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Gyro-free Attitude and Rate Estimation for a Small Satellite Using SVD and EKF

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Abstract— **This paper describes the development of a gyroless attitude determination system that can rely on magnetometer and sun sensor measurements and achieve good accuracy. Vectors coming from the selected sensor data and developed models can be placed in Wahba's problem. The system uses Singular Value Decomposition (SVD) method to minimize the Wahba's loss function and determine the attitude of the satellite. In order to obtain the attitude of the satellite with desired accuracy an extended Kalman filter (EKF) for satellite's angular motion parameter estimation is designed. The EKF uses this attitude information as the measurements for providing more accurate attitude estimates even when the satellite is in eclipse. The "attitude angle error covariance matrix" calculated for the estimations of the SVD method are regarded as the measurement noise covariance for the EKF.**

The SVD and EKF algorithms are combined to estimate the attitude angles and angular velocities, respectively. Besides, the proposed algorithm and traditional approach using nonlinear measurements are compared and concluded that SVD/EKF gives more accurate results for most of the time intervals. The algorithm can be used for low-cost small satellites where using high power consuming, expensive, and fragile gyroscopes for determining spacecraft attitude are not reasonable.

Keywords—Attitude angles, magnetometer, sun sensor, singular value decomposition, extended Kalman filter, angular velocity

1. Introduction

Sun sensors and magnetometers are common attitude sensors for small satellites missions; they are cheap, simple, light and available as commercial of-the-shelf equipment. However the overall achievable attitude determination accuracy is limited with these sensors mainly as a result of their inherent limitations and unavailability of the sun sensor data when the satellite is in eclipse. Vectors coming from the selected sensor data and developed models can be placed in Wahba's problem [1, 2]. Coordinate systems used as reference frame and body frame can be transformed to each other with necessary input parameters. The system uses Singular Value Decomposition (SVD) method to minimize the Wahba's loss function and determine the attitude of the satellite. As a reference direction, the unit vectors toward the Sun, and the Earth's magnetic field are used. Thus, magnetometers and Sun sensors are used as measuring devices.

Additionally, an attitude determination system is the subsystem of the spacecraft avoiding from high power consuming, expensive, and fragile gyroscopes. Developed micro electrical-mechanical systems (MEMS) are low power consuming and cheap sensors but they have inaccurate and with inadequate resolution for providing the desired performance. Besides gyroscopes have a tendency to degrade or fail in orbit with time because of their nature. Three types of rate gyros are used in today's Inertial Measurement Unit (IMU) systems which are ring laser gyro (RLG), fiber optic gyro (FOG) and MEMS. RLGs can have got "locked-in" condition at very slow rotation rates. FOGs in comparison to RLGs require no mechanical burden for their operation and thus eliminate a troubled noise source. The drawback is that the sensed angular velocity is limited with respect to the phase difference due to the Sagnac effect. Solid-state inertial sensors, such as MEMS devices, have potentially significant cost, size, and weight advantages on the contrary they have a disadvantage which their accuracy and resolution are lower than expected mission requirements mostly. If a satellite does not have any back up mode for gyro failure the

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whole mission may fail in the mentioned case. A couple of examples for the gyroscope failures can be given. International Ultraviolet Explorer (IUE) launched in 1978 had six gyroscope for the designed inertial system. In 1985, fourth gyro failed and IUE used only two gyros for the mission. To continue operations and achieve all scientific goals of the spacecraft, innovative redesign of specific systems was developed on the ground [3]. Hubble Space Telescope had six gyros including redundant three gyros, also its components have a chance to be replaced with the new equipment by the astronauts. In 1999, third gyro of the telescope failed so telescope started to use redundant, spare gyroscopes [4]. Failures of the gyroscopes can be caused by chemical, mechanical and electrical effects. Information about satellite operations of European Space Agency (ESA) can be obtained in which one of them, ESA Remote Sensing Satellites (ERS)-2, had an orbital rescue by reason of the gyro failure. In 2000, one gyro design for attitude and orbit control system is improved to minimize the necessity of the number of gyros without affecting from gyroscope failures [5]. In 2001, after the last gyroscope failed, a method of operating the ERS-2 sensors and actuators in a new way was developed for gyroless ERS-2.

Gyroless attitude estimation with magnetometer and sun sensor measurements has been addressed in many researches [6-11] and various algorithms that intend improving the estimation accuracy have been proposed. A basic solution for the problem is to use a Kalman filtering algorithm for integrating the measurements under the propagation model of the satellite dynamics and estimate the attitude of the satellite possibly along with the sensor biases. The main drawback of the existing algorithms is the degradation in the estimation results when the satellite is in eclipse and sun sensor data is not available.

The aim of the study is to design an attitude determination system that gives attitude knowledge with desired accuracy within the whole orbit. In order to obtain the attitude of the satellite with desired accuracy an extended Kalman filter (EKF) for satellite's angular motion parameter estimation can be used.

The traditional approaches to design of Kalman filter for satellite attitude and rate estimation use the nonlinear measurements of reference directions (Earth magnetic field, Sun, etc.) [2, 12-14].

In an approach based on the linear measurements the attitude angles are first found by using the vector measurements and applying a suitable single-frame (point-by-point) attitude determination method [15] at each step. Then these attitude angles are directly used as measurement input for the Kalman filter. Hence measurement model is linear in this case, since the states are measured directly. Integrated satellite attitude determination system based on the linear measurements is presented by [16, 17], in which the algebraic method and EKF algorithms are combined to estimate the attitude angles and angular velocities respectively. Attitude determination system uses the algebraic method (two-vector algorithm). This method is based on computing any two analytical vectors in the reference frame and measuring the same vectors in the body coordinate system. The magnetometers, Sun sensors, and horizon scanners/sensors are used as measurement devices and three different two-vector algorithms based on the Earth's magnetic field, Sun vector, and nadir vector are proposed. In order to obtain the satellite's angular motion parameters with the desired accuracy, an EKF is designed, the measurement inputs for which are the attitude estimates obtained using two-vector algorithms.

In Ref. [18] an EKF is proposed for real-time estimation of the orientation of human limb segments. The filter processes data from small inertial/magnetic sensor modules containing tri-axial angular rate sensors, accelerometers, and magnetometers. Quaternion representation is used for representing the rotation in the filter instead of the Euler angles. QUEST algorithm that solves the attitude based on the acceleration and magnetometer measurements gives the input for the EKF. Thus, dimension of the state vector reduces and measurement equations become linear.

In Ref. [19] the q-method for quaternion estimation has been integrated into an EKF to produce the novel qEKF filter for attitude estimation, which is capable of treating both attitude and non-attitude states without additional numerical iterations. Within the filter, attitude vector measurements are first processed using the q-method, which solves the nonlinear Wahba problem directly without any linearizing assumptions. Remaining measurements are processed to update the non-attitude states using the standard multiplicative EKF algorithm.

In general, the papers that are using untraditional approach techniques did not consider possibility of attitude determination in eclipse and the investigation of the accuracy in eclipse. Furthermore, those considered integrated algorithms did not investigate a comparative study with traditional methods. In this study, attitude determination algorithm is developed for whole orbital period including eclipse. A two-phased estimation algorithm is proposed for a small satellite which has magnetometers and sun sensors as the attitude sensors onboard. In the first phase,

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Wahba's problem, a well-known approach for single frame attitude estimation with vector sun sensor and magnetometer measurements, is solved by the Singular Value Decomposition (SVD) method and Euler angle-estimations are obtained for the satellite's attitude. Obtained Euler angle-estimations are used as measurement inputs for an Extended Kalman Filter (EKF), which forms the second phase of the algorithm. The covariance estimation of the SVD, is used as the measurement noise covariance matrix of the EKF; this is how the filter is tuned specifically in the eclipse period. The results of the proposed algorithm are compared with traditional approach using nonlinear measurements.

2. SVD Method

In this section, SVD method which is the pre-step for non-traditional approach and the models used in SVD method are explained briefly. Magnetic field and sun direction model with their sensor definitions are expressed in the subsections. After Wahba's optimization problem definition, two or more vectors can be used in statistical methods to minimize the loss [1]. In the equation 1, the loss can be seen as the difference between the models and the measurements which are found in unit vectors.

$$L(A)=\frac{1}{2}\sum_i a_i\,|\,\mathrm{b}_i-\mathrm{Ar}_i\,|^2 \tag{1}$$

$$B=\sum a_i b_i r_i^T \tag{2}$$

$$L(A)=\lambda_0-tr(\mathrm{AB}^T) \tag{3}$$

where b_i (set of unit vectors in body frame) and r_i (set of unit vectors in reference frame) with their a_i (non-negative weight) are the loss function variables obtained for instant time intervals and λ_0 is the sum of non-negative weights. Also, 'B' matrix is defined to reduce the loss function into the equation (3). Here, maximizing the trace ($tr(\mathrm{AB}^T)$) means minimizing the loss function (L). In this study, Singular Value Decomposition (SVD) Method is chosen to minimize the loss function [20].

$$\mathrm{B=USVT=U}\mathit{diag}|\mathbf{S_{11}}\ \mathbf{S_{22}}\ \mathbf{S_{33}}|\mathrm{VT} \tag{4}$$

$$A_{opt}=Udiag[1\quad 1\quad \det(U)\det(V)]V^T \tag{5}$$

The matrices U and V are orthogonal left and right matrices respectively and the primary singular values (**S11, S22, S33**) can be calculated in the algorithm. To find the rotation angles of the satellite, transformation matrix should be found in the equation (5) first with the determinant of one. "diag" operator returns a square diagonal matrix with elements of the vector on the main diagonal.

Rotation angle error covariance matrix (P) is necessary for determining the instant times which gives higher error results than desired.

$$P_{SVD}=Udiag[(\mathrm{s}_2+\mathrm{s}_3)^{-1}\quad (\mathrm{s}_3+\mathrm{s}_1)^{-1}\quad (\mathrm{s}_1+\mathrm{s}_2)^{-1}]U^T \tag{6}$$

where s_1 = $\mathbf{S_{11}}$, s_2 = $\mathbf{S_{22}}$, s_3 = det(U)det(V) $\mathbf{S_{33}}$.

The satellite only has two sensors (e.g. sun and magnetic field sensor), thus the SVD-method fails when the satellite is in eclipse period and when the two observations are parallel.

2.1. Modeling of Earth Magnetic Field Vector

IGRF model defines the series in nT seen below which depends on 4 input variables (r,θ,ϕ,t), using numerical Gauss coefficients (g, h) - the global variables in the algorithm [21].

$$B(r,\theta,\phi,t)=-\nabla\left\{a\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}[\mathrm{g}_n^m(\mathrm{t})\cos\mathrm{m}\phi+\mathrm{h}_n^m(\mathrm{t})\sin\mathrm{m}\phi]\times P_n^m(\cos\theta)\right\} \tag{7}$$

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Here, r is the distance between center of the Earth and satellite (km), a=6371.2 km (magnetic reference spherical radius), θ colatitude (deg), ϕ longitude (deg). The inputs are coming from the LEO satellite which has an orbit propagation algorithm only regarding to J2 effects for the selected time period. Major axis of the Earth accepted as 6378.137 km. IGRF 11 model makes the calculations at N=13th degree for 5-year intervals. Thus, coefficients of the model are updated at the years of the multiples of five (2010, 2015, etc.). The time dependence of the Gauss coefficients can be denoted as:

$$g_n^m(t) = g_n^m(T_0) + \dot{g}_n^m(T_0)(t - T_0) \tag{8}$$

$$h_n^m(t) = h_n^m(T_0) + \dot{h}_n^m(T_0)(t - T_0) \tag{9}$$

Here, T_0 is the epoch times multiple of five proceeding t and t is in the units of years for the selected time. IGRF-11 model uses predictive secular variation coefficients for 2010-2015 and main field coefficients for 1900-2010.

$$B_o = \begin{bmatrix} B_{x_o} \\ B_{y_o} \\ B_{z_o} \end{bmatrix} = \frac{1}{\sqrt{B_1^2 + B_2^2 + B_3^2}} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \tag{10}$$

Equation (10) shows the direction cosines for magnetic field model changing between -1 and +1, which only aim to determine the direction of the vector.

Three onboard magnetometers of the satellite measure the components of the magnetic field vector in the body frame. Therefore, for the measurement model, which characterizes the measurements in the body frame, gained magnetic field terms must be transformed by the use of direction cosine matrix, A. Overall measurement model may be given as;

$$B_m(k) = A(k)\mathrm{B}_o(k) + v_H(k), \tag{11}$$

where $B_m(k)$ is the measured Earth magnetic field vector as the direction cosines in body frame, $v_H(k)$ is the magnetometer measurement noise.

2.2. Modeling of Sun Direction Vector

To determine Sun direction vector in ECI (Earth Centered Inertial) frame, Julian Day (T_{TDB}) should be defined from the satellite's initial data and reference epoch. The first constant is the mean anomaly of the Sun (M_{Sun}) at epoch and the second constant is the change of the mean anomaly during Julian Day that generates. After the calculations, the ecliptic longitude of the Sun ($\lambda_{ecliptic}$) and the obliquity of the ecliptic (ε) can be determined by only the input of date in years, months, days and time in hours, minutes, seconds [22].

$$M_{Sun} = 357.5277233^0 + 35999.05034 T_{TDB} \tag{12}$$

$$\begin{aligned} \lambda_{ecliptic} = \lambda_{M_{Sun}} &+ 1.914666471^0 \sin(M_{Sun}) \\ &+ 0.019994643 \sin(2M_{Sun}) \end{aligned} \tag{13}$$

$$\varepsilon = 23.439291^0 - 0.0130042 T_{TDB} \tag{14}$$

Finally, the unit Sun vector ($\boldsymbol{S}_{ECI}$) can be found in the inertial frame.

$$\boldsymbol{S}_{ECI} = \begin{bmatrix} \cos\lambda_{ecliptic} \\ \sin\lambda_{ecliptic}\cos\varepsilon \\ \sin\lambda_{ecliptic}\sin\varepsilon \end{bmatrix} \tag{15}$$

The Sun direction vector measurements can be expressed in the following form:

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$$S_m(k) = A(k)S_o(k) + v_S(k), \tag{16}$$

where $S_m(k)$ is the measured Sun direction vector as the direction cosines in body frame, $S_0(k)$ represent the Sun direction vector in the orbit frame as a function of time and orbit parameters, $v_S(k)$ is the sun sensor measurement noise.

3. EKF Based Satellite Attitude Estimation on Linear Euler Angle Measurements

3.1. Satellite Mathematical Model

If the kinematics of the small satellite is derived in the base of Euler angles, then the mathematical model can be expressed with a 6 dimensional system vector which is made of attitude Euler angles (φ is the roll angle about x axis; θ is the pitch angle about y axis; ψ is the yaw angle about z axis) vector and the body angular rate vector with respect to the inertial axis frame,

$$x = \begin{bmatrix} \varphi & \theta & \psi & \omega_x & \omega_y & \omega_z \end{bmatrix}^T . \tag{17}$$

Also for consistency with the further explanations, the body angular rate vector with respect to the inertial axis frame should be stated separately as;

$$\omega_{BI} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T , \tag{18}$$

where $\overline{\omega}_{BI}$ is the angular velocity vector of the body frame with respect to the inertial frame. Besides, dynamic equations of the satellite can be derived by the use of the angular momentum conservation law;

$$J_x \frac{d\omega_x}{dt} = N_x + \left(J_y - J_z\right)\omega_y\omega_z, \tag{19}$$

$$J_y \frac{d\omega_y}{dt} = N_y + \left(J_z - J_x\right)\omega_z\omega_x, \tag{20}$$

$$J_z \frac{d\omega_z}{dt} = N_z + \left(J_x - J_y\right)\omega_x\omega_y, \tag{21}$$

where J_x, J_y and J_z are the principal moments of inertia and N_x, N_y and N_z are the terms of the external moment affecting the satellite. For a Low Earth Orbit (LEO) small satellite as in case, gravity gradient torque should be taken into consideration while the other disturbance torque terms such as aerodynamic torques, magnetic disturbance torques and torques caused by the solar radiation pressure may be neglected [13]. Only the gravity gradient torque is taken into account for the satellite, these torque terms in Equations (19-21) can be written as

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = -3\frac{\mu}{r_0^3}\begin{bmatrix} \left(J_y - J_z\right)A_{23}A_{33} \\ \left(J_z - J_x\right)A_{13}A_{33} \\ \left(J_x - J_y\right)A_{13}A_{23} \end{bmatrix} . \tag{22}$$

Here μ is the gravitational constant, r_0 is the distance between the center of mass of the satellite and the Earth and A_{ij} represents the corresponding element of the direction cosine matrix of [15];

$$A = \begin{bmatrix} c(\theta)c(\psi) & c(\theta)s(\psi) & -s(\theta) \\ -c(\varphi)s(\psi)+s(\varphi)s(\theta)c(\psi) & c(\varphi)c(\psi)+s(\varphi)s(\theta)s(\psi) & s(\varphi)c(\theta) \\ s(\varphi)s(\psi)+c(\varphi)s(\theta)c(\psi) & -s(\varphi)c(\psi)+c(\varphi)s(\theta)s(\psi) & c(\varphi)c(\theta) \end{bmatrix}. \tag{23}$$

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In matrix A, $c(\cdot)$ and $s(\cdot)$ are the cosines and sinus functions successively. Kinematic equations of motion of the picosatellite with the Euler angles can be given as,

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s(\varphi)t(\theta) & c(\varphi)t(\theta) \\ 0 & c(\varphi) & -s(\varphi) \\ 0 & s(\varphi)/c(\theta) & c(\varphi)/c(\theta) \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \tag{24}$$

Here $t(\cdot)$ stands for tangent function and p, q and r are the components of $\bar{\omega}_{BR}$ vector which indicates the angular velocity of the body frame with respect to the reference frame. $\bar{\omega}_{BI}$ and $\bar{\omega}_{BR}$ can be related via,

$$\omega_{BR} = \omega_{BI} - A \begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix} \tag{25}$$

where ω_0 denotes the angular velocity of the orbit with respect to the inertial frame, found as $\omega_0 = \left(\mu / r_0^3\right)^{1/2}$.

3.2. The Model of Attitude Measurements

In this study, SVD has been used as the observation model in the EKF framework. In case of EKF design based on linear Euler angle measurements, determination model of the angles that characterizes satellite's attitude, can be given as,

$$\begin{aligned} z_\varphi(k) &= \varphi(k) + \mathrm{v}_\varphi(k), \\ z_\theta(k) &= \theta(k) + \mathrm{v}_\theta(k), \\ z_\psi(k) &= \psi(k) + \mathrm{v}_\psi(k) \end{aligned} \tag{26}$$

where $\varphi(k)$, $\theta(k)$ and $\psi(k)$ are the attitude angles determined by SVD method, $\mathrm{v}_{(\cdot)}(k)$ is the measurement noise of the attitude angles. The mathematical expectations and variances of the measurement noises are

$E\left[\mathrm{v}_{(\cdot)}(k)\right] = 0$, $E\left[\mathrm{v}_\varphi^2(k)\right] = Var\left(\mathrm{v}_\varphi(k)\right)$, $E\left[\mathrm{v}_\theta^2(k)\right] = Var\left(\mathrm{v}_\theta(k)\right)$ and $E\left[\mathrm{v}_\psi^2(k)\right] = Var\left(\mathrm{v}_\psi(k)\right)$.

It is assumed that both measurement and system noise vectors $\mathrm{v}(k) = \left[\mathrm{v}_\varphi(k) \quad \mathrm{v}_\theta(k) \quad \mathrm{v}_\psi(k)\right]^T$ and $w(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean and the corresponding covariances:

$$E\left[w(i)w^T(j)\right] = Q(i)\delta(ij), \qquad E\left[\mathrm{v}(i)\mathrm{v}^T(j)\right] = R(i)\delta(ij), \tag{27}$$

where $\delta(ij)$ is the Kronecker symbol.

It is assumed that process and measurement noises are uncorrelated, i.e.,

$$E\left[w(i)\mathrm{v}^T(j)\right] = 0, \ \forall i, j \tag{28}$$

3.3. EKF for Satellite Attitude Estimation

If the state vector is arranged as (17) and the mathematical model of the LEO satellite's rotational motion about its center of mass, is linearized using quasi-linearization method. We will consider a real-time linear Taylor approximation of the system function at the previous state estimate. The Kalman Filter which is obtained will be called the Extended Kalman Filter (EKF). Filter algorithm, in this case as, is given below:

Equation of the estimation value,

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$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1) \times \{z(k+1) - H\hat{x}(k+1/k)\} \tag{29}$$

Here $z(k+1) = \begin{bmatrix} z_\varphi(k+1) & z_\theta(k) & z_\psi(k) \end{bmatrix}$ is the measurement vector, H is the measurement matrix. In the investigated case the measurement matrix can be written as

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Equation of the extrapolation value,

$$\hat{x}(k+1/k) = f[\hat{x}(k), k] \tag{30}$$

Filter-gain of EKF

$$K(k+1) = P(k+1/k)H^T(k+1) \times \left[H(k+1)P(k+1/k)H^T(k+1) + R(k) \right]^{-1} \tag{31}$$

The covariance matrix of the extrapolation error is,

$$P(k+1/k) = \frac{\partial f[\hat{x}(k), k]}{\partial \hat{x}(k)} P(k/k) \frac{\partial f^T[\hat{x}(k), k]}{\partial \hat{x}(k)} + Q(k) \tag{32}$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1) = \left[I - K(k+1)H(k+1) \right] P(k+1/k) \tag{33}$$

R is the covariance matrix of measurement noise, which has diagonal elements built of the variances of angle and angle rate measurement noises and Q is the covariance matrix of the system noises.

Equations given as (29)-(33) represent the Extended Kalman Filter (EKF), which fulfils recursive estimation of the satellite's rotational motion parameters about its mass center on the linear attitude measurements.

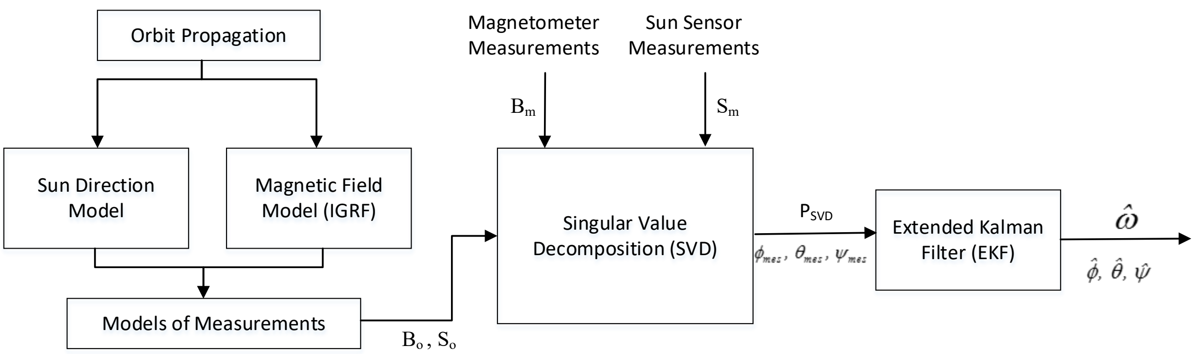

Fig.1. Attitude and rate estimation via SVD/EKF algorithm

In Fig.1, attitude and rate estimation scheme can be seen. B represents the magnetic field and S is the Sun direction. Accurate models for magnetic field and Sun direction vectors in addition to the sensor measurements are included in the SVD method in order to obtain angles and covariance matrix as input information for EKF. By this knowledge, attitude and rate can be estimated more accurate in the algorithm-EKF. This scheme can be called integrated SVD/EKF and separable from the traditional approach with the aspect of the linear measurements coming from the point-by-point method. Traditional approach is described in the Appendix A.

4. Simulation Results

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Simulations are realized with a sampling time of $T_s = 1\sec$. As an experimental platform a cube-sat model is used. Nonetheless the orbit of the satellite is a circular orbit with an altitude of $r = 550km$. The time interval 2000-4000s correspond to the eclipse. The satellite is assumed to have an approximate inertia matrix of $J = diag\left(2.1\times10^{-3} \quad 2.0\times10^{-3} \quad 1.9\times10^{-3}\right)$. For the magnetometer measurements, the sensor noise is characterized by zero mean Gaussian white noise with a standard deviation of $\sigma_m = 300\ nT$. The standard deviation for the sun sensor noise is taken as $\sigma_s = 0.002$ (for unit vector measurements). The diagonal terms of the process noise covariance matrix for SVD/EKF are taken as 1×10^{-4} *and* 1×10^{-4}, respectively for the attitude and attitude rate terms.

4.1. Simulation Results of The SVD/EKF Attitude Determination System

Portion of simulation results are given in Figs.2-8. Absolute errors and variance changes of attitude angles when SVD and SVD+EKF are used are given in Figs.2-4. Here, SVD/EKF attitude estimation results are superior at outside of the eclipse because of the coming covariance knowledge as an adaptation to the filter from SVD method. Measurement model is linear in this case, since the states are measured directly in SVD.

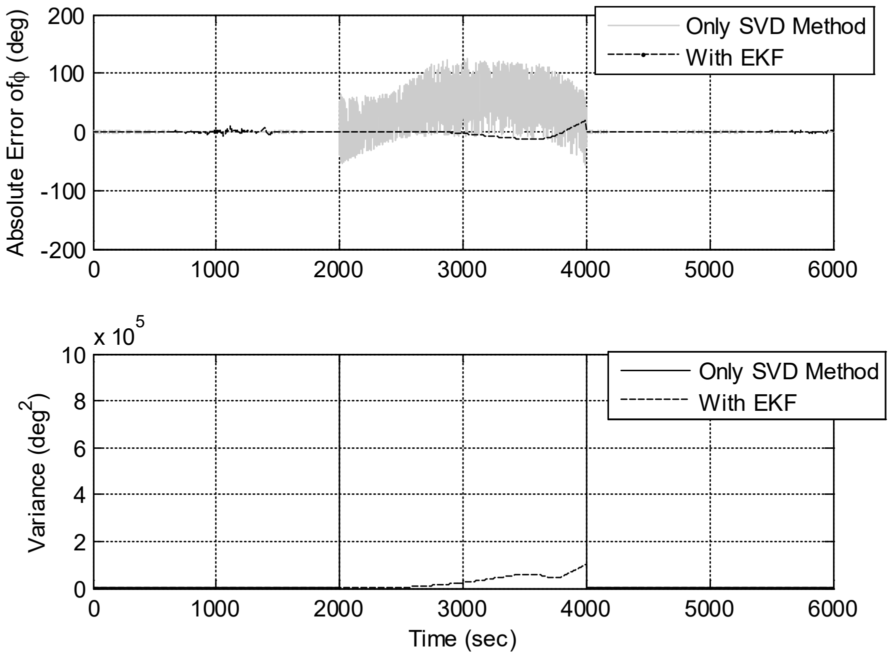

Fig.2. Absolute errors and variance changes of roll angle

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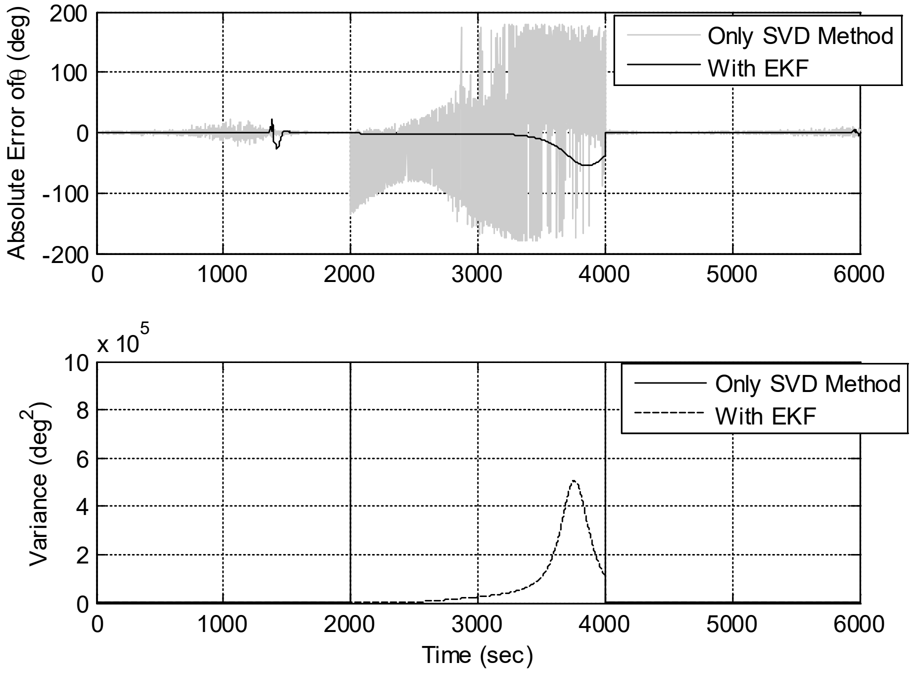

Fig.3. Absolute errors and variance changes of pitch angle

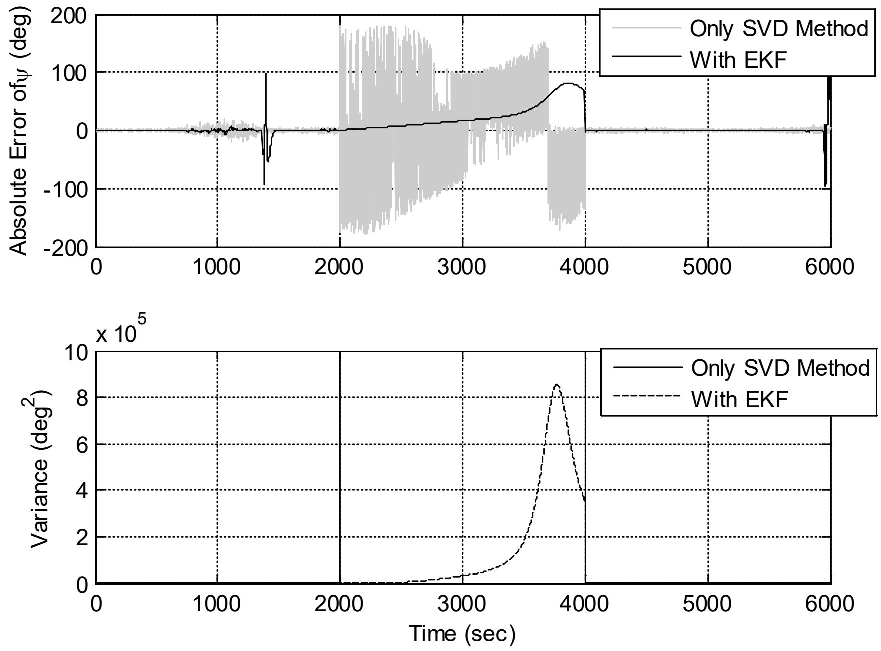

Fig.4. Absolute errors and variance changes of yaw angle

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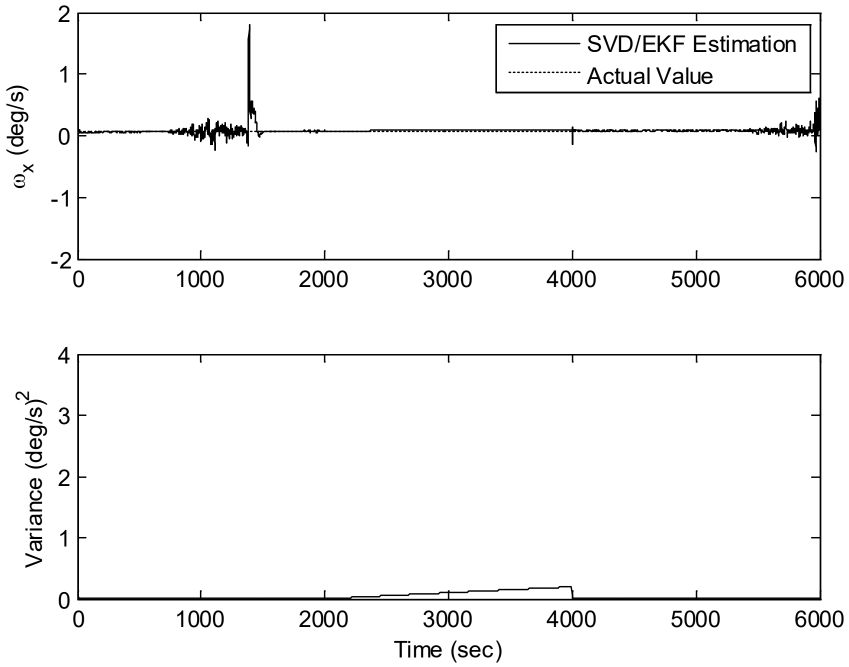

Fig.5. X-axis angular rate estimation results

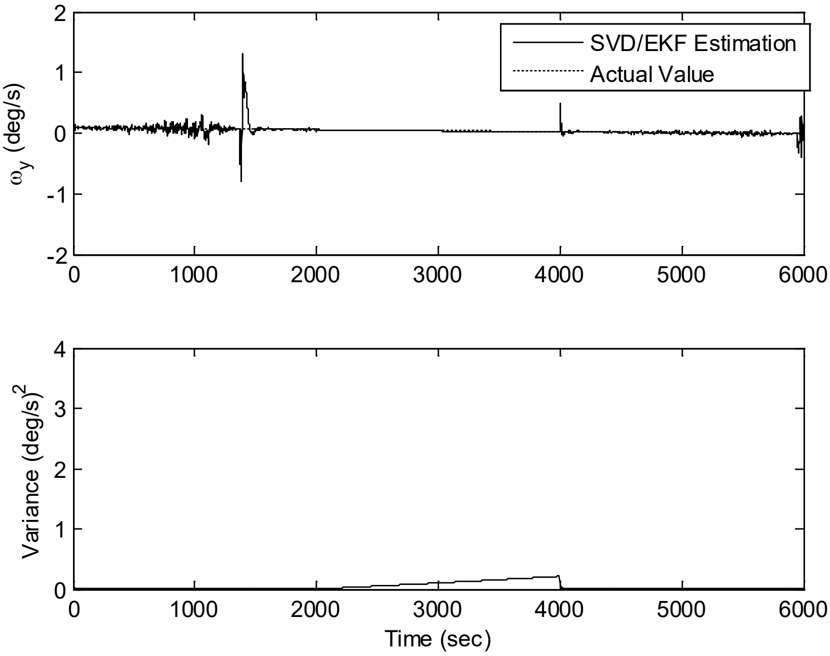

Fig.6. Y-axis angular rate estimation results

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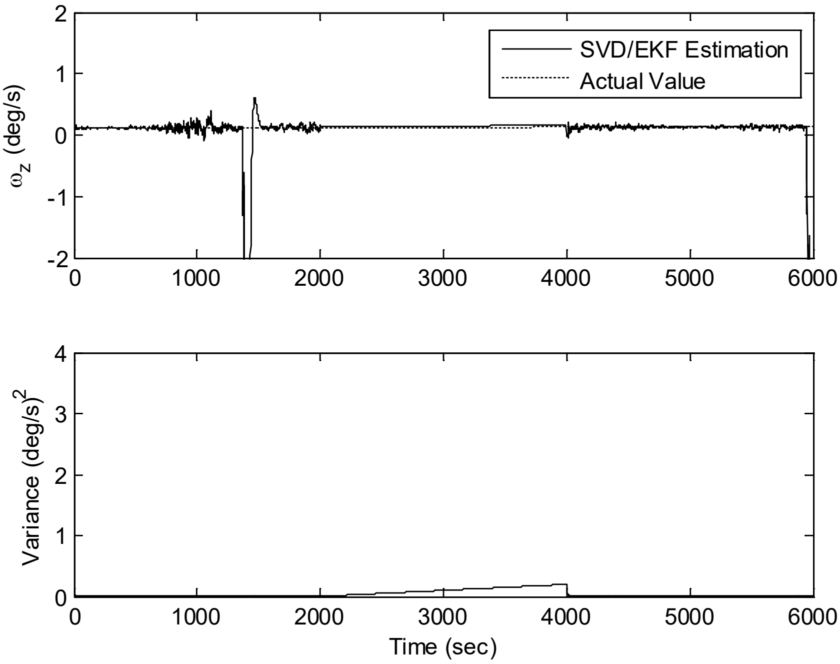

Fig.7. Z-axis angular rate estimation results

The X, Y and Z-axes angular rate estimation results are shown in Figs. 5, 6 and 7 respectively. The angular rates are estimated accurately. The SVD method fails in eclipse since there are no observations coming from the Sun. In this period the EKF gain decreases to very low values, the state update term of the EKF becomes insignificant and the predicted states contribute more to the estimations. In the figures, integrated SVD/EKF attitude estimation error increases through the eclipse. This situation is natural because EKF prediction errors are accumulated in time and after a while, they can get larger errors. In this case, another method can be used for attitude estimation e.g. EKF based on only magnetometer measurements or magnetometer and rate gyro measurements.

4.2. Comparison of the SVD/EKF and Traditional EKF

The traditional EKF for satellite attitude estimation on nonlinear magnetic field and sun direction measurements is given in Appendix A. In traditional approach, measurement noise covariance matrix is constant. However, error covariance of the SVD method changes depending on the angle between sun and magnetic field vectors. Therefore, measurement noise covariance matrix of EKF changes with time. Consequently, the gain of the EKF changes too. Thus, this algorithm adapts to the changes in the measurement noise in this study.

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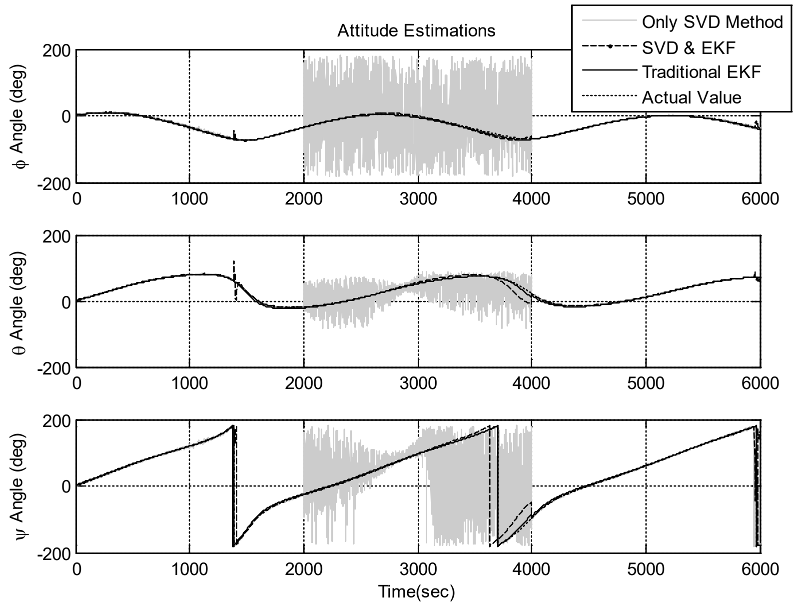

Fig.8. Comparison of the results of the algorithms (SVD, SVD&EKF, Traditional EKF) and actual values of Euler angles.

The SVD, SVD/EKF and Traditional EKF results can be seen in Fig.8. To see the results more clear the short interval is zoomed in Fig.9. As a consequence of the linear measurement usage in the integrated filter, the SVD/EKF algorithm has more accurate results.

If whole orbital period is separated into several time intervals seen in Table.1 performances of both algorithms can be investigated. As it can be seen from Table 1, integrated SVD/EKF algorithm results are superior in most of the time intervals. When the SVD has any jumps on the attitude angle determination or covariance (1000-2000 sec), SVD/EKF will be affected and the affected NRMS error results can be seen in Table.1. Integrated SVD/EKF can achieve more accurate attitude results than the traditional approach because of its adaptive way for the covariance values, explained deeply in section 3.3.

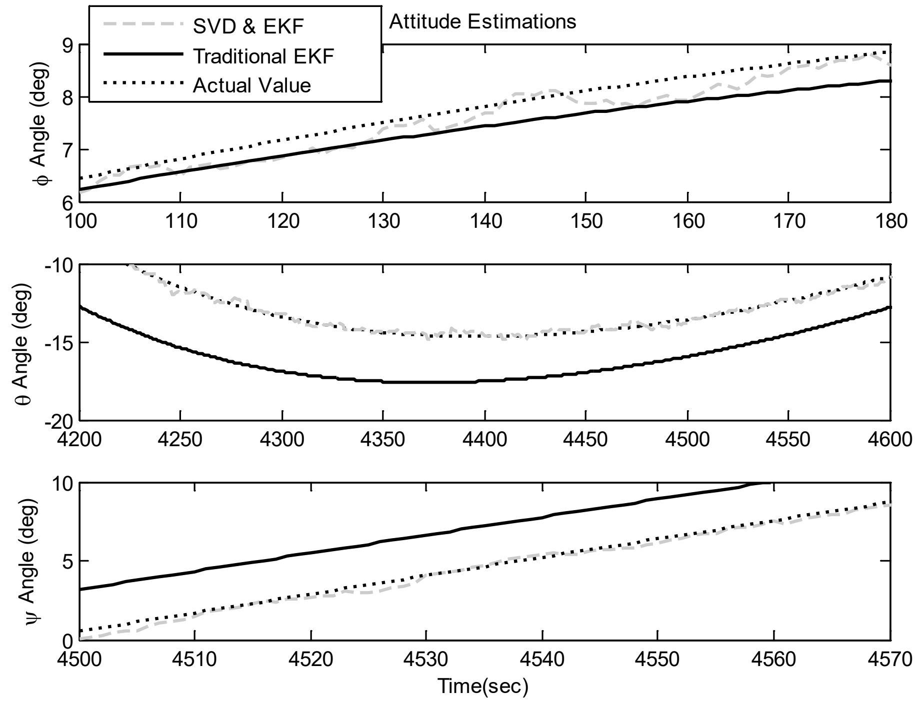

Fig.9. Comparison of the results of the algorithms (SVD&EKF, Traditional EKF) and actual values of Euler angles for specific time intervals.

Table. 1. NRMS errors for EKF/SVD and traditional EKF algorithms

Time Interval (sec)	Normalized RMS Errors for Angles					
	SVD/EKF			**Traditional EKF**		
	Phi	Theta	Psi	Phi	Theta	Psi
0-1200	0,0557	0,0044	0,0076	0,1273	0,0269	0,01822
1000-2000	0,0353	0,2143	4,3246	0,0189	0,1326	4,0551
2000-4000	0,0979	0,0414	0,0307	0,0462	0,0287	0,0542
4000-5500	0,0146	0,1604	0,0641	0,0321	0,5350	0,0879

Table. 2. NRMSE and Gain with Different Process Noise Covariance

Process Noise Covariance for Attitude	**NRMSE**		**Gain (6000th sec)**
	Outside of the Eclipse	Eclipse	
Low Noise (10^{-9})	0.12	1.19	$\begin{bmatrix} 0,0071 & -0,0003 & -0,0001 \\ -0,0003 & 0,0095 & -0,0009 \\ -0,0005 & -0,0042 & 0,0052 \end{bmatrix}$
Medium Noise (10^{-4})	0.07	1.18	$\begin{bmatrix} 0,1091 & -0,0056 & 0,0024 \\ -0,0056 & 0,1329 & -0,0078 \\ 0,0115 & -0,0373 & 0,0918 \end{bmatrix}$
High Noise (10^{-3})	0.17	4.03	$\begin{bmatrix} 0,2037 & -0,0001 & 0 \\ -0,0001 & 0,2264 & -0,0096 \\ -0,0002 & -0,0481 & 0,1483 \end{bmatrix}$

Gain and the normalized RMS error results can be seen in Table.2. The SVD/EKF performance varies depending on the filter tuning such as the process noise covariance matrix. If the filter rely mostly on the measurements with increasing noise, so when the measurements are deteriorated during the eclipse, the SVD/EKF estimation can even get worse. In addition, the filter is slow at converging to the actual states when the noise is decreased.

5. Conclusions

In this study an integrated SVD/EKF attitude determination system is presented, in which the SVD and EKF algorithms are combined to estimate the attitude angles and angular velocities, respectively. SVD has been used as the observation model in the EKF framework. As a difference from the other studies in this area, attitude determination algorithm for whole orbital period including the eclipse duration is investigated.

Preprocessing of the sun sensor and magnetometer measurements using the SVD algorithm produces a computed Euler angles and their variances input for the EKF. The EKF uses this attitude information as the measurements for providing more accurate attitude estimates even when the satellite is in eclipse. The “attitude angle error covariance matrix” calculated for the estimations of the SVD method are regarded as the measurement

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noise covariance for the EKF. In comparison to more traditional approaches, this preprocessing step significantly reduces the complexity of filter design by allowing the use of linear measurement equations.

The simulation results show that this method has high estimation precision for the attitude and rate errors in the attitude determination system. SVD method fails in eclipse period because of no Sun sensor measurements. However, integrated SVD/ EKF can estimate the attitude in this duration with increase in measurement covariance. Also, the results of the proposed algorithm are compared with traditional approach using nonlinear measurements. In general, proposed algorithm gives better results comparing with traditional approach.

For future work, a robust adaptive EKF will be developed therefore the jumps in the SVD/EKF caused by parallelism of the vectors and covariance raise in SVD method can be eliminated.

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Appendix

EKF for Satellite Attitude Estimation on Nonlinear Measurements-Traditional Approach

Consider the nonlinear mathematical model of the satellite's rotational motion about its mass center driven by white noise with white noise-corrupted measurements defined by

$$\begin{aligned} x(k+1) &= \varphi\left[x(k),k\right]+w(k) \\ z(k) &= h\left[x(k),k\right]+v(k) \end{aligned} \tag{A.1}$$

where $x(k+1)=\left[\varphi(k+1) \quad \theta(k+1) \quad \psi(k+1) \quad \omega_x(k+1) \quad \omega_y(k+1) \quad \omega_z(k+1)\right]^T$ is the state vector, $z(k)=\left[H_{mx}(k) \quad H_{my}(k) \quad H_{mz}(k) \quad S_{mx}(k) \quad S_{my}(k) \quad S_{mz}(k)\right]^T$ is the measurement at time k, $w(k)$ is the system noise, $v(k)=\left[v_{Hx}(k) \quad v_{Hy}(k) \quad v_{Hz}(k) \quad v_{Sx}(k) \quad v_{Sy}(k) \quad v_{Sz}(k)\right]^T$ is the measurement noise, $\varphi\left[x(k),k\right]$ is the nonlinear state transition function mapping the previous state to the current state, $h\left[x(k),k\right]$ is a nonlinear measurement model mapping current state to measurements.

It is assumed that both noise vectors $v(k)$ and $w(k)$ are linearly additive Gaussian, temporally uncorrelated with zero mean, which means

$$E\left[w(k)\right]=E\left[v(k)\right]=0, \ \forall k, \tag{A.2}$$

with the corresponding covariances:

$$\begin{aligned} E\left[w(i)w^T(j)\right] &= Q(i)\delta(ij), \\ E\left[v(i)v^T(j)\right] &= R(i)\delta(ij), \end{aligned} \tag{A.3}$$

where $\delta(ij)$ is the Kronecker symbol.

It is assumed that process and measurement noises are uncorrelated, i.e.,

$$E\left[w(i)v^T(j)\right]=0, \ \forall i,j. \tag{A.4}$$

We will consider a real-time linear Taylor approximation of the system function at the previous state estimate and that of the observation function at the corresponding predicted position. The Kalman Filter so obtained will be called the Extended Kalman Filter (EKF). Filter algorithm in this case as is given below [23].

Equation of the estimation value,

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$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1) \times \left\{ z(k+1) - h\left[\hat{x}(k+1/k), k+1 \right] \right\} \tag{A.5}$$

Equation of the extrapolation value,

$$\hat{x}(k+1/k) = \varphi\left[\hat{x}(k), k \right] \tag{A.6}$$

Filter-gain of EKF

$$K(k+1) = P(k+1/k)H^T(k+1) \times \left[H(k+1)P(k+1/k)H^T(k+1) + R(k) \right]^{-1} \tag{A.7}$$

where $H(k+1) = \dfrac{\partial h[\hat{x}(k+1/k), k+1]}{\partial \hat{x}(k+1/k)}$ is the measurement matrix constituted of partial derivatives.

The covariance matrix of the extrapolation error is,

$$P(k+1/k) = \frac{\partial \varphi[\hat{x}(k), k]}{\partial \hat{x}(k)} P(k/k) \times \frac{\partial \varphi^T[\hat{x}(k), k]}{\partial \hat{x}(k)} + Q(k) \tag{A.8}$$

The covariance matrix of the filtering error is,

$$P(k+1/k+1) = \left[I - K(k+1)H(k+1) \right] P(k+1/k) \tag{A.9}$$

The filter expressed by the formulas (A-5)-(A-9) is called the EKF based on traditional approach.

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