Homework 2
Theory of a Complex Variable Functions
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QUESTIONS

1. Construct an analytic function whose real part is u(x,y).

$$u(x,y) = x^3 - 3xy^2 + x^2 - y^2 + x + 1$$

2. Show that following functions are harmonic and determine their harmonic conjugates.
   (a) $u(x,y) = 2x(1-y)$
   (b) $u(x,y) = \frac{y}{x^2+y^2}$
   (c) $u(x,y) = y(3x^2 - y^2)$
   (d) $u(z) = ln|(z)|$ for $Re(z) > 0$

3. Show that $s(z) = x^3 + 3xy^2 - 3x + i(y^3 + 3x^2y - 3y)$ is differentiable on the coordinate axes but is nowhere analytic.

4. If u and v are expressed in terms of polar coordinates (r, θ), find the Cauchy-Riemann equations in polar form.

5. Verify that the real and imaginary parts of the following analytic functions satisfy Laplace's equation.
   (a) $f(z) = z^2 + 2z + 1$
   (b) $g(z) = \frac{1}{z}$
   (c) $h(z) = e^z$

6. Consider two non-concentric circles $C_1 : |z| = R$ and $C_2 : |z-a| = r$ as shown in Figure 1. Find a bilinear transformation that maps these non-concentric circles into two concentric circles.

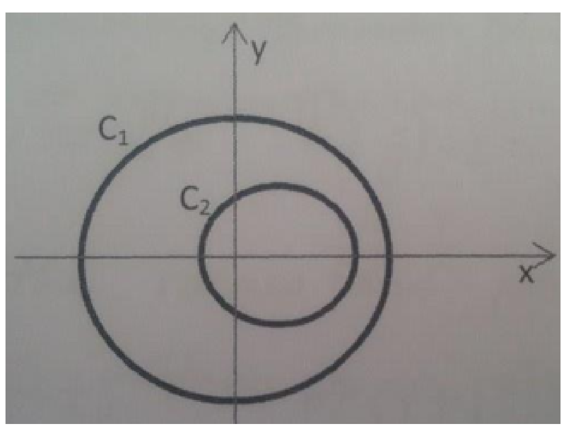

Figure 1