CONSTITUTIVE FAILURE MODELLING AND ANALYSIS OF STEEL WIRE ROPE STRUCTURES SUBJECTED TO IMPACT LOADING

PhD Thesis

Advisor: Prof. C. Erdem İmrak

ADEM CANDAŞ

FEBRUARY 23, 2021

Outline

INTRODUCTION

- Motivation

METHODOLOGY

- Peridynamic Theory

KALTHOFF WINKLER EXPERIMENT

WIRE ROPES REVIEW

MODELLING OF WIRE ROPES USING PERIDYNAMICS

- Results and Discussion

CONCLUSIONS

Fracture Mechanics

A field of mechanics concerned with the study of propagation of cracks in materials.

Crack: A line or surface that split a without breaking into separate parts.

Fracture (or damage): Local separation of a body into two or more pieces.

Field of Fracture Mechanics (FM): Linear Elastic FM, Elastic-Plastic FM, Dynamic FM, Viscoelastic FM and, Viscoplastic FM

Modelling Approaches

- Finite Element Method
- Discrete Element Method
- Molecular Dynamics
- Boundary Element Method
- PERIDYNAMICS

Motivation

- Pre-defined micro crack interaction with macro crack propagation
  - Kalthoff-Winkler Experiment

- Wire rope modelling using Peridynamic
  - Parameter studies
  - Failure mechanism

- Impact loading

What is Peridynamics?

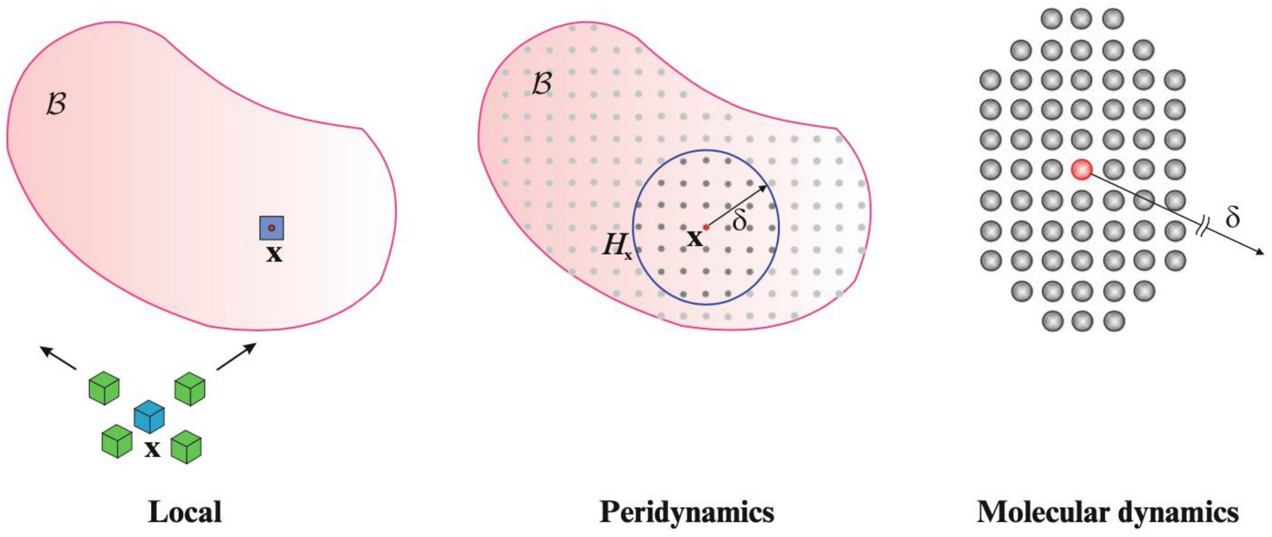
Peridynamics is a continuum formulation

- Peridynamic (PD) theory uses integral equations
  - No spatial derivatives
- Equations apply everywhere regardless of discontinuities
- No need for external supplied «crack growth law»
- Multiple crack paths can evolve in complex patterns and not known in advance.

Classical Continuum Mechanics

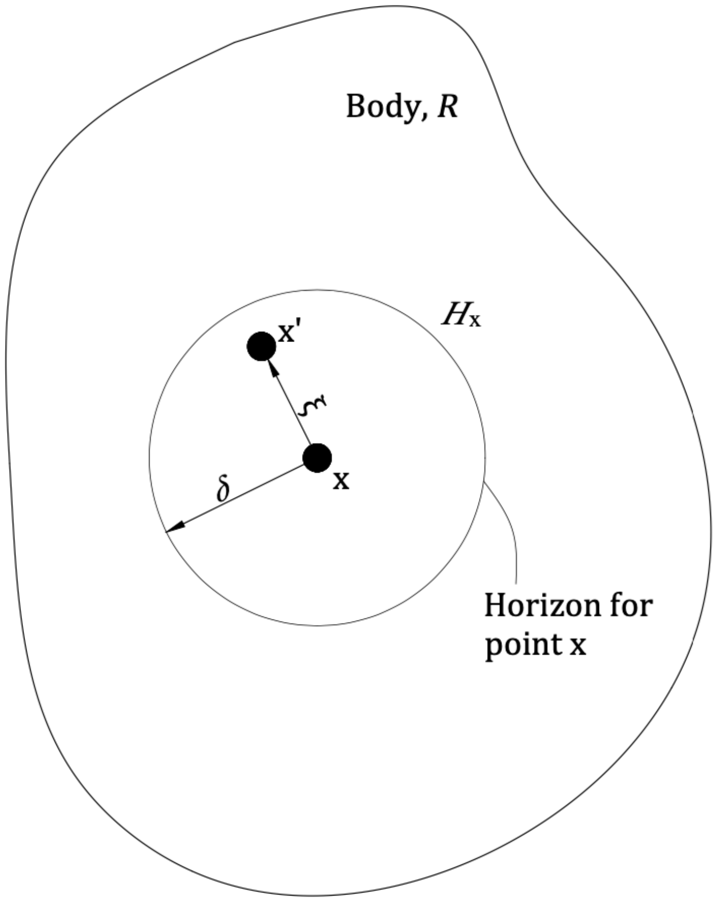
Interaction between material points are expressed in terms of traction vectors.

- Local interaction
- Partial derivatives are not defined along discontinuities.



Peridynamic Theory

The peridynamic theory is a reformulation of the equation of motion in solid mechanics that is better suited for modeling bodies with discontinuities, such as cracks.



Peridynamic Theory

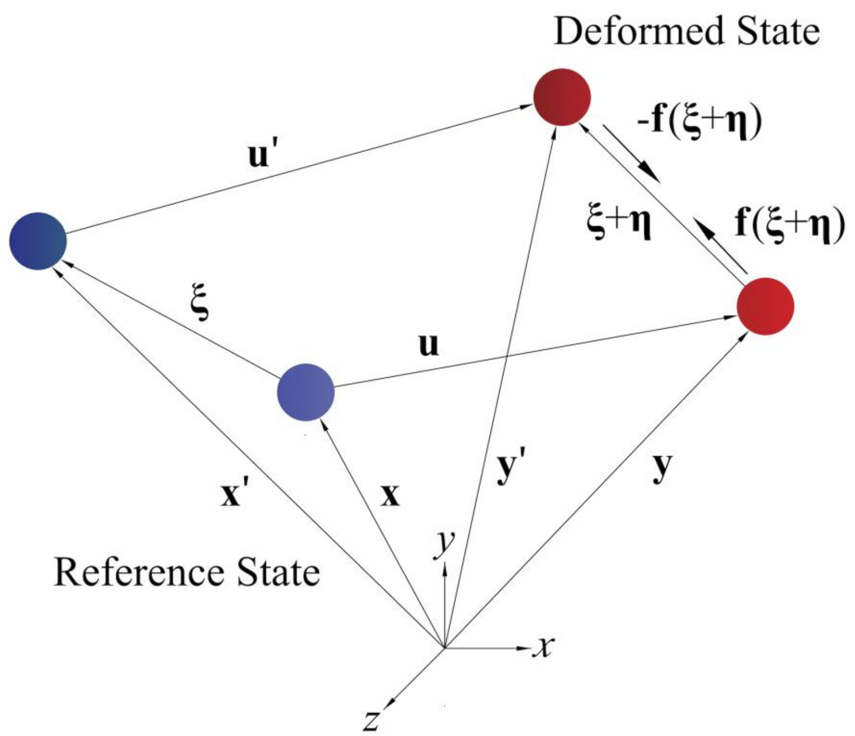
The equation of motion in PD theory:

$$\rho\ddot{\mathbf{u}}(\mathbf{x}, t) = \int_{\mathcal{H}_\mathbf{x}} \mathbf{f}(\mathbf{u}(\mathbf{x}', t) - \mathbf{u}(\mathbf{x}, t), \mathbf{x}' - \mathbf{x})\mathrm{d}V_{\mathbf{x}'} + \mathbf{b}(\mathbf{x}, t)$$

Bond Force: $\mathbf{f}(\boldsymbol{\eta},\boldsymbol{\xi}) = \dfrac{\boldsymbol{\xi}+\boldsymbol{\eta}}{|\boldsymbol{\xi}+\boldsymbol{\eta}|} c\, s$

Stretch: $s = \dfrac{|\boldsymbol{\xi}+\boldsymbol{\eta}| - |\boldsymbol{\xi}|}{|\boldsymbol{\xi}|}$

Bond constant: $c = \dfrac{12\,E}{\pi\,\delta^4}$



Failure in Peridynamics

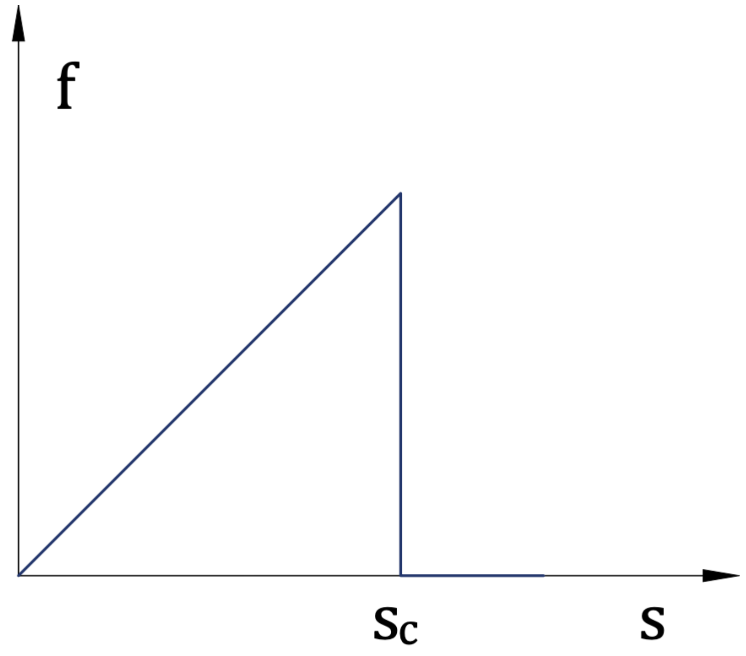
When the stretch between two material points is greater than its critical value, bond breakage occurs.

Modified Bond Force:

$$f(|\boldsymbol{\xi} + \boldsymbol{\eta}|, \boldsymbol{\xi}) = cs\mu(t, \boldsymbol{\xi})$$

Damage:

$$\mu(t, \boldsymbol{\xi}) = \begin{cases} 1 & \text{if } s(t', \boldsymbol{\xi}) < s_c \text{ for all } 0 \le t' \le t, \\ 0 & \text{otherwise} \end{cases}$$

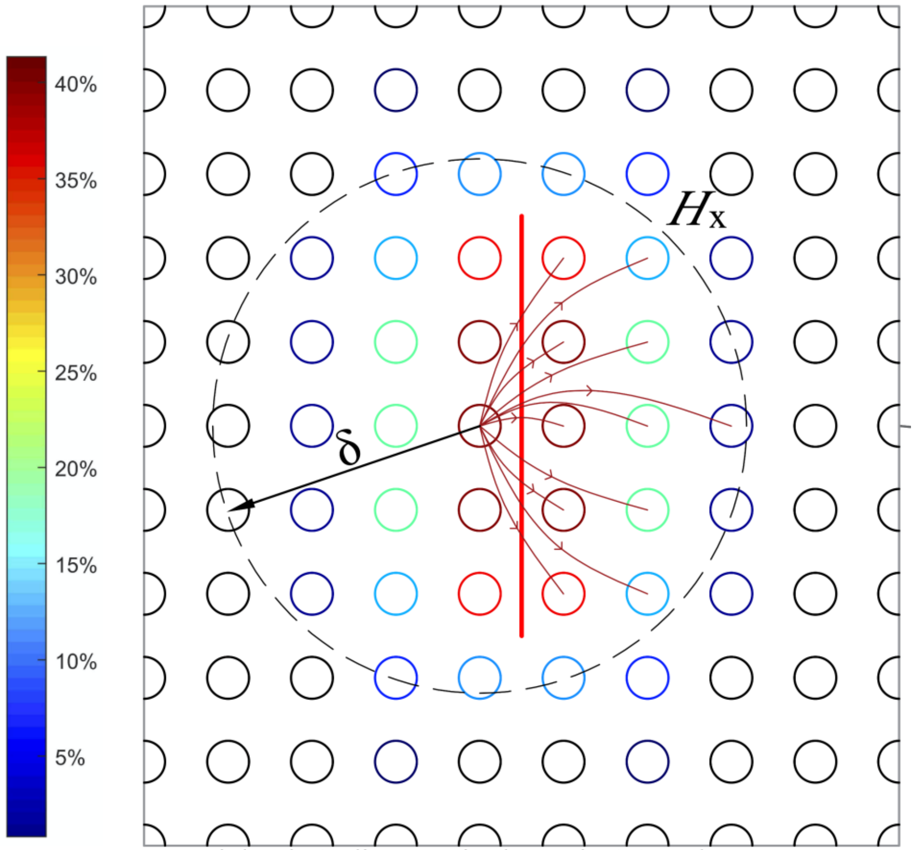


Damage in Peridynamics

The broken bonds of a material point between the family members in its horizon.

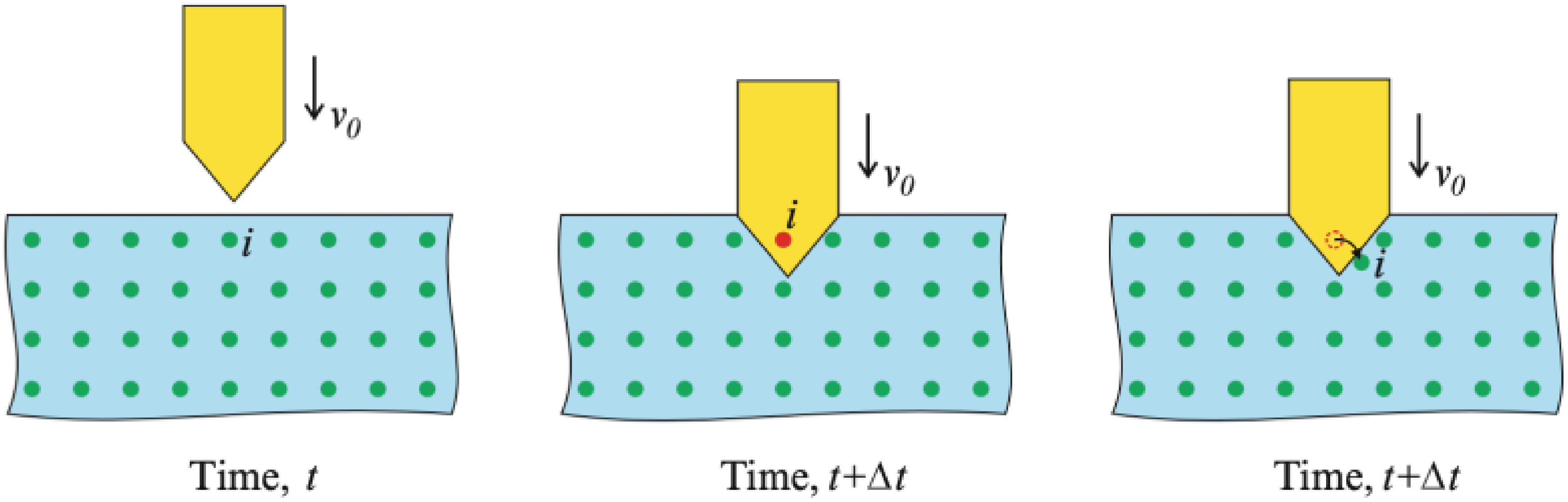
Damaged bonds: Red lines with arrows
Crack: Red thick line
Local Damage:

$$\varphi(\mathbf{x},t) = 1 - \frac{\int_{\mathcal{H}_{\mathbf{x}}} \mu(\mathbf{x},t,\boldsymbol{\xi})\, dV_{\boldsymbol{\xi}}}{\int_{\mathcal{H}_{\mathbf{x}}} dV_{\boldsymbol{\xi}}}$$



Impact Modelling

Contact between a rigid impactor and deformable target subjected to impact load.



Madenci, E. and Oterkus, E. (2014). Peridynamic Theory and Its Applications, Springer, New York

Numerical Solution Method

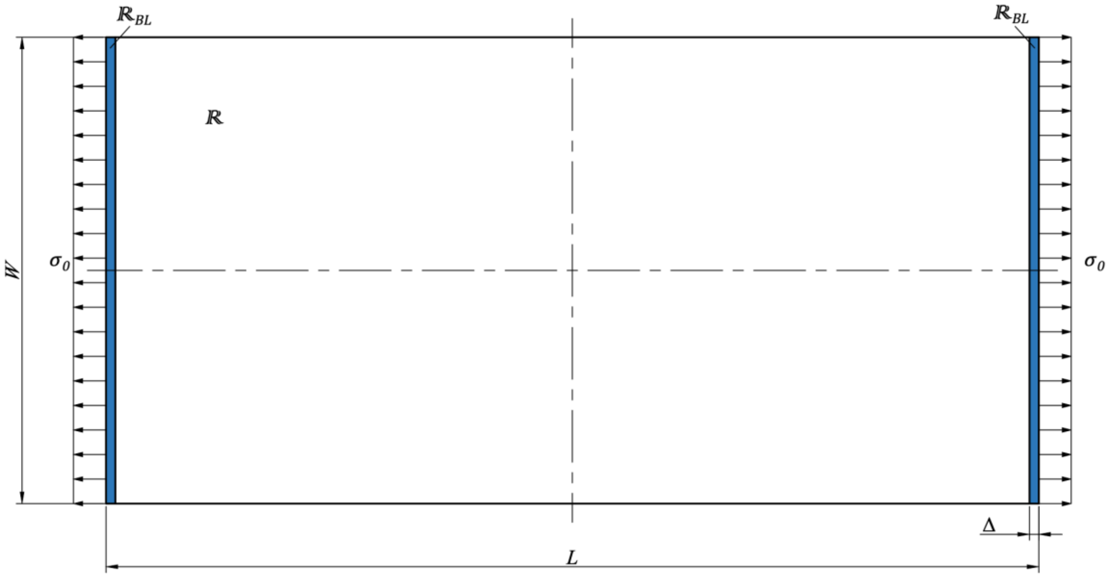
Discretization:

$$\rho_k \mathbf{u}_k^n = \sum_j \mathbf{f}\left(\mathbf{u}_j^n - \mathbf{u}_k^n, \mathbf{x}_j - \mathbf{x}_k\right) V_j + \mathbf{b}_k^n$$

Numerical convergence:

Some important parameters affect the computational process and analysis, such as the distance between material points and horizon radius.

Test Suit – Model Validation



The analytical solutions for $u_x(x, y=0)$ and $u_y(x=0, y)$ were given as,

$$u_x(x, y=0) = \frac{\sigma_0}{E}x \quad \text{and} \quad u_y(x=0, y) = -\nu\frac{\sigma_0}{E}y.$$

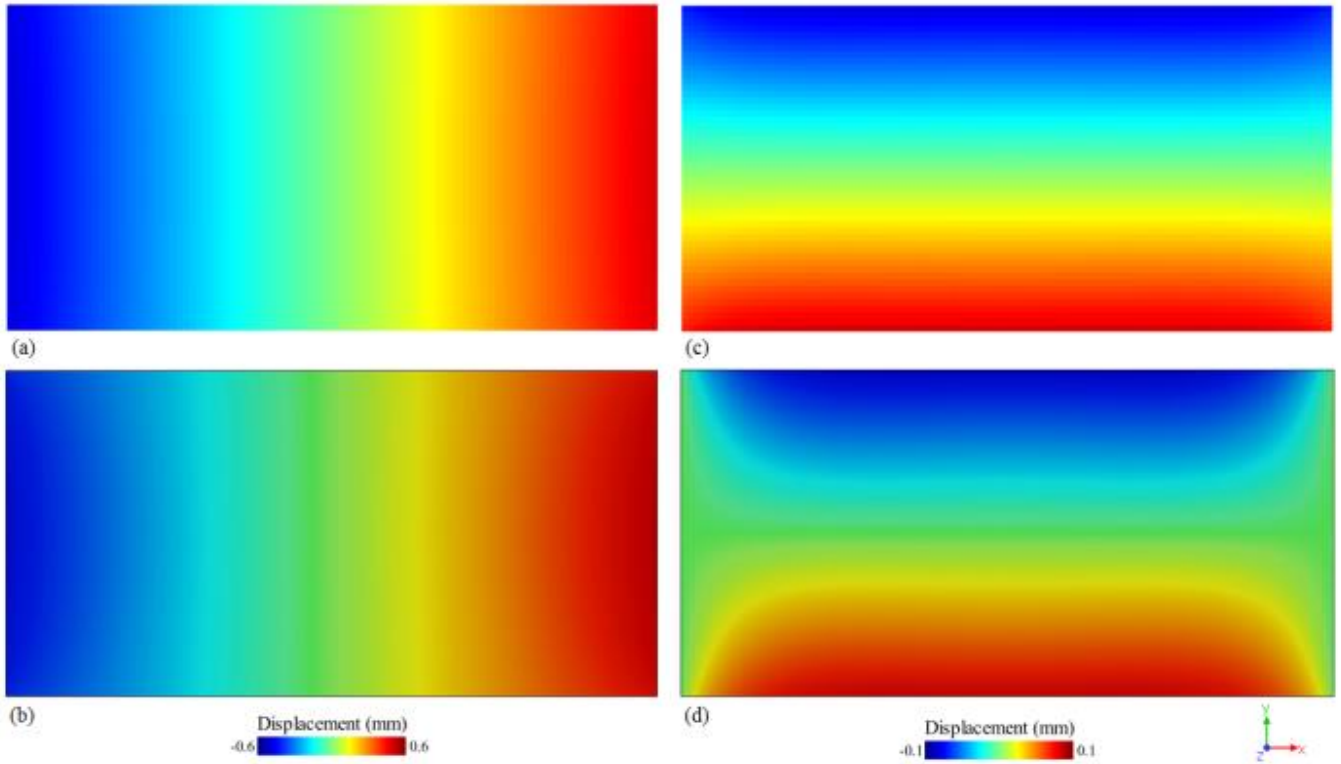


Figure 2.14 : Displacements (a) $u_x(x, y=0)$ for PD, (b) $u_y(x=0, y)$ for PD, (c) $u_x(x, y=0)$ for FEM, and (d) $u_y(x=0, y)$ for FEM.

Test Suit – Model Validation

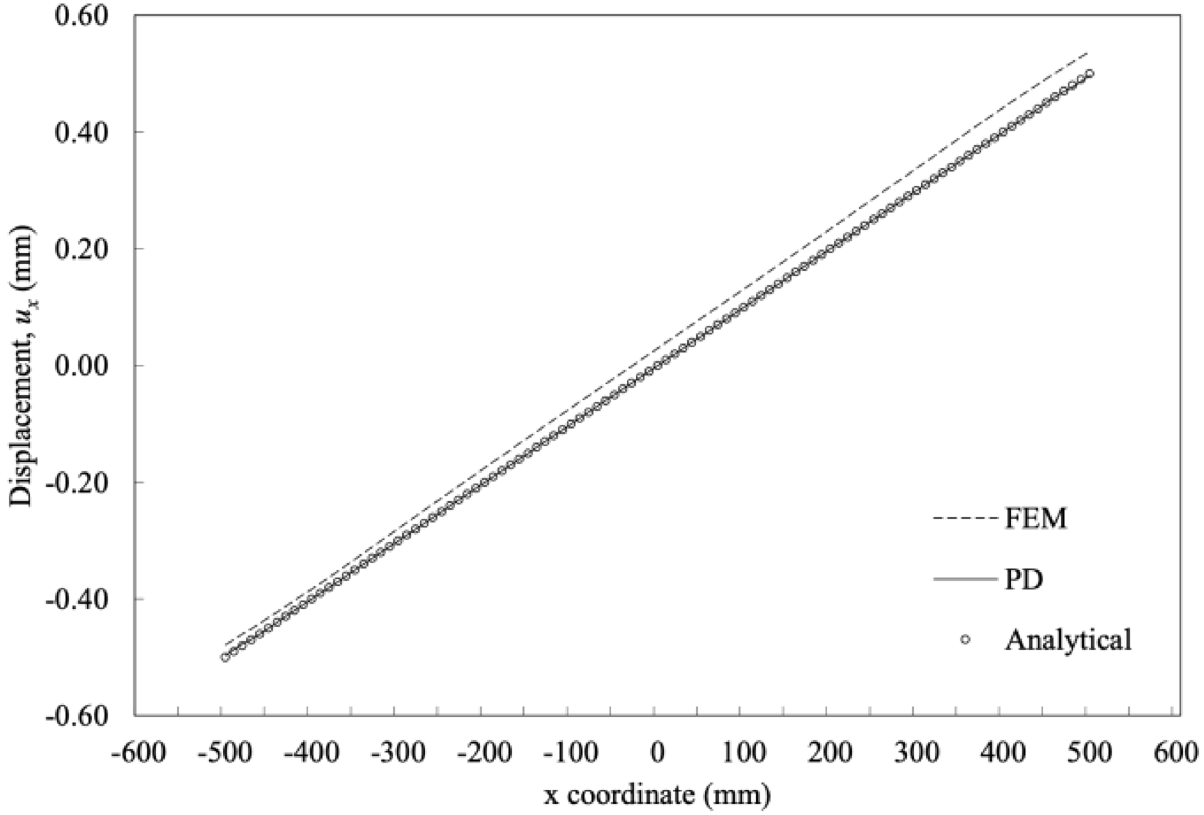


Figure 2.15 : Displacement along the centre *x*-axis.

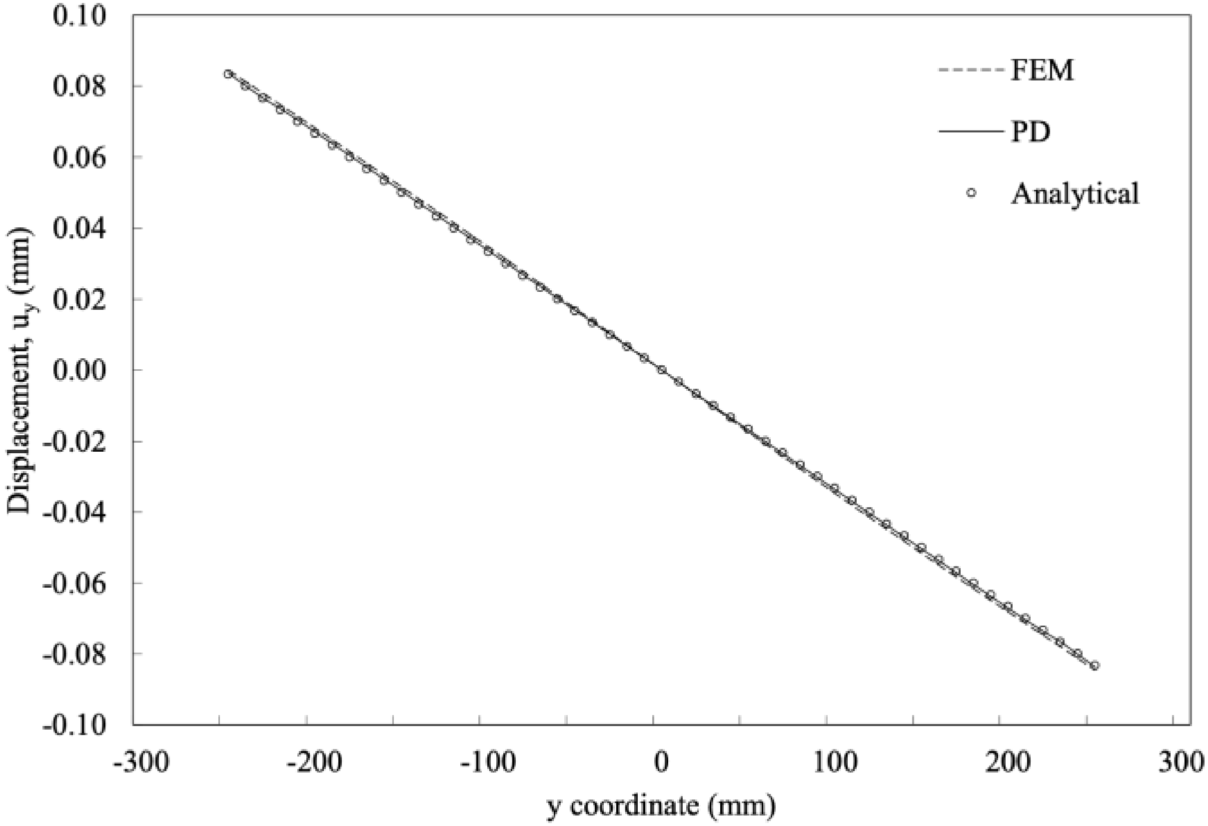


Figure 2.16 : Displacement along the centre *y*-axis.

Kalthoff – Winkler Experiment

Experimental setup

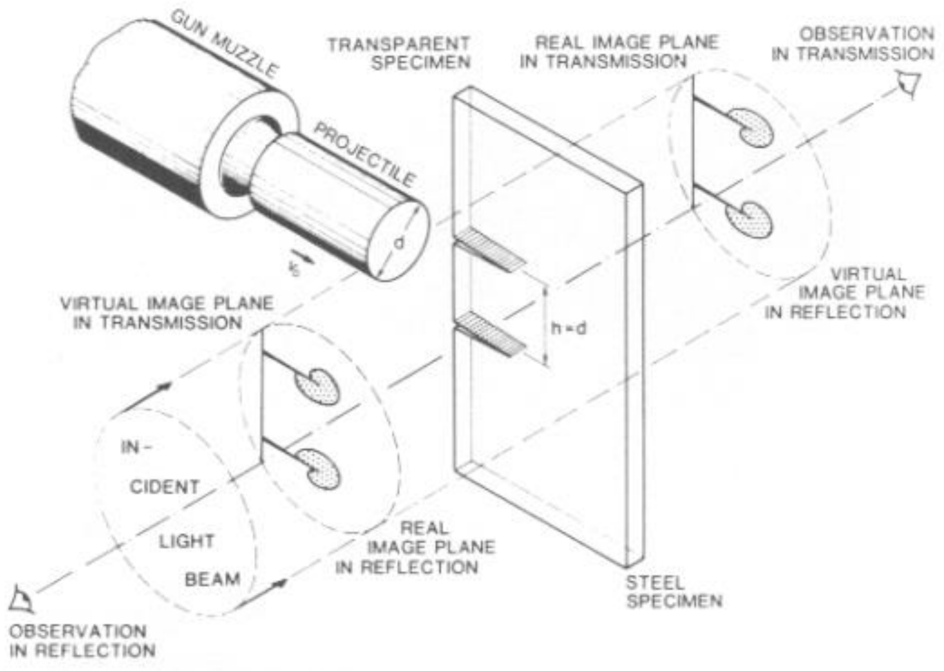
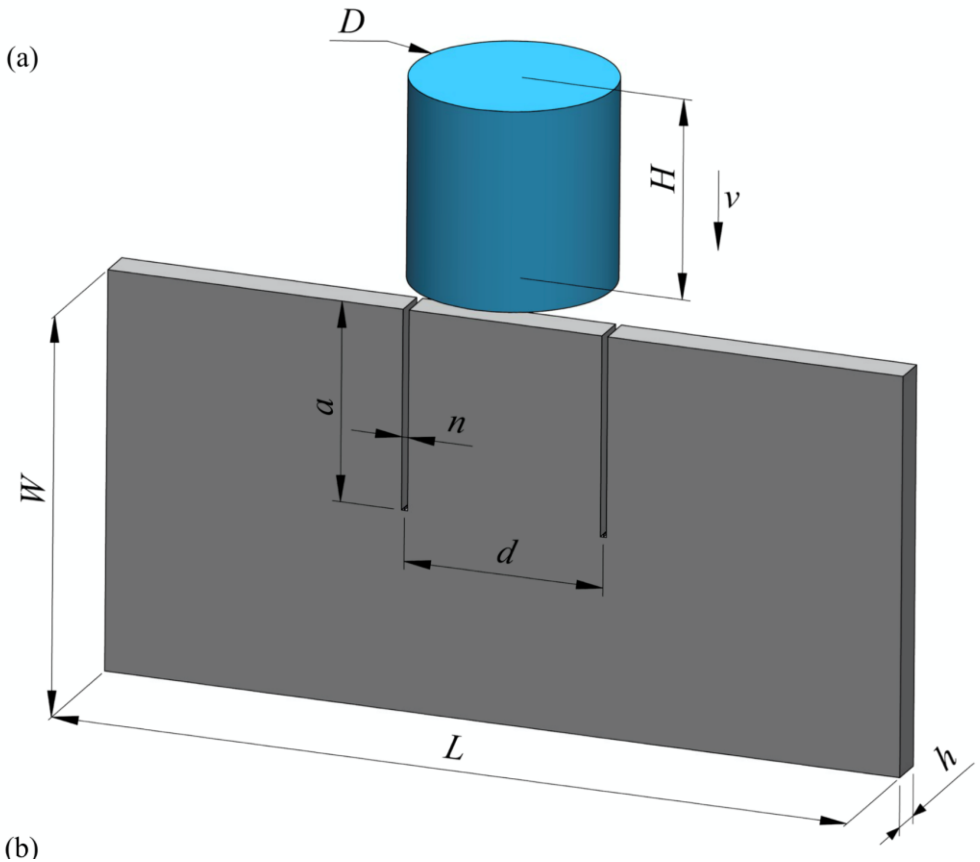


Fig. 1: Experimental set up (schematically)

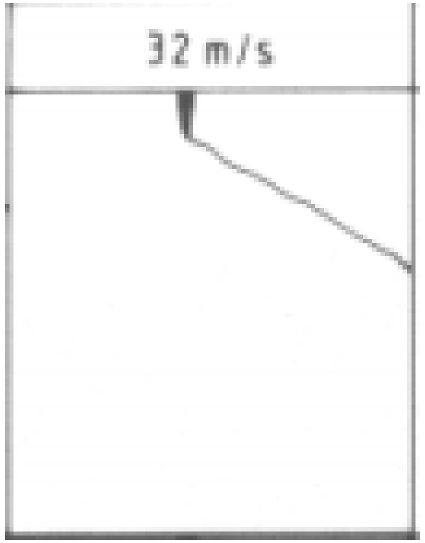


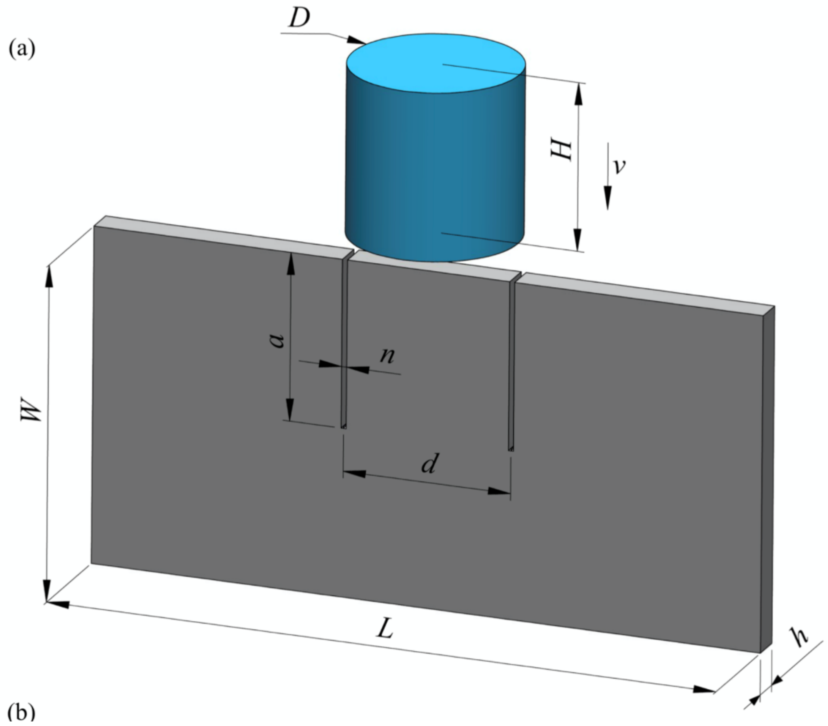
Kalthoff, J. F., and Winkler, S., 1988, “Failure Mode Transition of High Rates of Shear Loading,” Proceedings of the International Conference on Impact Loading and Dynamic Behavior of Materials, C. Y. Chiem, H.-D. Kunze, and L. W. Meyer, eds., Bremen, Germany, May 18–22, 1987, Deutsche Gesellschaft für Metallkunde, DGM-Verlag, Oberursel, pp. 185–195

Kalthoff – Winkler Experiment

High-strength maraging steel

E = 191 GPa, ν = 0.25, Density = 8000 kg/m





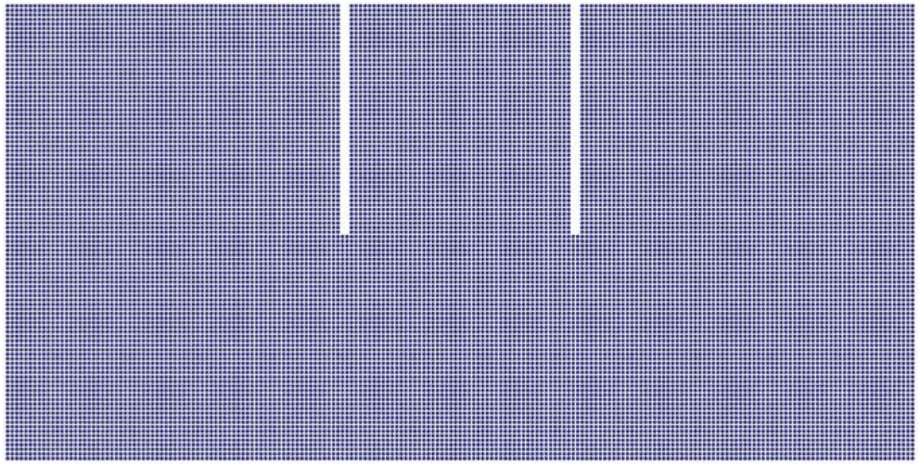
L = 0.2 m, W = 0.1 m, h = 0.009 m, d = 0.05 m, a = 0.05 m, n = 1.5 mm,

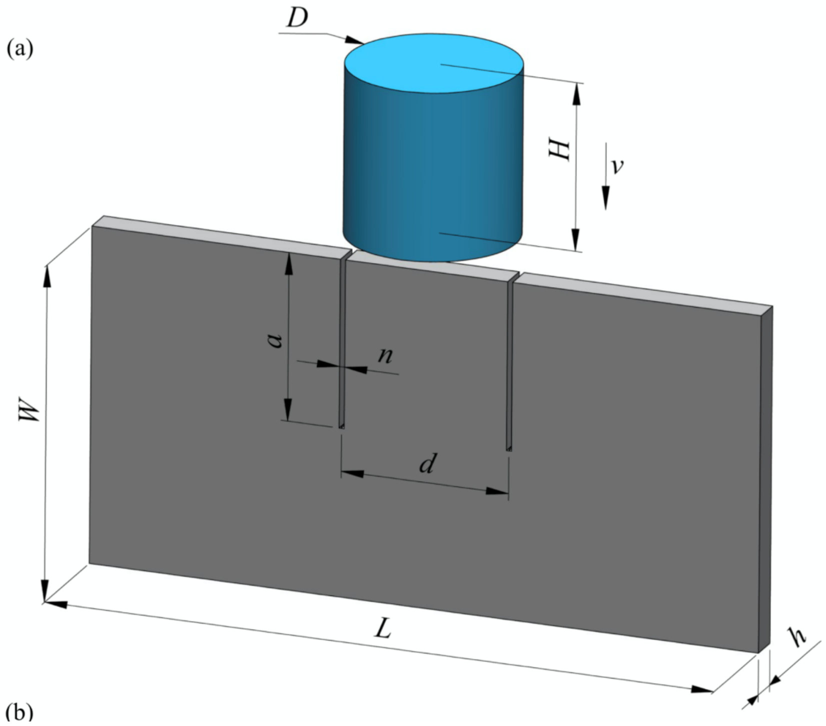
Kalthoff – Winkler Experiment

The discretized model 201 × 101 × 9

material points along *x*, *y*, and *z* axes.

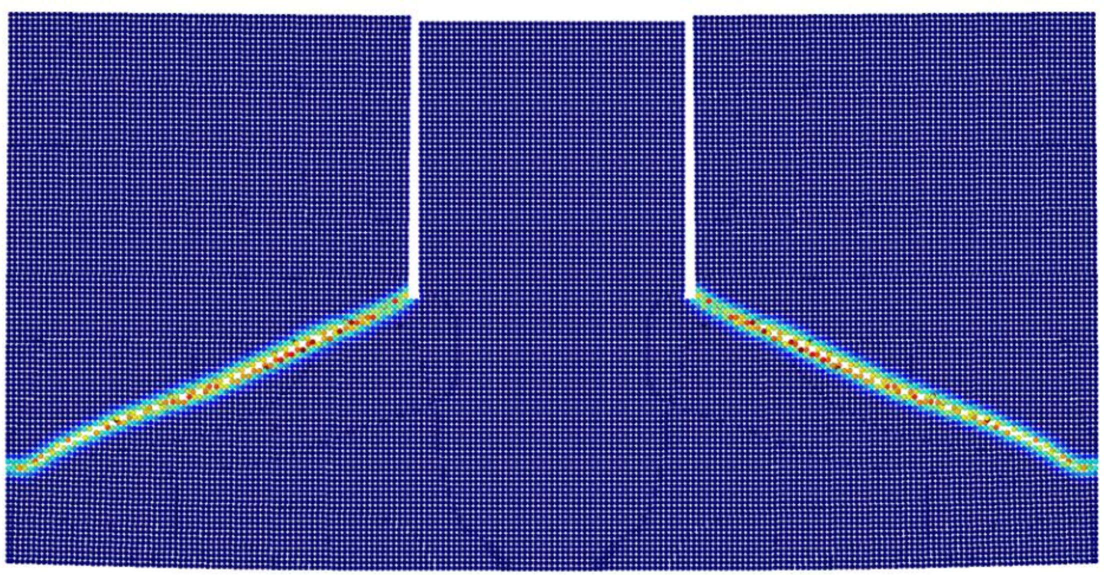
Δ = 0.001 m δ = 3.015 × Δ m

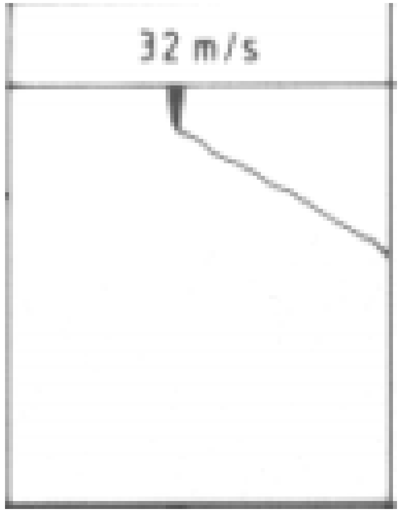




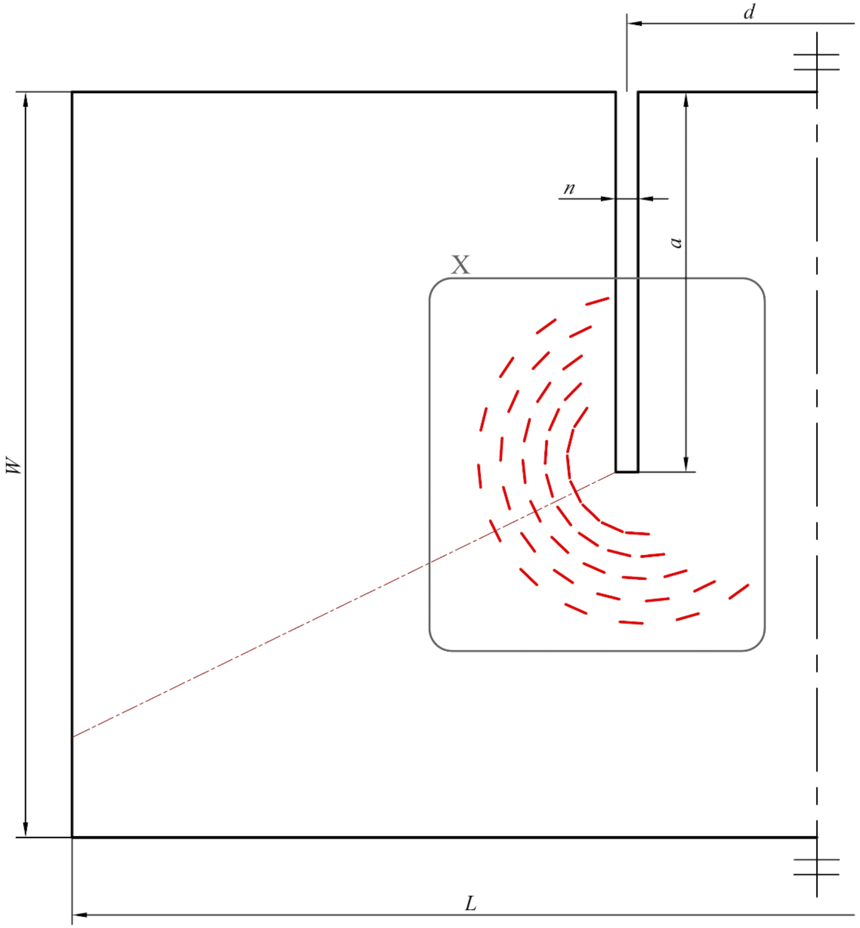
Kalthoff – Winkler Experiment

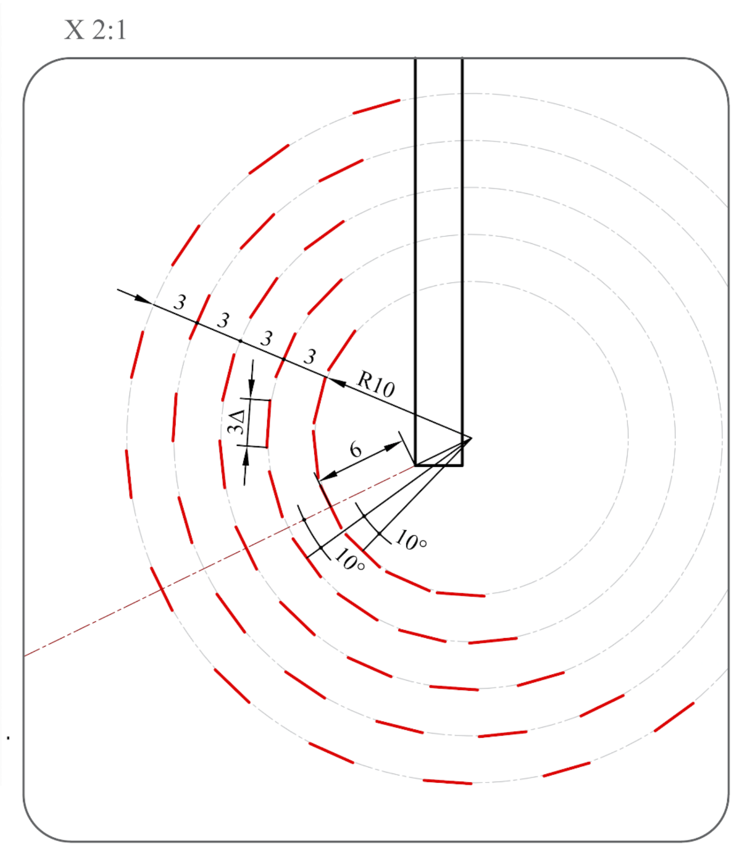
Crack growth in PD and Experiment



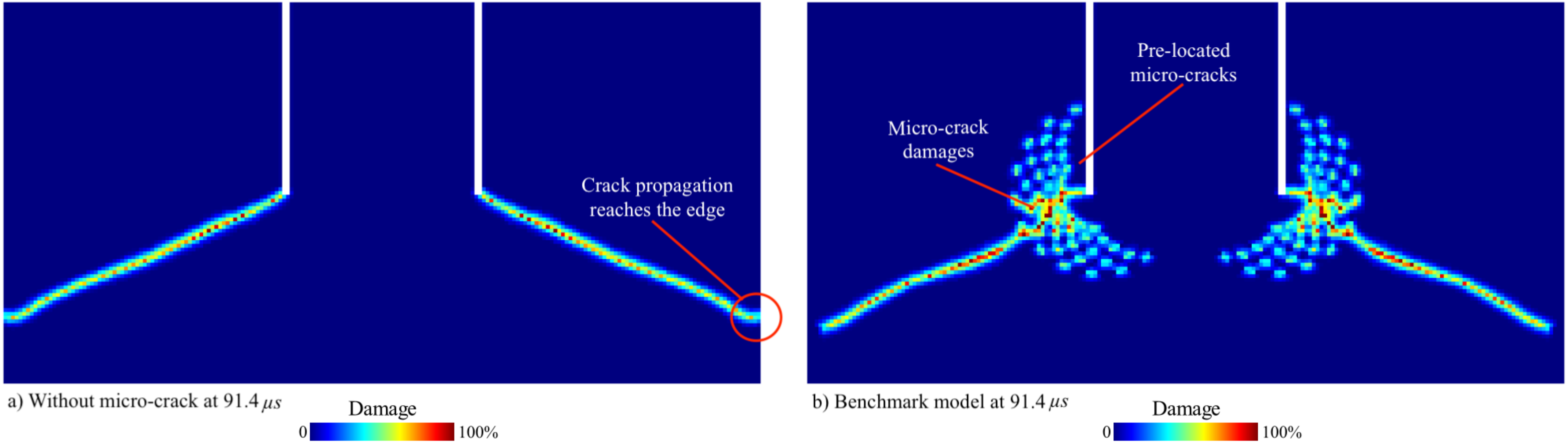


Micro-Crack Toughening Mechanism





Micro-Crack Toughening Mechanism



Average velocity: 1345 m/s

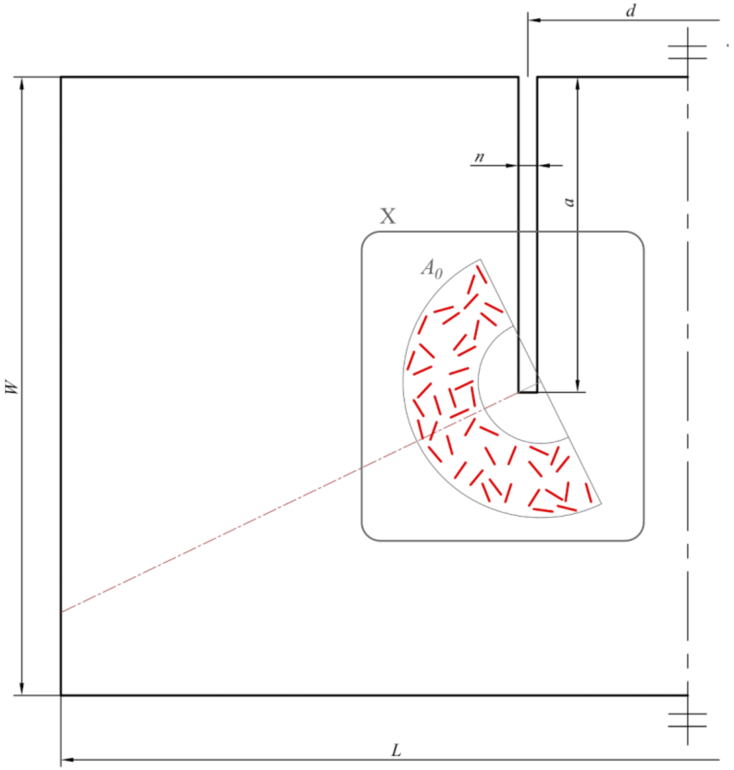
Average velocity: 1188 m/s

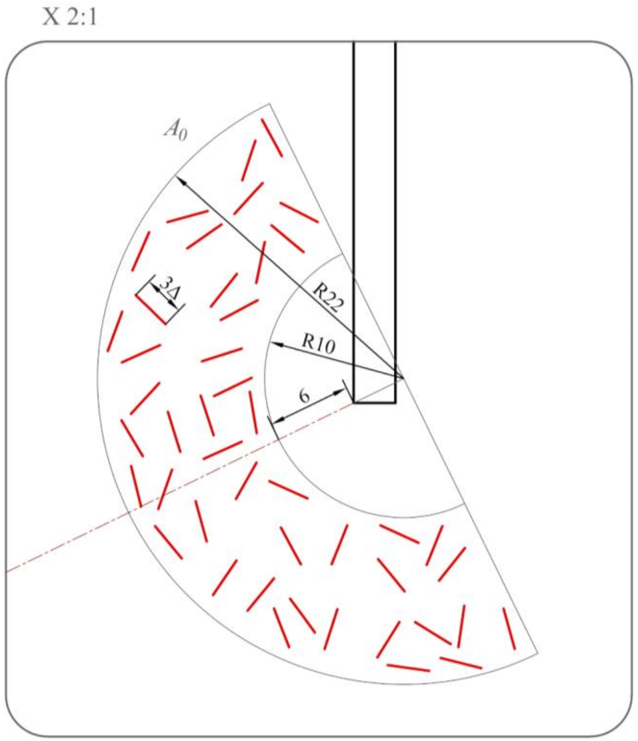
Stochastically Distributed Micro-Cracks

Micro-cracks with varying densities

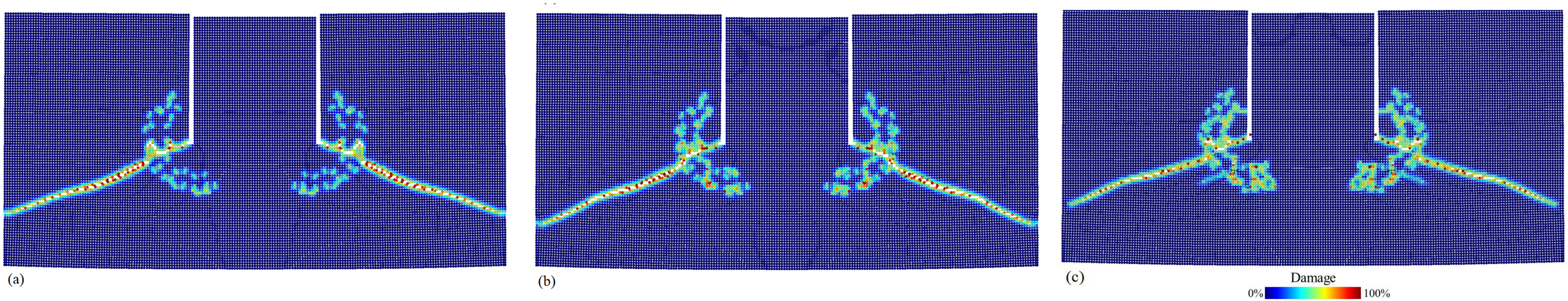
$$n_0 = N_0/A_0$$

$$n_0 = \{0.75, 1, 1.25\}$$





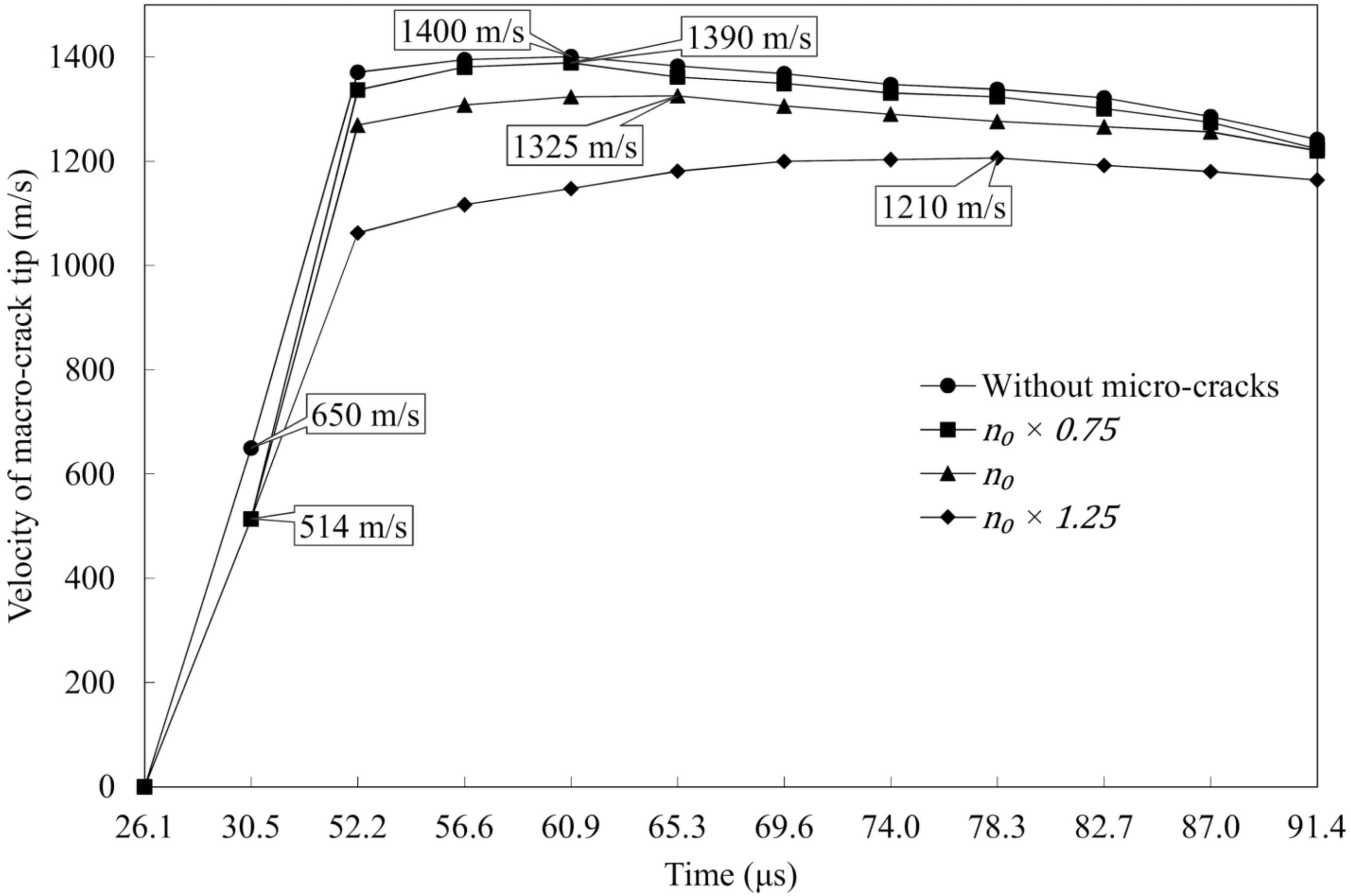
Micro-cracks with Varying Densities



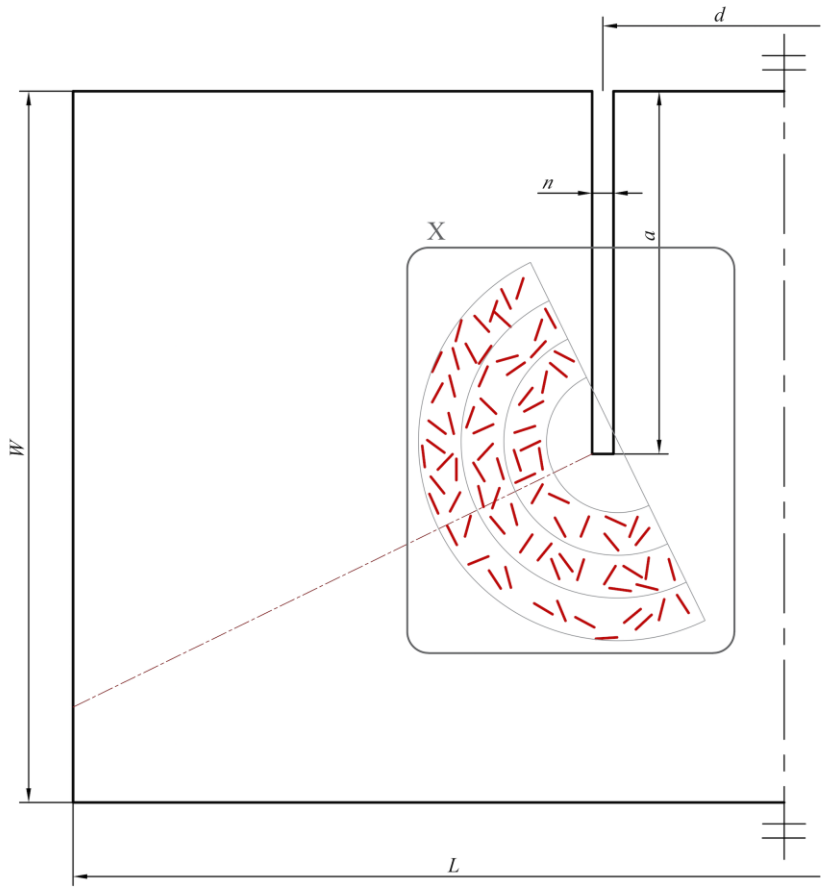
$n_0 = 0.75$ $n_0 = 1$ $n_0 = 1.25$

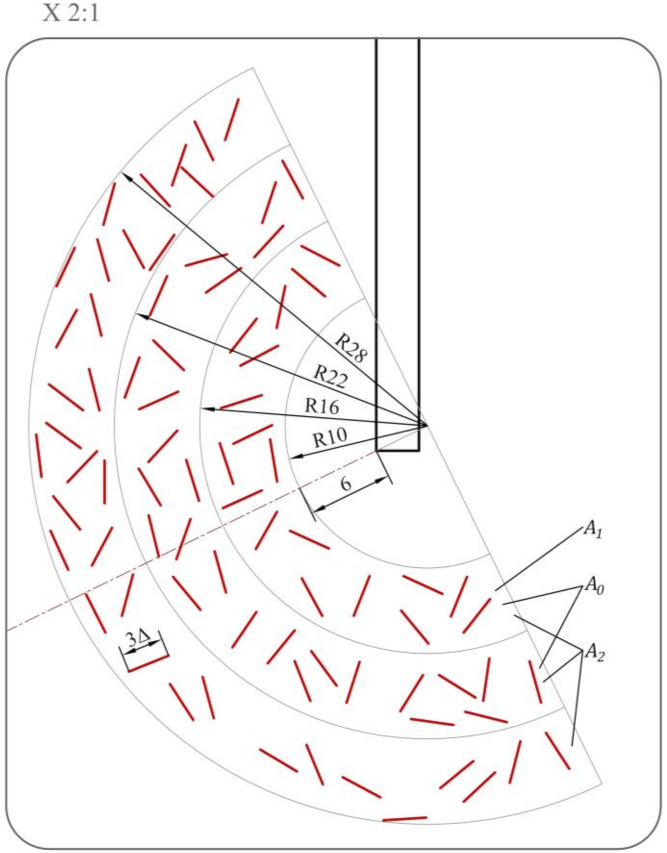
Average velocity: 1327 m/s 1284 m/s 1165 m/s

Micro-cracks with Varying Densities

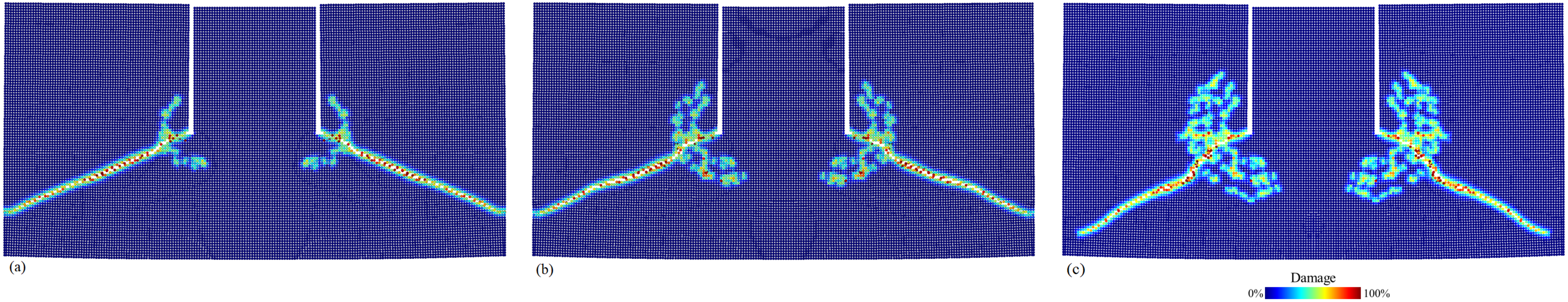


Micro-cracks with Various Number





Micro-cracks with Various Number



A_1

Average velocity: 1284 m/s

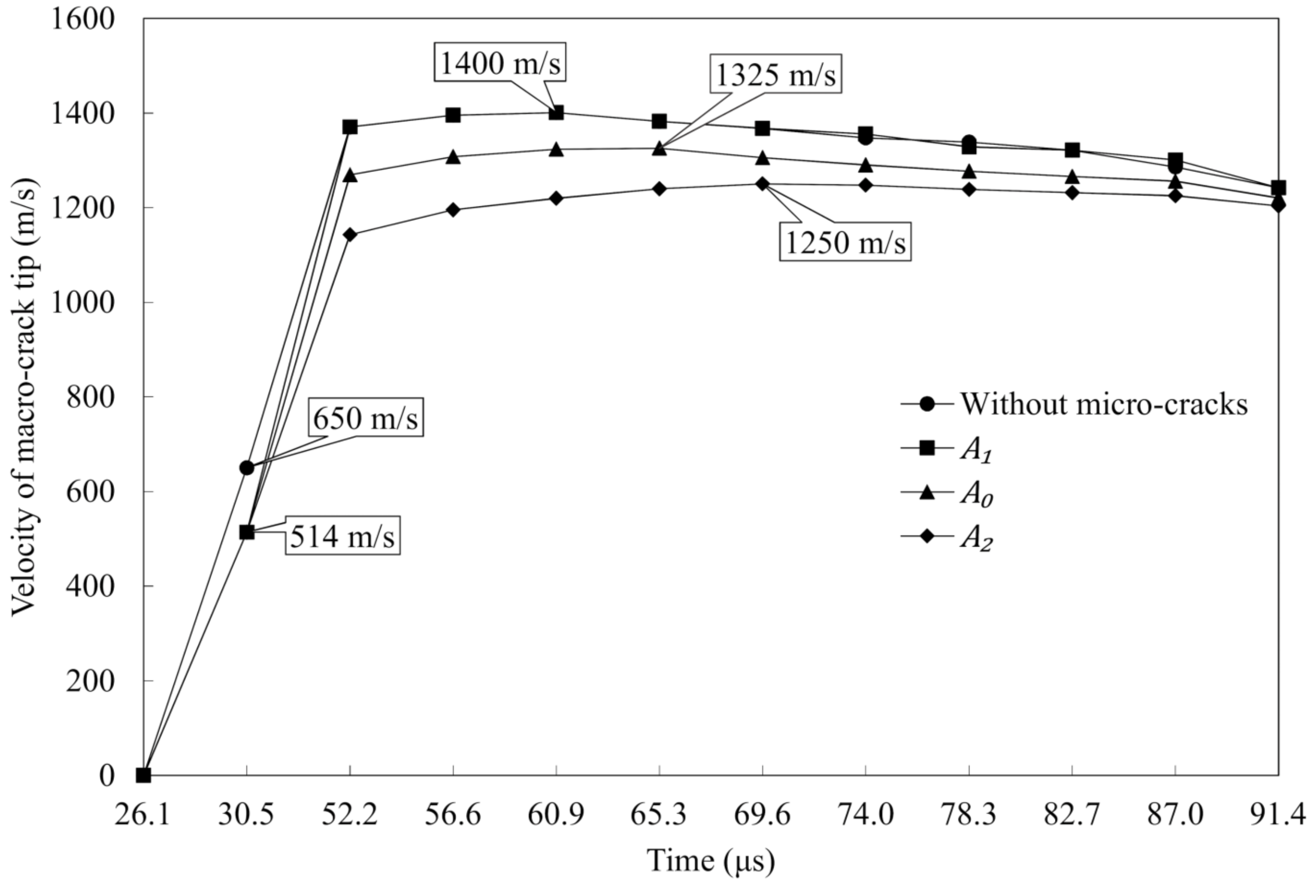
A_0

1345 m/s

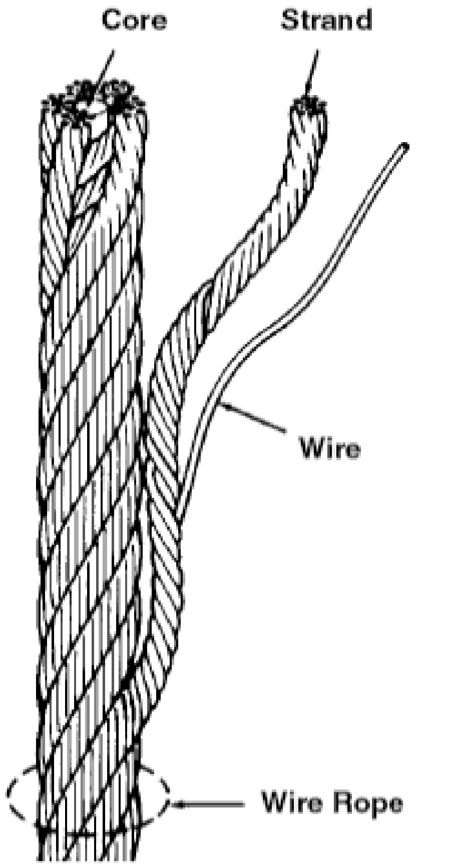
A_2

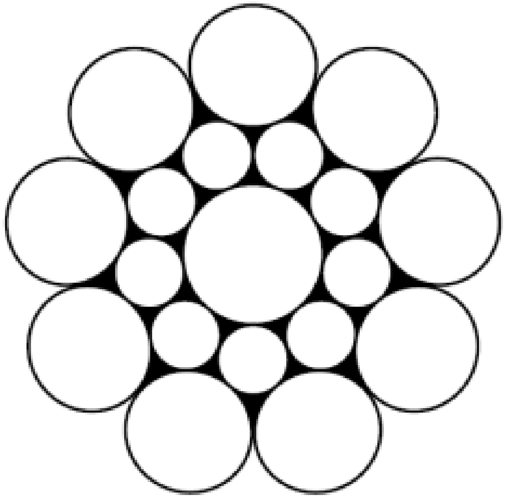
1219 m/s

Micro-cracks with Various Number

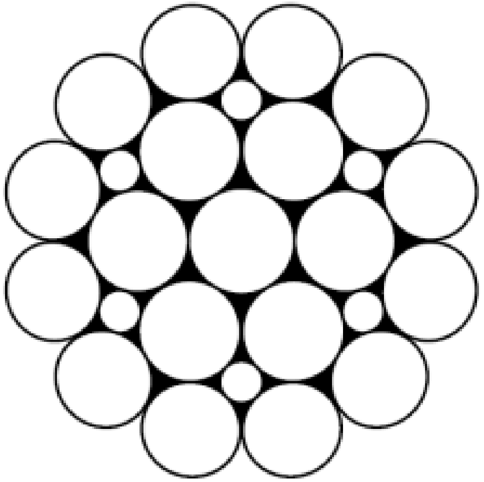


WIRE ROPES

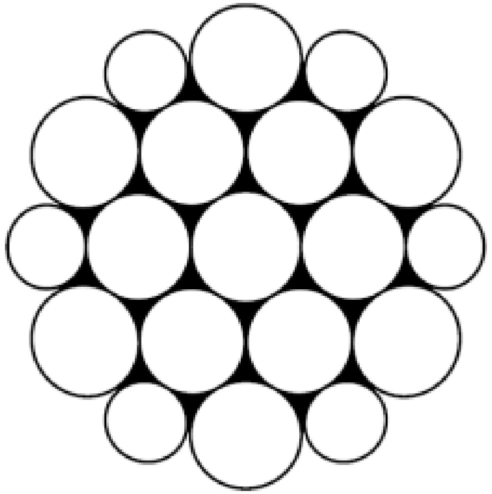




Seale



Filler



Warrington

Costello, G.A. and Miller, R.E. (1977). Lay Effect of Wire Rope., *Ill Univ Dep Theor Appl Mech TAM Rep*, (422).
Erdönmez, C. (2010). Mathematical Modeling And Stress Analysis Of Wire Ropes Under Certain Loading Conditions, Phd thesis, Istanbul Technical University

Recent Studies

Mahmoud, K.M. (2007). Fracture strength for a high strength steel bridge cable wire with a surface crack

Erdönmez, C. and Imrak, C.E. (2011). Modeling of nested helical structure based geometry

Fontanari, V. et al. (2015). Experimental investigation and numerical analysis.

Foti, F. and de Luca di Roseto, A. (2016). Analytical and FEM of the elastic–plastic behaviour of metallic strands under axial–torsional loads

Karathanasopoulos, N. et al. (2017). FEM of the elastoplastic axial-torsional response

Kastratovic, G. et al. (2020). Numerical Simulation of Crack Propagation in Seven-Wire Strand.

Recent Studies

Kastratović et al. 2020

FEM – Crack Propagation

- Bonded Contact
- Axial load
- The initial crack was a 0.8 mm in radius penny shaped crack.

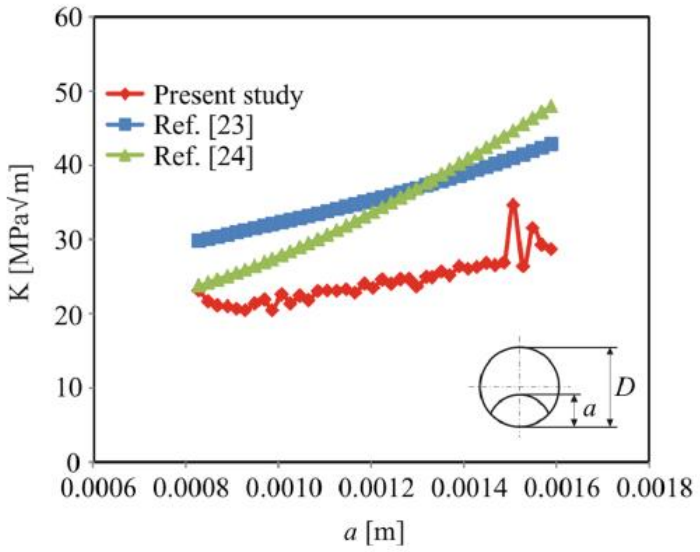


Fig. 11. Stress intensity factors along crack.

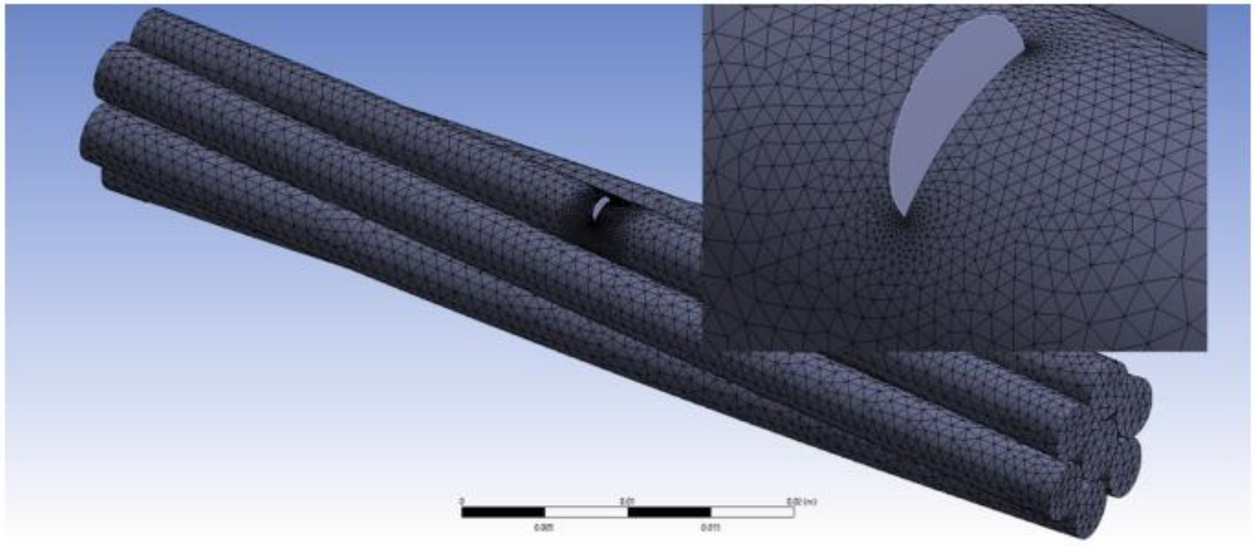


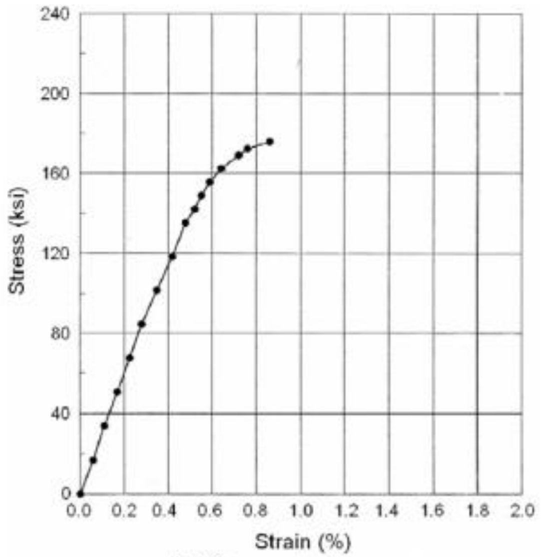
Fig. 8. FEM model of seven-wire strand

Recent Studies

Mahmoud 2007

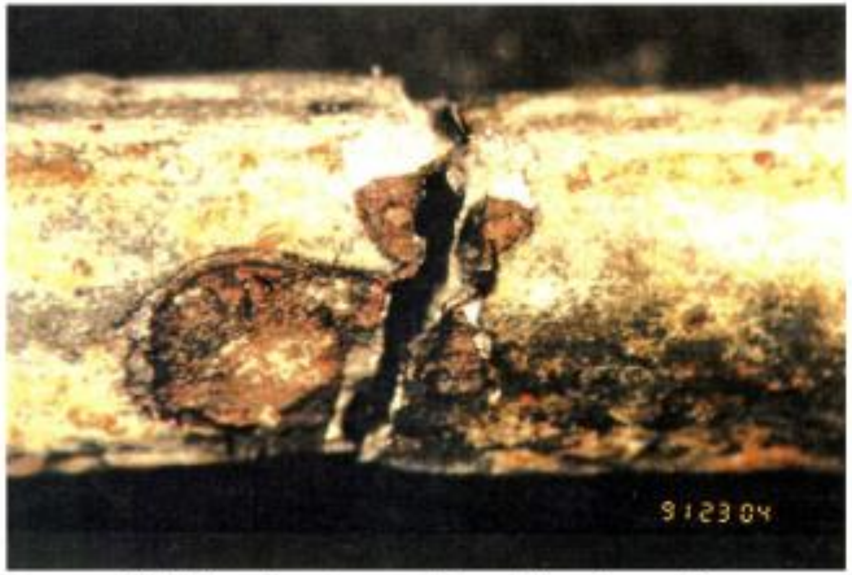
FEM – Crack Propagation

- Fracture strength for a high strength steel bridge cable wire with a surface crack
- Hydrogen embrittlement -> ductility loss
- The axial tensile stress.



(b) Stress strain curve

Fig. 1. Brittle fracture in a wire test specimen.



(a) Wire fracture with minimal necking

A Damaged Wire Rope

Modelling of Wire Ropes /w PD

FORTRAN

kalt_micro_crack.f90

```fortran
program main

implicit none

integer ndivx, ndivy, ndivz, ntotnode, nt, maxfam, nnum, cnode,
parameter(ndivx = 201)
parameter(ndivy = 101)
parameter(ndivz = 9)
parameter(nbnd = 0)
parameter (ntotnode = ndivx * ndivy * ndivz)
parameter(nt = 1350)
parameter(maxfam = 200)

!Microcrack integars
integer nmc, namc, zz, col

!Microcrack parameters
parameter(nmc = 200) !number of max micro crack
parameter(namc = 45) !number of actual micro crack on left side

real *8 length, width, thick, dx, delta, dens, emod, pratio, smo
```

MATLAB

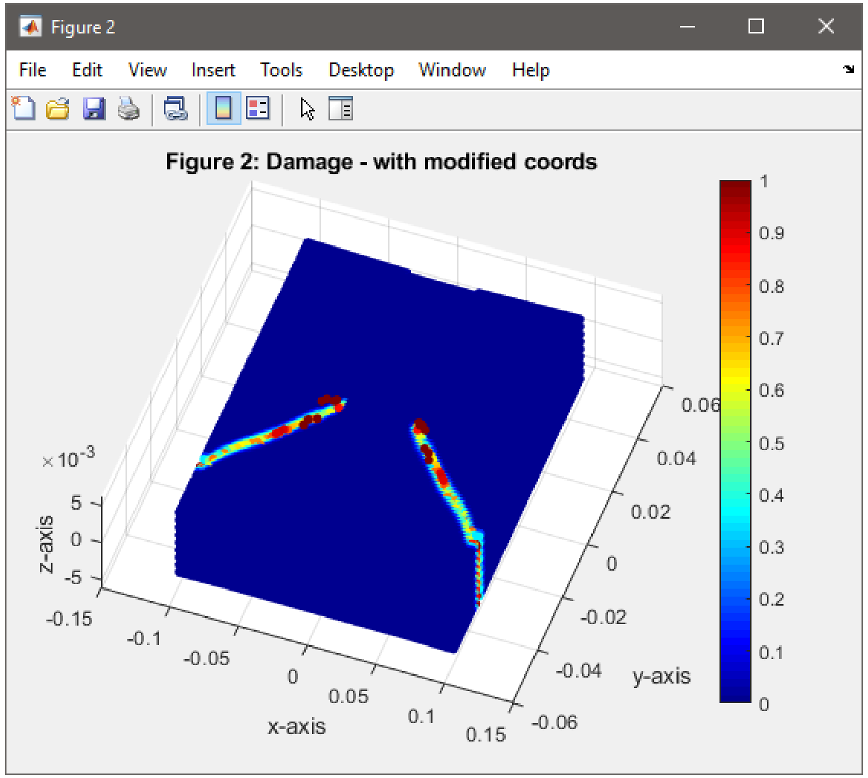
Editor - D:\Adem\MATLAB\kalt_or\kalt_or.m

kalt_or.m

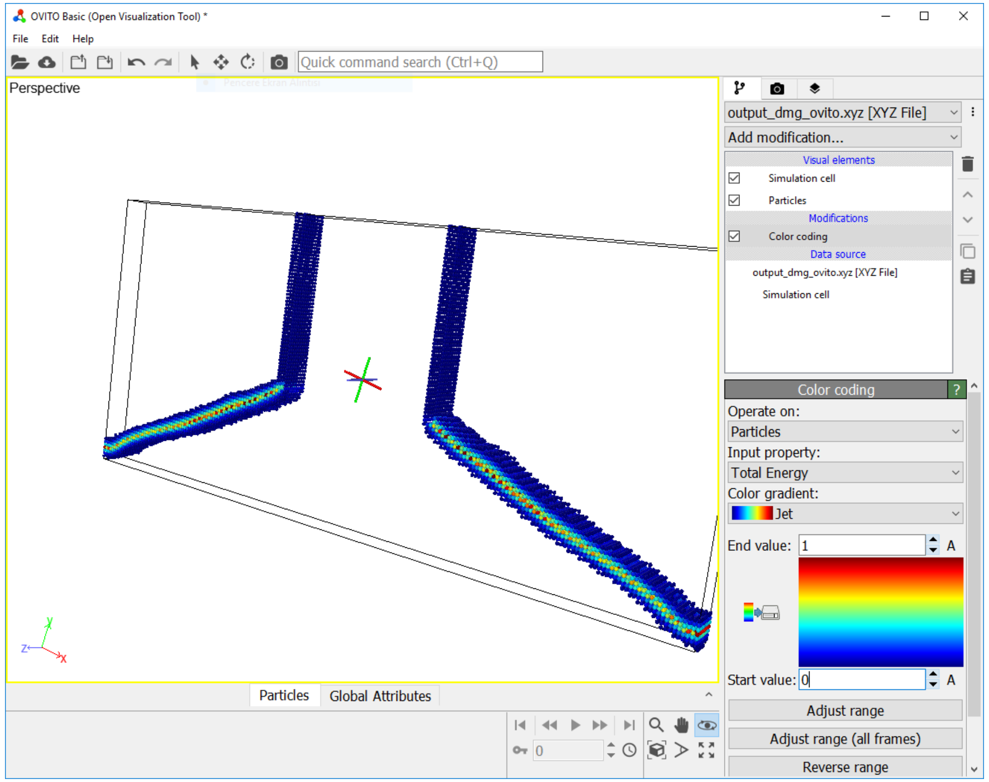
```matlab
clc
clear
ndivx = 201;
ndivy = 101;
ndivz = 9;
nbnd = 0;
ntotnode = ndivx * ndivy * ndivz;

nt = 1;
maxfam = 200;

    coord(1:ntotnode,1) = 0;
    coord(1:ntotnode,2) = 0;
    coord(1:ntotnode,3) = 0;
    numfam(1:ntotnode,1) = 0;
    numfamnew(1:ntotnode,1) = 0;
```

Handling with Outputs

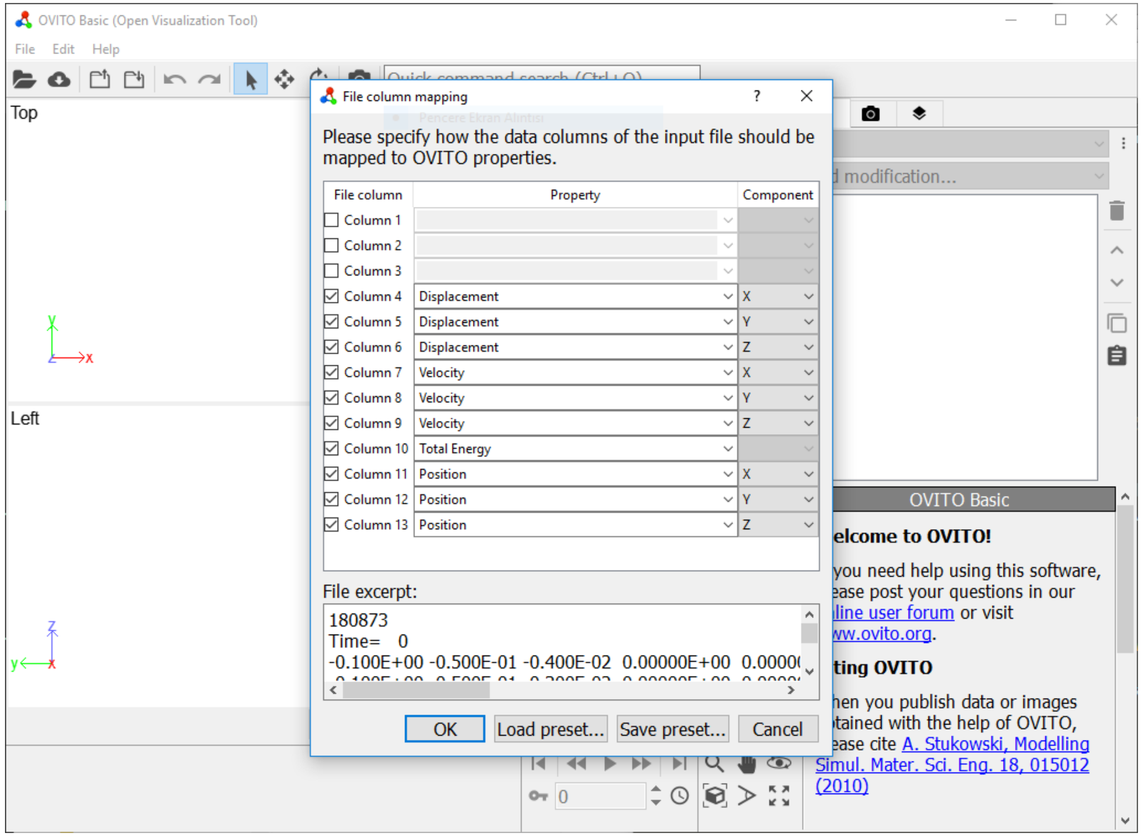
MATLAB

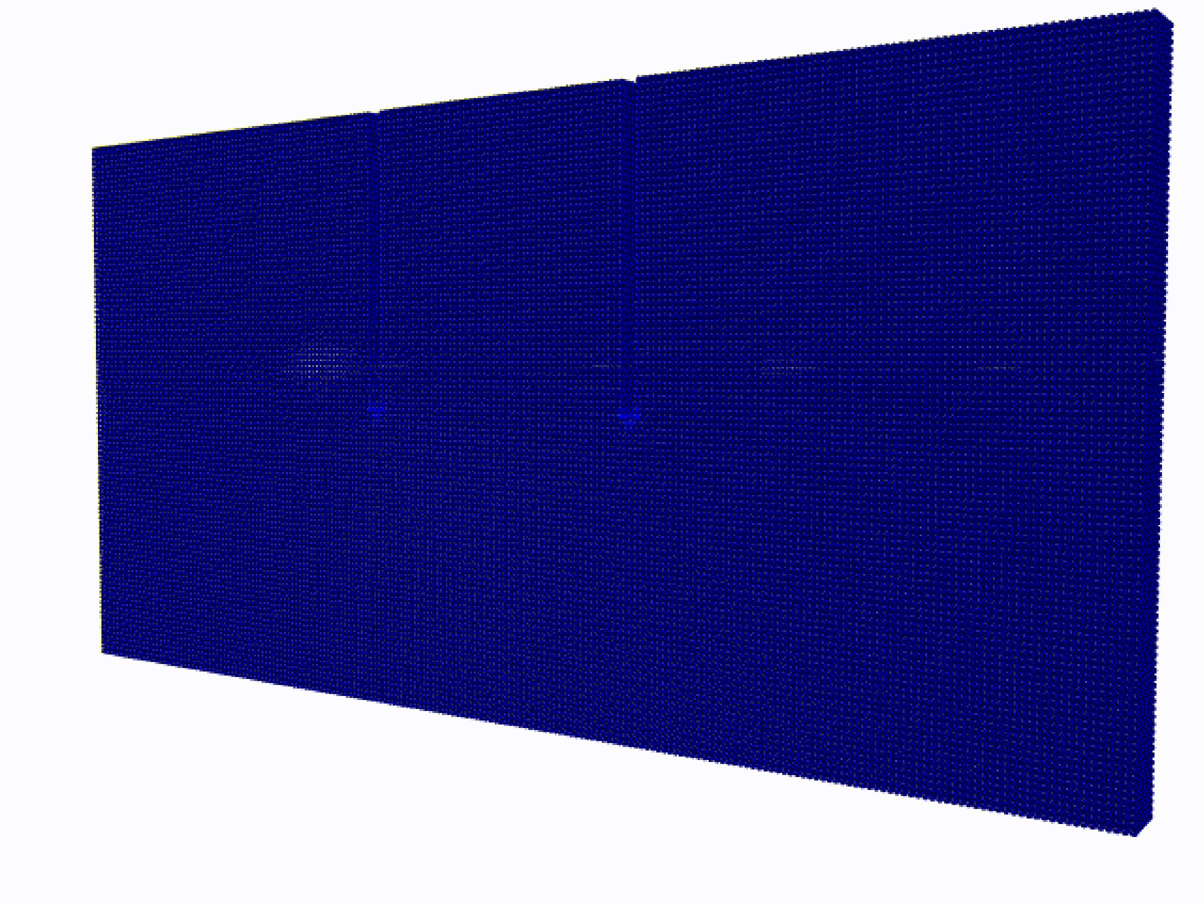


OVITO

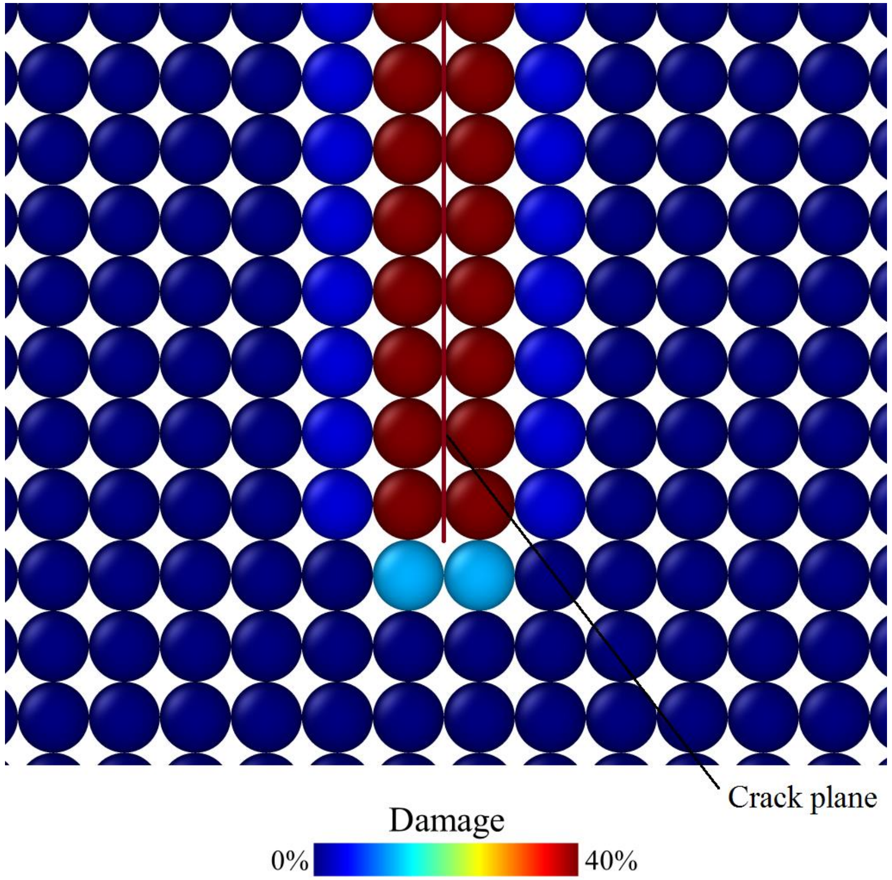


Handling with Outputs - OVITO

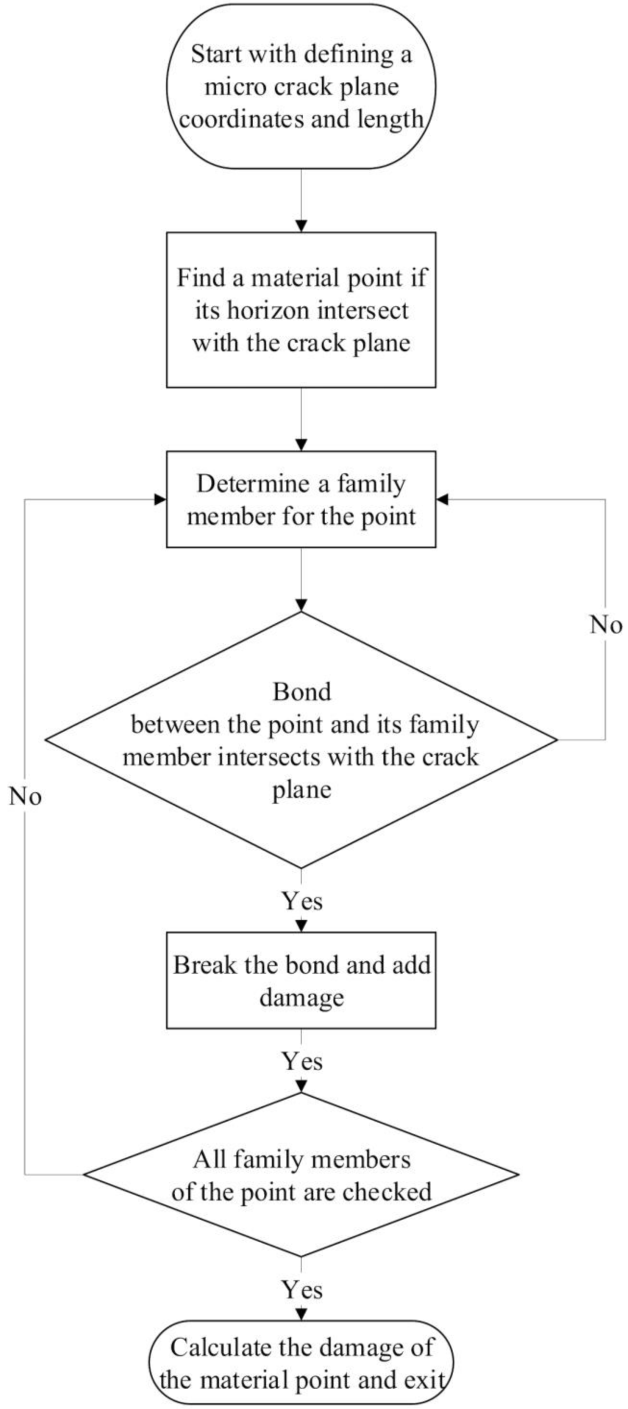




Crack Code

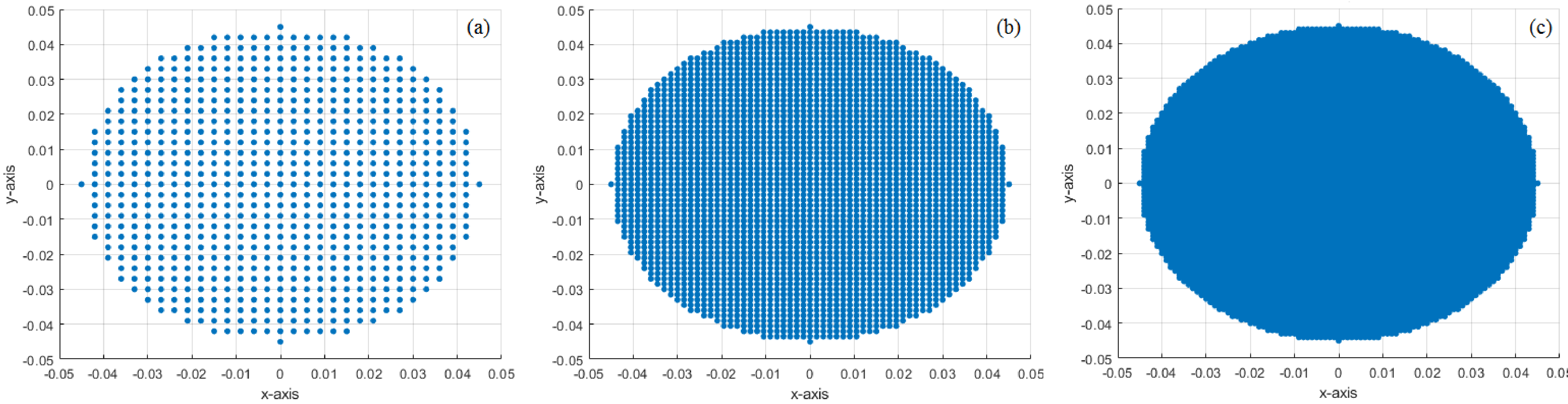


Crack plane
Damage
0%
40%

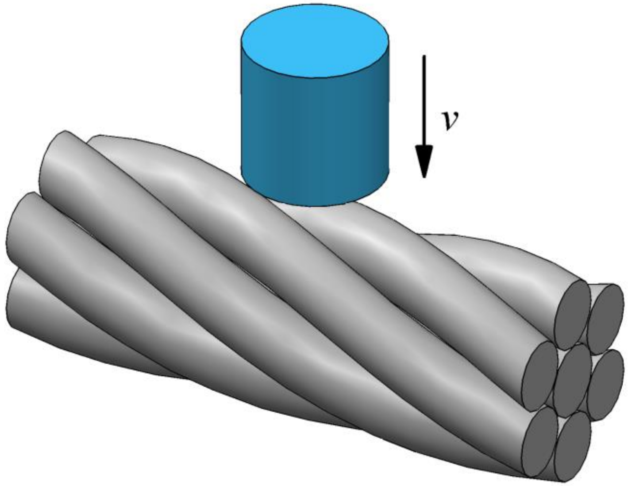


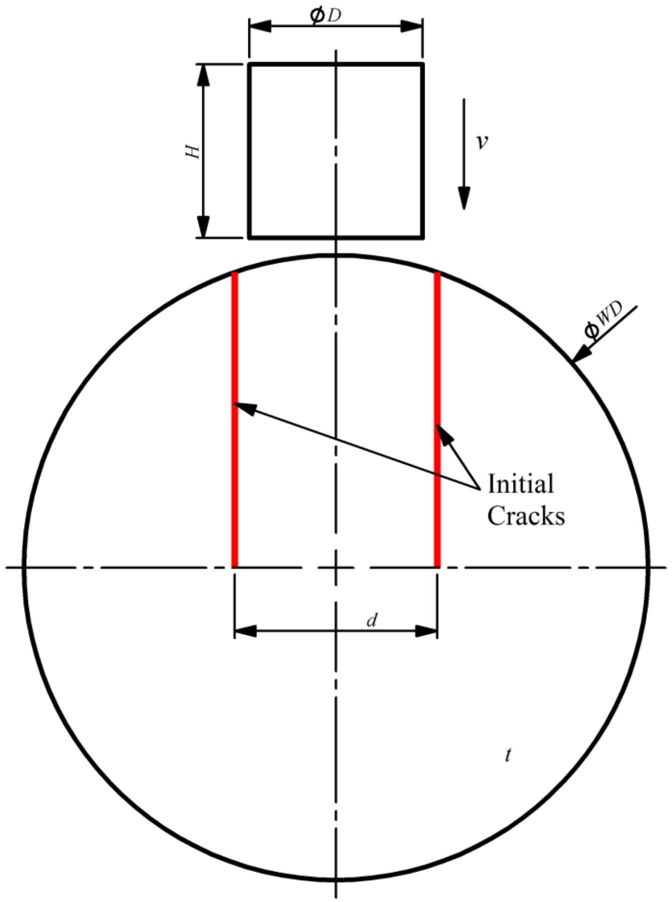
Start with defining a micro crack plane coordinates and length
Find a material point if its horizon intersect with the crack plane
Determine a family member for the point
Bond between the point and its family member intersects with the crack plane
No
No
Yes
Break the bond and add damage
Yes
All family members of the point are checked
Yes
Calculate the damage of the material point and exit

Wire Rope Modelling



Wire Rope Modelling in PD





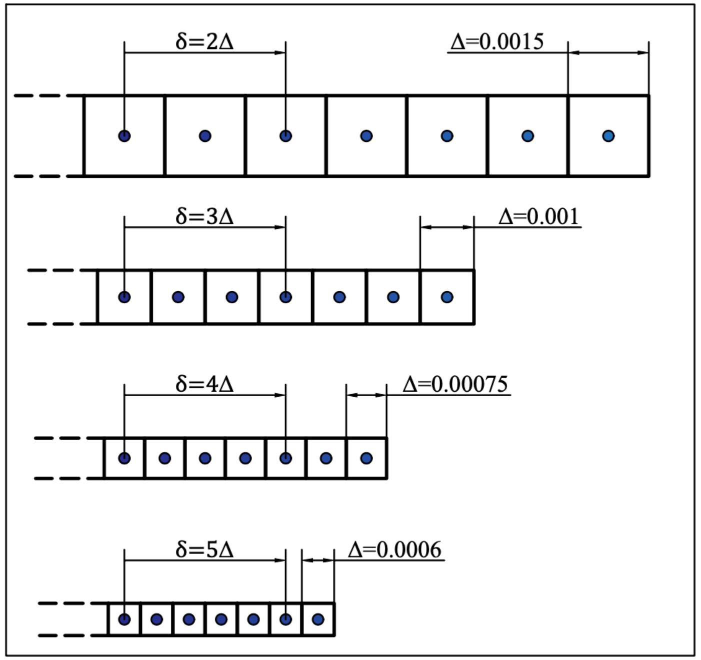
Dimensions and mechanical properties of the impactor and single wire model.

Parameter	Value
D	$\phi 0.025$ m
H	0.025 m
v	32 m/s
WD	$\phi 0.090$ m
d	0.029 m
t	0.003 m
Rigid impactor mass	0.785 kg
Poisson's ratio, ν	0.25
Young's modulus, E	191 GPa
Mass density, ρ	8000 kg/m^3

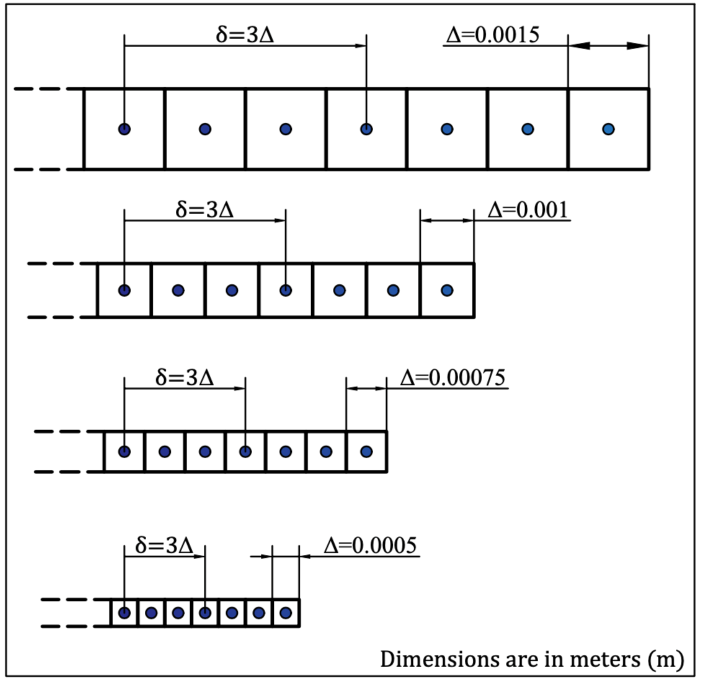
Convergence Tests

Table 5.3 : m-Convergence test setup and parameters.

m	2	3	4	5
Horizon, δ (m) - constant	0.003	0.003	0.003	0.003
ndivx	61	91	121	151
ndivy	61	91	121	151
ndivz	3	4	5	6
Diameter of the wire (m)	0.09	0.09	0.09	0.09
Thick in z direction (m)	0.003	0.003	0.003	0.003
Particle radius (m)	0.000750	0.000500	0.000375	0.000300
Δx, Δy, Δz (m)	0.001500	0.001000	0.000750	0.000600
m	2	3	4	5
Volume of a material point (m^3)	3.38E-09	1.00E-09	4.22E-10	2.16E-10
Total node number	8,460	25,440	56,440	105,984
Points in contact layer	15	19	21	25
Contact length (m)	0.0210	0.0180	0.0150	0.0144



(a) m-convergence (constant horizon radius, δ)



(b) δ-convergence: constant material points number in a horizon

m-convergence

Initial State with pre-cracks

Damage 0 - 100%:
The weighted ratio of the number of damaged bonds:

$$\varphi(\mathbf{x},t) = 1 - \frac{\int_{\mathscr{H}_\mathbf{x}} \mu(\mathbf{x},t,\boldsymbol{\xi})\, \mathrm{d}V_{\boldsymbol{\xi}}}{\int_{\mathscr{H}_\mathbf{x}} \mathrm{d}V_{\boldsymbol{\xi}}}$$

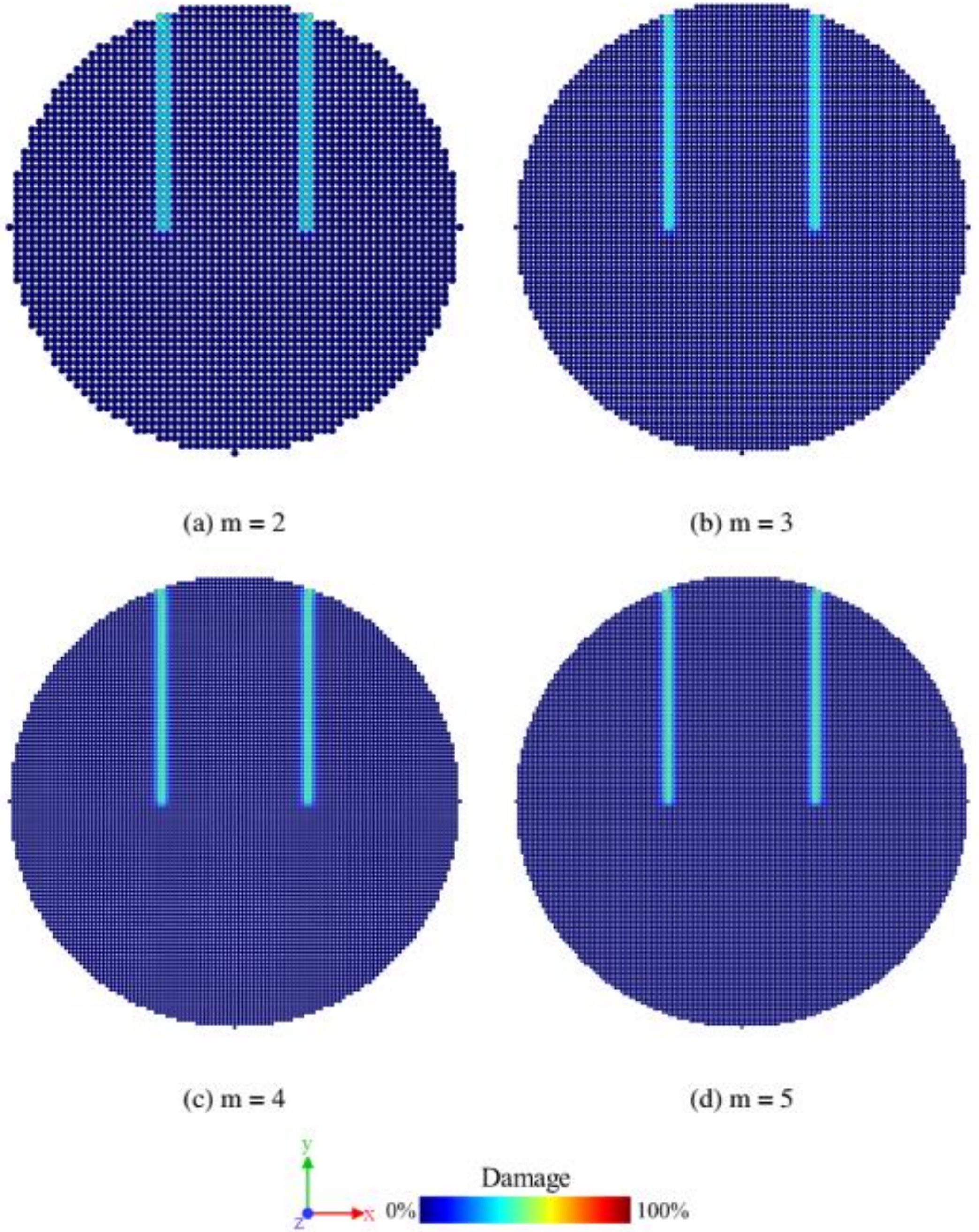


Figure 5.12 : The discretized wire section with two pre-cracks for horizon size $\delta =$ 0.003 m at the initial state.

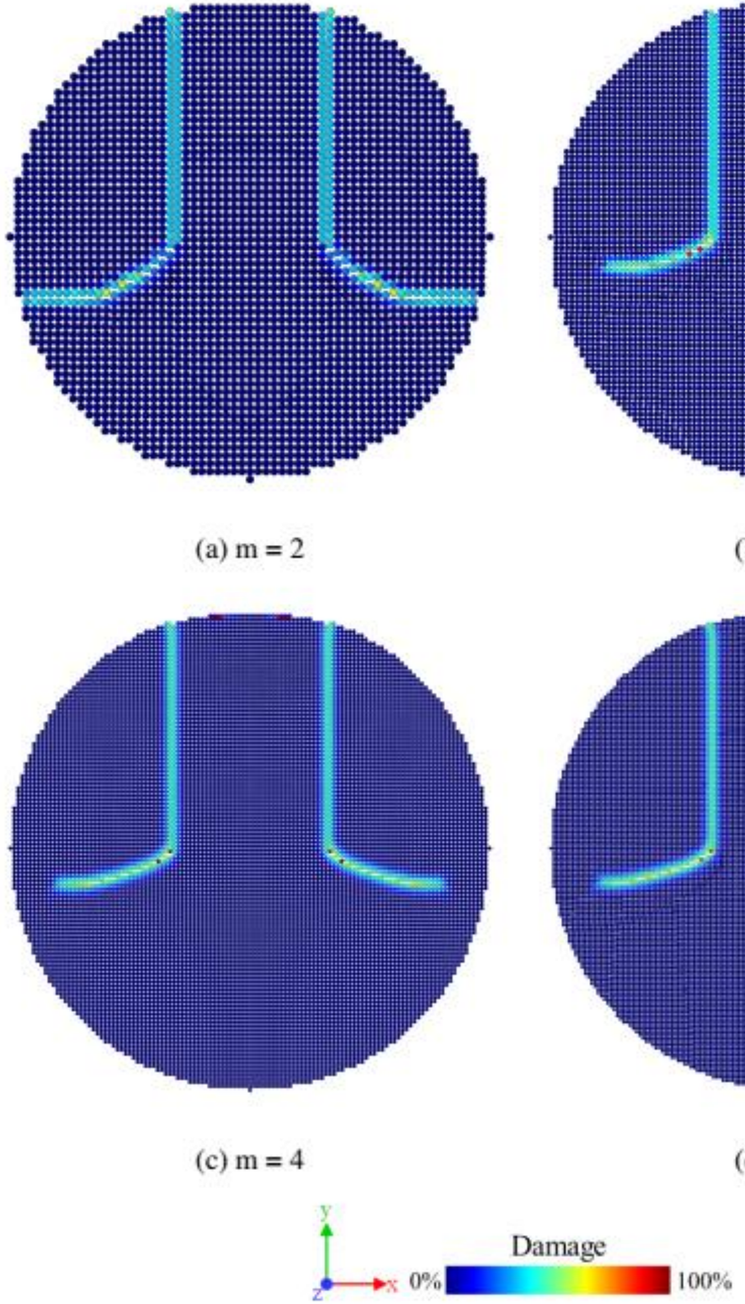


Figure 5.13 : The crack propagation for horizon size $\delta = 0.003$ m at 47.9 μs (550th time step).

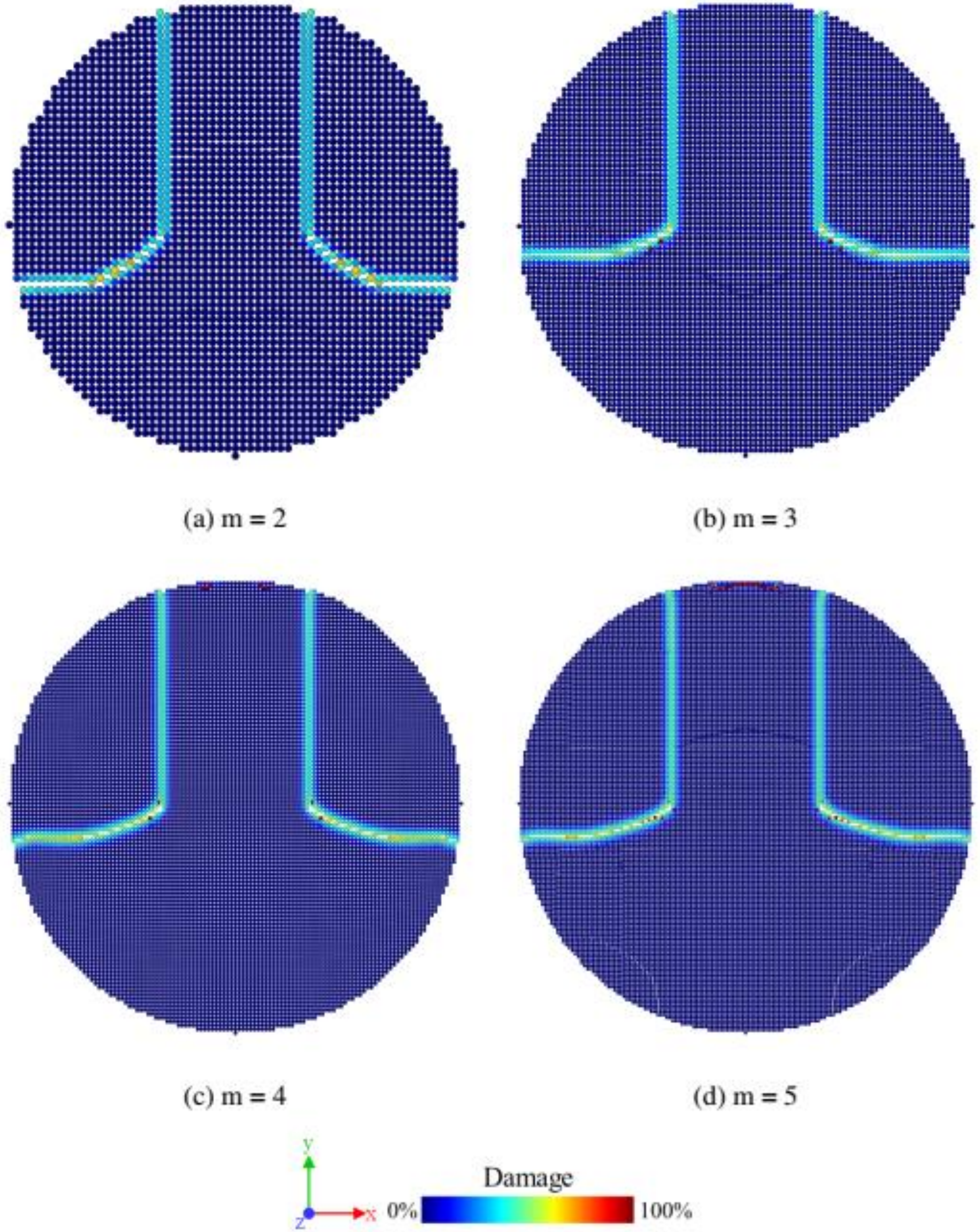


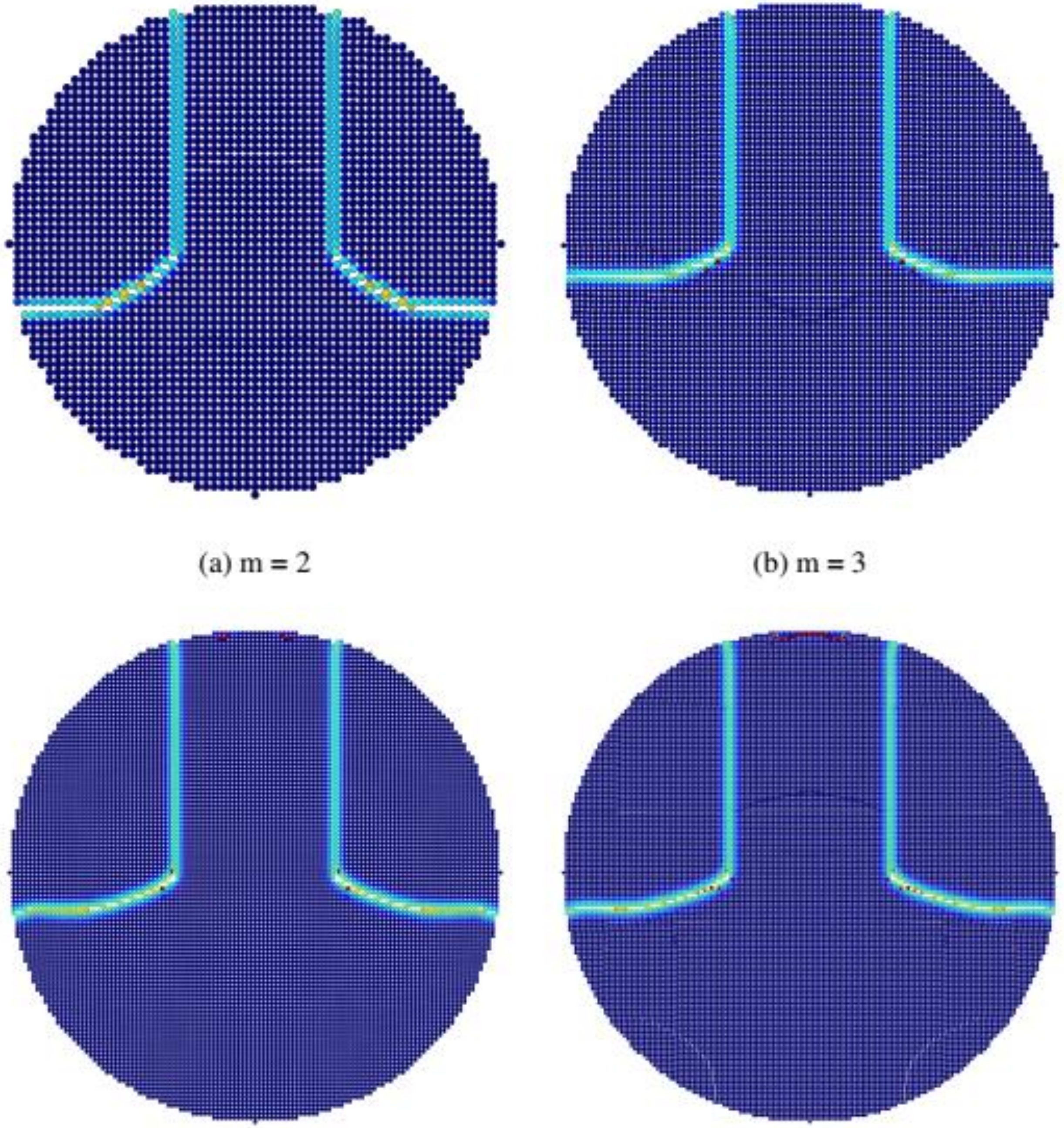
Figure 5.14 : The crack propagation for horizon size $\delta = 0.003$ m at 60.9 μs (700th time step).

m-convergence

The contact between the impactor and the top layer of the section decreases while the m number increases.

The model m = 2 has the largest contact line; this effect may have provided protection to particles at the top layer.

With decreasing the contact line, damages in the top layer become more visible.



(a) m = 2 (b) m = 3 (c) m = 4 (d) m = 5

m-convergence

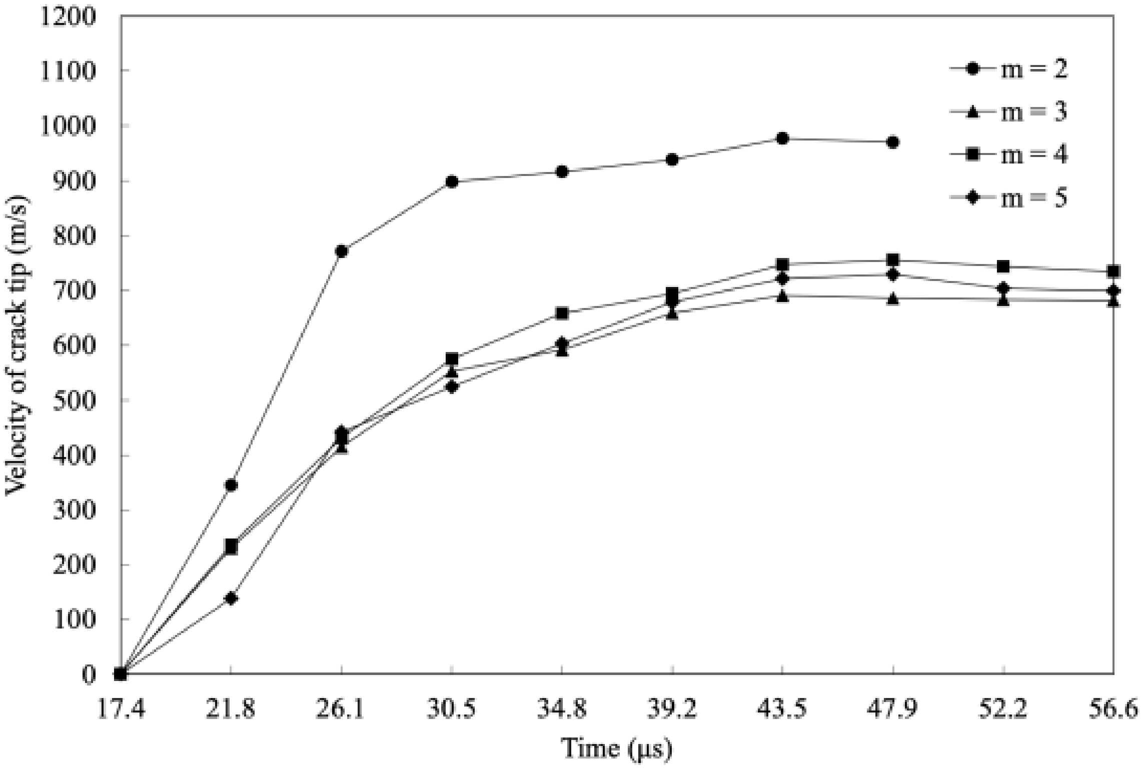


Figure 5.15 : The crack propagation velocities of m-convergence test cases between 17.4 and 56.6 μs.

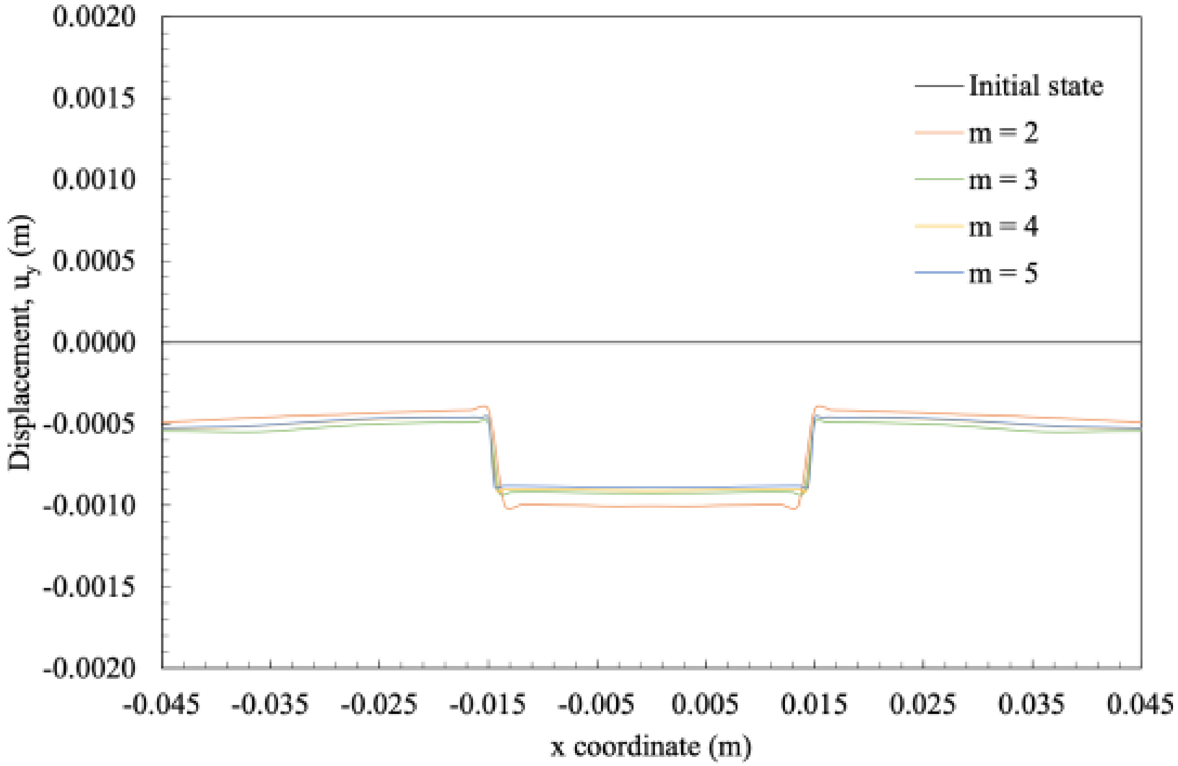
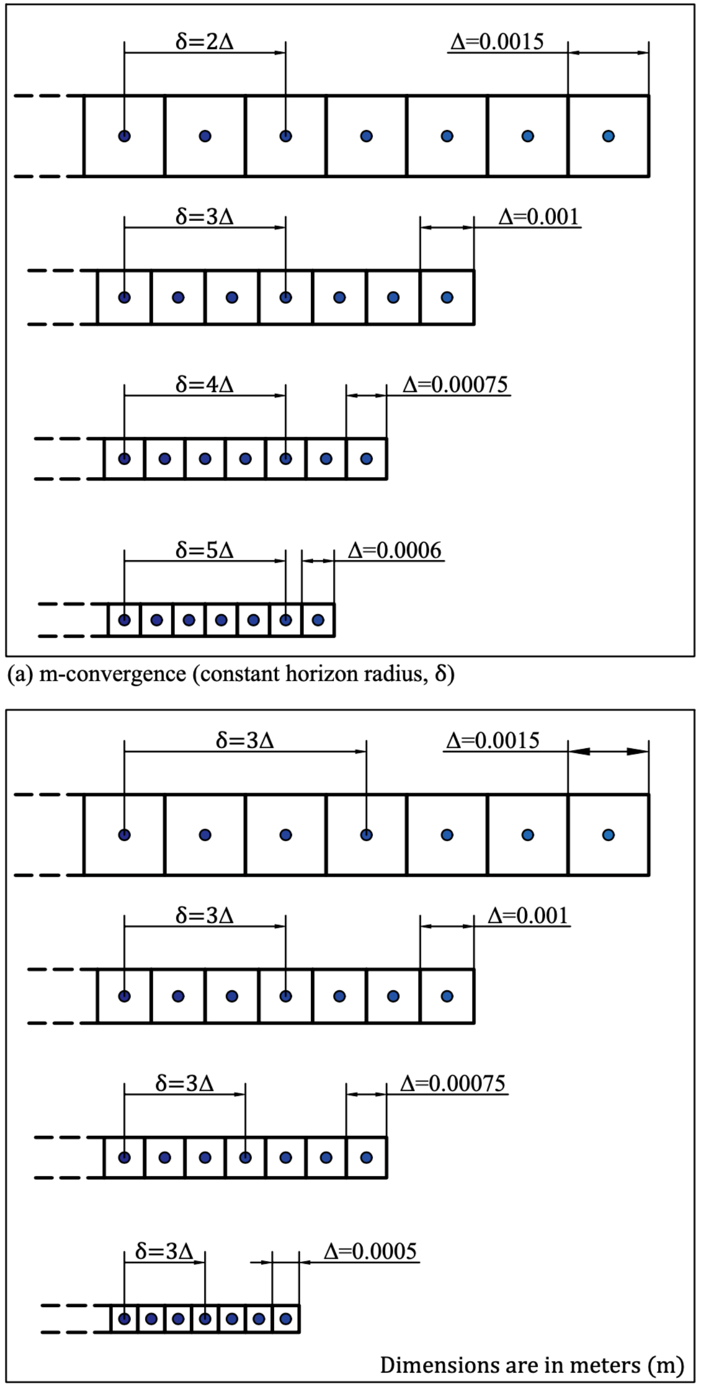


Figure 5.16 : Displacement in the y-direction of m-convergence tests along the central *x* axis at 56.6 μs .

δ-convergence tests

Table 5.4 : δ-Convergence test setup and parameters.

Horizon, δ (m)	0.0045	0.003	0.00225	0.0015
m - constant	3	3	3	3
ndivx	61	91	121	181
ndivy	61	91	121	181
ndivz	3	4	5	7
Diameter of the wire (m)	0.09	0.09	0.09	0.09
Thick in z direction (m)	0.003	0.003	0.003	0.003
Particle radius (m)	0.000750	0.000500	0.000375	0.000250
Δx, Δy, Δz (m)	0.001500	0.001000	0.000750	0.000500
Volume of a material point (m^3)	3.38E-09	1.00E-09	4.22E-10	1.25E-10
Total node number	8,460	25,440	56,440	178,108
Points in contact layer	15	19	21	27
Contact length (m)	0.0210	0.0180	0.0150	0.0130



(a) m-convergence (constant horizon radius, δ)

(b) δ-convergence: constant material points number in a horizon

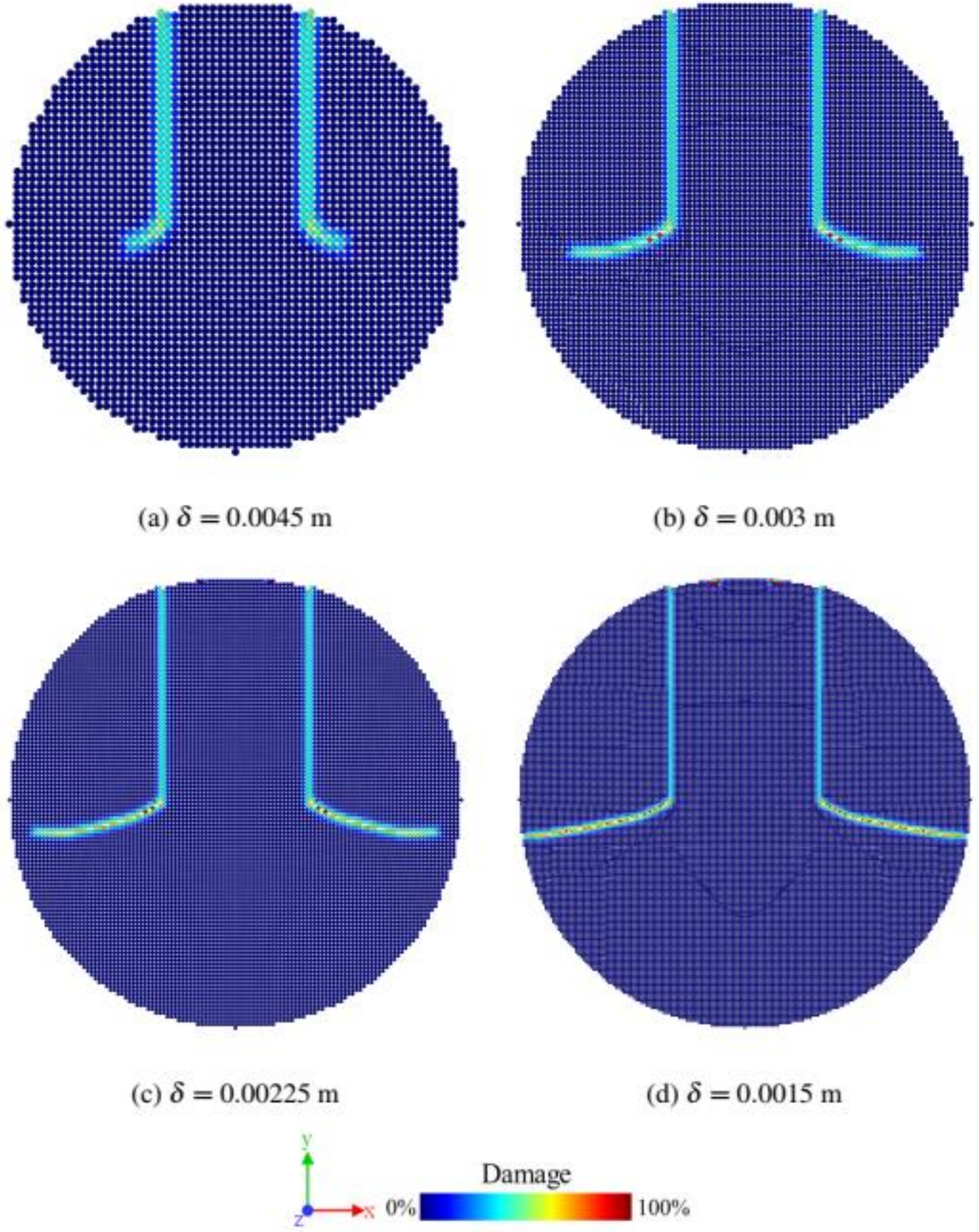


Figure 5.18 : The crack propagation for m = 3 at 47.9 μs (550th time step).

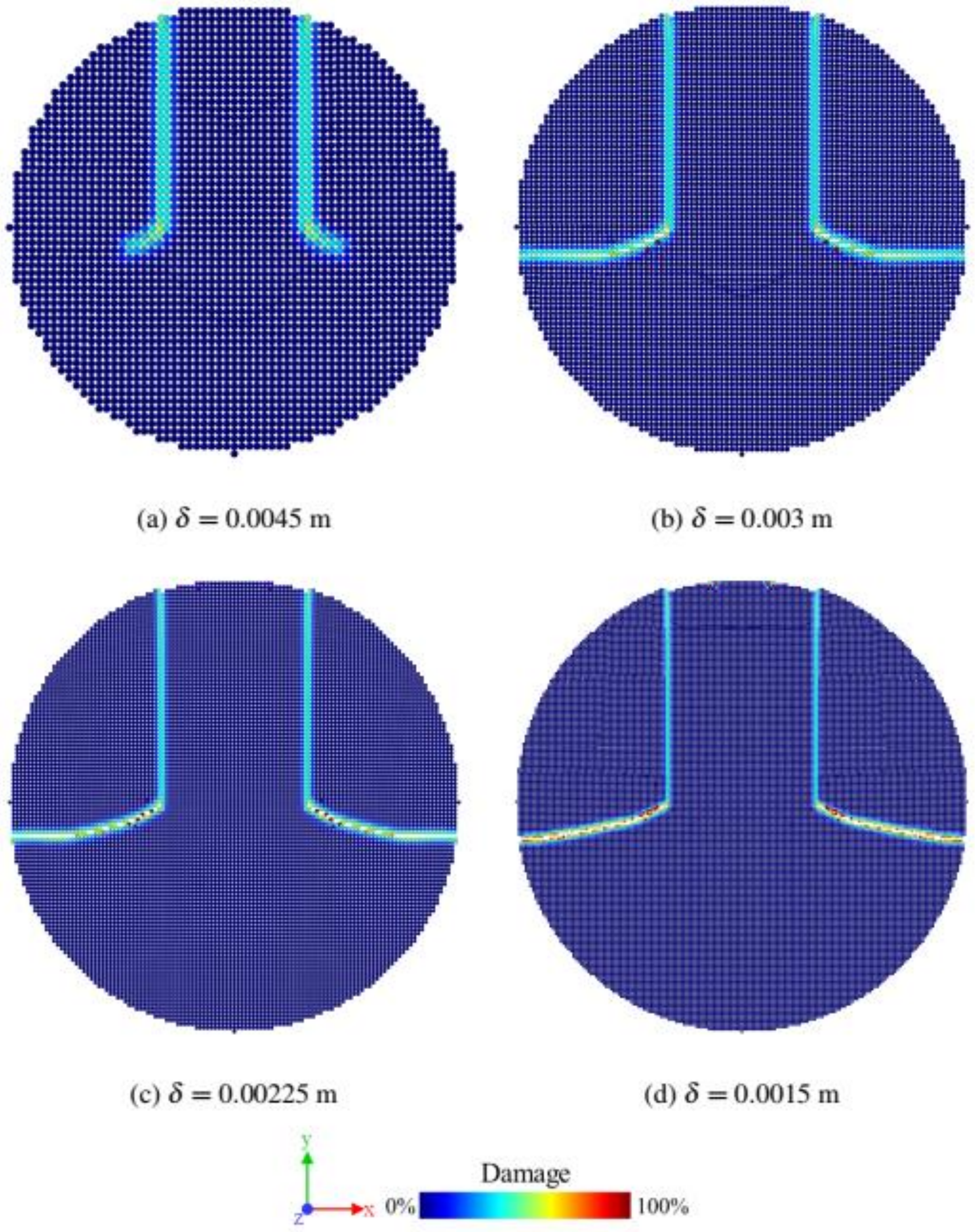


Figure 5.19 : The crack propagation for m = 3 at 60.9 μs (700th time step).

δ-convergence

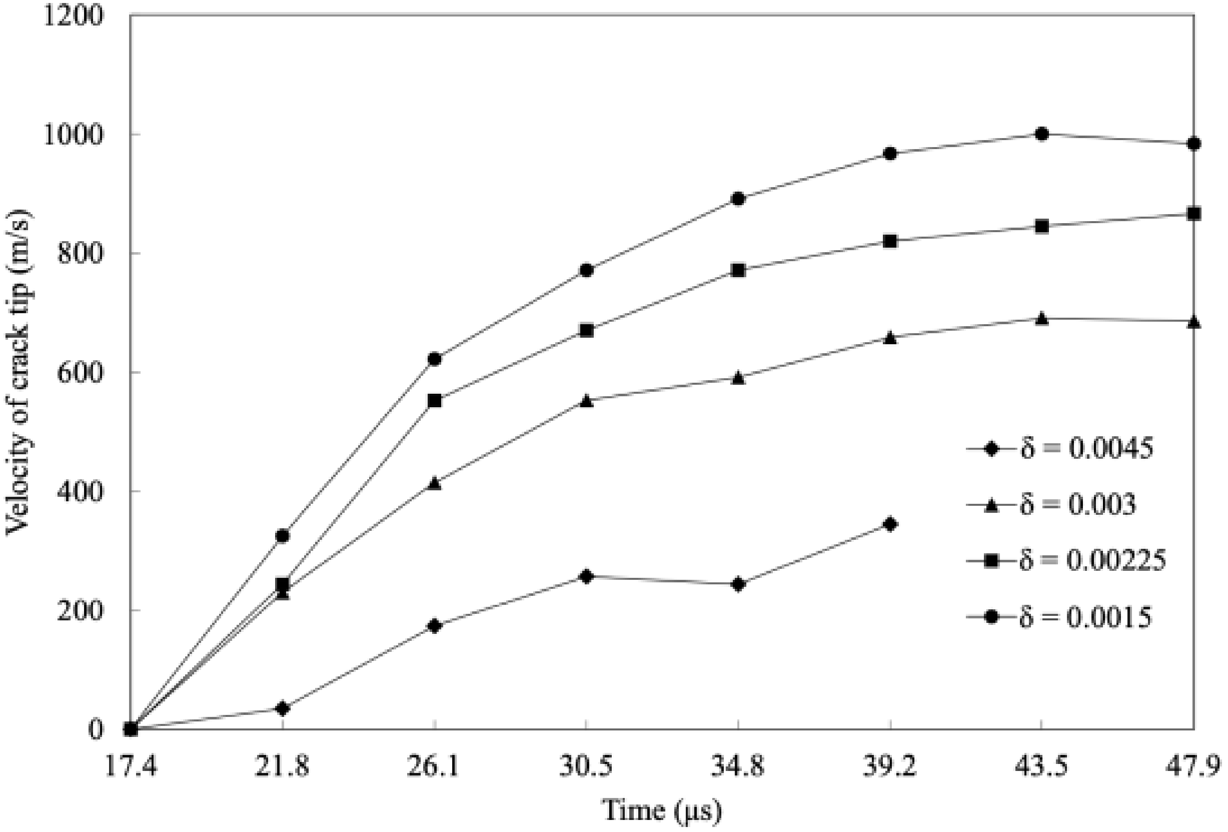


Figure 5.20 : The crack propagation velocities of δ-convergence test cases between 17.4 and 47.9 μs.

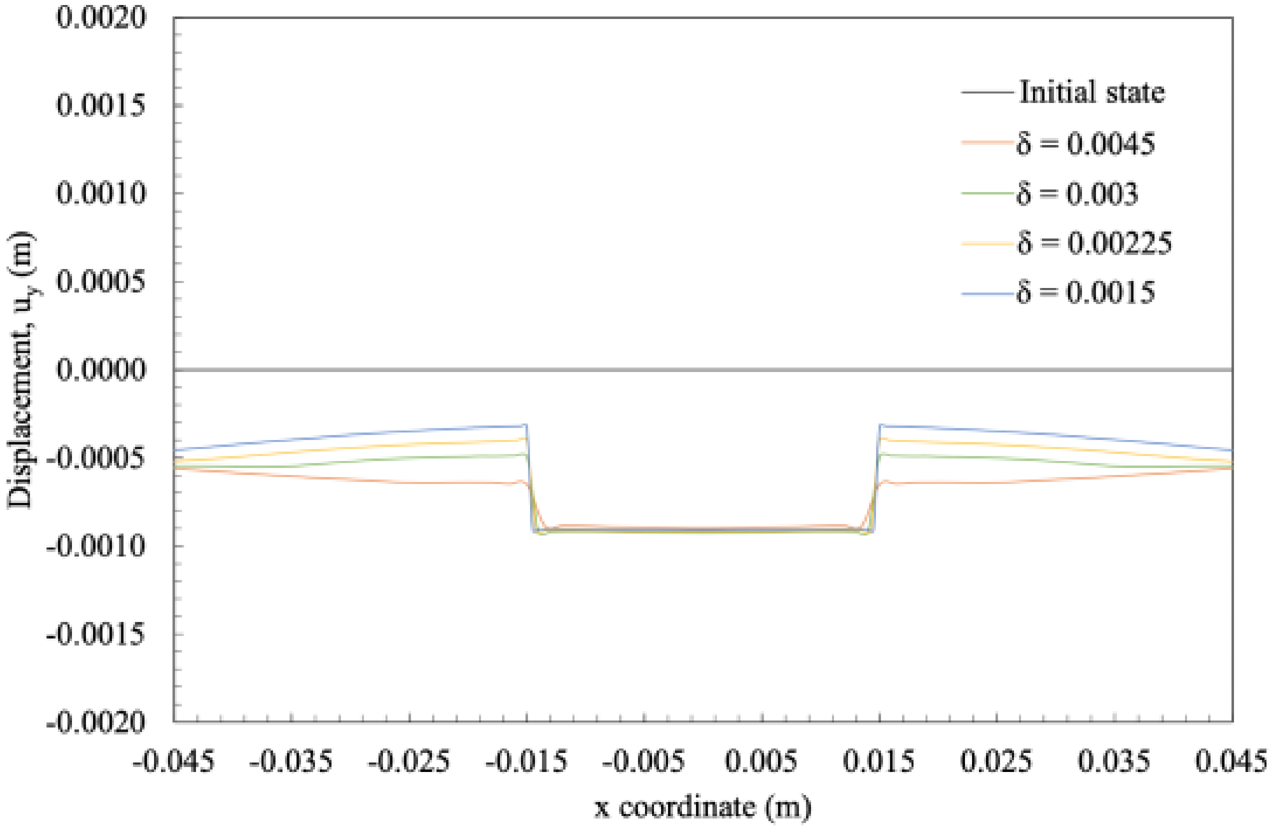
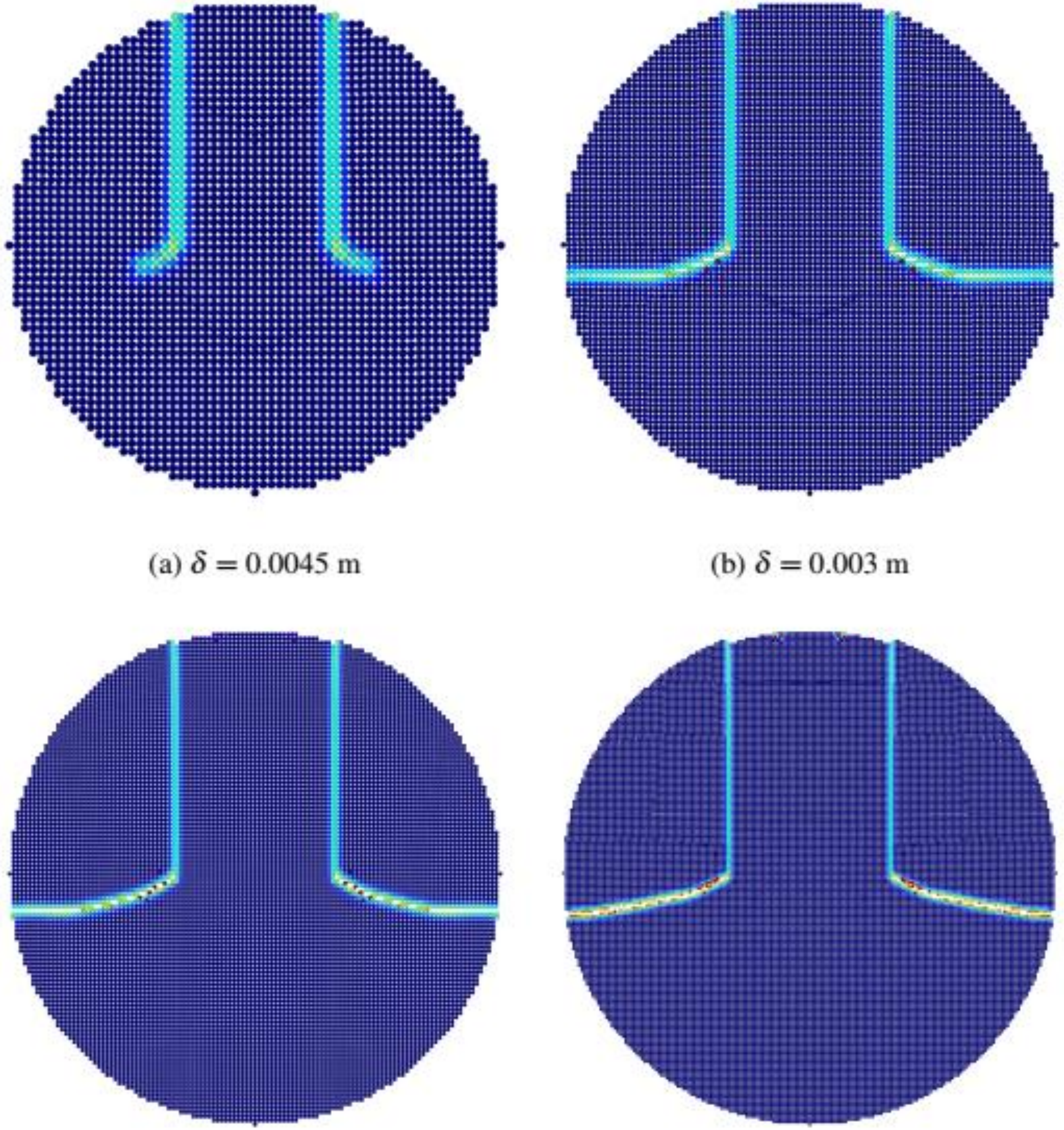
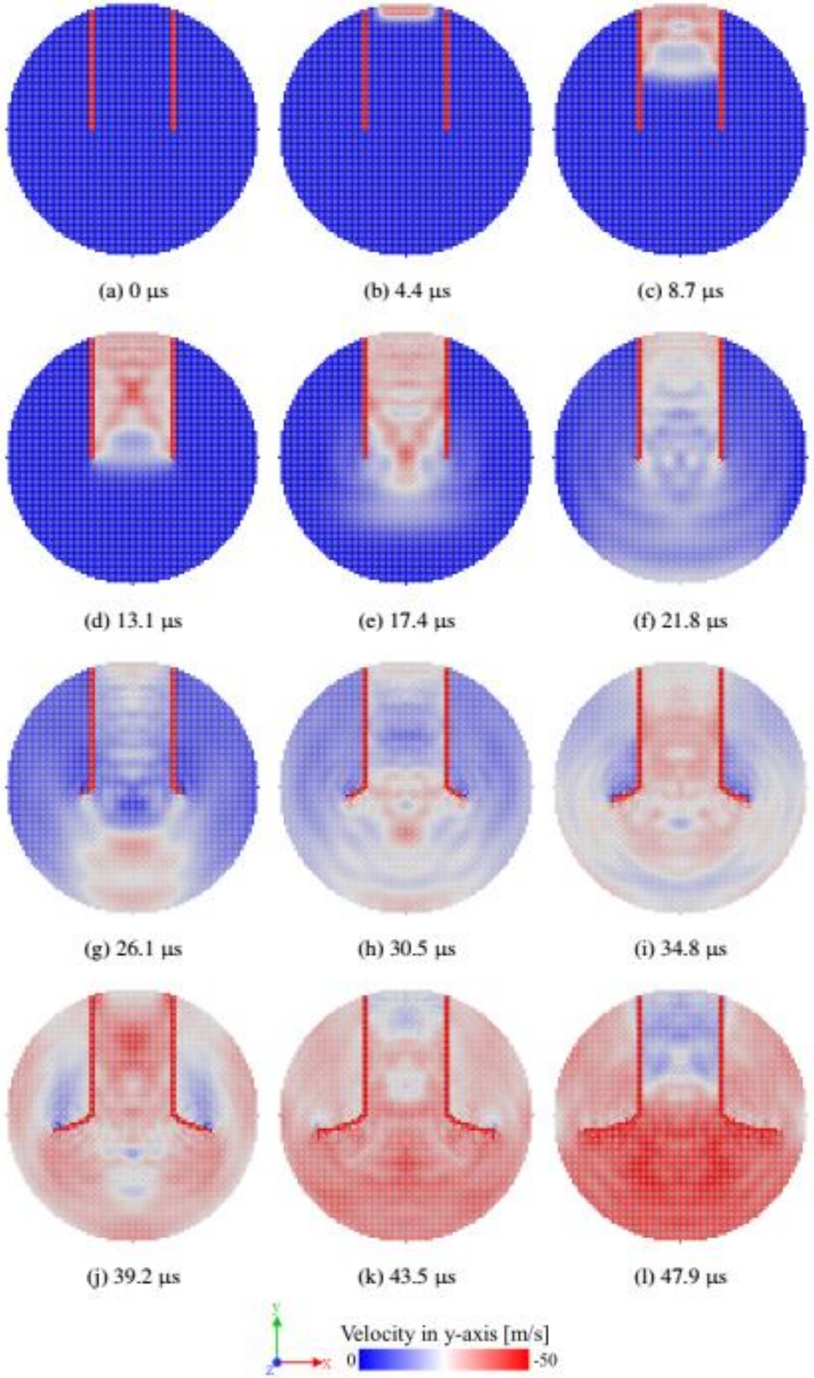


Figure 5.21 : Displacement in the y-direction of δ-convergence tests along the central x axis at 47.9 μs.



(a) $\delta = 0.0045$ m (b) $\delta = 0.003$ m

(c) $\delta = 0.00225$ m (d) $\delta = 0.0015$ m



(a) 0 µs (b) 4.4 µs (c) 8.7 µs

(d) 13.1 µs (e) 17.4 µs (f) 21.8 µs

(g) 26.1 µs (h) 30.5 µs (i) 34.8 µs

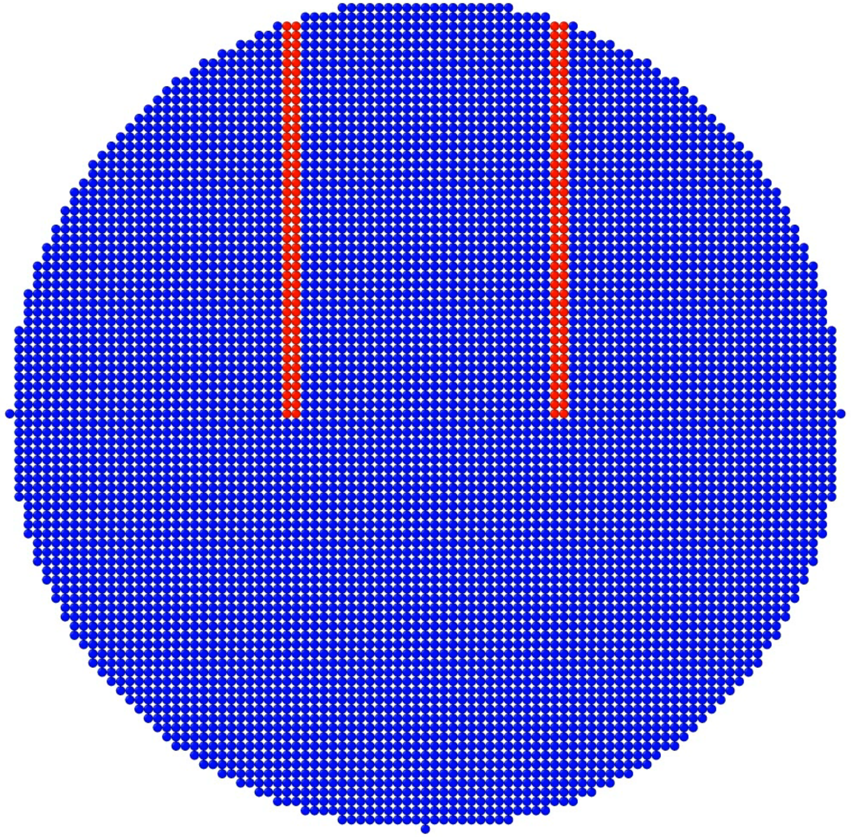
(j) 39.2 µs (k) 43.5 µs (l) 47.9 µs

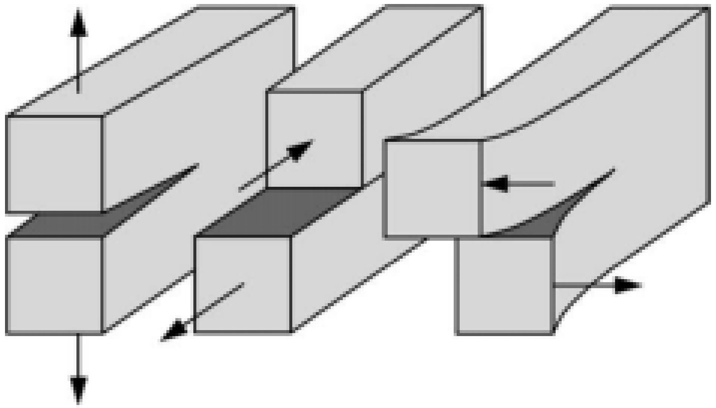
Figure 5.22 : The wave propagation on the case with m = 3 and $\delta = 0.003$ m.

Wave Progression

m = 3
δ = 0.003 m

Velocity in *y* direction (m/s)





Three crack modes with regard to the loading conditions in fracture mechanics: Mode I (Opening), Mode II (In-Plane Shear), Mode III (Out-of-Plane Shear).

Comparison δ and m-convergences

Table 5.5 : Average velocity data.

Model	Velocity (m/s)	Normalized to ref. (%)
m-tests		
m = 2	831	142
m = 3 (ref.)	587	100
m = 4	627	107
m = 5	592	101
δ-tests		
$\delta = 0.00450$	211*	36*
$\delta = 0.00300$ (ref.)	587	100
$\delta = 0.00225$	704	120
$\delta = 0.00150$	794	135

**Outlier because of non-propagating crack*

Conclusions

KALTHOFF WINKLER MODELLING

- PD can be applied on the simulation of micro-cracks' effect on the material toughness for an impact loading problem.

- Less density of located micro-cracks around the crack tip has no effect on toughening mechanism.

- Adding more micro-cracks in the same area can reduce the crack tip velocity and increase the toughness.

Conclusions

KALTHOFF WINKLER MODELLING

- An effective number of micro-cracks can cause the toughening.
- Insufficient number of micro-cracks are inadequate to slow down crack tip's propagation velocities.
- To obtain the toughening effect, a certain number of pre-defined micro-cracks should be located.

Conclusions

WIRE ROPE MODELLING

- Crack propagations in a wire section under transverse loading were examined.
- A basis for analysing wire ropes subjected to impact load with PD theory was proposed.
- One of the first investigations into how model the crack propagation and related failure mechanism of wire ropes using PD was carried out.

Conclusions

- Findings of the m-convergence test suit inferred that the minimum value of m (as an indicator of material points within a horizon) should be **3** for the given model with these parameters and dimensions.
- Average velocities in **m = 3, 4, and 5** models do not differ significantly.
- δ value has more effect on crack velocities than m.
- Horizon value, **δ = 0.00450** m is not applicable for the model with given parameters.
- The model **δ = 0.0015** m can be considered as a better parameter choice for the given model.
- The Mode I crack opening transition in the reference model indicates a routing of the crack in the horizontal direction.

Future Studies

- The complete analysis of a strand provides a more accurate analysis of fracture and failure in wire ropes.
- More test suit can be designed to obtain more accurate wire rope geometry.
- Complete structure of wire rope can be modelled.
- Wire sections can be modelled as elliptical in angular cross-section.
- PD model can be compared with experimental studies.

Publications

Candaş, A., Oterkus, E., İmrak, C.E., 2021. Dynamic Crack Propagation and Its Interaction With Micro-Cracks in an Impact Problem. Journal of Engineering Materials and Technology, Transactions of ASME, 143(1).

Adem Candaş[1]
Faculty of Mechanical Engineering,
Istanbul Technical University,
Beyoğlu, Istanbul 34437, Turkey
e-mail: candas@itu.edu.tr

Erkan Oterkus
Department of Naval Architecture, Ocean
and Marine Engineering,
University of Strathclyde,
Glasgow G4 0LZ, UK
e-mail: erkan.oterkus@strath.ac.uk

Cevat Erdem İmrak
Faculty of Mechanical Engineering,
Istanbul Technical University,
Beyoğlu, Istanbul 34437, Turkey
e-mail: imrak@itu.edu.tr

Dynamic Crack Propagation and Its Interaction With Micro-Cracks in an Impact Problem

The dynamic fracture behavior of brittle materials that contain micro-level cracks should be examined when material subjected to impact loading. We investigated the effect of micro-cracks on the propagation of macro-cracks that initiate from notch tips in the Kalthoff–Winkler experiment, a classical impact problem. To define predefined micro-cracks in three-dimensional space, we proposed a two-dimensional micro-crack plane definition in the bond-based peridynamics (PD) that is a non-local form of classical continuum theory. Randomly distributed micro-cracks with different number densities in a constant area and number in expending area models were examined to monitor the toughening of the material. The velocities of macro-crack propagation and the time required for completing fractures were considered in several predefined micro-cracks cases. It has been observed that toughening mechanism is only initiated by exceeding a certain number of micro-cracks; therefore, there is a positive correlation between the density of predefined micro-cracks and macro-crack propagation rate and, also, toughening mechanism.
[DOI: 10.1115/1.4047746]

Keywords: dynamic fracture, brittle materials, micro-cracks, peridynamics

CONSTITUTIVE FAILURE MODELLING AND ANALYSIS OF STEEL WIRE ROPE STRUCTURES SUBJECTED TO IMPACT LOADING

PhD Thesis

Advisor: Prof. C. Erdem İmrak

ADEM CANDAŞ

FEBRUARY 23, 2021