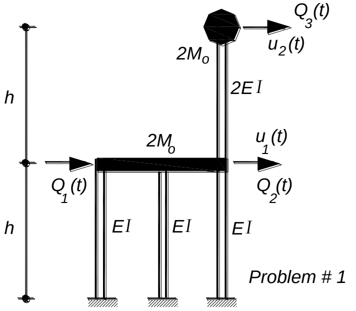
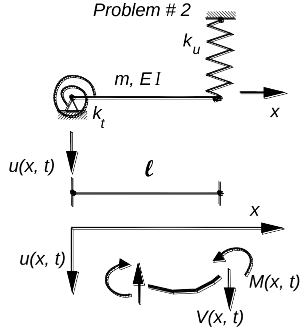
DYNAMICS OF STRUCTURES / Midterm Exam / December 13, 2011 *Z.Celep*

Problem # 1:

Consider the system of two degrees-of-freedom shown where the first story is rigid plate having a mass of $2M_o$ and the second story consists of a cantilever column. a. Evaluate the flexibility $\mathbf{d}$ matrix, the mass matrix $\mathbf{m}$ and the rigidity matrix $\mathbf{k}=\mathbf{d}^{-1}$ and the load vector $\mathbf{p}$. b. Determine the circular frequencies and the periods of the free vibration ω_i and T_i in terms of EI, m and h. c. Obtain the corresponding two mode shapes ϕ_i and give their graphical representation ($i=1,2$).d. Check the orthogonality of the modes with respect to the mass matrix and the stiffness matrix $\boldsymbol{\phi}_1^T\,\mathbf{m}\boldsymbol{\phi}_2$, and $\boldsymbol{\phi}_1^T\,\mathbf{k}\boldsymbol{\phi}_2$. e. Evaluate the generalized masses and stiffness $M_i=\boldsymbol{\phi}_i^T\,\mathbf{m}\boldsymbol{\phi}_i$ and $K_i=\boldsymbol{\phi}_i^T\,\mathbf{k}\boldsymbol{\phi}_i$ and assess the relationship $\omega_i^2=K_i/M_i$ ($i=1,2$).





Problem # 2:

Consider the distributed parameter system shown where m is the mass per unit length and EI is the bending rigidity of the cross section. The left end of the beam is fixed and has a rotational spring k_t. The right end of the beam is free and has a lateral spring k_u. Write down the boundary conditions for the free vibration of the beam. Obtain the frequency determinant in terms of $\beta^4=m\ell^4\omega^2/(EI)$ $\beta=m\ell^4\omega^2/(EI)$ by assuming $M_o=m\ell$, $k_u=EI/\ell^3$ and $k_t=EI/\ell$.

$$\mathbf{m}\,\ddot{\mathbf{u}}(t)+\mathbf{k}\,\mathbf{u}(t)=\mathbf{p}(t)\quad \mathbf{u}(t)=\begin{bmatrix}u_1(t) & u_2(t)\end{bmatrix}^T\quad \mathbf{p}(t)^T=\begin{bmatrix}P_1(t) & P_2(t)\end{bmatrix}\quad \omega_i=2\,\pi/T_i$$

$$(\mathbf{k}-\omega_i^2\,\mathbf{m})\,\boldsymbol{\phi}_i=0\quad (\mathbf{I}-\omega_i^2\,\mathbf{d}\,\mathbf{m})\,\boldsymbol{\phi}_i=0\quad \left|\mathbf{k}-\omega_i^2\,\mathbf{m}\right|=0\quad \left|\mathbf{I}-\omega_i^2\,\mathbf{d}\,\mathbf{m}\right|=0\quad M_i=\boldsymbol{\phi}_i^T\mathbf{m}\,\boldsymbol{\phi}_i$$

$$K_i=\boldsymbol{\phi}_i^T\mathbf{k}\,\boldsymbol{\phi}_i\quad M_i\,\ddot{Y}_i(t)+K_i\,Y_i(t)=\boldsymbol{\phi}_i^T\mathbf{p}(t)\quad Y_i(t)=\sum_{i=1}^{2}\boldsymbol{\phi}_i^T\,\mathbf{m}\,\mathbf{v}/M_i\quad k=\frac{3EI}{h^3}\quad k=\frac{12EI}{h^3}$$

$$Y_i(t)=\frac{\sin\omega_i t}{M_i\,\omega_i}\left[\boldsymbol{\phi}_i^T\int_o^{t_o}\mathbf{p}(\tau)\,d\tau\right]\quad L_i=\phi_i^T\mathrm{m}\,1\quad \Gamma_i=L_i/M_{\mathrm{i}}\quad M_i^*=\Gamma_i\,L_i$$

$$1^{\mathrm{T}}=\begin{bmatrix}1 & 1\end{bmatrix}\quad u(x,t)=\sum\boldsymbol{\phi}_i(x)\,Y_i(t)\quad \ddot{Y}_i(t)+\omega_i^2\,Y_i(t)=0\quad M(x,t)=-EI\frac{\partial^2u}{\partial x^2}$$

$$\phi(x)=A_1\sin ax+A_2\cos ax+A_3\sinh ax+A_4\cosh ax\quad a^4=\frac{m\,\omega^2}{EI}\quad V(x,t)=-EI\frac{\partial^3u}{\partial x^3}$$

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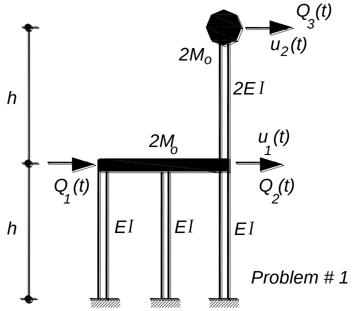

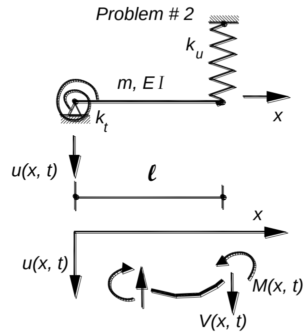

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