INS 311E - Theory of Structures I

Directions, Signs and Sign Conventions

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Introduction

We need to follow certain conventions for signs and directions when performing structural analysis. These conventions can be best explained considering two stages of the analysis process as follows:

Stage 1: Finding the Values and Directions of Unknown Forces

Conventions for:

- Showing Unknown Forces in a FBD
- Positive Direction of Forces in Equations of Equilibrium
- Identifying Actual Directions of Unknown Forces

Stage 2: Reporting the Findings to Other Engineers

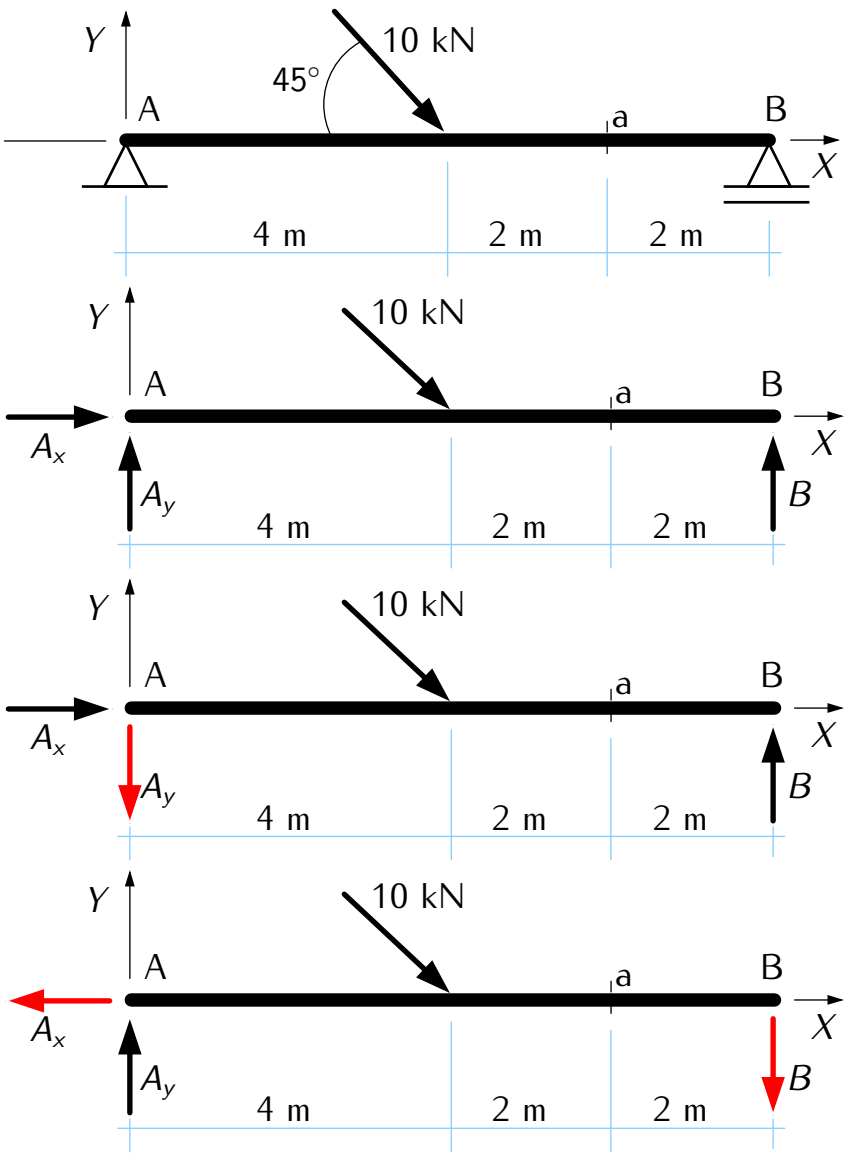
Conventions for:

- Reporting Reactions and Internal Forces

Outline

Outline

Reaction Forces in a FBD: General Idea

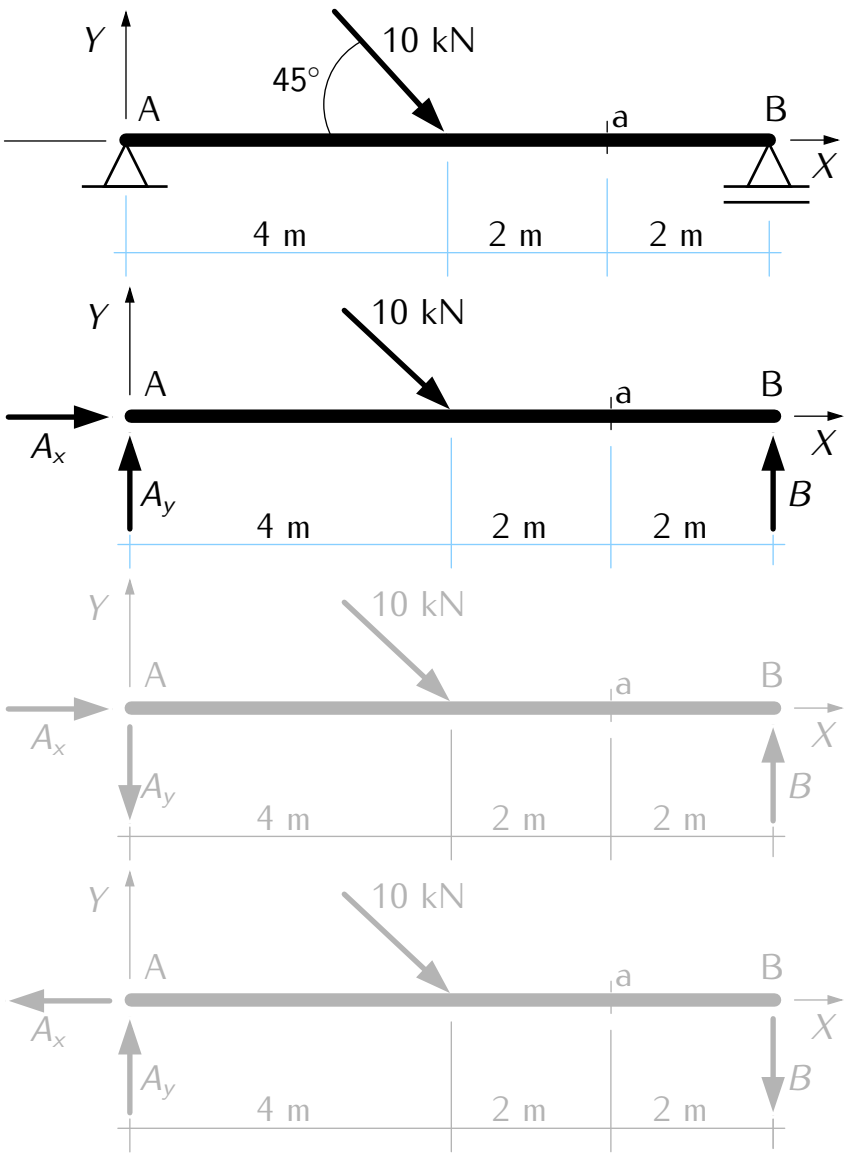


When drawing FBD, we have to make assumptions on the directions of the unknown reaction forces.

These directions can be arbitrary provided that the reaction forces represent the support conditions correctly.

In the example shown, several choices for the direction of the reaction forces at supports A and B are presented. All of these FBDs are valid for analysis.

Reaction Forces in a FBD: Practical Approach



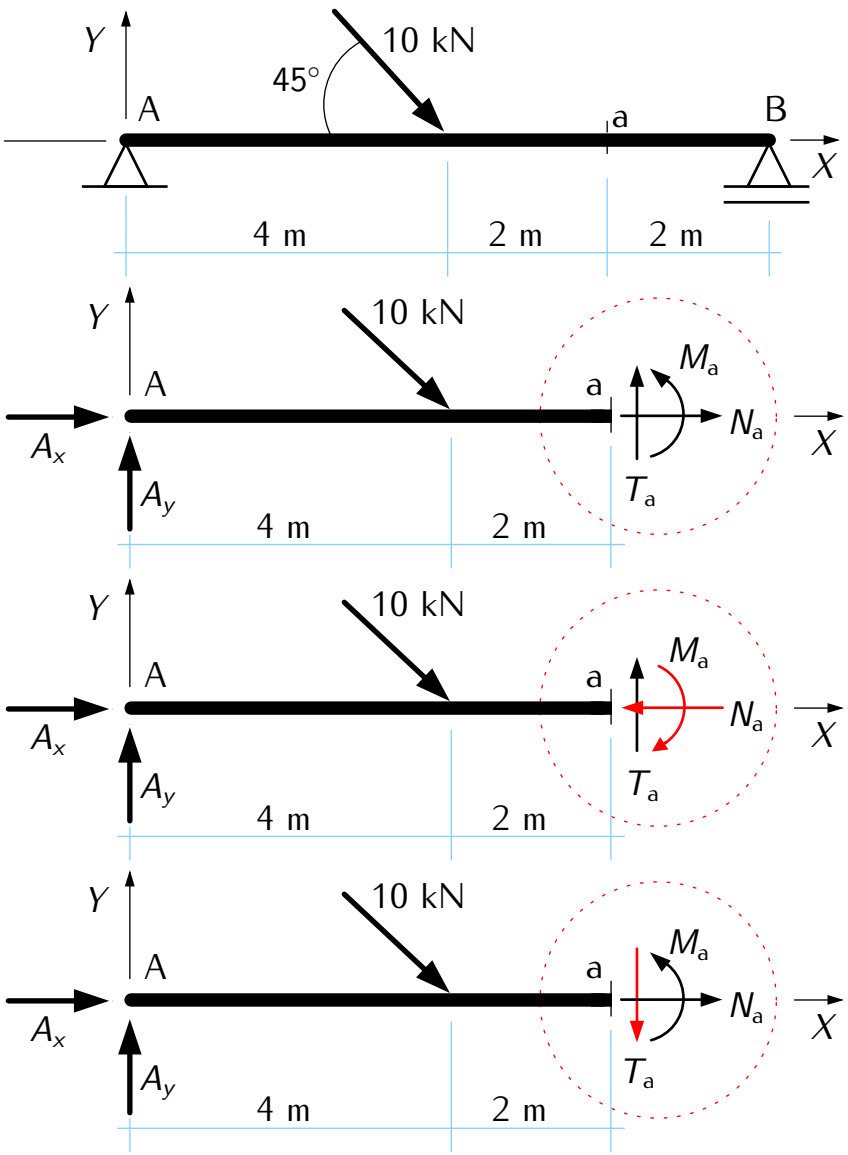
When it comes to practical applications, instead of setting the directions of the unknown reaction forces randomly, it is preferred to use a fixed set of directions for consistency.

Most engineers prefer to use **global X- and Y-directions** for the directions of the unknown reactions.

Therefore, the **first FBD** shown on the left, is a common way of representing the unknown reactions forces in a FBD.

Outline

Internal Forces in a FBD: General Idea

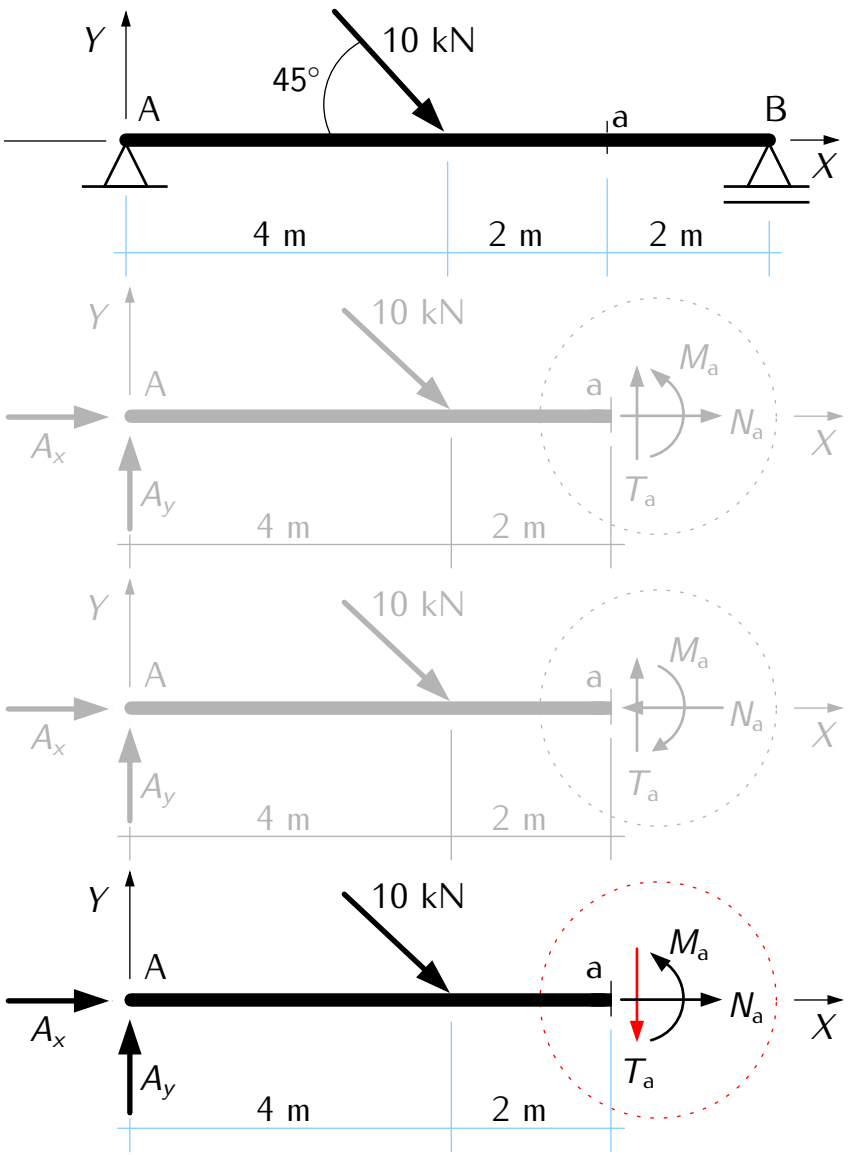


Similar to reactions, we have to make assumptions on the directions of the unknown internal forces when drawing a FBD.

As in the case of the reactions, directions of the unknown internal forces in a FBD is at engineer's discretion.

In the example shown, several choices for the directions of the unknown internal forces at section "a" are presented. All these FBDs are valid for analysis.

Internal Forces in a FBD: Practical Approach



When it comes to practical applications, there are several cases where it is more convenient to use a set of **predefined directions** for the unknown internal forces, instead of setting them randomly. Some of these cases are:

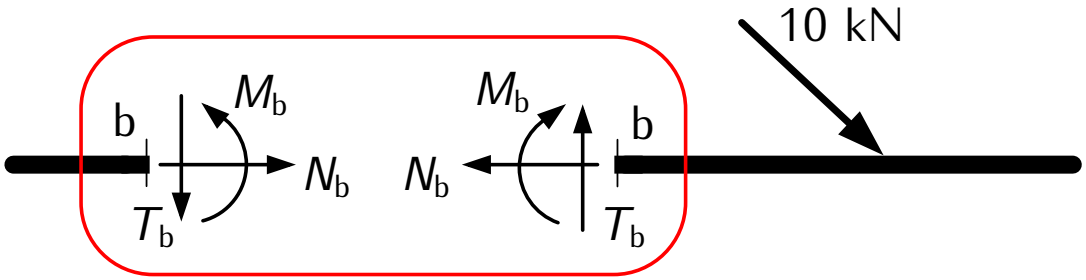
- General Sections
- Slope-Deflection Method
- Stiffness Method

For General Sections, it is preferred to use the "Sign Convention for Reporting M,N,T" (please see slide 29). In the example shown, the **third FBD** uses this convention.

All three cases are summarized in the following slides .

Internal Forces in a FBD: General Sections

- General Sections:



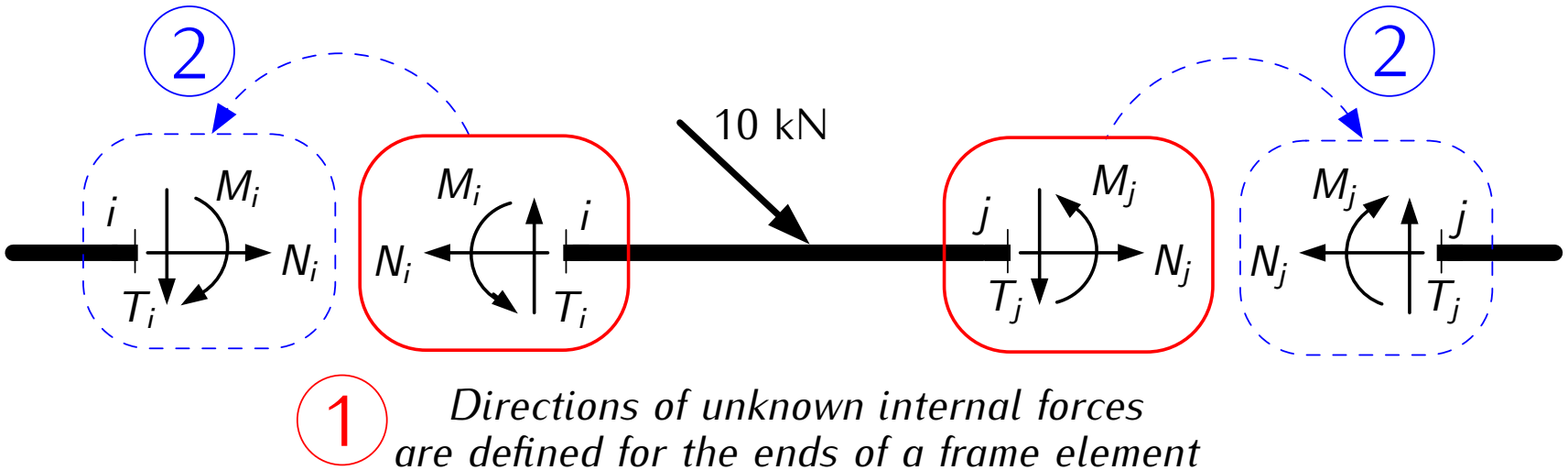
Directions of unknown internal forces are defined for a given section

Directions of Unknown Internal Forces for a General Section

(same as "Sign Convention for Reporting M, N and T," please see slide 29)

Internal Forces in a FBD: Slope Deflection Method

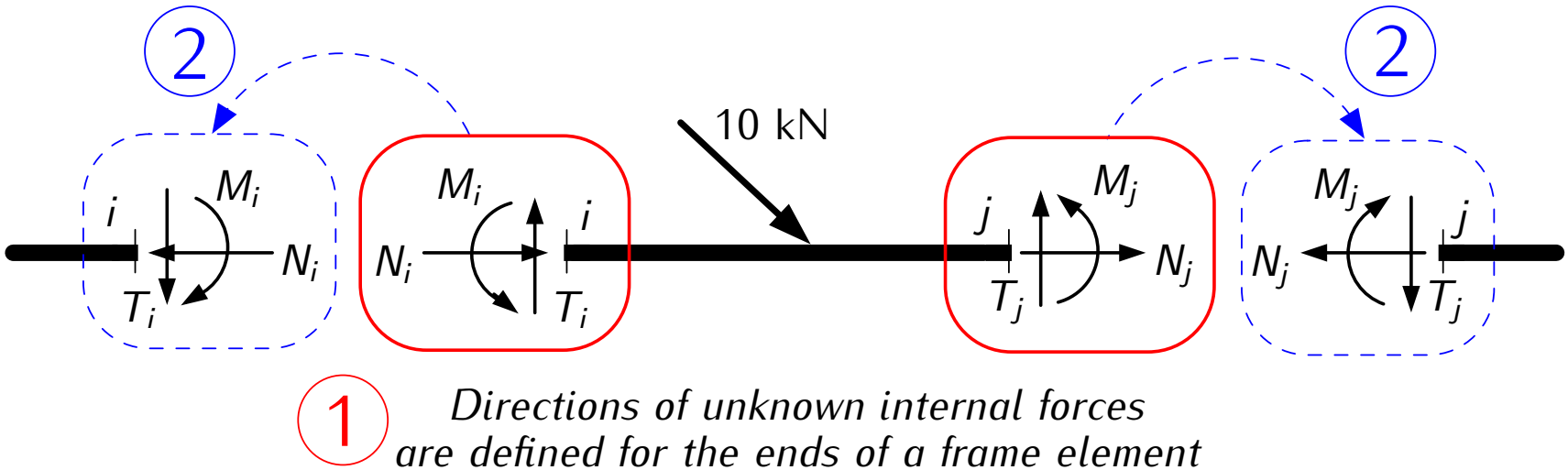
- Slope-Deflection Method (*not scope of INS311E*):



Directions of Unknown Internal Forces in the Slope-Deflection Method

Internal Forces in a FBD: Stiffness Method

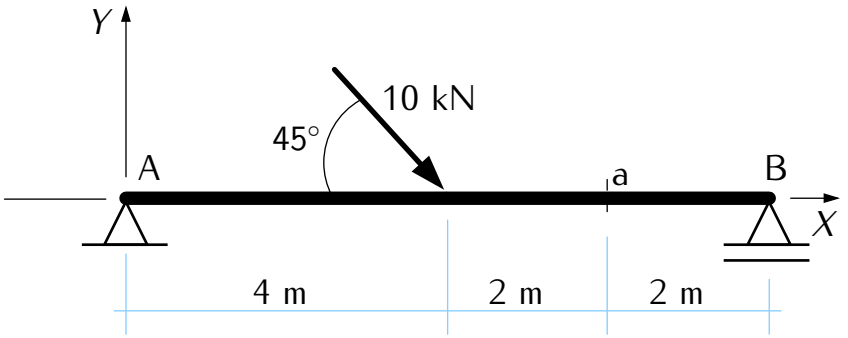
- Stiffness Method (*not scope of INS311E*):

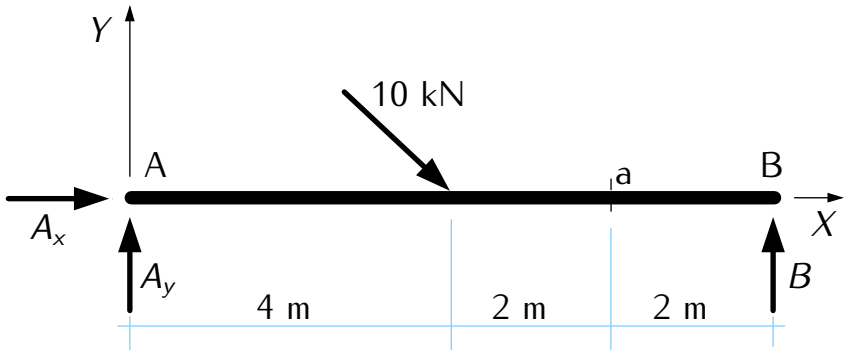


Directions of Unknown Internal Forces in the Stiffness Method

Outline

Positive Direction of Forces in Equations of Equilibrium





Free Body Diagram

Consider the equation of static equilibrium in the X-direction:

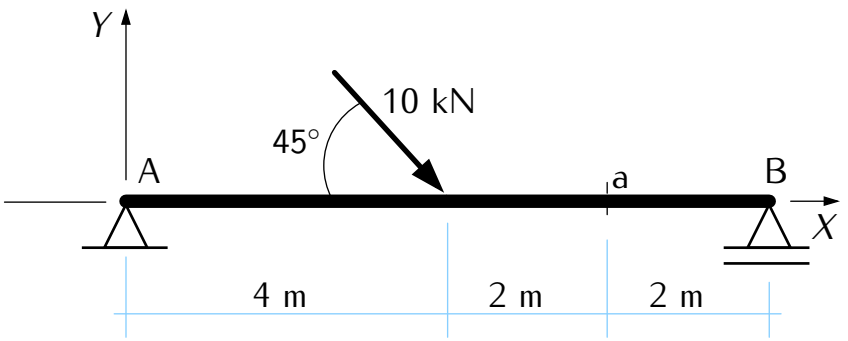
$$\sum F_X = 0$$

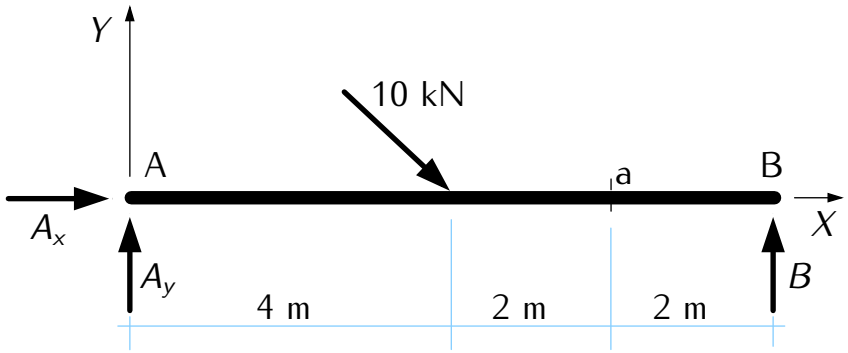
All the forces shown in the FBD in the X-direction will be placed on the LHS of the equation. For consistency of the directions, we need to specify a direction for positive values of the forces. For example:

$$\sum F_X = 0 \quad (\rightarrow +)$$

What this means is; if the direction of a force in the FBD is $(\rightarrow)$, it will appear as positive $(+)$ on the LHS of the equation.

Positive Direction of Forces in Equations of Equilibrium





Free Body Diagram

In the FBD shown, the forces in the X-direction will appear in the equation as follows:

$$(\rightarrow +) \quad \sum F_X = 0$$

$$+A_X + (10\,\text{kN})\cos 45^\circ = 0$$

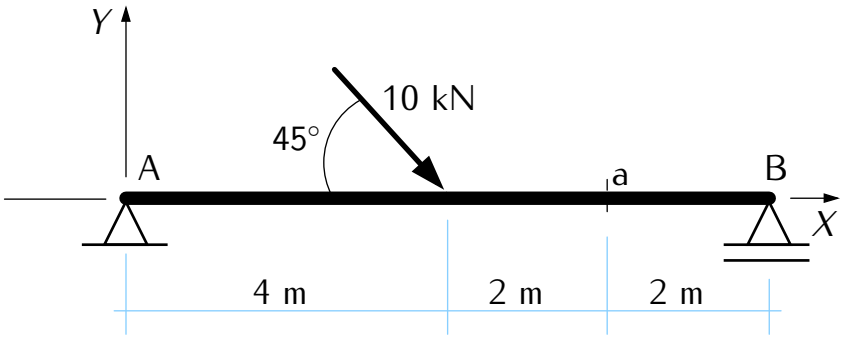
If we select a different positive direction, the equation will be:

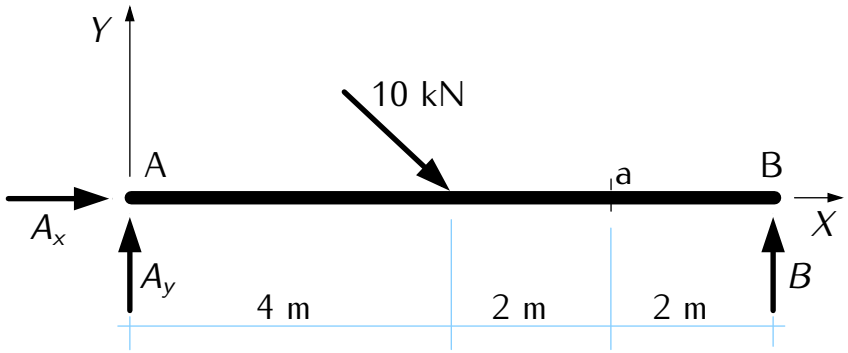
$$(\leftarrow +) \quad \sum F_X = 0$$

$$-A_X - (10\,\text{kN})\cos 45^\circ = 0$$

As can seen, both equations are same. **Therefore, final equation does not depend on the positive direction selected.**

Positive Direction of Forces in Equations of Equilibrium





Free Body Diagram

Let's look at the equation in the Y-direction:

$$(\uparrow +) \quad \sum F_Y = 0$$

$$+A_Y - (10\,\text{kN})\cos 45^\circ + B = 0$$

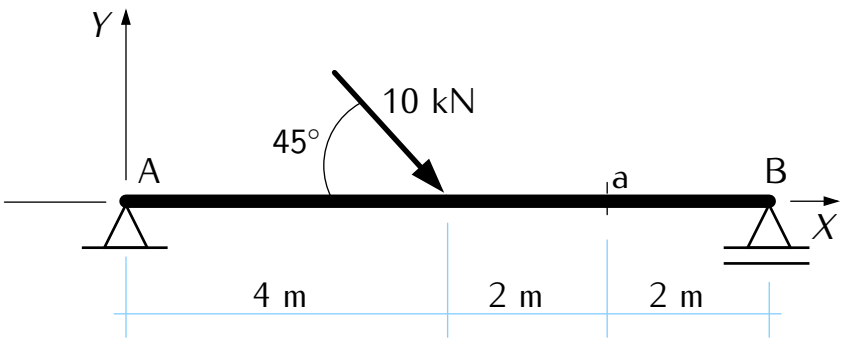
If we select a different positive direction, the equation will be:

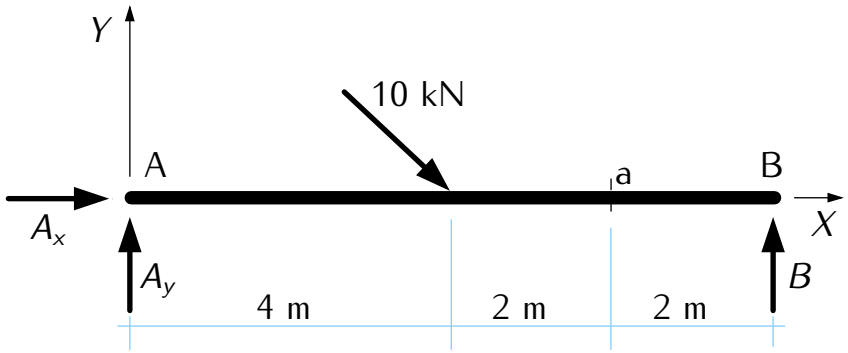
$$(\downarrow +) \quad \sum F_Y = 0$$

$$-A_Y + (10\,\text{kN})\cos 45^\circ - B = 0$$

Again, both equations are same.

Positive Direction of Forces in Equations of Equilibrium





Free Body Diagram

Let's consider the moment equation wrt to point A:

$$(\curvearrowleft +) \quad \sum M_A = 0$$

$$+(B)(8\,\text{m}) - (10\,\text{kN})(\cos 45^\circ)(4\,\text{m}) = 0$$

If we select a different positive direction, the equation will be:

$$(\curvearrowright +) \quad \sum M_A = 0$$

$$-(B)(8\,\text{m}) + (10\,\text{kN})(\cos 45^\circ)(4\,\text{m}) = 0$$

Again, both equations are same.

Outline

Identifying Actual Directions of Unknown Forces

After solving the equations of equilibrium, actual directions of the unknowns forces should be identified.

- If the unknown force value turns out to be positive, this means that assumed direction is correct. In this case:

  $\rightarrow$ The direction of the unknown force is placed beside the value of the force: Eg. $A_y = +3.5\,\text{kN} \quad (\uparrow)$

  $\rightarrow$ No changes made to FBD. Only the positive value of the force is placed beside the force in the FBD.

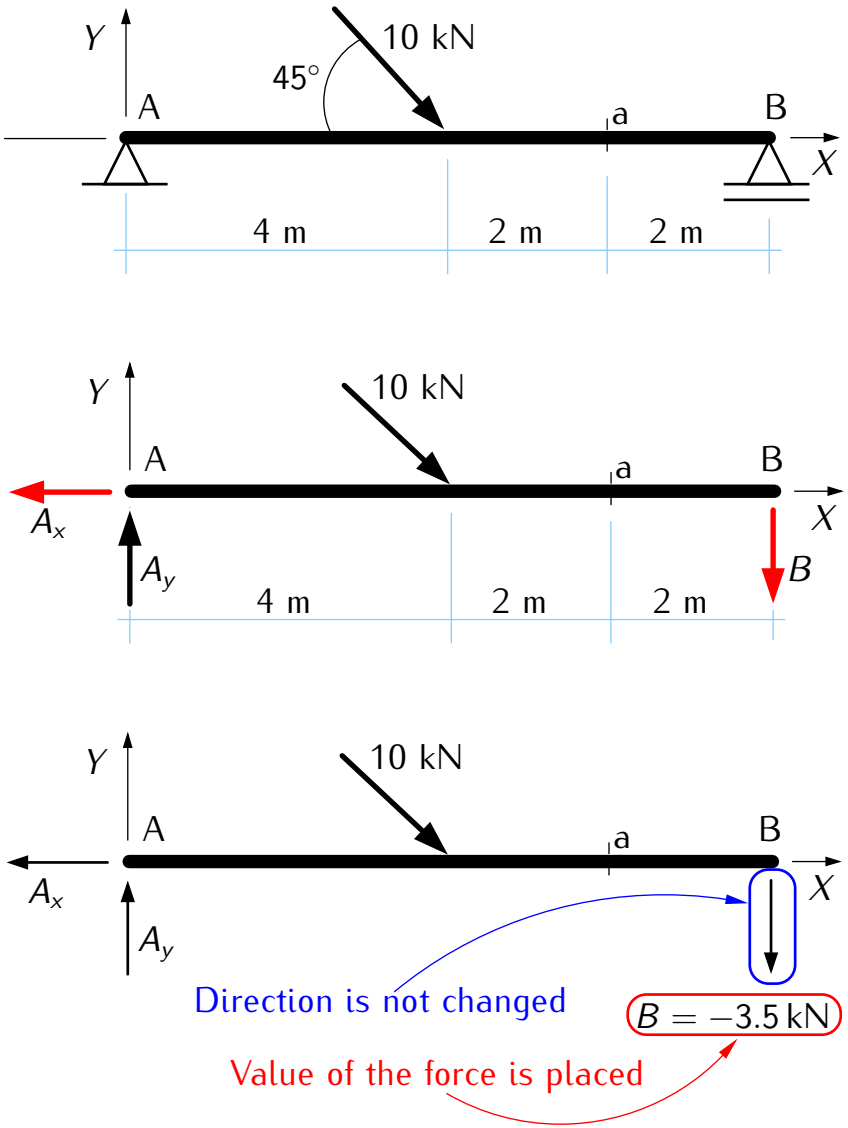
- If the unknown force value turns out to be negative, this means that assumed direction is not correct. In this case:

  $\rightarrow$ Actual direction of the unknown force is the opposite of the assumed direction.

  $\rightarrow$ The correct direction of the unknown force is placed beside the value of the force: Eg. $B = -3.5\,\text{kN} \quad (\uparrow)$

  $\rightarrow$ No changes made to FBD. Only the negative value of the force is placed beside the force in the FBD.

Identifying Actual Directions of Unknown Forces



In the example shown, solution of the following equation gives the reaction B:

$$(\curvearrowleft +) \quad \sum M_A = 0$$

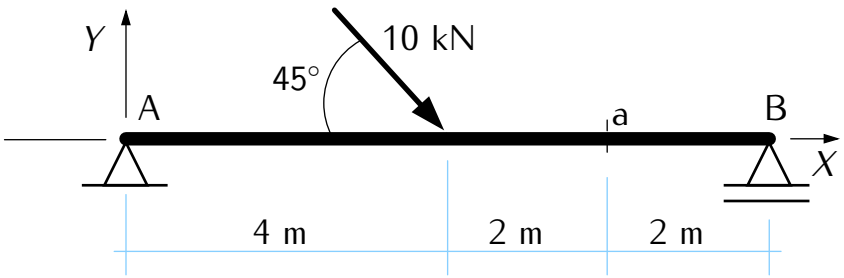
$$-(B)(8\,\text{m}) - (10\,\text{kN})(\cos 45^\circ)(4\,\text{m}) = 0$$

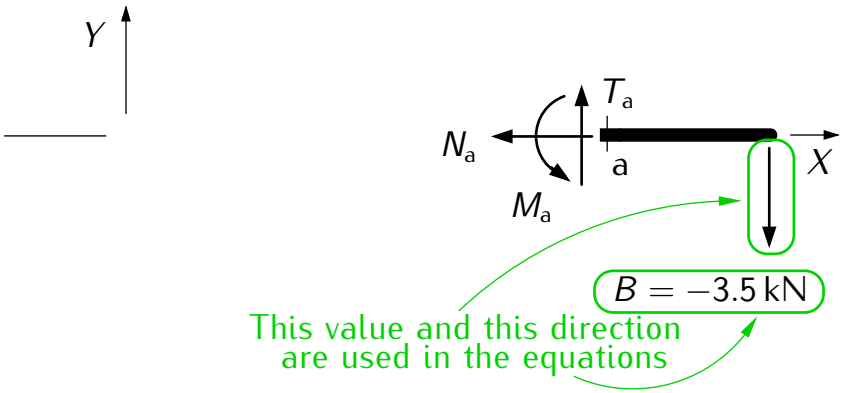
$$\Rightarrow \quad B = -3.5\,\text{kN} \quad (\uparrow)$$

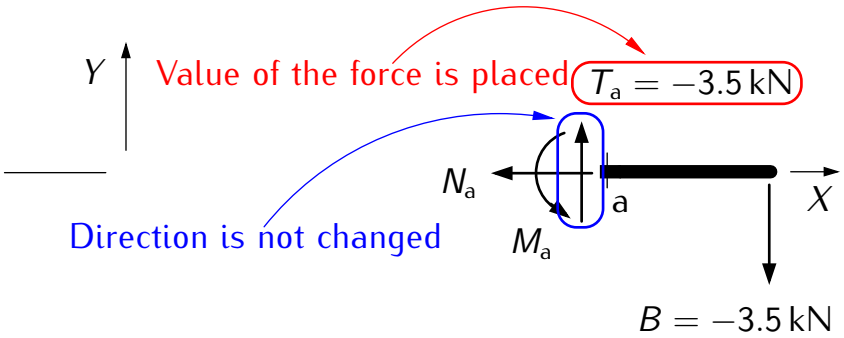
Although it is found that the actual direction of the reaction B is ($\uparrow$), the direction in the FBD is not corrected; only the value of the reaction B (with the negative sign) is added to the FBD.

Question: *Why it is not suggested to make changes on the directions of the unknown forces in the FBD?*

Identifying Actual Directions of Unknown Forces







In the example shown, consider the section "a" and the right portion of the structure. Let's find the internal force T_a:

$$(\downarrow +) \quad \sum F_Y = 0$$

$$-T_a + (-3.5\,\text{kN}) = 0$$

$$\Rightarrow \quad T_a = -3.5\,\text{kN} \quad (\downarrow)$$

Answer: *There may be several other equations that might have been written based on the original FBD.*

$\rightarrow$ *A matter of Consistency!*

Outline

Outline

Reporting Reaction Forces: First Method

- When reporting the reaction forces at supports, we may follow two methods.
- In the first method, the result is simply stated with a positive value along with the actual direction of the reaction force.
- Examples are:

First Method

$$A_x = 7.1\,\text{kN} \quad (\leftarrow)$$
$$A_y = 3.5\,\text{kN} \quad (\uparrow)$$
$$B = 3.5\,\text{kN} \quad (\uparrow)$$

- Note that the values of the reaction forces are always positive.
- This is the method used for the graphical outputs of the computer structural analysis programs.
- While this method is simple and easily understood by others, it may be cumbersome to report many reaction forces using this method.

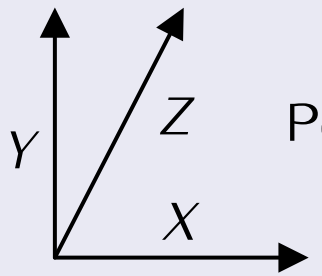
Reporting Reaction Forces: Second Method

- When reporting many support reaction forces, it is more convenient to use signs (+ or −) along with a "Sign Convention" that shows the positive (+) directions for identification.
- Most of the time, global $+X$, $+Y$, and $+Z$ coordinates are used as the positive "Sign Convention."
- Examples are:

Second Method

$$A_x = +7.1\,\text{kN}$$
$$A_y = +3.5\,\text{kN}$$
$$B_y = +3.5\,\text{kN}$$

Sign Convention

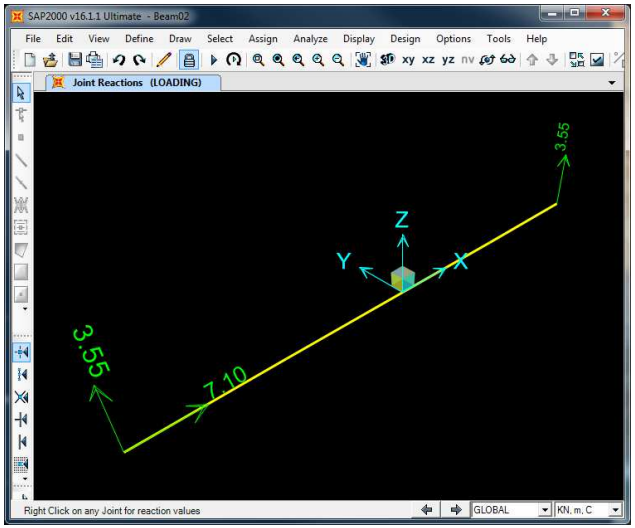


Positive (+) Directions of Reactions

- This is the method used for the tabular outputs of the computer structural analysis programs.

Reporting Reaction Forces: Computer Program Example

- Reporting the Reaction Forces in the form of Graphical Output using the First Method



- Reporting the Reaction Forces in the form of Tabular Output using the Second Method

Joint Reactions

File View Format-Filter-Sort Select Options

Units: As Noted — Joint Reactions

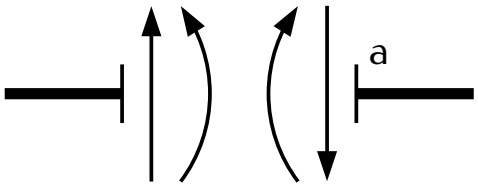
	Joint Text	OutputCase Text	CaseType Text	F1 KN	F2 KN	F3 KN	M1 KN-m	M2 KN-m	M3 KN-m
▶	1	LOADING	LinStatic	7.1	0	3.55	0	0	0
	2	LOADING	LinStatic	0	0	3.55	0	0	0

Record: 1 of 2 — Add Tables... — Done

Outline

Reporting Internal Forces: Sign Convention for M, N, T

- Internal force values are stated with signs (+ or −) along with a "Sign Convention" that shows the positive (+) directions for identification.
- Consider the following result of an analysis, where actual directions of the internal forces are shown:



$T_a = 3.5\ kN$

$M_a = 3.5\ kN$

- For the sign convention shown, M,N, and T values are reported as follows:

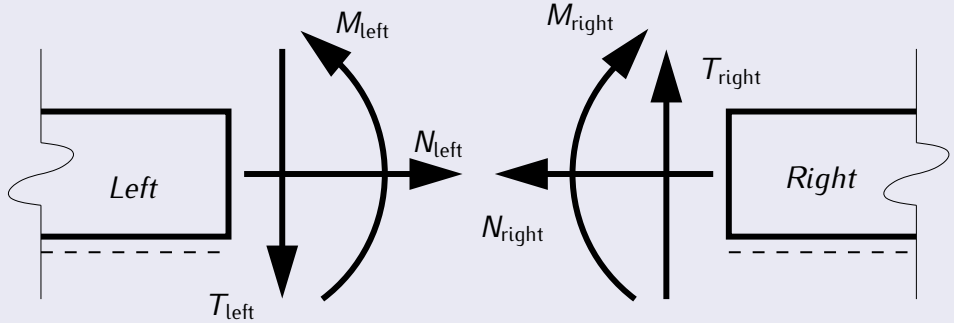
Reporting M,N,T

$M_a = +7.1\,\text{kNm}$

$N_a = +0.0\,\text{kN}$

$T_a = -3.5\,\text{kN}$

Sign Convention

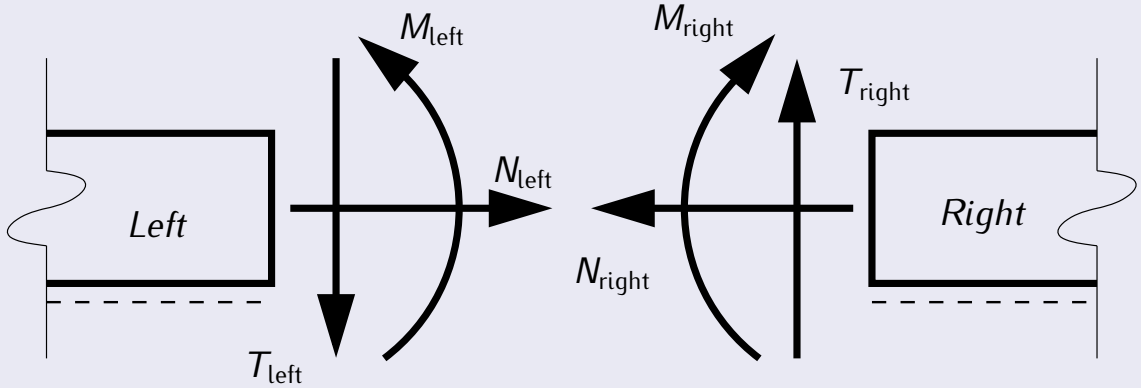


Positive (+) Directions of M, N, and T

Reporting Internal Forces: Sign Convention for M, N, T

- The Sign Convention shown below is, in fact, a global sign convention that is used worldwide. Almost all of the engineering practice, including computer programs, uses this sign convention for reporting internal forces.

Sign Convention for M, N, and T



Positive (+) Directions of M, N, and T

- Also note that -while it is not compulsory- if this Sign Convention is used for the directions of unknown internal forces during the analysis, reporting the internal forces becomes more convenient (please see slide 9).

End of Presentation