ADDITIONAL HOMEWORK 2

1)

Perform the indicated operations.

(a) $(2-3i)+(4+5i)$ (b) $(6+5i)-(-2+3i)$

(c) $(3-i)(4+3i)$ (d) $\dfrac{1-2i}{1+3i}$

11)

Show in two ways that the function $\ln(x^2+y^2)$ is harmonic in every domain that does not contain the origin.

13)

Show that

(*a*) $\overline{\cos(iz)} = \cos(i\bar{z})$ for all z;

(*b*) $\overline{\sin(iz)} = \sin(i\bar{z})$ if and only if $z = n\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$.

14)

Show that

(*a*) $\sinh(z+\pi i) = -\sinh z$; (*b*) $\cosh(z+\pi i) = -\cosh z$;

(*c*) $\tanh(z+\pi i) = \tanh z$.

16)

find $f'(z)$ when

(*a*) $f(z) = 3z^2 - 2z + 4$; (*b*) $f(z) = (1-4z^2)^3$;

(*c*) $f(z) = \dfrac{z-1}{2z+1} \quad (z \neq -1/2)$; (*d*) $f(z) = \dfrac{(1+z^2)^4}{z^2} \quad (z \neq 0)$.

18)

show that $f'(z)$ and its derivative $f''(z)$ exist everywhere, and find $f''(z)$ when

(*a*) $f(z) = iz + 2$; (*b*) $f(z) = e^{-x}e^{-iy}$;

(*c*) $f(z) = z^3$; (*d*) $f(z) = \cos x \cosh y - i \sin x \sinh y$.

Ans. (*b*) $f''(z) = f(z)$; (*d*) $f''(z) = -f(z)$.

20)

verify that each of these functions is entire:

(a) $f(z) = 3x + y + i(3y - x)$; (b) $f(z) = \sin x \cosh y + i \cos x \sinh y$;

(c) $f(z) = e^{-y} \sin x - ie^{-y} \cos x$; (d) $f(z) = (z^2 - 2)e^{-x}e^{-iy}$.

22)

In each case, determine the singular points of the function and state why the function is analytic everywhere except at those points:

(a) $f(z) = \dfrac{2z+1}{z(z^2+1)}$; (b) $f(z) = \dfrac{z^3+i}{z^2-3z+2}$; (c) $f(z) = \dfrac{z^2+1}{(z+2)(z^2+2z+2)}$.

Ans. (a) $z = 0, \pm i$; (b) $z = 1, 2$; (c) $z = -2, -1 \pm i$.

24)

Suppose that, in a domain D, a function v is a harmonic conjugate of u and also that u is a harmonic conjugate of v. Show how it follows that both $u(x, y)$ and $v(x, y)$ must be constant throughout D.

34)

Find the image of the semi-infinite strip $x > 0$, $0 < y < 2$ when $w = iz + 1$. Sketch the strip and its image.

36)

Find the bilinear transformation that maps the points $z_1 = \infty$, $z_2 = i$, $z_3 = 0$ onto the points $w_1 = 0$, $w_2 = i$, $w_3 = \infty$.

40)

Discuss the image of the circle $|z - 2| = 1$ and its interior under the following transformations.

(a) $w = z - 2i$ (b) $w = 3iz$ (c) $w = \dfrac{z-2}{z-1}$

(d) $w = \dfrac{z-4}{z-3}$ (e) $w = \dfrac{1}{z}$