. The adaptive structure of the SVD/EKF algorithm owing to the usage of measurement noise covariance of attitude from SVD directly in the EKF makes the nontraditional approach robust against measurement faults.

Gyroless attitude determination methods are used widely in aerospace, particularly in attitude determination and control systems of microsatellites.

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Figure Captions

Figure 1. Star observation and catalog view.

Figure 2. GPS signal and baseline.

Figure 3. Attitude and rate estimation using the traditional approach.

Figure 4. Attitude and rate estimation using nontraditional approach.

Figure 5. Comparison of the results of the traditional and nontraditional algorithms for specific time intervals [21].

Table Captions

Table 1. Gyroless attitude determination methods.

Table 2 RMS results for attitude angles using different single-frame methods.

Table 3. GPS measurements.

Table 4. Absolute values of error for EKF and UKF in the case of measurement malfunction [70].

Table 5. NRMS errors for SVD/EKF and traditional EKF algorithms [21].

Table 6. Absolute values of error for REKF and RUKF with SMNSF in the case of measurement malfunction [70].

Table 7. The RMSE for the UKF, EKF, RUKF and REKF in the case of continuous bias fault; the data sampled between the 5000th and 7000th s [83].