

MAT 205E – Theory of a Complex Variable Functions

Homework 3

1. Evaluate following integrals.

a)
$$\int_{-\pi}^{\pi} \frac{\cos \theta}{5 + 4 \cos \theta} d\theta = -\frac{\pi}{3}$$

b)
$$\int_{-\pi}^{\pi} \frac{d\theta}{1 + \sin^2 \theta} = \pi\sqrt{2}$$

c)
$$\int_0^{2\pi} \frac{d\theta}{1 + k \sin \theta} = \frac{2\pi}{\sqrt{1 - k^2}}, \quad (k^2 < 1)$$

d)
$$\int_0^{2\pi} \frac{d\theta}{1 + k \cos \theta} = \frac{2\pi}{\sqrt{1 - k^2}}, \quad (k^2 < 1)$$

e)
$$\int_0^{2\pi} \frac{\cos^2 3\theta}{5 - 4 \cos 2\theta} d\theta = \frac{3\pi}{8}$$

2. Evaluate following integrals.

a)
$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$$

b)
$$\int_0^{\infty} \frac{\log x}{x^2 + 1} dx = 0$$

c)
$$\int_0^{\infty} \frac{\log x}{(x^2 + 1)^2} dx = -\frac{\pi}{4}$$

3. Evaluate following integrals.

a)
$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 2x + 2)} = -\frac{\pi}{5}$$

b)
$$\int_{-\infty}^{\infty} \frac{\sin x}{x^2 + 4x + 5} dx = -\frac{\pi}{e} \sin 2$$

4. Compute residues of following functions at $z = 0$.

a)
$$\operatorname{cosec}^2 z$$

b)
$$\frac{\operatorname{cosec} z^2}{z^3}$$

c)
$$z \cos\left(\frac{1}{z}\right)$$

5. Show that singular points of following functions, are also poles of that functions. Determine orders of poles and compute residues.

a)	$\frac{z + 1}{z^2 - 2z}$
b)	$\frac{\tanh z}{1 - e^{2z}}$
c)	$\frac{z^4}{e^{2z}}$
d)	$\frac{z}{(z - 1)^2}$
e)	$\frac{\cos z}{e^z}$
f)	$\frac{z}{z^2 + \pi^2}$

Note: *Demonstrate every steps clearly in your answers. Do not give the solution directly. If you miss any step between your solutions you won't get any point from that question.*

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