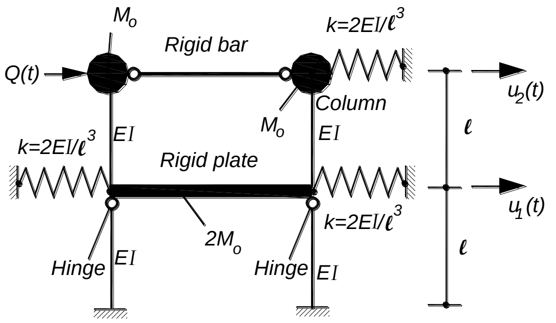
DYNAMICS OF STRUCTURES / Final Exam / January 15, 2013

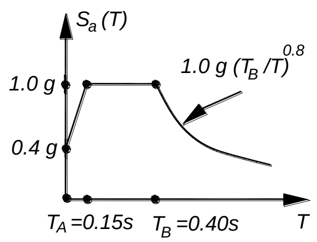
Problem # 1:

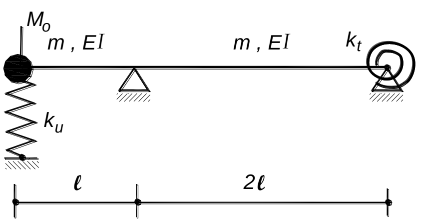
a. Consider the system of two degree-of-freedom shown where the first story is rigid plate having a mass of $2M_o$ and the second story consists of two cantilever columns having a tip mass M_o each connected to each other by a rigid bar. The two stories are connected by springs to the fixed supports. (a) Write down equations of motion by considering the free body diagram of the two stories separately. (b) Evaluate the mass matrix $\mathbf{m}$, and the rigidity matrix $\mathbf{k}$ and the load vector $\mathbf{p}$. (c) Determine the circular frequencies ω_i and the periods T_i of the free vibration in terms of EI, M_o and ℓ. (d) Obtain the corresponding two mode shapes ϕ_i and give their graphical representation ($i=1,2$). (e) Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T\,\mathbf{m}\,\boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T\,\mathbf{k}\,\boldsymbol{\phi}_2$. (e). Evaluate the generalized masses and stiffness $M_i=\boldsymbol{\phi}_i^T\,\mathbf{m}\boldsymbol{\phi}_i$ and $K_i=\boldsymbol{\phi}_i^T\,\mathbf{k}\,\boldsymbol{\phi}_i$ and assess the relationship $\omega_i^2=K_i\,/\,M_i$ ($i=1,2$). Determine the effective modal masses M_1^* and M_2^*, and assess $M_1^*+M_2^*=4M_o$

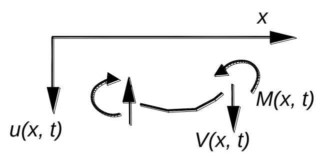
b. The heights of the stories are $\ell=3meter$, the columns have cross section of $b/h=0.30m/0.50m$, the first period of the system is $T_1=0.20s$ and $E=30GPa$. Find the numerical values the parameter M_o and the second period T_2 of the system.

c. Evaluate the base shear forces V_{b1} and V_{b2} corresponding to the two mode shapes, the equivalent forces applied to the system at the story levels for both cases and the story shear forces by using the acceleration spectrum given. Obtain the shear forces and the bending moments at the columns by using the SRSS combination rule.









Problem # 2:

Consider the continuous elastic beam having two spans shown where m is the mass per unit length and EI is the bending rigidity of the cross sections of the beam, The left end of the beam has a concentrated mass M_o and a translational spring k_u and is simply supported having a rotational spring k_t. Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant in terms of $\beta^4=m\,\ell^4\omega^2\,/\,(EI)$ by assuming $M_o=2\,m\,\ell$, $k_u=2EI\,/\,\ell^3$ and $k_t=3EI\,/\,\ell$.

$$\mathbf{m}\,\ddot{\mathbf{u}}(t)+\mathbf{k}\,\mathbf{u}(t)=\mathbf{p}(t)\quad \mathbf{u}(t)=\begin{bmatrix}u_1(t) & u_2(t)\end{bmatrix}^T\quad \mathbf{p}(t)^T=\begin{bmatrix}P_1(t) & P_2(t)\end{bmatrix}\quad \omega_i=2\,\pi\,/\,T_i$$

$$(\mathbf{k}-\omega_i^2\ \mathbf{m})\ \boldsymbol{\phi}_i=0\quad (\mathbf{I}-\omega_i^2\ \mathbf{d}\,\mathbf{m})\ \boldsymbol{\phi}_i=0\quad \left|\mathbf{k}-\omega_i^2\ \mathbf{m}\right|=0\quad \left|\mathbf{I}-\omega_i^2\ \mathbf{d}\,\mathbf{m}\right|=0\quad M_i=\boldsymbol{\phi}_i^T\mathbf{m}\,\boldsymbol{\phi}_i$$

$$K_i=\boldsymbol{\phi}_i^T\mathbf{k}\,\boldsymbol{\phi}_i\quad M_i\,\ddot{Y}_i(t)+K_i\,Y_i(t)=\boldsymbol{\phi}_i^T\mathbf{p}(t)\quad Y_i(t)=\sum_{i=1}^{2}\ \boldsymbol{\phi}_i^T\ \mathbf{m}\ \mathbf{v}\,/\,M_i\quad k=\frac{3EI}{h^3}\quad k=\frac{12EI}{h^3}$$

$$Y_i(t)=\frac{\sin\omega_i t}{M_i\ \omega_i}\left[\boldsymbol{\phi}_i^T\int_o^{t_o}\ \mathbf{p}(\tau)\,d\tau\right]\quad L_i=\boldsymbol{\phi}_i^T\mathbf{m}\,\mathbf{1}\quad \Gamma_i=L_i\,/\,M_i\quad M_i^*=\Gamma_i\ L_i\quad \mathbf{1}=\begin{bmatrix}1 & 1\end{bmatrix}^T\quad V_{bj}=M_j^*\,S_a(T_j)$$

$$u(x,t)=\sum\boldsymbol{\phi}_i(x)\,Y_i(t)\quad \ddot{Y}_i(t)+\omega_i^2\ Y_i(t)=0\quad M(x,t)=-EI\,\frac{\partial^2 u}{\partial x^2}\quad V(x,t)=-EI\,\frac{\partial^3 u}{\partial x^3}\quad f_{nj}=V_{bj}\,\frac{m_n\,\phi_{nj}}{\sum_k m_k\,\phi_{kj}}$$

$$\phi(x)=A_1\sin ax+A_2\cos ax+A_3\sinh ax+A_4\cosh ax\qquad a^4=\frac{m\,\omega^2}{EI}$$

Prof.Dr. Zekai Celep (http://web.itu.edu.tr/celep/