Summary of Structure Equations

n: total number of DOF in the structure *m*: unconstrained DOF *k*: constrained DOF *n* = *m* + *k*

F is the $n \times 1$ force vector **D** is the $n \times 1$ displacement vector **K** is the $n \times n$ stiffness matrix **P** is the $n \times 1$ fixed end force vector

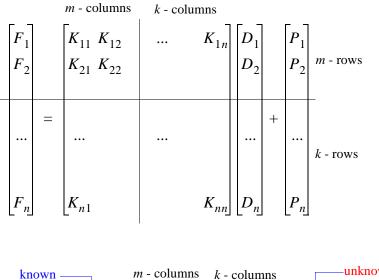
$$\mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \\ ... \\ F_n \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} D_1 \\ D_2 \\ ... \\ D_n \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ ... \\ P_n \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} K_{11} & K_{12} & ... & K_{1n} \\ K_{21} & K_{22} & ... \\ ... & ... \\ K_{n1} & ... & K_{nn} \end{bmatrix}$$

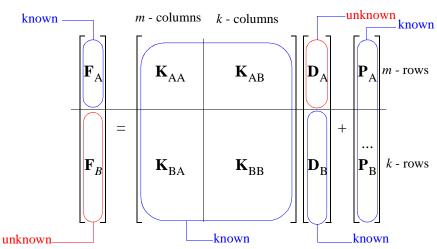
 $\mathbf{F} = \mathbf{K}\mathbf{D} + \mathbf{P}$

$$\begin{bmatrix} F_1 \\ F_2 \\ \dots \\ F_n \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & \\ \dots & \dots & \dots & \\ K_{n1} & \dots & K_{nn} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ \dots \\ D_n \end{bmatrix} + \begin{bmatrix} P_1 \\ P_2 \\ \dots \\ P_n \end{bmatrix}$$

 $F_{1} = K_{11}D_{1} + K_{12}D_{2} + \dots + K_{1n}D_{n} + P_{1}$ $F_{2} = K_{21}D_{1} + K_{22}D_{2} + \dots + K_{2n}D_{n} + P_{2}$ \dots $F_{n} = K_{n1}D_{1} + K_{n2}D_{2} + \dots + K_{nn}D_{n} + P_{n}$ 1

Partitioning





$$\begin{bmatrix} \mathbf{F}_{A} \\ \mathbf{F}_{B} \end{bmatrix} = \begin{bmatrix} \mathbf{K}_{AA} & \mathbf{K}_{AB} \\ \mathbf{K}_{BA} & \mathbf{K}_{BB} \end{bmatrix} \begin{bmatrix} \mathbf{D}_{A} \\ \mathbf{D}_{B} \end{bmatrix} + \begin{bmatrix} \mathbf{P}_{A} \\ \mathbf{P}_{B} \end{bmatrix}$$

$$\mathbf{F}_{A} = \mathbf{K}_{AA}\mathbf{D}_{A} + \mathbf{K}_{AB}\mathbf{D}_{B} + \mathbf{P}_{A}$$
(1)
$$\mathbf{F}_{B} = \mathbf{K}_{BA}\mathbf{D}_{A} + \mathbf{K}_{BB}\mathbf{D}_{B} + \mathbf{P}_{B}$$
(2)

Note that when there is no support settlement and support rotation, $\mathbf{D}_{\rm B} = \mathbf{0}$ (displacement and rotations at unconstrained DOF).

Solution of the Equations

- (1) gives $\mathbf{D}_{A} = \mathbf{K}_{AA}^{-1}(\mathbf{F}_{A} \mathbf{K}_{AB}\mathbf{D}_{B} \mathbf{P}_{A})$ (displacements and rotations at unconstrained DOF)
- (2) gives $\mathbf{F}_{B} = \mathbf{K}_{BA}\mathbf{D}_{A} + \mathbf{K}_{BB}\mathbf{D}_{B} + \mathbf{P}_{B}$ (support reactions reactions at constrained DOF)

Member end displacements in global coordinates, d, can be found from D.

Member end displacements in local coordinates and member end forces in local and global coordinates can be found using **d**.