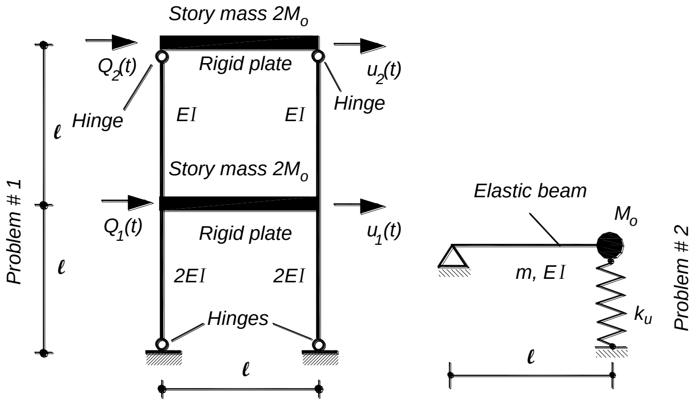
DYNAMICS OF STRUCTURES / Midterm Exam / December 12, 2012

Problem # 1:

Consider the system of two degrees-of-freedom shown. (a). Evaluate the flexibility $\mathbf{d}$ matrix, the mass matrix $\mathbf{m}$ and the rigidity matrix $\mathbf{k} = \mathbf{d}^{-1}$ and the load vector $\mathbf{p}$. (b). Determine the circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI, m and ℓ. (c). Obtain the corresponding two mode shapes ϕ_i and give their graphical representation ($i = 1,2$). (d). Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T \mathbf{m} \boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T \mathbf{k} \boldsymbol{\phi}_2$. (e). Evaluate the generalized masses and stiffness $M_i = \boldsymbol{\phi}_i^T \mathbf{m} \boldsymbol{\phi}_i$ and $K_i = \boldsymbol{\phi}_i^T \mathbf{k} \boldsymbol{\phi}_i$ and assess the relationship $\omega_i^2 = K_i / M_i$ ($i = 1,2$).

Problem # 2:

Consider the elastic beam shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The left end of the beam is simply supported and the right end of the beam is free and has a concentrated mass M_o and a lateral spring k_u. Write down the boundary conditions for the free vibration of the beam.



Obtain the frequency determinant in terms of $\beta^4 = m\ell^4\omega^2 / (EI)$ by assuming $M_o = m\ell$, and $k_u = 2EI/\ell^3$.

$$\mathbf{m}\,\ddot{\mathbf{u}}(t) + \mathbf{k}\,\mathbf{u}(t) = \mathbf{p}(t) \quad \mathbf{u}(t) = \begin{bmatrix} u_1(t) & u_2(t) \end{bmatrix}^T \quad \mathbf{p}(t)^T = \begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix} \quad \omega_i = 2\pi / T_i$$

$$(\mathbf{k} - \omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad (\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i = 0 \quad \left|\mathbf{k} - \omega_i^2\,\mathbf{m}\right| = 0 \quad \left|\mathbf{I} - \omega_i^2\,\mathbf{d}\,\mathbf{m}\right| = 0 \quad M_i = \boldsymbol{\phi}_i^T \mathbf{m}\,\boldsymbol{\phi}_i$$

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$$Y_i(t) = \frac{\sin\omega_i t}{M_i\ \omega_i}\left[\boldsymbol{\phi}_i^T \int_o^{t_o} \mathbf{p}(\tau)\,d\tau\right] \quad L_i = \phi_i^T \mathrm{m}\,1 \quad \Gamma_i = L_i / M_\mathrm{i} \quad M_i^* = \Gamma_i\,L_i \quad 1^\mathrm{T} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$u(x,t) = \sum \boldsymbol{\phi}_i(x)\,Y_i(t) \quad \ddot{Y}_i(t) + \omega_i^2\ Y_i(t) = 0 \quad M(x,t) = -EI\frac{\partial^2 u}{\partial x^2} \quad V(x,t) = -EI\frac{\partial^3 u}{\partial x^3}$$

$$\phi(x) = A_1 \sin ax + A_2 \cos ax + A_3 \sinh ax + A_4 \cosh ax \quad a^4 = \frac{m\,\omega^2}{EI}$$

(Prof.Dr. Zekai Celep (http://web.itu.edu.tr/celep/)

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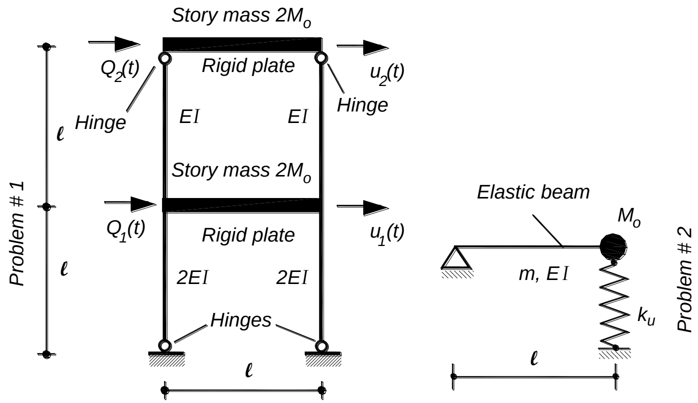

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