3 PHASE RECTIFIERS

UNCONTROLLED

RESISTIVE
INDUCTIVE

CONTROLLED

RESISTIVE

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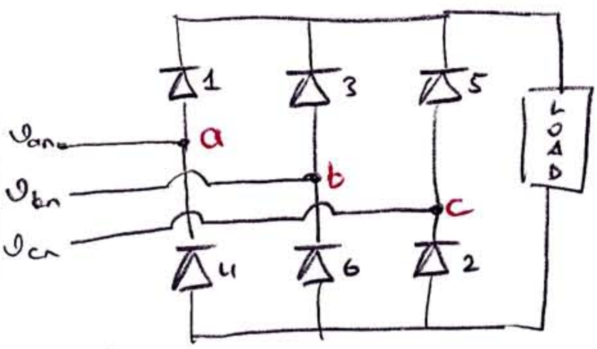
— 3 PHASE RECTIFIERS —

UNCONTROLLED



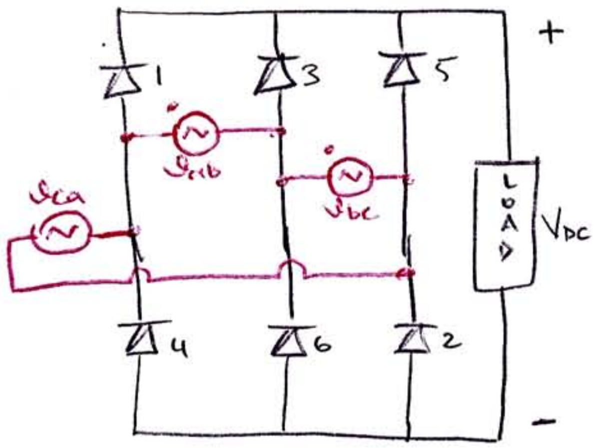
$v_{an} = V_{max} \cdot \sin(\omega t)$

$v_{bn} = V_{max} \cdot \sin(\omega t - 120^\circ)$

$v_{cn} = V_{max} \cdot \sin(\omega t - 240^\circ)$

Actually, between a-b, b-c and c-a points, phase-to-phase voltages can be seen.

Rectifier is modified...



$v_{ab} = v_{an} - v_{bn} = V_{max}\sqrt{3}\,\sin(\omega t + 30^\circ)$

$v_{bc} = v_{bn} - v_{cn} = V_{max}\sqrt{3}\,\sin(\omega t + 150^\circ)$

$v_{ca} = v_{cn} - v_{an} = V_{max}\sqrt{3}$

All phase-to-phase voltages are rectified and the greatest of three rectified voltages appear on the load.

From the figure; phase voltages, phase-to-phase voltages, rectified phase-to-phase voltages and load voltage can be seen respectively.

For rectified voltages;

- v_{ab} is greatest for $30^\circ \sim 90^\circ$ and $210^\circ \sim 270^\circ$
- v_{bc} is greatest for $150^\circ \sim 210^\circ$ and $330^\circ \sim 390^\circ\,(-30^\circ)$
- v_{ca} is greatest for $90^\circ \sim 150^\circ$ and $270^\circ \sim 330^\circ$

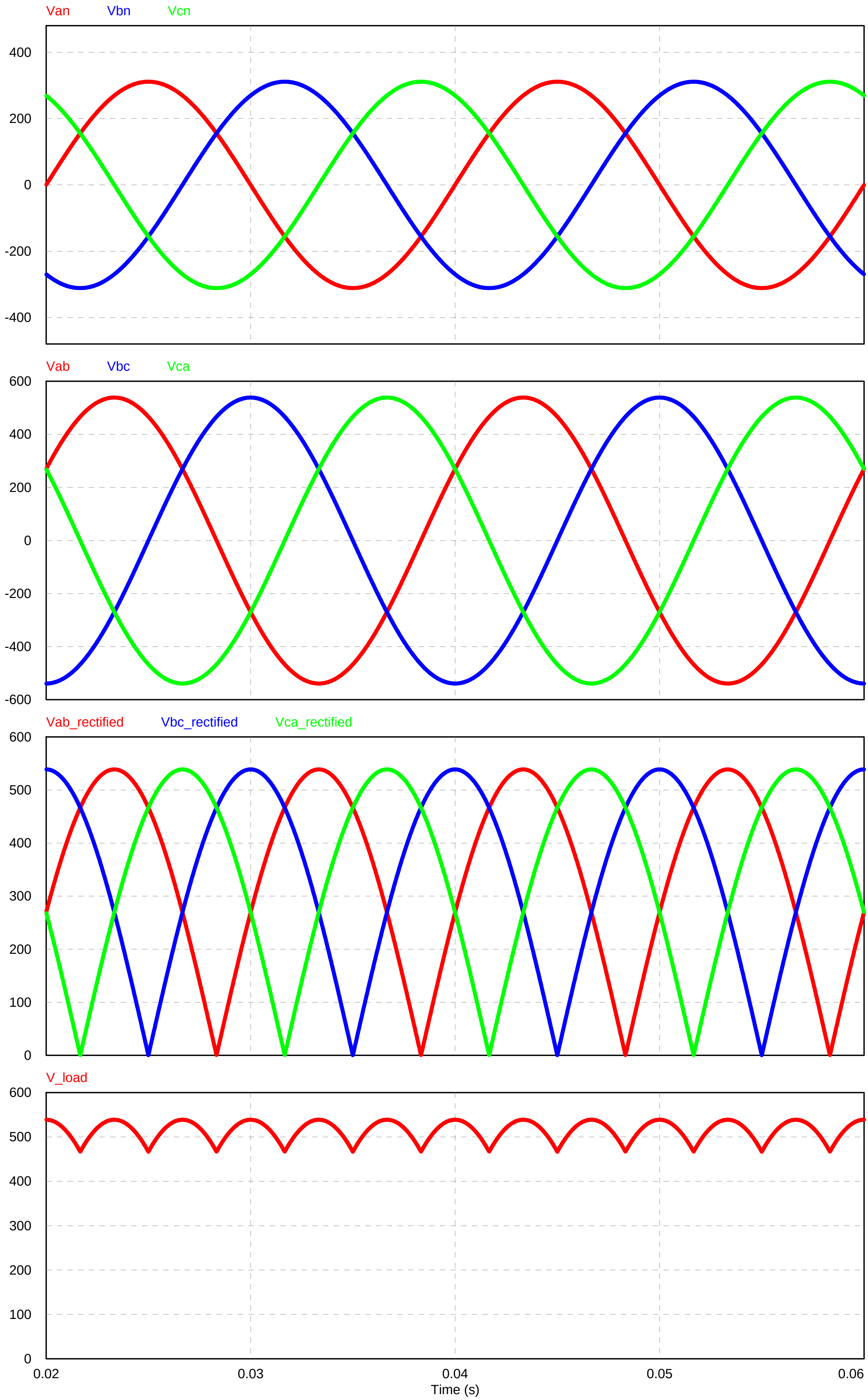
Diode conduction times are determined by the greatest and lowest phase voltages for higher and lower side diodes respectively.

For higher side diodes;

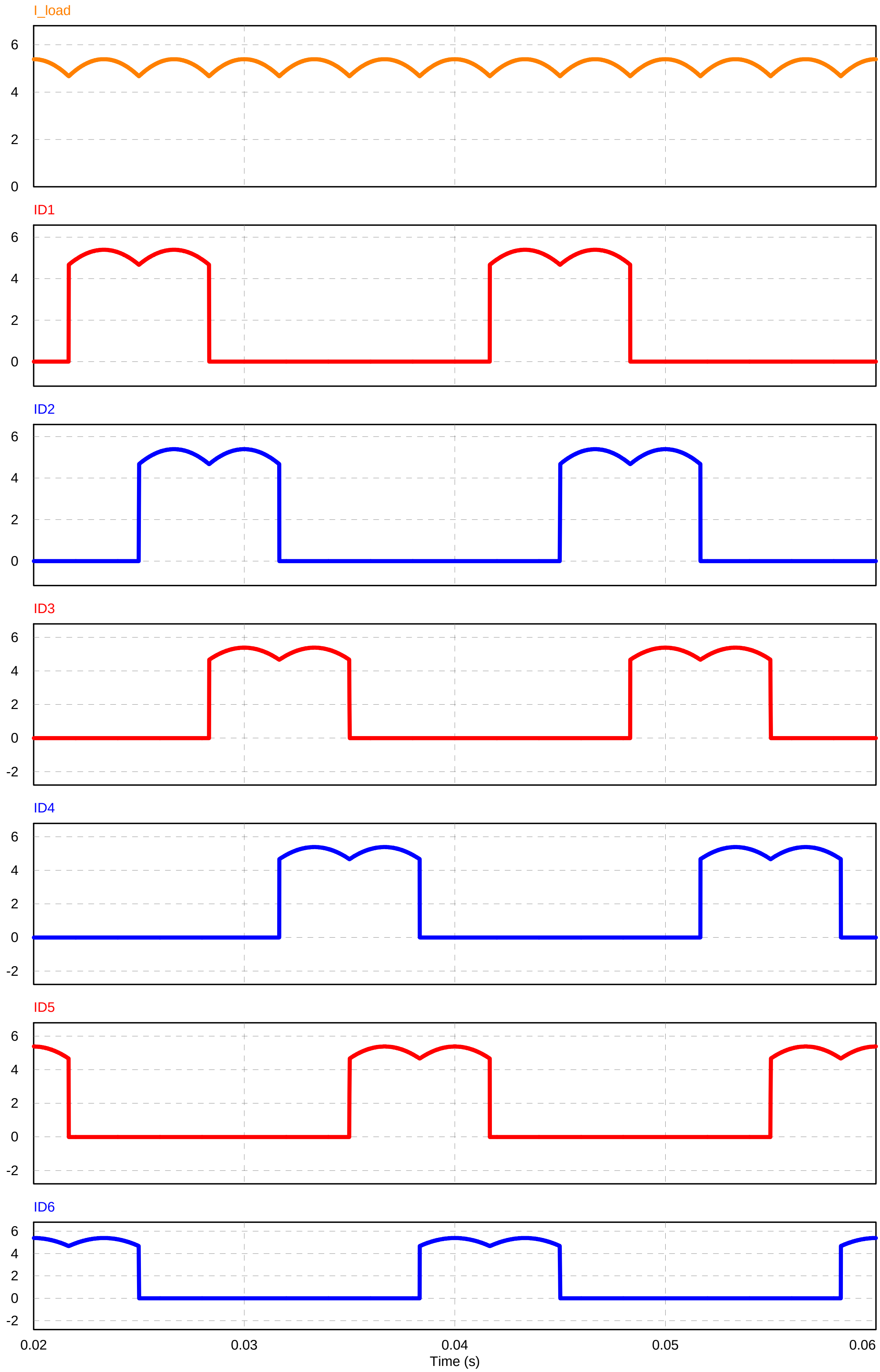
- D1 conducts for $30^\circ \sim 150^\circ$
- D3 conducts for $150^\circ \sim 270^\circ$
- D5 conducts for $270^\circ \sim 390^\circ\,(-30^\circ)$

For lower side diodes;

- D4 conducts for $210^\circ \sim 330^\circ$
- D6 conducts for $-30^\circ \sim 90^\circ$
- D2 conducts for $90^\circ \sim 210^\circ$



Van
Vbn
Vcn
400
200
0
-200
-400
Vab
Vbc
Vca
600
400
200
0
-200
-400
-600
Vab_rectified
Vbc_rectified
Vca_rectified
600
500
400
300
200
100
0
V_load
600
500
400
300
200
100
0
0.02
0.03
0.04
0.05
0.06
Time (s)


I_load
ID1
ID2
ID3
ID4
ID5
ID6
6
4
2
0
-2
0.02
0.03
0.04
0.05
0.06
Time (s)

Load Voltage Average Value

Since the load voltage is repetative for 60° time intervals, the average (DC) load voltage can be calculated as below.

$$\frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{max}\sqrt{3}\cdot\sin(\omega t)\cdot d\omega t = \frac{V_{max}\,3\sqrt{3}}{\pi}\Big[(-\cos(\omega t))\Big]_{\pi/3}^{2\pi/3}$$

$$= \frac{V_{max}\,3\sqrt{3}}{\pi}\Big(-(-0{,}5)-(-0{,}5)\Big) = \frac{V_{max}\,3\sqrt{3}}{\pi}$$

$$= 1{,}654\,V_{max}.$$

Load Voltage RMS Value

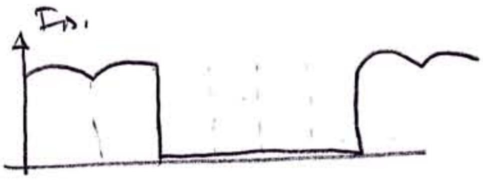
$$\sqrt{\frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3}(V_{max}\sqrt{3})^2\cdot\sin^2(\omega t)\cdot d\omega t} = \sqrt{\frac{9V_{max}^2}{\pi}\cdot\Big[\frac{1}{2}\Big(\omega t-\frac{\sin 2\omega t}{2}\Big)\Big]_{\pi/3}^{2\pi/3}}$$

$$= \sqrt{\frac{9V_{max}^2}{2\pi}\Big(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\Big)} = 1{,}65544\,V_{max}$$

For Resistive Load Condition

Average Load Current $\quad I_{DC} = \dfrac{V_{DC}}{R}$

Average Diode Current $\quad I_D = \dfrac{I_{DC}}{3} = \dfrac{V_{DC}}{3R}$



RMS Diode Current

$$I_{D,rms} = \sqrt{\frac{2}{2\pi}\int_{\pi/3}^{2\pi/3}\Big(\frac{V_{max}\sqrt{3}}{R}\Big)^2\sin^2(\omega t)\cdot d\omega t} = \sqrt{\frac{3V_{max}^2}{\pi R^2}\Big[\frac{1}{2}\Big(\omega t-\frac{\sin 2\omega t}{2}\Big)\Big]_{\pi/3}^{2\pi/3}}$$

$$= \sqrt{\frac{3V_{max}^2}{2\pi R^2}\Big(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\Big)} = 0{,}9558\cdot\frac{V_{max}}{R}$$

For Inductive Load Condition

Load average and RMS voltages are same with resistive load condition.
But current waveforms are different.
Since the output voltage is nearly DC, the current is always continous.

$V_{DC} = 1{,}654\ V_{max}$

$V_{rms} = 1{,}65544\ V_{max}$

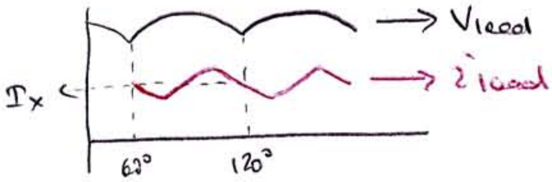
$I_{DC} = 1{,}654\ V_{max}/R$

General solution of a AC excited RL load current is given;

$$i_{load} = \frac{V_x}{Z}\sin(\omega t - \phi) + A \cdot e^{-(R/L)t}$$

- $V_x = V_{max}\sqrt{3}$ for our notation
- $Z = \sqrt{R^2 + \omega^2 L^2}$
- $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ rad

"A" is a constant which can be calculated from load voltage waveform.
For a RL loaded 3ϕ uncontrolled rectifier;



When $\omega t = 60° = \pi/3$, it is assumed that $i_{load} = I_x$, "A" is;

$$A = \left[I_x - \frac{V_x}{Z}\cdot\sin\left(\frac{\pi}{3} - \phi\right)\right] e^{(R/L)\pi/3\omega}$$

i_{load} is written again;

$$i_{load} = \frac{V_x}{Z}\sin(\omega t - \phi) + \left[I_x - \frac{V_x}{Z}\cdot\sin\left(\frac{\pi}{3} - \phi\right)\right] \overbrace{e^{(R/L)\frac{\pi}{3\omega}} \cdot e^{-(R/L)t}}^{e^{(R/L)\left(\frac{\pi}{3\omega} - t\right)}}$$

under steady-state; $i_{load}\left(\omega t = \frac{\pi}{3}\right) = i_{load}\left(\omega t = \frac{2\pi}{3}\right) = I_x$

$$I_x = \frac{V_x}{Z} \cdot \frac{\sin\left(\frac{2\pi}{3} - \phi\right) - \sin\left(\frac{\pi}{3} - \phi\right) e^{-(R/L)(\pi/3\omega)}}{1 - e^{-(R/L)(\pi/3\omega)}}$$

I_x is inserted into i_{load} and i_{load} is rewritten.

$$i_{load} = \frac{V_x}{Z}\left[\sin(\omega t - \phi) + \frac{\sin\left(\frac{2\pi}{3} - \phi\right) - \sin\left(\frac{\pi}{3} - \phi\right)}{1 - e^{-(R/L)(\pi/3\omega)}} \cdot e^{(R/L)\left(\frac{\pi}{3\omega} - t\right)}\right] \text{ for } \frac{\pi}{3} < \omega t < \frac{2\pi}{3}$$

$60° < \omega t < 120°$

Example: A 3ϕ uncontrolled bridge rectifier is supplying the 20Ω resistive load from line voltage of 380V / 50Hz.

- Calculate mean load voltage and current. (V_{DC} and I_{DC})

$V_{max} = V_{rms} \cdot \sqrt{2} = 380 \cdot \sqrt{2} = 537{,}4$ V.

$V_{DC} = 1{,}654 \cdot V_{max} = 1{,}654 \cdot 537{,}4 = 888{,}86$ V

$I_{DC} = V_{DC}/R = 888{,}86/20 = 44{,}44$ A.

- Calculate average and rms diode current (per diode)

$$I_{D,av} = \frac{I_{DC}}{3} = \frac{44{,}44}{3} = 14{,}81\text{A}$$

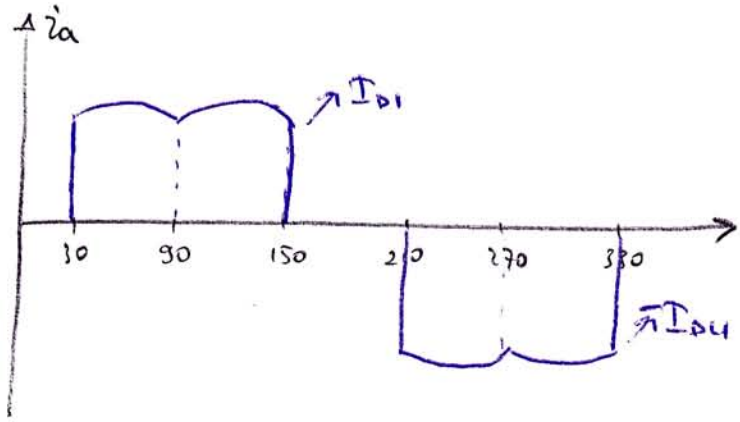
$$I_{D,rms} = 0{,}9558 \cdot \frac{V_{max}}{R} = 0{,}9558 \cdot \frac{537{,}4}{20} = 25{,}68\text{A}$$

- Draw the source current for one phase, calculate rms phase current.

$i_a \rightarrow$ source current from "phase a"

I_{D1} (diode 1 current is drawn from "phase a")

I_{D4} (diode 4 current is supplied to "paheze a")



$I_{a,rms} = \sqrt{2} \cdot I_{D1,rms} = \sqrt{2} \cdot 25{,}68 = 36{,}32$ A.

- Calculate the DC power of the circuit.

$P_o = V_{DC} \cdot I_{DC} = 888{,}86 \cdot 44{,}44 = 39501$ W

- Calculate the power factor of the circuit.

$$PF = \frac{P_{o,rms}}{P_{i,rms}} = \frac{V_{rms}^2/R}{3 \cdot V_{a,rms} \cdot I_{a,rms}} = \frac{(1{,}65544 \cdot 537{,}4)^2/20}{3 \cdot 380 \cdot 36{,}32} = 0{,}955743$$

(power factor is defined as power consumed at output resistor over input rms power)

Example: A 3ϕ uncontrolled bridge rectifier is supplying an inductive load with $R=60$ and $L=25mH$, from 220V/50Hz utility.

- Calculate average load voltage and current.

$$V_{max} = 220\sqrt{2} = 311,13\,V$$

$$V_{DC} = 1,654 \cdot V_{max} = 1,654 \cdot 311,13 = 514,6\,V$$

$$I_{DC} = V_{DC}/R = 8,577\,A$$

- Write the expression for load current.

$$V_x = V_{max}\sqrt{3} = 311,13\sqrt{3} = 538,89\,V$$

$$Z = \sqrt{w^2L^2+R^2} = \sqrt{4\pi^2 50^2 0,025^2 + 60^2} = 60,51\,\Omega$$

$$\phi = \tan^{-1}\left(\frac{wL}{R}\right) = 7,458^\circ = 0,13\text{ rad}$$

$60^\circ < wt < 120^\circ$

$\frac{\pi}{3} < wt < \frac{2\pi}{3}$

$\frac{\pi}{3w} < t < \frac{2\pi}{3w}$

$$i_{load} = \frac{538,89}{60,51}\cdot\left[\sin(wt-7,458^\circ) + \frac{\sin(120^\circ-7,458^\circ)-\sin(60-7,458^\circ)}{1-e^{-\left(\frac{60}{0,025}\right)\left(\frac{\pi}{6\pi 50}\right)}}\cdot e^{\left(\frac{60}{0,025}\right)\left(\frac{\pi}{6\pi 50}-t\right)}\right]$$

$$= 8,906\left[\sin(wt-7,458^\circ) + \frac{0,9236-0,7938}{1-e^{(-8)}}\cdot e^{(8-2400t)}\right]$$

$$= 8,906\left[\sin(wt-7,458^\circ) + 0,12984\cdot e^{(8-2400t)}\right]$$

- calculate i_{load} for $wt = 60^\circ, 70^\circ, 80^\circ, 90^\circ, 100^\circ, 110^\circ, 120^\circ$

$$i_{load}(60^\circ) = 8,906\left[\sin(60^\circ-7,456^\circ) + 0,12984\, e^{\left(8-2400\cdot\frac{\pi}{3w}\right)}\right] = 8,226\text{ A}$$

$i_{load}(70^\circ) = 8,2076A$ $\quad i_{load}(80^\circ) = 8,5761\,A$ $\quad i_{load}(90^\circ) = 8,8518\,A$

$i_{load}(100^\circ) = 8,9028\,A$ $\quad i_{load}(110^\circ) = 8,6949\,A$ $\quad i_{load}(120^\circ) = 8,226\,A.$

- calculate rms load current with discrete current values.

$$\sqrt{\frac{i^2_{load}(60^\circ)+i^2_{load}(70)+i^2_{load}(80)+i^2_{load}(90^\circ)+i^2_{load}(100)+i^2_{load}(110)}{6}} = i_{RMS}$$

$$i_{RMS} = \sqrt{\frac{441,7968}{6}} = 8,581\text{ A}$$

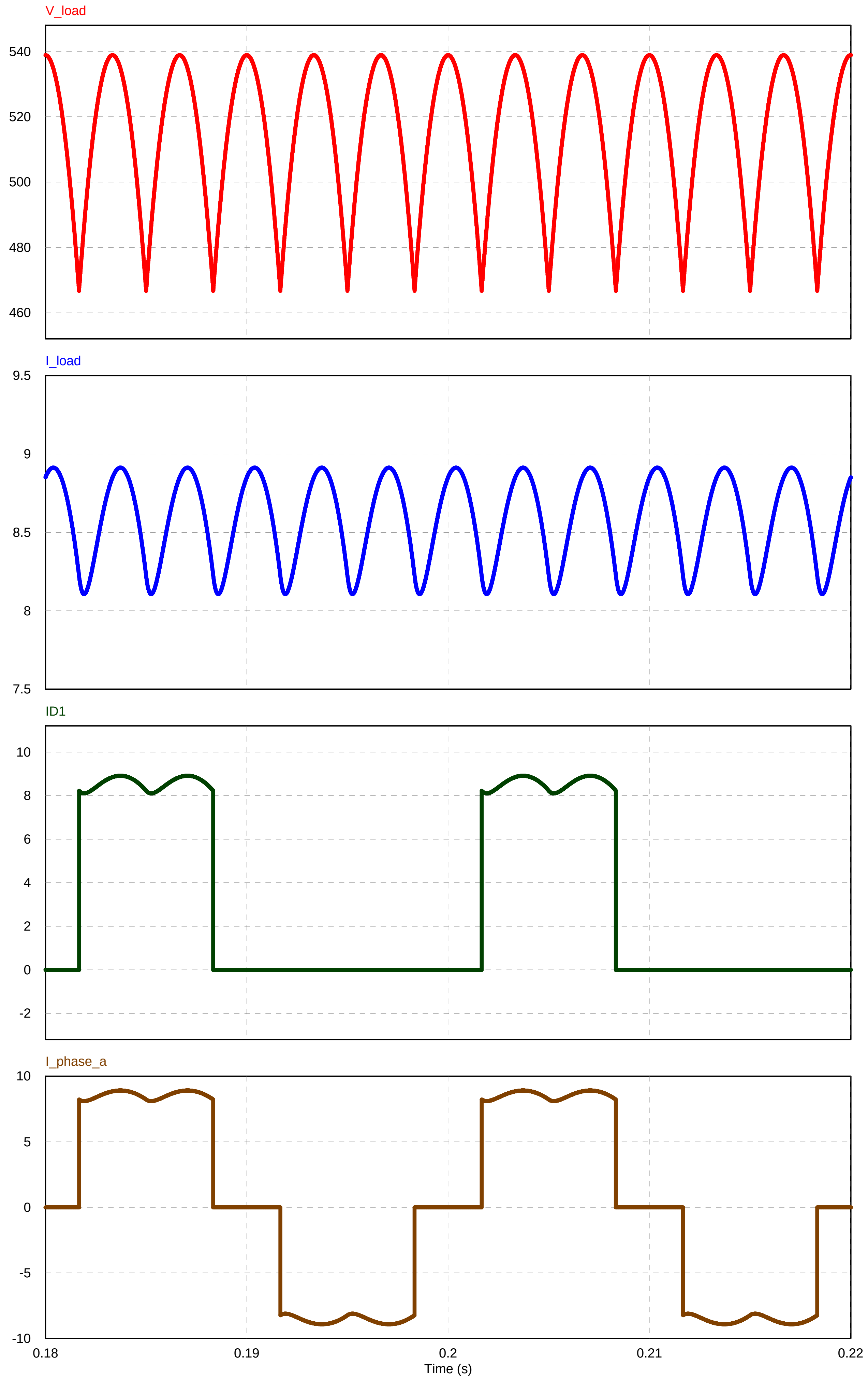
$$i_{D,RMS} = i_{RMS}\cdot\sqrt{1/3}$$

$$i_{a,RMS} = i_{D,RMS}\sqrt{2}$$

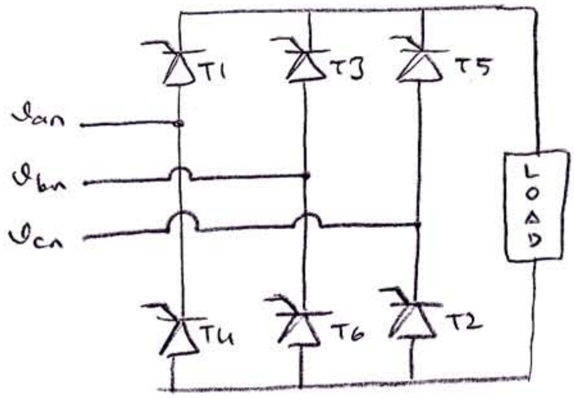
$$i_{a,RMS} = i_{RMS}\sqrt{2/3}$$

- Calculate power factor of the rectifier.

$$PF = \frac{i^2_{RMS}\cdot R}{3\cdot 220\cdot i_{a,RMS}} = \frac{8,581^2\cdot 60}{3\cdot 220\cdot 7,006} = 0,9554$$


V_load
540
520
500
480
460
I_load
9.5
9
8.5
8
7.5
ID1
10
8
6
4
2
0
-2
I_phase_a
10
5
0
-5
-10
0.18
0.19
0.2
0.21
0.22
Time (s)

- CONTROLLED RECTIFIERS -



α angle is the control parameter.

As it is seen from uncontrolled rectifiers, if there are thyristors replaced with diodes, conduction angles become;

$T_1 \rightarrow 30° \sim 150°$

$T_2 \rightarrow 90° \sim 210°$

$T_3 \rightarrow 150 \sim 270°$

$T_4 \rightarrow 210° \sim 330°$

$T_5 \rightarrow 270° \sim 390°$

$T_6 \rightarrow -30° \sim 90°$

α angle is defined for T_1, T_1 starts to conduct at $30°$.
if $\alpha = 0°$, gate signal must be applied to thyristor 1 (α_1) at $\alpha + 30°$.

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other thyristors requires gate signal $60°$ later from α_1.

$\alpha_2 = T_2 \rightarrow \alpha + 30° + 60° = \alpha + 90° = \alpha_2$

$\alpha_3 = T_3 \rightarrow \alpha + 30° + 120° = \alpha + 150° = \alpha_3$

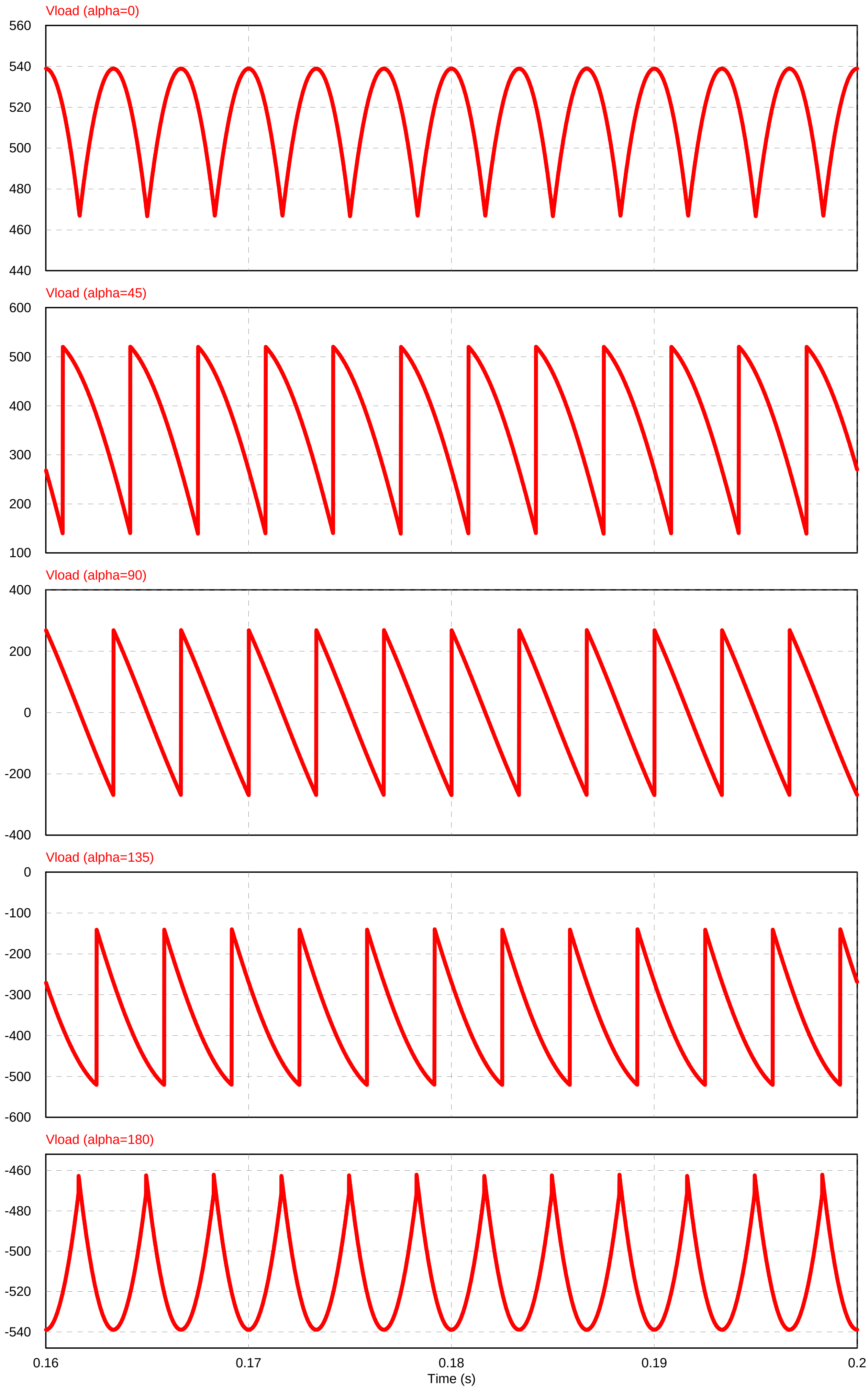
$\vdots \quad \vdots \quad \vdots$

This converter operates as rectifier for $\alpha \leq 90°$
and operates as inverter for $\alpha \geq 90°$

At $\alpha = 90°$, average output voltage is zero.

for $\alpha \leq 90°$, average output voltage is positive

for $\alpha \geq 90°$, average output voltage is negative.


Vload (alpha=0)
560
540
520
500
480
460
440
Vload (alpha=45)
600
500
400
300
200
100
Vload (alpha=90)
400
200
0
-200
-400
Vload (alpha=135)
0
-100
-200
-300
-400
-500
-600
Vload (alpha=180)
-460
-480
-500
-520
-540
0.16
0.17
0.18
0.19
0.2
Time (s)

Load Voltage Average Value

Phase to phase voltages are seen on the load. It is known that, $60°$ of the phase-to-phase voltage was the starting point at uncontrolled rectifiers. For the controlled rectifiers, the starting point becomes $60° + \alpha$ $\left(\frac{\pi}{3}+\alpha\right)$. Also every switch conducts current for $60°$ $\left(\frac{\pi}{3}\right)$. Therefore; the finish point becomes $60°+\alpha+60°$ $\left(\frac{\pi}{3}+\alpha+\frac{\pi}{3}=\frac{2\pi}{3}+\alpha\right)$.

$$V_{DC} = \frac{1}{\pi/3}\int_{\pi/3+\alpha}^{\frac{2\pi}{3}+\alpha} V_{max}\sqrt{3}\cdot \sin(\omega t)\cdot d\omega t = \frac{V_{max}\,3\sqrt{3}}{\pi}\Big[\left(-\cos(\omega t)\right)\Big]_{\pi/3+\alpha}^{2\pi/3+\alpha}$$

$$= \frac{V_{max}\,3\sqrt{3}}{\pi}\Big[\underbrace{\left(-\cos\left(\tfrac{2\pi}{3}+\alpha\right)\right)-\left(-\cos\left(\tfrac{\pi}{3}+\alpha\right)\right)}_{\left(\cos\frac{\pi}{3}\cdot\cos\alpha-\sin\frac{\pi}{3}\cdot\sin\alpha\right)-\left(\cos\frac{2\pi}{3}\cdot\cos\alpha-\sin\frac{2\pi}{3}\cdot\sin\alpha\right)}\Big]$$

$$= \frac{V_{max}\,3\sqrt{3}}{\pi}\left[\frac{\cos\alpha}{2}-\cancel{\frac{\sin\alpha\sqrt{3}}{2}}+\frac{\cos\alpha}{2}+\cancel{\frac{\sin\alpha\sqrt{3}}{2}}\right] = \frac{V_{max}\,3\sqrt{3}}{\pi}\cos\alpha$$

Load Voltage RMS value

$$V_{rms} = \sqrt{\frac{1}{\pi/3}\int_{\pi/3+\alpha}^{2\pi/3+\alpha}(V_{max}\sqrt{3})^2\cdot\sin^2(\omega t)\cdot d\omega t} = \sqrt{\frac{9V_{max}^2}{\pi}\left[\frac{1}{2}\left(\omega t-\frac{\sin 2\omega t}{2}\right)\right]_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}+\alpha}}$$

$$= \frac{3V_{max}}{\sqrt{2\pi}}\sqrt{\underbrace{\left(\frac{2\pi}{3}+\alpha-\frac{\sin\left(\frac{4\pi}{3}+2\alpha\right)}{2}\right)-\left(\frac{\pi}{3}+\alpha-\frac{\sin\left(\frac{2\pi}{3}+2\alpha\right)}{2}\right)}_{\frac{\pi}{3}+\frac{\sin\left(\frac{2\pi}{3}+2\alpha\right)-\sin\left(\frac{4\pi}{3}+2\alpha\right)}{2}=\frac{\pi}{3}+\frac{\cancel{2}\cdot\sin\left(-\frac{\pi}{3}\right)\cdot\cos(\pi+2\alpha)}{\cancel{2}}}}$$

$$= \frac{3V_{max}}{\sqrt{2\pi}}\sqrt{\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\cos 2\alpha\right)}$$

For Resistive Load Condition

Load current waveform is identical to load voltage waveform.

Therefore;

$$I_{DC} = V_{DC}/R = \frac{V_{max}\,3\sqrt{3}}{\pi R} \cdot \cos\alpha$$

$$I_{RMS} = V_{RMS}/R = \frac{3V_{max}}{R\sqrt{2\pi}} \sqrt{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right)}$$

Current of per diode

$$I_{D,DC} = \frac{I_{DC}}{3} = \frac{V_{max}\sqrt{3}}{\pi R} \cdot \cos\alpha$$

$$I_{D,RMS} = I_{RMS} \cdot \sqrt{1/3} = \frac{3V_{max}}{R\sqrt{6\pi}} \cdot \sqrt{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right)}$$

RMS current per phase

$$i_{a,RMS} = \sqrt{2} \cdot I_{D,RMS} = \frac{3V_{max}}{R\sqrt{3\pi}} \sqrt{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right)}$$

Example: At a factory room, pure resistive heater is used with $R = 3\Omega$. Supply of the factory is 3ϕ 110V/60Hz utility. The heater is supplied with 3ϕ controlled rectifier to control the temperature.

- calculate V_{DC} and V_{RMS} for $\alpha = 0°, 30°, 60°$.

$$V_{DC} = \frac{V_{max} 3\sqrt{3}}{\pi} \cos\alpha \qquad V_{RMS} = \frac{3V_{max}}{\sqrt{2\pi}} \sqrt{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right)}$$

$$V_{DC}(0°) = \frac{110\sqrt{2}\cdot 3\sqrt{3}}{\pi}\cdot\cos 0 = 257{,}3\,V \qquad V_{RMS}(0°) = \frac{3\cdot 110\sqrt{2}}{\sqrt{2\pi}} \sqrt{\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos 0} = 257{,}5V$$

$$V_{DC}(\alpha = 30°) = 222{,}83V \qquad V_{RMS}(\alpha = 30°) = 226{,}52V$$

$$V_{DC}(\alpha = 60°) = 128{,}65V \qquad V_{RMS}(\alpha = 60°) = 145{,}91V$$

- calculate P_o for $\alpha = 0°, 30°, 60°$.

$$P_o(\alpha = 0°) = V_{RMS}^2(0°)/R = 257{,}5^2/3 = 22{,}1\,kW$$

$$P_o(\alpha = 30°) = V_{RMS}^2(30°)/R = 226{,}52^2/3 = 17{,}1\,kW$$

$$P_o(\alpha = 60°) = V_{RMS}^2(60°)/R = 145{,}91^2/3 = 7{,}1\,kW$$

- calculate one phase current (RMS) for $\alpha : 0°, 30°, 60°$.

$$i_{a,RMS}(\alpha = 0°) = \frac{3V_{max}}{R\sqrt{3\pi}} \sqrt{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right)} = \frac{3\cdot 110\sqrt{2}}{3\sqrt{3\pi}} \cdot \sqrt{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos(2\cdot 0)\right)} = 70{,}01\,A$$

$$i_{a,RMS}(\alpha = 30°) = 61{,}65A \qquad i_{a,RMS}(\alpha = 60°) = 39{,}71A.$$

- calculate power factor for $\alpha = 0°, 30°, 60°$

$$pf(\alpha = 0°) = \frac{V_{RMS}^2/R}{3\cdot V_{a,rms}\cdot i_{a,rms}} = \frac{22{,}1\,kW}{3\cdot 110\cdot 70{,}01} = 0{,}9566$$

$$pf(\alpha = 30°) = \frac{V_{RMS}^2/R}{3\cdot V_{a,rms}\cdot i_{a,rms}} = \frac{17{,}1\,kW}{3\cdot 110\cdot 61{,}65} = 0{,}8405$$

$$pf(\alpha = 60°) = \frac{V_{RMS}^2/R}{3\cdot V_{a,rms}\cdot i_{a,rms}} = \frac{7{,}1\,kW}{3\cdot 110\cdot 39{,}71} = 0{,}5418$$

- if the heater is wanted to give 10kW to the room, what must the α be?

$$10kW = \frac{V_{RMS}^2}{R} \qquad \frac{3V_{max}}{\sqrt{2\pi}} \sqrt{\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right)} = V_{RMS}$$

$$V_{RMS} = 173{,}21V. \qquad \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\cos 2\alpha\right) = \left(\frac{173{,}21\sqrt{2\pi}}{3\cdot 110\sqrt{2}}\right)^2 \;\ldots\; \alpha = 51{,}06°$$