On the Accuracy of Method of Moments for Solution of Full 3D Vectorial Electromagnetic Forward Scattering Problem

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Abstract— In this communication, an analysis on the accuracy of the method of moments solution of full 3D vectorial electromagnetic forward scattering problem is presented. Although different mathematical techniques are developed for determination of the error rate of method of moments [1–6], this paper presents a numerical approach to this problem. In contrast to weak formulations of method of moments as in [7], we use a dyadic Green function based approach. We adopted the pulse functions as basis functions and obtained equations are weighted by the dirac-delta functions. In fact such a choice obliges us to calculate the hypersingular integrals of the components of the well known dyadic Green function. We utilize from [8] for the Cauchy principal value of these singular integrals. To be able to decrease memory requirement and computational complexity, bi-conjugate gradient method is applied togather with the fast fourier transform (FFT) algorithm for the matrix multiplication. An accuracy analysis is made by comparing the simulated fields with the analytical expressions of the scattering field from a dielectric sphere. The results show that such an dyadic Green function based implementation of the method of moments works sufficiently well for a wide range of various parameters.

1. INTRODUCTION

Microwave technologies has become an important ingredient of today's science. Researches regarding microwaves can range from inverse problems [9–14] to different applications in electronics [15, 16]. Although each of these areas require different approaches, their common point is perhaps that: all of them depends on solving Maxwell equations.

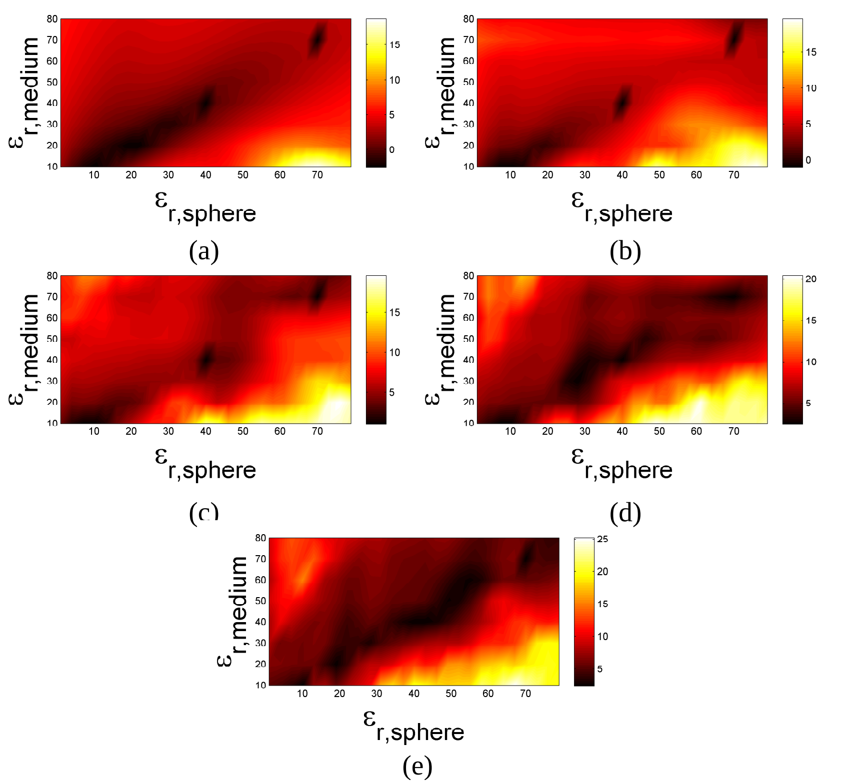

Figure 1: Error defined in (1) for (a) 0.50 GHz, (b) 0.75 GHz, (c) 1.00 GHz, (d) 1.25 GHz, (e) 1.50 GHz when the field is measured on the cylinder having radius of $R = 7$ cm. (ϵ_r stands for the dielectric permittivity of the corresponding medium.).

One of the effective forward electromagnetic solution technique is the Method of Moments (MoM) [7, 17, 18]. The accuracy rate of MoM procedure is a significant issue, since this method is extensively used in different applications of electromagnetics. There are a few papers that present some analytical approaches for the accuracy of the MoM [1–6].

In this paper, we analyze the accuracy of MoM numerically. Our implementation is based on evaluation of hypersingular integrals, which are coming from dyadic Green function [8]. To be able to reduce the computational burden bi-conjugate gradient fast fourier transform (BiCG-FFT) method is employed [19]. For the discretization purposes the investigation region is divided into cubes with an edge length of one tenth of the minimum wavelength. The accuracy analysis is based on comparing the scattering from a dielectric sphere. For this aim, the root of the mean square error between the evaluated scattered electric fields and the analytical solution is computed for different cases. In particular, the accuracy analysis includes the following parts:

- Accuracy rate vs. the dielectric contrast between sphere and medium (Here the frequency of illumination and the size of the sphere are constant).
- Accuracy rate vs. the frequency of illumination (Here the dielectric contrast between sphere and medium and the size of the sphere are constant).
- Accuracy rate vs. the size of the sphere (Here the dielectric contrast between sphere and medium, the frequency of illumination are constant).

Obtained results show that MoM procedure is feasible for various scattering problems.

2. RESULTS AND DISCUSSION

As explained above the comparisons are made for an plane wave incidence on a dielectric sphere. Here two distinct case is considered. In the first case the radius of the sphere is 5 cm. For this case, the measurements are taken on three different horizontal circles, which belong to a cylinder Γ having a radius of $R = 7$ cm. The heights of the circles are chosen as -5 cm, 0 cm, 5 cm respectively.

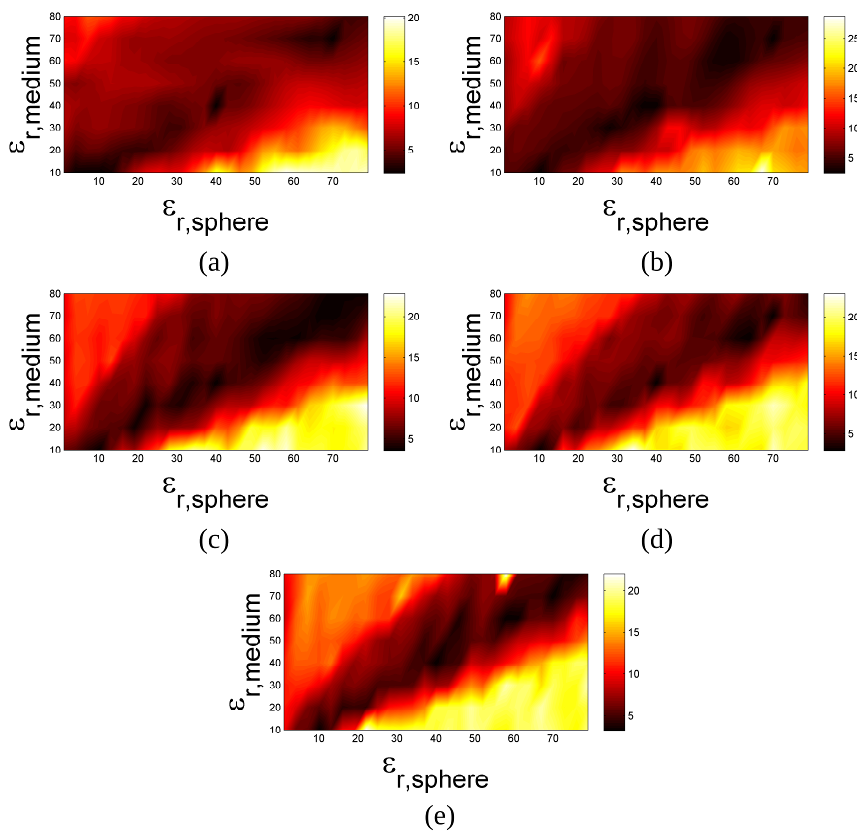

Figure 2: Error defined in (1) for (a) 0.50 GHz, (b) 0.75 GHz, (c) 1.00 GHz, (d) 1.25 GHz, (e) 1.50 GHz when the field is measured on the cylinder having radius of $R = 12$ cm. (ϵ_r stands for the dielectric permittivity of the corresponding medium.).

On each circle, all three components of the electric fields are sampled at 12 uniformly distributed points, which results in 36×3 measurements in total. The accuracy rate of the simulated field is assessed as:

$$\text{Log-RMS} = 10 \log_{10} \left(\frac{100}{3} \times \left(\sum_{j \in \{x,y,z\}} \frac{||E_{anal,j} - E_{MoM,j}||_\Gamma}{||E_{anal,j}||_\Gamma} \right) \right) \text{ [dB]} \tag{1}$$

where $||\cdot||_\Gamma$ denotes the Euclidean norm defined over surface Γ and $E_{\text{anal},j}, j \in \{x,y,z\}$, $E_{\text{MoM},j}, j \in \{x,y,z\}$ stand for the Cartesian components of the analytical, simulated fields, respectively. As can be seen from these result in Figure 1 as the frequency gets larger the accuracy rate of the solutions decreases. Furthermore, it can also be stated that the solution becomes more accurate when the dielectric permittivities of the medium and the sphere is close to each other.

Finally, the same setup is repeated when the radius of the sphere is 10 cm and the radius of the measurement cylinder Γ is $R = 12$ cm. The logarithmic errors are given in Figure 2 for this case. By comparing Figure 1 with Figure 2, one can conclude that the errors gets higher when the radius of the scatterer gets larger.

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