Theory of Structures

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    - ∗ Only show external case, do now show internal force and stiffness

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Part III

Statically Indeterminate Structures

Chapter 10

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- External (Reactions) and Internal (Member Forces) Redundant Forces, Total Degree of Static Indeterminacy (Unknowns)
- Multiple Choices of Primary Stable Determinate Structure
- Use of Compatibility for Developing Equations

10.2 Compatibility Method of Analysis

- Set of Equations are Based on Compatibility
- Unknowns are Redundant Forces
- Superposition
- Coefficients are Displacement Quantities (Flexibility Coefficients)
- Other Names: Force Method, Flexibility Method

10.3 Kinematic Indeterminacies, Redundant Deflections, Equilibrium

- Nodes of Structure (joints and supports)
- Deflections (Displacements and Rotations) that Define the Deflected Shape of a Structure
- Single Choice of Redundant Deflections
- Use of Equilibrium for Developing Equations

10.4 Equilibrium Method of Analysis

- Forces Represented in terms of Deflections
- Unknowns are Deflections
- Equilibrium Equations for Each Node
- Coefficients are Force Quantities (Stiffness Coefficients)
- Other Names: Displacement Method, Stiffness Method

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- Redundant Reaction (One Unknown) and Its Application to the Primary Structure
- Deflection of Indeterminate Structure at the Application Point of Reduntant Reaction is Zero
- Deriving Compatibility Equation by Manual Superposition of Deflections
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- Sign Convention

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- Redundant Reactions (Two Unknowns) and Their Application to the Primary Structure
- Deflections of Indeterminate Structure at the Application Points of Reduntant Reactions are Zero
- Deriving Compatibility Equation by Manual Superposition of Deflections
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- Superposition of Internal Forces and Reactions
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  - Identify Deflections of Indeterminate Structure at Redundant Force Locations as Zero
  - Apply One Redundant Force to the Primary Structure as External Loading
  - Define Virtual Displacements and Deformations as the Displacements and Deformations of Indeterminate Structure (Internal Forces under These Deformations are Designated)
  - Apply Virtual Displacements and Deformations to the Primary Structure on Top of the Redundant Force
  - Two Quantities of Work are Estimated
    * Work Done by Internal Forces Due to Virtual Deformations
    * Work Done by Redundant Forces Over Virtual Deflections at the Redundant Force Location. This is Zero if There is No Support Settlement.
  - Two Quantities of Work are Equal to Each Other Due to Virtual Work Principle
  - This Procedure Gives "Closed-Form" of Compatibility Equations

- Derivation of Equations of Superposition of Internal Forces

- Insertion of Equations of Superposition of Internal Forces into Closed-Form of Compatibility Equations Gives "Open-Form" of Compatibility Equations.

- Coefficients of Open-Form of Compatibility Equations are Displacements of Primary Stable Determinate Structure Under Unit Loadings that are Applied to Redundant Force Locations

- Maxwell's Reciprocal Theory and Betti's Law

11.4 General Procedure of Force Method

- Selection of Primary Stable Determinate Structure and Redundant Forces (Reactions and Internal Forces)

- Stating Primary Structure with Original Loading and Redundant Loading

  - Primary Stable Determinate Structure with Original Loading. Internal Forces are Designated as: M_0, N_0, T_0
  - Primary Stable Determinate Structure with Redundant Loading (Generally Unit Load) for the i^{th}Redundant Force. Internal Forces are Designated as: M_i, N_i, T_i

- Deflections of Primary Structures. These Deflections are Coefficients of Open-Form of Compatibility Equations. Open-Form of Compatibility Equations are Derived.

- Equations are Solved for Unknown Redundant Forces

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